Please print single-sided with each problem on its own pages. List any collaborators or sources (including yourself) at the end of your submission.

0 (optional ungraded practice)

(a) Prove the following languages are not regular:
   i. \( \{a+b=c \mid a, b, c \in \{0, 1\}^* \text{ and interpreted as binary representations of integers, } a + b = c \} \)
   ii. \( \{1^p \mid p \text{ prime} \} \)

(b) Give regular expressions for the following languages:
   i. \( \{x \in \{c, a, b\}^* \mid x \text{ contains the substring } "cab" \} \)
   ii. \( \{x \in \{a, b, c\}^* \mid x \text{ contains an even number of } b's \} \)
   iii. \( \{x \in \{a, b, c\}^* \mid \text{ every } a \text{ in } x \text{ is followed by a } b, \text{ and } x \text{ ends with a } c \} \)

1 Deterministic Infinite Automata

We define a DIA = \((Q, \Sigma, q_0, \delta, F)\) identically as for DFAs, except the set \(Q\) of states must be infinite. What set of languages do DIAs recognize? Prove your claim.

2 Blow-Up

Using the Rabin-Scott powerset construction from lecture, an NFA with \(n\) states can be made into an equivalent DFA with \(2^n\) states, but this is usually much larger than the minimal equivalent DFA. Prove that for all \(n > 0\), there is an \(n\)-state NFA whose minimal equivalent DFA has at least \(2^{n-1}\) states.

3 Converse Pumping Lemma

Consider the language \(A = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k.\}\)

(a) Show that \(A\) is not regular.

(b) Show that the pumping lemma is insufficient to prove \(A\) is not regular, that is, we have some \(P\) where if \(w \in A\) and \(|w| \geq P\), we can write \(w = xyz\) with \(|y| > 0, |xy| \leq P\), and for every \(i \geq 0\), \(xy^i z \in A\).

(c) Do (a) and (b) contradict the pumping lemma? Explain.