Coordination of a Multi-Commodity Supply Chain with Multiple Members using Flow Networks

Kaveh Fayazbakhsh1, Mohammadreza Razzazi2
Department of Computer Engineering and IT, Amirkabir University of Technology
Hafez Ave., Tehran, Iran
1kfayazbakhsh@gmail.com
2razzazi@aut.ac.ir

Abstract—A successful supply chain responds to customer’s requirements with minimum cost. In general, organizations which are members of a supply chain are independent of each other and try to maximize their own benefits. Appropriate behavior of each member is necessary but does not suffice to have a competitive supply chain. A supply chain coordination mechanism leads members to behave such that the entire supply chain satisfies customer demand with minimum costs. Previous research works which provided formally analyzed coordination mechanisms have made assumptions about restrictions on the number of supply chain stages and members in each stage. Moreover, multi-commodity case has not been investigated in the context of supply chain coordination yet. This paper exploits flow networks concept to propose a coordination mechanism which is applicable to supply chains with multiple members, multiple stages and presence of multi-commodity case while each commodity is composed of several basic components. In the proposed solution an entity named supply chain optimizer (SCO) which is actually an e-hub, gathers supply chain state information and after necessary computations, informs members of supply chain of optimal decisions about orders. The goal is to minimize operational costs of the whole supply chain while satisfying customer demand.

I. INTRODUCTION

A supply chain is composed of distinct businesses which are related to each other directly or indirectly to satisfy customer demand. A supply chain is defined as a collection of suppliers, manufacturers, distributors and retailers (supply chain stages) along with all inter-relationships [1]. Fig. 1 depicts a typical supply chain. Bidirectional arrows represent material, information and financial flows. The main goal of every supply chain is to maximize its value added. Value added is difference between value of final products delivered to the customer and total costs imposed to members of supply chain who have participated in producing and selling them. The value added is one of the most significant indicators of performance of any supply chain. From definition of values added we can conclude that in study of supply chain performance, profitability of the whole supply chain (not only every single member) is important. Since the only source of revenue in supply chain is customer, profitability of supply chain is directly affected by satisfying of customer demand with minimum possible overall cost.

Coordination is one of the major challenging issues in area of supply chain management. Lack of coordination causes several disruptive effects such as long lead times, increase in operational (production, transportation, human resources and inventory) costs, customer service level degradation and negative impact on relationships among members of supply chain. Hence lack of coordination reduces competitiveness of the whole supply chain. Achieving coordination is possible only if each member considers the effects of its decisions on the entire supply chain.

The idea behind the proposed mechanism in this paper is to model supply chain as a flow network. The supply chain can be composed of several stages and any number of members in each stage. Different types of commodities can be produced in the supply chain. Existences of multiple members and multi-commodity case have not been studied in previous research works by using simplifying assumptions. In this work supply chain optimizer (SCO) which is actually an electronic hub (e-hub), gathers necessary information about operational costs and capacities from members of supply chain and after solving two linear programming models of corresponding flow network (as described later in the paper), informs members of supply chain about order decisions imposing minimum overall cost on the entire supply chain. Without such mechanism every member makes decisions on order quantities based on its local and accessible information which results in non-optimal performance of the supply chain.

The paper is organized as follows: section II describes the background and related work. Section III gives an overview of flow networks. In section IV we demonstrate how a supply chain can be modeled as a flow network. Section V clarifies how supply chain optimizer plays a major role in the solution. In section VI a numerical example of applying the proposed mechanism is provided. We conclude the paper in section VI.
II. RELATED WORK
Coordination between a buyer and a seller has been of great interest for several past decades. Since the number of cooperating businesses providing a single product has increased in recent few decades, coordination issue has been immensely complicated. It is possible to classify supply chain coordination mechanisms into three types: coordination contracts, information sharing and negotiation. The most common mechanism for bilateral relations between a buyer and a seller is to agree on a contract. Supply chain contracts include quantity discount, return, channel rebate and revenue sharing [2]. Effective contracts in bilateral relations fail to be useful to coordinate multilateral relations. For example quantity discount provides incentives for a buyer to place extra orders. As a result of such misleading order information, upstream members in supply chain will not be capable of forecasting customer demand for future periods based on previous demand information precisely. In summary, increase in the number of members of supply chain, transforms traditional contracts into inefficient coordination mechanisms. Li and Wang [3] provide a survey of traditional coordination mechanisms for supply chain taking an inventory control approach.

Decision making based on shared information by members of supply chain is the second major type of coordination mechanisms. Utilizing customer relationship management (CRM), supplier relationship management (SRM), e-marketplaces and trading agents are usual choices for managing and sharing business information (e.g. see [4], [1], [5], [6]). An absolutely significant question is that while members of supply chain do not trust each other completely, why they should accept to share their own strategically important information via such information systems? Therefore it is essential to restrict shared information as much as possible.

Lastly, some recent studies have focused on negotiation-based mechanisms for supply chain coordination. Negotiation can be considered as a special type of information sharing in which less information is shared and a defined protocol conducts negotiation process. Dudek and Studdler [7] study a two-member supply chain and by defining members’ mathematical operational model, propose a negotiation mechanism to reduce total costs. Ding and Chen [8] consider return policy using negotiation to coordinate a three-stage (with a single member in each stage) supply chain. Fink [9] proposes using a mediator software agent to conduct a bilateral negotiation process until both firms accept a contract. Limitation on the number of members of supply chain is a major obstacle to apply current negotiation-based mechanisms to supply chains with multiple members.

III. FLOW NETWORKS
Electrical networks, urban transportation networks, telecommunications networks, railroad networks and oil product pipeline networks are examples of flow networks. A flow network is a directed graph in which each vertex can produce, consume or pass a flow. Each directed edge is a conduit for the flow with a defined capacity. Vertices are conjunction points of flow paths and can only pass the flow except for two special vertices: a source vertex and a sink vertex. Source vertex has only outgoing edge(s) and produces the flow, while sink vertex has only incoming edge(s) and consumes the flow. Ahuja, Magnanti, and Orlin [10] and Goldberg, Tardos, and Tarjan [11] provide nice surveys of algorithms for network-flow problems.

A flow network $G = (V, E)$ is a directed graph in which each edge $(u, v) \in E$ has a nonnegative capacity $c(u, v) \geq 0$. If $(u, v) \notin E$, it is assumed that $c(u, v) = 0$. In flow network, two vertices are distinguished as source vertex $s$ and sink vertex $t$. It is assumed that every vertex lies on some path from the source to the sink. A flow is a real-valued function $f : V \times V \rightarrow \mathbb{R}$ that satisfies the following properties:

a) Capacity constraint: for all $u, v \in V$, we require $f(u, v) \leq c(u, v)$.

b) Skew symmetry: for all $u, v \in V$, we require $f(u, v) = -f(v, u)$.

c) Flow conservation: for all $u \in V - \{s, t\}$, we require $\sum_{v \in V} f(u, v) = 0$.

Given that price of a product is determined by the law of demand, each member of supply chain follow fixed order interval (FOI) policy for inventory control. Hence each member has determined a fixed minimum quantity discount provides incentives for a buyer to place extra orders. As a result of such misleading order information, upstream members in supply chain will not be capable of forecasting customer demand for future periods based on previous demand information for each type of commodity for the customer. Each type of commodity is made up of a set of basic components. As a result of this approach, two simplifying assumptions that exist in previous works containing formal analysis, i.e. limited number of supply chain members and single-commodity case, are removed and a more realistic situation is investigated.

As mentioned before, the supply chain can provide $k$ different types of commodities for the customer. Raw materials or basic components of every commodity type are provided by suppliers. Every manufacturer has the potential for producing all types of commodities according to its operational capacities. Distributors are able to distribute finished products by manufacturers among retailers. Finally, retailers sell products to the customer. The proposed mechanism aims to minimize operational costs of the whole supply chain. Less cost of providing a commodity for end-customer results in higher competitiveness of supply chain.

Given that price of a product is determined by the law of supply and demand in competitive markets and it is not controlled significantly by members of supply chain, we exclusively focus on costs of providing commodities and consider revenues independent of existence of the coordination mechanism. It is assumed that members of supply chain follow fixed order interval (FOI) policy for inventory control. Hence each member has determined a fixed
order placement period for every type of commodity and should make decisions about quantities and destination organization of its orders. Since each member of supply chain places one order in every order placement period, cost of placing orders with a member in previous stage is not affected by the coordination mechanism. Moreover every unit of shortage of inventory in a retailer imposes a constant cost to it. Customer demand is deterministic in each period and is forecasted by retailers. Obviously, most real-world scenarios involve more intricate and complicated characteristics such as using different inventory management systems and stochastic nature of demand. However, in this paper we aim to study the principal potential of the proposed coordination mechanism for a straightforward situation as a primary step.

B. Formulation of the Problem

In this subsection flow network model for supply chain is described. Consider a graph $G = (V,E)$ that satisfies three properties of a flow network discussed in section III. In this model each vertex represents a member of supply chain and each directed edge represents a potential relationship between two members. Every directed edge $(u,v)$ shows possibility of providing and sending basic components, raw materials or finished products from organization $u$ to organization $v$. Edge capacities are considered as capacities for supply, production, transportation or delivery (depending on nature of a relationship) from an organization to another for the planning time horizon. Moreover, a cost factor is assigned to each edge representing costs of supply, production, transportation or delivery (depending on nature of a relationship) for each unit of a commodity. These costs are imposed to source member of a relation (i.e. organization $u$).

In the model, sources and sinks of flow network are equivalent to suppliers and retailers of supply chain respectively.

Consider $k$ types of commodities which are produced from $p$ different basic components or raw materials. Set $A_s = \{a_{1s}, a_{2s}, \ldots, a_{ps}\}$ is set of initial components (or raw materials) composing every unit of commodity type $i$ ($i=1, 2, \ldots, k$), where $a_{is}$ is the quantity (or amount) of components of type $j$ necessary to produce every unit of commodity type $i$ ($j=1, 2, \ldots, p$). For example if $A_s = \{0, 2, 1\}$ then every unit of forth type of commodities contains two units of component type 2 and one unit of component type 3. It is obvious that component type 1 is not needed to produce this type of commodity.

Suppose that $Sset$, $Mset$, $Dset$ and $Rset$ represent sets of suppliers, manufacturers, distributors and retailers in the supply chain respectively. We have:

- $Sset = \{sp_s, \forall s = 1, 2, \ldots, S\}$,
- $Mset = \{manu_m, \forall m = 1, 2, \ldots, M\}$,
- $Dset = \{dist_d, \forall d = 1, 2, \ldots, D\}$,
- $Rset = \{ret_r, \forall r = 1, 2, \ldots, R\}$.

According to the nature of flow, we decompose the original flow network into two parts: network (I) includes manufacturers, distributors and retailers and network (II) includes suppliers and manufacturers. $V_1$ is set of vertices of network (I) and $V_2$ is set of vertices of network (II). It is obvious that $V_1 \cap V_2 = Mset$. In network (I) commodities flow, while in network (II) components (or raw materials) flow. Model (I) is the model for network (I) and model (II) is model for network (II). They are specified in Fig. 2 and Fig. 3 respectively.

$$\min \, z_1 = \sum_{u, v \in V_1} \sum_{i=1}^{k} a_i(u, v) f_i(u, v)$$

subject to

$$f_i(u, v) \leq c_i(u, v) \quad \text{for each } i=1, 2, \ldots, k$$

and for each $u, v \in V_1$,

$$f_i(u, v) = -f_i(v, u) \quad \text{for each } i=1, 2, \ldots, k$$

and for each $u, v \in V_1$,

$$\sum_{v \in V_2} f_i(u, v) = 0 \quad \text{for each } i=1, 2, \ldots, k$$

and for each $u \in V_1$,

$$\sum_{u \in V_1} f_i(u, v) = 0 \quad \text{for each } i=1, 2, \ldots, k$$

and for each $v \in V_1$.

Fig. 2. Model (I)

In model (I) manufacturers and retailers are considered as sources and consumers (sinks) of commodities respectively. Model (I) is a linear programming model and takes as input demand for commodities in retailers, capacities and costs of edges. $d_{ir}$ is quantity of demand for commodity type $i$ in retailer $r$ (where $r = 1, 2, \ldots, R$ and $i = 1, 2, \ldots, k$). As stated before, for each edge $(u, v)$ two properties are defined: capacity and cost. $c_i(u, v)$ is the capacity of edge $(u, v)$ for flow of commodity $i$ and $o_i(u, v)$ is cost of flow of each unit of commodity $i$ through edge $(u, v)$. These properties may take different values in each planning period. $c_i(u, v)$ is interpreted as maximum feasible capacity of organization $u$ for providing (i.e. manufacturing or distributing) commodity $i$ and delivering it to organization $v$ incurring cost $o_i(u, v)$. Value of the flow of each commodity through each edge is determined by solving the model (I). $f_i(u, v)$ is value of flow of commodity type $i$ through edge $(u, v)$. Objective function $z_1$ is total operational costs of the supply chain in network (I) section and it is to be minimized. Six categories of constraints in form of linear equalities and inequalities are considered in the model. First three categories of constraints are equivalent to capacity constraint, skew symmetry and flow conservation properties of flow networks respectively. Fourth category of constraints guarantees satisfying demand in retailers. Fifth category of constraints assures that manufacturers produce enough commodities. Finally, sixth group of constraints are non-negativity constraints on flows’ values. Solving model
(I) gives values for flows \( f_j(u,v) \) such that network (I) satisfies customer demand with minimum possible cost.

Fig. 3 specifies model (II) which should be solved after model (I).

\[
\min z_2 = \sum_{u \in V_2} \sum_{j=1}^p \alpha_j(u,v) f_j(u,v) \\
\text{subject to} \\
f_j(u,v) \leq c_j(u,v) \text{ for each } j = 1,2,\ldots, p \\
\quad \text{and for each } u,v \in V_2, \\
\sum_{m \in V_2} f_j(u,manu_m) = \sum_{i=1}^k (a_{ui} \sum_{v \in V_1} f_i(manu_m,v)) \\
\quad \text{for each } j = 1,2,\ldots, p, \\
\quad \text{and for each } m = 1,2,\ldots,M, \\
\sum_{j=1}^p \sum_{v \in V_2} f_j(sp_s,v) = \sum_{j=1}^k (a_{sp_s} \sum_{v \in V_1} f_j(manu_m,v)) \\
\quad \text{for each } j = 1,2,\ldots, p, \\
\quad \text{and for each } v \in V_2. \\
\]

Fig.3. Model (II)

\( c_j(u,v) \) is the capacity of edge \((u,v)\) for flow of basic component (or raw materials) type \(j\) and \(o_j(u,v)\) is cost of flow of component type \(j\) through edge \((u,v)\) \((j = 1,2,\ldots, p)\). \( c_j(u,v) \) can be interpreted as maximum feasible capacity of supplier \(u\) for providing component \(j\) and delivering it to manufacturer \(v\) incurring cost \(o_j(u,v)\). Note that \( f_j(manu_m,v) \) values have been determined in model (I) and are inputs of model (II) along with \( c_j(u,v)\) and \( o_j(u,v)\). Model (II) determines optimal flow of different components in network (II) (i.e. \( f_j(u,v) \) values). Objective function \( z_2 \) is total operational costs of the supply chain in network (II) section and it is to be minimized. Four groups of linear constraints in model (II) are as follows. First group of constraints are equivalent to capacity constraint of flow networks. Second group of constraints guarantees satisfying manufacturers demand for basic components to be able to produce sufficient commodities. Third group of constraints assures that suppliers supply enough basic components for manufacturers. Lastly, Fourth group of constraints are non-negativity constraints on flows values. Solving model (II) gives values for flows \( f_j(u,v) \) such that network (I) satisfies customer demand with minimum possible cost.

The supply chain is composed of network (I) and network (II). Therefore operational costs of the supply chain are sum of costs of these two networks and optimal cost of supply chain equals \( z_1 + z_2 \). Since both model (I) and model (II) are linear programming models, polynomial-time algorithms such as Karmarkar’s algorithm [12] exist to solve them. By solving the model and informing supply chain members of related flow values, they will be able to make order decisions such that result in optimal situation for the whole supply chain. It may be necessary to include another group of constraints to ensure that some or all of decision variables (i.e. \( f_j(u,v) \) and \( f_j(u,v) \)) take integer values. Such additional constraints may be necessary when a commodity or component is countable and can not be quantified by fractions. In this case mixed-integer programming [13] methods are applicable.

V. Supply Chain Optimizer

The idea behind the proposed coordination mechanism is to construct and solve models (I) and (II) and using their optimal solutions. To achieve this goal a central entity named supply chain optimizer (SCO) takes the major role. Suppose that suppliers and manufacturers have provided \( A_k \) sets information for the SCO. At the beginning of the planning horizon (for example at the beginning of each month), retailers provide demand forecast information for the SCO. Every supplier, manufacturer and distributor in supply chain provides the SCO with names of connected organizations in its next stage along with associated capacity and cost parameters. In other words retailer \( r_{ei} \) sends \( d_{ir} \) and every non-retailer member of supply chain such as \( u \) sends edges \((u,v)\) and values for \( c_j(u,v)\) and \( o_j(u,v)\) (or \( c_j(u,v)\) and \( o_j(u,v)\) in the case of suppliers) to the SCO. Using gathered information the SCO will be able to make and solve models (I) and (II) and send flow values to supply chain members. Receiving values from the SCO, each distributor or retailer organization \( v \) places an \( f_j(u,v) \)-unit order for commodity \(i\) with organization \(u\). Manufacturer \( manu_m\) places \( f_j(u,manu_m)\)-unit order with supplier \(u\). Therefore, the entire supply chain will operate with minimum costs. Fig. 4 and Fig. 5 depict the coordination mechanism schematically. Gathering information about connections, capacities and costs as shown Fig. 4, the SCO will be able to form models (I) and (II). Fig. 5 on the next page specifies order information that will be sent from the SCO to manufacturers, distributors and retailers after solving models (I) and (II). Therefore they will be able to place orders such that assure supply chain operations with minimum feasible costs and satisfying customer demand.
The SCO can be implemented in form of an e-hub. E-hubs or B2B marketplaces can be classified into four categories: MRO (maintenance, repair and operating) hubs, yield managers, exchanges and catalog hubs [14]. MRO hubs and yield managers give buyers access to industry-independent commodities (such as office supplies). Conversely, exchanges and catalog hubs provide raw materials and components that vary considerably from industry to industry. While exchanges enable fulfillment of an immediate need at the lowest possible cost, using catalog hubs involves negotiated contracts with qualified seller organizations. In this case the buyers and the sellers often develop close relationships. We conclude that catalog hub is the right choice to implement the SCO; because members of supply chain tend to have long-term relationships and shared benefits.

If the proposed coordination mechanism is not used, each member of supply chain attempts to fulfill its demand in a locally optimal style. In this case beginning at retailer stage each member of supply chain considers capacity and cost factors of incoming edges to place orders such that its demand will be satisfied with minimum cost. Since every member makes decisions based on limited accessible information and there is no comprehensive coordination mechanism, retailers should encounter shortage of inventory and members of supply chain will operate with higher costs compared to optimal situation.

VI. Illustrating the Mechanism

Since the proposed mechanism relies on linear programming models and finding optimal solution for these models (if existed) is mathematically guaranteed, there is not any other better solution that imposes lower costs to supply chain. We provide a numerical example to illustrate the mechanism. Consider a supply chain consisted of three suppliers, two manufacturers, two distributors and four retailers as depicted in Fig. 6. The supply chain provides three types of commodities for the customer and there are five types of basic components. We have $A_1 = \{1,0,1,0,2\}$, $A_2 = \{0,1,0,0,1\}$ and $A_3 = \{3,1,0,2,0\}$. In this figure edges are labeled with capacities and costs. Numbers before / symbol on edges are capacities and numbers after it are costs for different types of commodities or basic components. Demand information is as follows: $d_{11} = 2$, $d_{22} = 5$, $d_{33} = 3$, $d_{13} = 0$, $d_{23} = 0$, $d_{31} = 4$, $d_{14} = 3$, $d_{24} = 0$, $d_{34} = 2$. Each unit of lost sales due to shortage of inventory imposes 25 units of cost on supply chain.

Using proposed coordination mechanism model (I) consisted of 66 decision variables and 87 constraints and model (II) with 25 decision variables and 40 constraints have been formed. Note that these are small models in the linear programming area. To solve models we used What’s Best! Version 9 [15] package which is able to solve linear (and some non-linear) models consisted of several hundred variables and several hundred constraints. Optimal values of decision variables and total cost of model (I) and model (II) are provided in TABLE I and TABLE II respectively. Optimal total costs of supply chain denoted by $z^*$, is sum of $z_1$ and $z_2$. In this example $z^*$ is 659.

![Fig. 5. Sending optimal order quantities by the SCO](image)

**TABLE I**

<table>
<thead>
<tr>
<th>$f_1(u,v)$</th>
<th>$f_2(u,v)$</th>
<th>$f_3(u,v)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4,6)</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>(4,7)</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>(5,6)</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>(5,7)</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>(6,8)</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>(6,9)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(6,10)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(6,11)</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>(7,8)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>(7,9)</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>(7,11)</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

| $z_1$ | 152 |

![Fig. 6. Example of a 3-commodity supply chain](image)

**TABLE II**

<table>
<thead>
<tr>
<th>$f_1(u,v)$</th>
<th>$f_2(u,v)$</th>
<th>$f_3(u,v)$</th>
<th>$f_4(u,v)$</th>
<th>$f_5(u,v)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>6</td>
<td>0</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>2</td>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>3</td>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td>19</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

| $z_2$ | 507 |
VII. Conclusion

Despite the vital importance of coordination issue in supply chain management, previous works in this area face two shortcomings. Assumption of limited number of members of supply chain and considering only single commodity case have been two simplifying assumptions that prevents them from being applicable to real world supply chains. In this paper we have proposed an IT-based coordination mechanism for multi-commodity supply chains with multiple members. Every type of commodity is produced from a set of basic components or raw materials. We have modeled supply chain as a flow network which makes it possible to consider operational capacities and costs for all members of supply chain. By forming and solving a linear programming model, members will be able to make decisions that result in totally optimal situation with minimum cost for the whole supply chain. To achieve this goal a central entity named supply chain optimizer (SCO) receives information about connections, capacities and costs from members of supply chain at the beginning of the planning time horizon. The SCO then forms and solves a linear programming model and sends optimal order quantities to members. The SCO can be implemented as a catalog hub which is an e-hub suitable for long-term relationships and industry-specific transactions. We believe that the proposed coordination mechanism is a new approach to have more competitive supply chains.

REFERENCES