1 Cartography

Let \( L = \{ \langle M \rangle \mid M \text{ is a Turing machine and on some input } w, M \text{ changes the input portion of its tape} \} \). Prove that \( L \) is semidecidable. Prove that it is undecidable by giving a mapping reduction (see Sipser p. 235) from some undecidable language to \( L \).

2 Shortlexly Enumerable

Prove that a language is decidable iff some enumerator enumerates it in shortlex order.

3 Pushup Automata

A pushup automaton is similar to a deterministic pushdown automaton, except:

1. a pop operation returns the least (instead of most) recently pushed element that has not yet been popped,

2. the machine accepts by entering a special accept state that has immediate effect (a la Turing machines), and

3. the input is followed by a special delimiter character.

Prove that the class of languages recognized by pushup automata is the set of Turing-recognizable languages. (To do this, you should prove that Turing machines and pushup automata are mutually simulable.)