Show your work!

Problems are worth 5 points unless otherwise specified, for a total of 30 points.

In addition to the homework, I recommend that you work through:

7.4: Problems 1, 3, 7, 9, 11, 39
7.7: Problems 7, 9, 11

1. You do **NOT** need to find any integral in this problem.

   (a) (2 points) Perform long division on the rational function
   
   \[ \frac{x^4 - 2x^2 + 2}{(x - 1)(x^2 + 1)} \]
   
   i.e. use long division to find polynomials \(p(x), q(x)\) such that
   
   \[ \frac{x^4 - 2x^2 + 2}{(x - 1)(x^2 + 1)} = p(x) + \frac{q(x)}{(x - 1)(x^2 + 1)} \]
   
   where the degree of \(q(x)\) is at most 2.

   (b) (3 points) Find the partial fraction decomposition of your resulting \( \frac{q(x)}{(x - 1)(x^2 + 1)} \).

2. Evaluate the indefinite integral

   \[ \int \frac{x^3}{x^3 - 3x^2 - 6x + 8} \, dx. \]

   **Hint:** In order to factor the denominator you need to find a solution to \(x^3 - 3x^2 - 6x + 8 = 0\). Try some simple values of \(x\).

3. Evaluate the indefinite integral

   \[ \int \frac{x - 2}{(x^2 - 1)(x - 1)} \, dx. \]

4. Evaluate the indefinite integral

   \[ \int \frac{2x^2}{(x + 2)(x^2 - 2x + 2)} \, dx. \]

5. The 'bell curve' \( \phi(x) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left\{ -\frac{(x - \mu)^2}{2\sigma^2} \right\} \) (where \(\mu, \sigma\) are constants equal to the mean and standard deviation of a statistical measurement) describes a normal distribution and plays a central role in statistics. No elementary antiderivative of \(\phi\) exists, so integration of \(\phi\) has to be done numerically. Consider the integral

   \[ I = \int_{0}^{4} e^{-x^2} \, dx. \]

   (Please turn over)
Use a calculator to calculate $e^{-x^2}$ when needed. Round your answers to five decimal places.

(a) (3 points) Use the midpoint rule over $n = 5$ intervals to approximate $I$, i.e. calculate $M_5$.

(b) (3 points) Use the trapezoidal rule with $n = 8$ to approximate $I$.

6. (4 points) Suppose you want to use Simpson’s rule to approximate

$$\int_1^3 e^{3x} + 3x^2 - 4x + 7 \, dx.$$

How large does $n$ need to be to guarantee that your error is smaller than 0.001? In other words, find the smallest number $N$ such that if $n \geq N$ then $|E_S| \leq 0.001$. Your answer may be algebraic or numerical.