1. **Problem:** (1 point each) Consider the parametric curve

\[ x = f(t) = 4t^2 - 1, \quad y = g(t) = t + 1, \quad -1 \leq t \leq 3. \]

For each of the following points, decide if the point is on the curve. If a point \( P_0 = (x_0, y_0) \) is on the curve, find a \( t_0 \) such that \( P_0 = (f(t_0), g(t_0)) \).

(a) \( P_1 = (3, 1) \).
(b) \( P_2 = (0, 3/2) \).
(c) \( P_3 = (3, 0) \).
(d) \( P_4 = (15, 3) \).
(e) \( P_5 = (-1, 1) \).

**Solution:**

(a) \( P_1 \) is not on the curve.
(b) \( P_2 \) is on the curve, and corresponds to \( t_2 = 1/2 \).
(c) \( P_3 \) is on the curve, \( t_3 = -1 \).
(d) \( P_4 \) is on the line, \( t_4 = 2 \).
(e) \( P_5 \) is on the line, \( t_5 = 0 \).

2. **Problem:** Find a Cartesian equation (i.e. eliminate \( t \)) for the curve traced by

\[ x = (\ln t)^2 + \ln \sqrt{t}, \quad y = \ln t, \quad t > 0. \]

**Solution:** We have

\[ x = (\ln t)^2 + \frac{1}{2} \ln t = y^2 + \frac{1}{2} y. \]

3. **Problem:** Find the slope of the tangent to

\[ x = e^t - 1, \quad y = e^{3t} \]

at the point (1, 8).

**Solution:** The point (1, 8) corresponds to \( t = \ln 2 \). We have

\[ \frac{dx}{dt} = e^t, \quad \frac{dy}{dt} = 3e^{3t} \]

so at \( t = \ln 2 \),

\[ \frac{dx}{dt} = 2, \quad \frac{dy}{dt} = 24 \]

and so

\[ \frac{dy}{dx} = \frac{24}{2} = 12. \]
4. **Problem:** Find an equation of the form \(y = kx + m\) for the tangent to 

\[ x = \cos t, \quad y = \cosh \left( \frac{t}{\pi} \right) \]

at the point \((0, \cosh(1/2))\).

**Solution:** The point \((0, \cosh(1/2))\) corresponds to \(t = \pi/2\). At this point,

\[ \frac{dx}{dt} = -\sin t = -1, \quad \frac{dy}{dt} = \frac{1}{\pi} \sinh \left( \frac{t}{\pi} \right) = \frac{1}{\pi} \sinh \left( \frac{1}{2} \right) \]

so

\[ \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = -\frac{1}{\pi} \sinh \left( \frac{1}{2} \right). \]

So the tangent is given by

\[ y = -\frac{1}{\pi} \sinh \left( \frac{1}{2} \right) x + m \]

and plugging in \((x, y) = (0, \cosh(1/2))\) we get

\[ m = \cosh \left( \frac{1}{2} \right) \]

so the tangent is

\[ y = -\frac{1}{\pi} \sinh \left( \frac{1}{2} \right) x + \cosh \left( \frac{1}{2} \right) \]

5. **Problem:** Find the area under the parametric curve 

\[ x = \frac{1}{6} t^3, \quad y = 4 - t^2, \quad -2 \leq t \leq 2. \]

**Solution:** The area is given by (in the notation we used in class)

\[ A_x = \int_{-2}^{2} g(t) f'(t) dt = \int_{-2}^{2} (4 - t^2) \frac{1}{2} t^2 dt \]

\[ = 2 \int_{0}^{2} (4 - t^2) \frac{1}{2} t^2 dt \]

\[ = \int_{0}^{2} 4t^2 - t^4 dt \]

\[ = \left[ \frac{4t^3}{3} - \frac{t^5}{5} \right]_{t=0} \]

\[ = \frac{32}{3} - \frac{32}{5} \]

6. **Problem:** Find the arc length of the parametric curve 

\[ x = e^t + e^{-t}, \quad y = 5 - 2t, \quad 0 \leq t \leq 3. \]

**Solution:** We have

\[ \frac{dx}{dt} = e^t - e^{-t}, \quad \frac{dy}{dt} = -2 \]
so the arc length is given by

\[ L = \int_0^3 \sqrt{\left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2} \, dt \]

\[ = \int_0^3 \sqrt{(e^t - e^{-t})^2 + (-2)^2} \, dt \]

\[ = \int_0^3 \sqrt{e^{2t} - 2e^t e^{-t} + e^{-2t} + 4} \, dt \]

\[ = \int_0^3 \sqrt{e^{2t} + 2 + e^{-2t}} \, dt \]

\[ = \int_0^3 (e^t + e^{-t})^2 \, dt \]

\[ = \int_0^3 e^t + e^{-t} \, dt \]

\[ = \left[ e^t - e^{-t} \right]_{t=0}^3 \]

\[ = e^3 - e^{-3} \]