Show your work!

In addition to the homework, I recommend that you work through:

7.8: Odd-numbered problems 5–15, 29–35.

1. (5 points) Determine whether the improper integral
   \[ \int_{1}^{\infty} \frac{1}{x(x+1)} \, dx \]
   converges or diverges. If it converges, calculate the integral.

2. (5 points) Determine whether the improper integral
   \[ \int_{1}^{e} \frac{1}{x \ln x} \, dx \]
   converges or diverges. If it converges, calculate the integral.

3. (10 points in total) The improper integral
   \[ \int_{2}^{\infty} \frac{2 + \sin x}{x \sqrt{x^2 - 2}} \, dx \]
   is type 1 (infinite bound) and type 2 (asymptote at \( x = 2 \)). Finding an antiderivative is not possible. We will show that the integral is convergent by writing
   \[ \int_{2}^{\infty} \frac{2 + \sin x}{x \sqrt{x^2 - 2}} \, dx = \int_{2}^{3} \frac{2 + \sin x}{x \sqrt{x^2 - 2}} \, dx + \int_{3}^{\infty} \frac{2 + \sin x}{x \sqrt{x^2 - 2}} \, dx \]
   and showing that each part is convergent by using the comparison test.

   (a) (2 points) Show that \( 0 \leq \frac{2 + \sin x}{x \sqrt{x^2 - 2}} \leq \frac{3}{2 \sqrt{x^2 - 2}} \) for \( 2 < x < 3 \).

   **Remark:** There are two inequalities in (a). Prove them separately.

   (b) (3 points) Show that the improper integral
   \[ \int_{2}^{3} \frac{3}{2 \sqrt{x^2 - 2}} \, dx \]
   converges, by showing that the limit
   \[ \lim_{t \to 2^+} \int_{t}^{3} \frac{3}{2 \sqrt{x^2 - 2}} \, dx \]
   exists and is finite.
(c) (5 points) To apply the comparison test to show that \( \int_{3}^{\infty} \frac{2 + \sin x}{x \sqrt{x - 2}} \, dx \) converges we need a function \( g(x) \) such that

- \( 0 \leq \frac{2 + \sin x}{x \sqrt{x - 2}} \leq g(x) \) for all \( x \geq 3 \), and
- \( \int_{3}^{\infty} g(x) \, dx \) converges.

Find such a \( g(x) \).

**Hint:** Try \( g(x) = \frac{c}{x^{3/2}} \) for some large constant \( c \).

4. (5 points) Find the arc length of the curve segment

\[ y = \frac{1}{3} x^{3/2}, \quad 1 \leq x \leq 4. \]

5. (5 points) The curve \( y = \sqrt{1 - x^2} \) forms a semicircle of radius 1. Use the arc length formula to show that the arc length of the semicircle is \( \pi \).