21-122: Integration and Approximation  
Summer session one, 2016  

Midterm Exam I

• Name: 

• You have **80 minutes (=1 hour and 20 minutes)**.

• There are 8 problems worth a total of 100 points.

• No calculators, notes, or electronic devices may be used.

• Show all work. Write your answer neatly, clearly indicating the final answer.

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During the exam an additional hint was given:

\[
\int \frac{1}{\sqrt{1+x^2}} \, dx = \ln \left| x + \sqrt{1+x^2} \right| + C.
\]
Problem 1
20 points

Indicate the correct choice. You do **NOT** need to motivate your answer.

(a) (4 points) The definite integral \( \int_{0}^{1} 2xe^{x^2+1}dx \) is equal to

(i) \( \int_{1}^{2} e^{u+1}du \)  
(ii) \( \int_{1}^{2} e^u du \)

(b) (4 points) The improper integral \( \int_{1}^{\infty} x^{-1.7}dx \) is

(i) convergent  
(ii) divergent

(c) (4 points) The polynomial \( x^2 - 3x + 2 \) is

(i) reducible  
(ii) irreducible

(d) (4 points) Consider the integral \( \int_{0}^{10} \sin(x)dx \). The midpoint rule approximation \( M_5 \) is

(i) 2(sin 1 + sin 3 + sin 5 + sin 7 + sin 9)  
(ii) 2(sin 2 + sin 4 + sin 6 + sin 8)

(e) (4 points) The recommended substitution for \( \int \frac{1}{\sqrt{x^2 - 4}}dx \) is

(i) \( x = 2 \tan \theta, \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \)  
(ii) \( x = 2 \sec \theta, \quad 0 \leq \theta < \frac{\pi}{2} \text{ or } \pi \leq \theta < \frac{3\pi}{2} \)
Problem 2
10 points

Show your work!

Evaluate the indefinite integral

\[ \int (3x^2 - 1)e^{-x} \, dx. \]
Problem 3
10 points

Show your work!

Evaluate the definite integral

\[ \int_{1/2}^{1} \frac{x^5}{\sqrt{1 - x^2}} dx. \]
Problem 4
10 points

Show your work!

Determine the arc length of the curve

\[ y = \frac{x^2}{4} - \frac{1}{2} \ln x, \quad 1 \leq x \leq 4 \]
Problem 5
10 points

Show your work!

Determine whether the improper integral

$$\int_{0}^{\pi/2} \tan x \, dx.$$ 

converges or diverges. If it converges, evaluate the integral.
Problem 6
10 points

Show your work!
Evaluate the indefinite integral
\[ \int \sin^2 x \cos^2 x \, dx. \]
Problem 7
15 points

Show your work!

NOTE: The subproblems are unrelated and can be solved independently of each other.

(a) (5 points) Find polynomials \( q(x), r(x) \), where the degree of \( r(x) \) is at most 1, such that

\[
\frac{x^4 - x^3 + 2x^2 + 1}{x^2 + x - 2} = q(x) + \frac{r(x)}{x^2 + x - 2}
\]

(i.e. perform long division on the rational function in the left-hand side).

(b) (5 points) Determine the values of constants \( A, B, C, D \) so that

\[
\frac{2x + 1}{(x^2 + 1)x^2} = \frac{Ax + B}{x^2 + 1} + \frac{C}{x} + \frac{D}{x^2}
\]

holds for all \( x \neq 0 \).

(c) (5 points) Evaluate the indefinite integral

\[
\int \frac{3x + 2}{x^2 + 4} dx.
\]
Show your work!

Consider a lamina of density \( \rho = 2 \) bounded by the four curves

\[ y = \frac{1}{\sqrt{1 + x^2}}, \quad y = 0, \quad x = 1, \quad x = 3. \]

Calculate

(a) (5 points) \( M_x \),

(b) (5 points) \( M_y \),

(c) (5 points) the total mass \( m \) of the lamina.

Hint:

\[ M_x = \rho \int_a^b \frac{1}{2} f(x)^2 \, dx, \quad M_y = \rho \int_a^b x f(x) \, dx. \]