Show your work!

Each problem is worth 5 points, for a total of 30 points.

Remark: When the question asks for a solution for a differential equation, try to get an explicit solution (an expression like $y = f(x)$ for some function $f(x)$). Sometimes you have to settle with an implicit solution (an expression like $g(y) = f(x)$). When the answer is implicit, I can’t give a clear rule for what your answer should look like. The only thing I can really say is simplify your answer as much as possible. For example, if you have

$$(y + 1)^2 = 3x^2 - x + C$$

(which is technically an implicit solution by itself) your answer should be one of the following:

$$|y + 1| = \sqrt{3x^2 - x + C} \quad \text{or} \quad y = -1 \pm \sqrt{3x^2 - x + C}.$$ 

Note that the latter is not explicit since there is a ±, and a function $f$ can’t involve any ±. For $f$ to be called a function, it must be an unambiguous rule for how to calculate $f(x)$, but a ± gives two possible values.

In addition to the homework I recommend that you work through:


9.4: Problems 1, 3, 5.

1. Use Euler’s method with step size 0.2 to approximate $y(0.8)$, where $y$ satisfies

$$y' = x - 2y, \quad y(0) = 1.$$ 

2. Find a general solution for the differential equation

$$y' = -\frac{y^4}{x^4}.$$ 

Your solution must be explicit, i.e. of the form $y = f(x)$.

3. Solve the differential equation

$$\frac{dy}{d\theta} = \frac{e^y \sin \theta}{y \sec \theta}.$$ 

4. Solve the differential equation

$$(y^2 + xy^2)y' = 1, \quad y(0) = 2.$$
5. Find the particular solution for the differential equation

\[ \frac{dy}{dx} = \frac{\cos x}{y - 2}, \quad y(0) = 3. \]

**Hint:** Your general solution should be an implicit solution of the form \(|y - 1| = g(x)|, or equivalently \(y = 1 \pm g(x)\). If you can show that \(g(x) = 0\) never happens then you can conclude that either \(\pm = +\) or \(\pm = -\) for all \(x\), since \(y\) must be continuous.

6. Suppose a population \(P(t)\) satisfies

\[ \frac{dP}{dt} = 0.4P - 0.001P^2, \quad P(0) = 50, \]

where \(t\) is measured in years.

(a) (2 points) What is the carrying capacity?

(b) (1 point) What is \(P'(0)\)?

(c) (2 points) When will the population reach 50% of the carrying capacity?