0 (ungraded practice)

Draw DFAs for the following languages. Briefly justify why your DFAs recognize the correct language.

1. The language of strings in \{a, b, c\} containing the string \textit{cab} at least twice.
2. The language of strings in \{a, b, c\} containing the string \textit{cab} at most twice.
3. The language of strings in \{a, b, c\} whose length is either even or divisible by 5 (or both!)
4. The language of strings in \{a, b, c\} containing at least one \textit{a} and an even number of \textit{b}'s.

1

Given a string \(w \in \{0, 1, 2\}^*\), let \(\text{Int}(w)\) be \(w\) interpreted as a base-3, most-significant-digit first integer, so:

\[
\text{Int}(002) = \text{Int}(2) = 2 \\
\text{Int}(012) = \text{Int}(12) = 5 \\
\text{Int}(20) = 6 \\
\text{Int}(21) = 7 \\
\text{Int}(120) = 15
\]

and so on.

Let \(L = \{w \in \{0, 1, 2\}^* \mid \text{Int}(w) \equiv 0 \mod (5)\}\).

Prove that \(L\) is regular by providing a DFA that recognizes it.

2

For any language \(A\), we define

\[
\text{Del}(A) = \{wx \in \Sigma^* \mid \exists \sigma \in \Sigma \text{ such that } w\sigma x \in A\}
\]

In other words, the set of strings from \(A\) with one symbol removed.

Prove that if \(A\) is regular, \(\text{Del}(a)\) is regular.

Hint: Assume you have a DFA that recognizes \(A\). How would you modify it to recognize \(\text{Del}(A)\)?

3

For any languages \(L_1, L_2\), we define

\[
\text{Cut}(L_1, L_2) = \{xyz \in \Sigma^* \mid xz \in L_1, y \in L_2\}
\]

Prove that if \(L_1\) and \(L_2\) are regular, \(\text{Cut}(L_1, L_2)\) is regular.