Containing errors in computations for neural sensing: does a hierarchical-referencing strategy lead to energy savings?

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ABSTRACT

Recent experimental evidence and theoretical results challenge the belief that ultra high-density EEG sensing will not yield higher spatial resolutions. It raises the exciting possibility that source-localization accuracy can be improved substantially with ultra high-density systems; however, these systems are hindered by implementation constraints in circuit volume and energy consumption. Recently, an information-theoretic hierarchical referencing mechanism has been proposed to exploit the decay of high-spatial frequencies during volume conduction from source to scalp -- and the induced high local spatial correlations -- to reduce the required circuit power and volume. In this paper, human EEG data is used to experimentally test and validate theoretical inter-electrode correlations and bit savings when employing a hierarchical referencing strategy. Extrapolating from electrodes that are at least 2 cm apart, we observe that on average savings can exceed 3 bits per electrode at inter-electrode distances of 3 mm.

***Index Terms*—**EEG, design and implementation of sensing systems, biopotentials, quantization, energy savings

1. INTRODUCTION

Current electroencephalography (EEG) systems excel in temporal resolution while remaining poor in spatial resolution. This is due to the decay of high-spatial frequency information during volume conduction from electrical sources in the brain to the electrodes on the scalp. This decay drastically decreases the ability to resolve the locations of the source of activity, and is generally considered a detriment to the accuracy of source localization in EEG. However, our recent research [1] noted that this decay of high-spatial frequencies, while indeed reducing source-localization accuracy, also induces strong local correlations in the EEG signals that can be exploited to reduce the requirements on the sensing circuitry. This led to the development of a "hierarchical referencing" based biopotential sensing strategies. These strategies are able to keep noise contained by using a tree structure instead of a sequential differential-sensing architecture that is known to be suboptimal through the information-theoretic work of Kim and Berger (ISIT '05 and '06). We note that information-theoretic strategies based on Differential Pulse Coded Modulation (DPCM) are unable to exploit the induced correlations because they require the reconstruction of the signal to be known at the transmitting end. Because the problem here is one of distributed sensing, an electrode may not be able to exploit the knowledge of the reconstruction of the signal sensed and quantized at the other electrode.

While the bit-savings estimated in [1] are based on idealized (spherical) EEG models from [2], this paper tests the savings predicted in [1] using experimentally obtained data.

An obvious question is: if signals get increasingly correlated as the sensor-distances are reduced, what is the utility of high-density sensing? Previous work has attempted to determine the electrode densities and coverage necessary to capture the spatial frequency spectrum on the scalp adequately [2-11]. While early spatial Nyquist-sampling based-results showed that the required inter-electrode distance can be as large as 2.5 cm [3] (see also recent corrections on these estimates [1]), amounting to a few hundred electrodes, the more recent estimates (including some of our own work [1,4]) deem it to be closer to 3 mm [5]. Such small distances require close to 10,000 electrodes! The difference between the two estimates of the sufficient number of electrodes comes from what is being reconstructed: if the goal is to simply recover the EEG potential at all points on the scalp faithfully (*i.e.,* within a small mean-squared error, as is implicitly the case in [3]), then large spacing between electrodes suffices. However, if the goal is to reconstruct the *source of activity* faithfully, then the number of sensors required can be much larger. This is because the high-resolution (high-spatial frequency) information about the source is exactly what decays the most during conduction from source to scalp. Thus ultra high-density systems are necessary to capture minute differences between nearby sensors to adequately resolve the remaining low order bit information.

Is the energy or volume required by sensing circuitry large enough to warrant a new technique? In terms of energy: Indeed, commonly used low-noise sensors often consume 5 mW or more power *per electrode.* Thus, the power consumption of 10,000 electrodes would scale to 50 W just for the sensing circuitry. Such power levels are, at best, too large to allow the system to be portable or to keep the scalp perspiration-free, and at worst, harmful for moderate-term use. In terms of circuit volume: Movement towards the design of EEG "tattoos" using stretchable electronics (e.g.

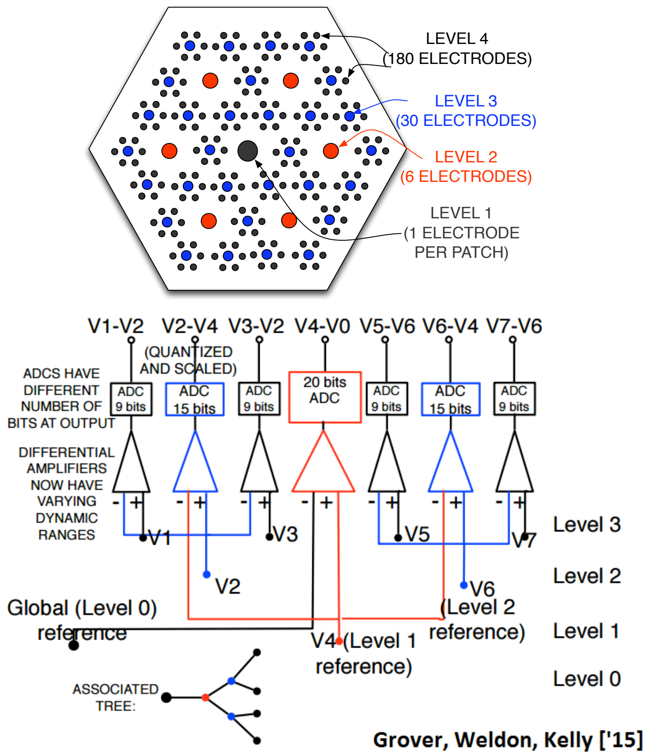

Figure 1 shows the electrode organization and hierarchical referencing scheme proposed for ultra high-density EEG in [1]. At the upper levels of the tree, higher resolution ADCs are used to quantize signals with higher variance. The lower levels do not require high resolution ADCs because the variance of the signal is reduced when referenced with respect to a nearby, and thus highly correlated, signal in the level above.

[12]) poses many practical advantages, but is naturally constrained by both circuit volume and energy. Thus it is necessary to formulate techniques that reduce both for successful development of ultra high-density EEG.

To reduce this energy consumption and the required circuit volume, in [1], we proposed a strategy to implement a 10,000 electrode system. The proposed "hierarchical referencing" sensing mechanism arranges sensors in a tree configuration (see Fig. 1). Each sensor at a lower level is referenced against a sensor (in close proximity) at one level higher. The lower level electrodes share higher order bits due to large correlation with the immediately higher level. Thus the difference signal has lower bit requirements. This allows for the use of low-resolution analog-to-digital converters (ADCs) at the lower levels. By reducing the required ADC bit resolution we directly decrease their energy consumption and circuit volume.

How much savings can there be? For a signal X_1 referenced against signal X_0, the required number of bits for quantizing the difference (X_1-X_0) can be expressed as:

$$\frac{1}{2}\log\left(\frac{2\sigma^2(1-\rho)}{q_e^2}\right),$$

where σ^2 is the variance of signal X_1 and X_0, ρ is the Pearson's correlation between X_1 and X_0, and q_e is the quantization error. It is clear from this expression that the required number of bits reduces as ρ increases and is less than the number of bits needed to quantize signal X_1 as long as ρ is greater than 0.5. In [1], using theoretical head-models (from [2]), it was shown that approximately 3 bits of ADCs can be saved at the lowest level electrodes. Using sigma-delta ADC power consumption models, this amount of bit savings translates to roughly 1/3 the power consumption for high-resolution ADCs, and potentially larger power savings for low-resolution ADCs, in comparison with classical global referencing schemes. The projected power savings are significant because it is the lowest-order nodes (the leaves of the tree) that are dominant in number, and because energy consumption of ADCs increases exponentially with the number of bits (for the sigma-delta-type ADCs frequently used for high-resolution quantization).

However, spherical head-models are too idealistic to provide an assurance of energy savings in the real world. Therefore, here we examine experimental data to (i) affirm the theoretically predicted relationship between electrode distance and signal correlation with actual EEG recordings, and (ii) calculate bit-savings in the lowest-electrode layer using hierarchical referencing in comparison with global referencing. Based on extrapolation of data recorded from a low-density system, we estimate that on average, 3 bits of ADC can be saved for each electrode. Thus, if this system is implemented with a 3-level tree (Levels 0, 1, and 2), then 6 out of every 7 electrodes will require an ADC of 3 bits smaller resolution. Finally, we discuss the implications of these results on the development of ultra high-density systems and assess under which scenarios hierarchical referencing allows for significant energy savings.

2. METHODS

EEG data were collected using a 128-electrode BioSemi Active Two system while subjects attempted to identify target images during rapid serial presentation of visual stimuli. The electrodes were placed according to the international standard 10-5 system [13] using an elastic head cap, which ensured maximum distributed coverage of the scalp. Samples were continuously collected at a rate of 512 Hz for 50 minutes and referenced online to the standard BioSemi reference electrodes. The EEGLAB toolbox [14] was used for offline preprocessing steps including referencing signals to electrodes positioned over both mastoids, a 1-100 Hz bandpass filter, and subsequent removal of large artifacts. After preprocessing, 1,829 seconds of signal (936,554 data points) remained.

Radial electrode distances were calculated from standard 10-5 electrode coordinates [13,14]. Pearson's correlation coefficients (ρ) were computed between all pairs of electrodes using all processed data points to give 8,128 ρ-values computed from 936,554 observations each. Finally, potential bit-savings were calculated by computing the following difference:

$$\log(2max[X_1 - X_0]) - \log(2max[X_1])$$

This estimate is more realistic than the Gaussian rate-distortion-function based estimate used in [1] as it computes the number of bits needed in the worst-case with uniform quantization-bin size. Importantly, this bit-savings estimate is independent of the desired quantization error as long as the higher quantization layers use a sufficient number of bits to adequately resolve X_0 [1]. In order to reduce the influence of outliers on common signal amplitudes, this calculation was performed on 1,000 randomly chosen 1 second intervals within the continuous EEG data. These calculations were meant to simulate referencing at a lower level of the tree, where bit savings for the entire tree is determined

3. RESULTS

3.1. Signal correlations

Figure 2 shows theoretical estimates [1] and experimentally obtained correlations versus angular distances. In the plot illustrating the experimentally obtained correlations, the average of the binned distribution is shown in black while 10th and 90th percentiles are indicated in red. From this figure, we can support the prediction that signals become increasingly correlated with decreasing angular distance. It is also the case that the variance of this correlation decreases with decreasing distance. The nearest electrodes for this analysis were 0.216 radians apart, equating to roughly 2 centimeters on the average skull, which has a radius of about 9 cm [15]. The nearest 80 electrodes (0.216-0.245 radians) had an average ρ of 0.918.

3.2. Projected bit savings at the lowest hierarchical level

Figure 3 shows observed bit savings when referencing signals X_1 & X_0 together compared to referencing X_1 to the global reference when X_1 & X_0 are at the given angular distance. These results are plotted on a logarithmic scale to allow for the extrapolation of bit savings to smaller angular distances. It is clear from this plot that, as angular distances increase, referencing between electrodes does not save bits, but more interesting are bit savings at small angular distances. At observed inter-electrode spacing of roughly 2 cm, savings exceed 1 bit on average. If we extend this logarithmic trend to smaller inter-electrode distances using the least-squares regression, then near the practical limit of minimum inter-electrode spacing (~3 mm) bit savings are projected to be 3 bits on average and at least 2 bits when disregarding the lower 10% of electrodes. One might notice

Figure 2. Pearson's correlation coefficient in response to decreasing angular distance

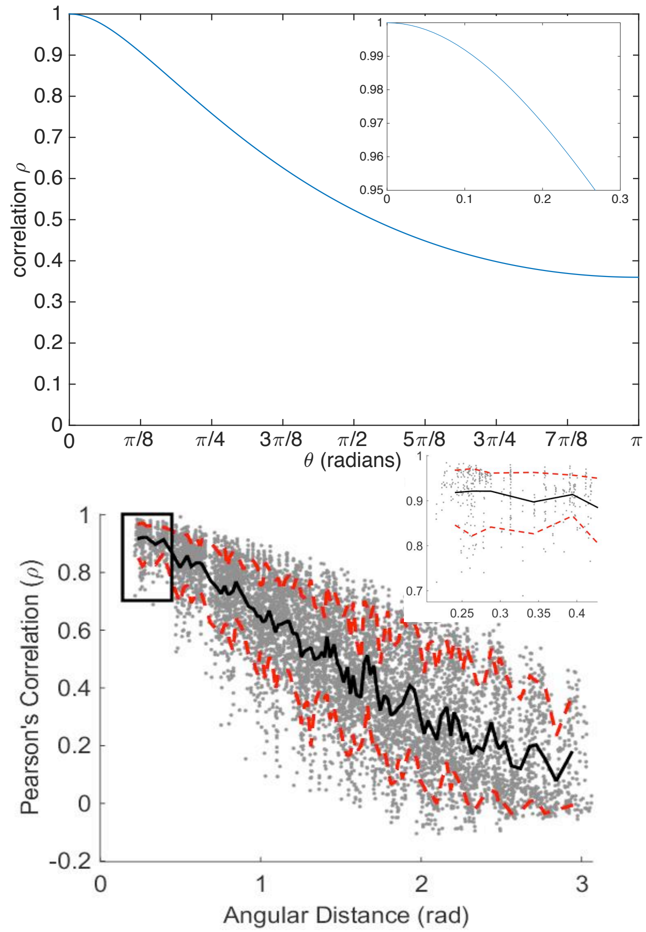

Figure 2. The top plot displays the theoretical predictions of correlations versus angular distance calculated in [1] using an idealized 4-sphere head model based on that in [2]. The bottom plot displays experimentally obtained correlations for all electrode pairs. Angular distance is given in radians as calculated from standard electrode positions. In black is the average correlation at a given distance, while the dotted red lines show the 10th and 90th percentiles at each point. The black rectangle is the portion of the graph displayed in the upper right-hand plot. It is clear from these plots that signal correlations increase with decreasing angular distance and that correlations are roughly 0.918 at the minimum distances observed.

that referencing X_1 to ρ X_0 instead of simply X_0 is the MMSE strategy; however, because of the large values of ρ (close to 1) at small angular distances, we did not notice any advantages of this strategy.

Figure 3. Actual and projected bit savings using hierarchical referencing

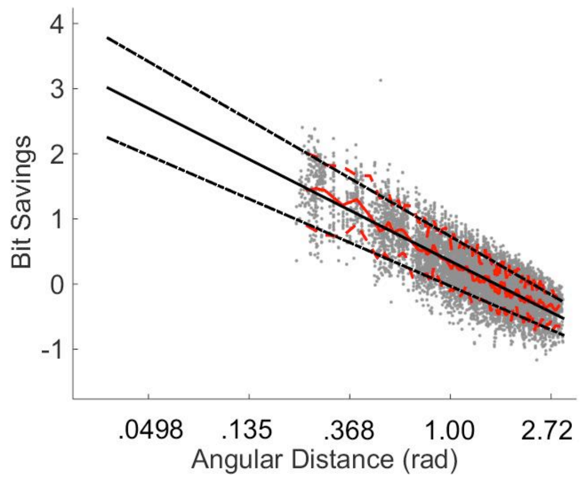

Figure 3 displays the average observed bit savings of all electrode pairs after logarithmic transformation along the x-axis. The red solid line represents the average bit savings at a given angular distance, while the dotted red lines are the 10th and 90th percentiles. The solid black line represents the least-squares regression of all the observed data values ($R^2 = 0.697$) [16], while the dotted black lines are the least-squares regressions of the binned 10th and 90th percentiles. Bit savings exponentially increase with decreasing angular distances. The smallest observation is at an angular distance 0.216 radians (~2 cm). Least-squares regressions are continued to 0.031 radians (roughly 3 mm distances), the practical limit for inter-electrode distances in ultra high-density EEG. This projection predicts bit savings between 2 and 3 bits per electrode at the lowest level of the referencing tree.

4. DISCUSSION

4.1. Shortcomings of our experimental evidence

The observed electrode correlations from Figure 2 were significantly less than the theoretical predictions in [1] which were 0.96 for angular distances of 0.23 rad [1]. Although this difference between observed (0.92) and predicted correlations (0.96) seems relatively small, ρ is related to bits logarithmically and thus, small changes can have large implications in potential bit savings using a hierarchical referencing strategy. Therefore, it is necessary to calculate potential bit savings from actual EEG data and to report any projected bit savings with caution.

At such small distances, the bit savings we project has the potential to be affected by trends that manifest themselves below our observation range. There is a possibility that correlations for nearby electrodes reach a ceiling that is caused by independent noise introduced by the circuit or small impedance differences between electrodes. Therefore, it is necessary to build higher density EEG systems to observe correlations of closer electrodes to determine if this ceiling exists and what implications it has on the proposed hierarchical referencing scheme.

It is also possible that the chosen experimental setup may influence the degree to which electrodes are correlated. It is thus important to consider these changes when designing systems and allowing for flexibility when designing referencing trees. If these considerations are not accounted for, it is possible to lose the resolution of large deviations from the expected waveform, which may be the most critical part of the signal being investigated. For all of these reasons, we feel that the estimate of bit savings obtained here should be viewed as a start. To fully understand the potential of the strategy, more measurements with larger number of (and closely spaced) electrodes, different head sizes and different animals, and with different stimuli, must be performed.

Beyond the immediate question of the utility of ultra-high-density EEG, the results of this paper are consistent with the theoretical observation in [4]: it is possible that there is high spatial frequency information that can be retrieved by sampling with > 128 electrodes.

4.2. Conclusions

The results presented in this paper suggest two scenarios where hierarchical referencing may have the potential to significantly reduce quantization energy and circuit volume. The first case is one in which low resolution ADCs (~6 bits) suffice to adequately resolve a given signal, of relevance in thin sensors such as those built using stretchable electronics [12]. Here, energy savings of 2-3 bits per moderately spaced electrode can equate to significantly less power consumption. The power savings can be larger in purely analog systems such as [12].

The second case is ultra high-density EEGs with electrode spacing of (~3 mm). In this situation, bit savings of 2-3 bits add up when it is employed at the lower end of a tree which comprises 80-90% of all electrodes. This inter-electrode spacing lies near the practical minimum when considering noise introduced by small surfaced electrodes and the necessity of preventing cross contamination between wet electrodes. This arrangement and spacing is also one which can adequately sample the Nyquist frequency of human EEG [5].

Clearly, commonly used EEG systems today would not benefit from a hierarchical referencing scheme, nor do they reach densities that challenge the limits to circuit volume or energy consumption that necessitates the adoption of such a scheme; however, it is widely believed that these systems are also unable to achieve source localization accuracies comparable to MEG. Therefore, the proposed techniques could enable higher resolution EEG systems, thereby reducing the need for expensive and bulky MEG systems.

Our team is currently implementing ultra-high density systems with a small number of electrodes to better test ideas related to reducing energy consumption while maintaining quantization resolutions that can answer tough neural questions. In addition, it is necessary to determine how EEG correlations and thus quantization noise is affected by the task or brain area being monitored to better inform bit allocation during signal acquisition. Finally, another area of interest is the development of circuit techniques to employ flexible ADCs which would allow differential bit allocation in a single channel to save bits when (X_1 - X_0) is small while conserving signal quality when it is large.

5. ACKNOWLEDGMENTS

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