List of Notation

$\circ$	entry-wise multiplication of vectors
∇	the gradient: $\nabla f(x) = (\mathrm{D}_1 f(x), \dots, \mathrm{D}_n f(x))$
$\neg$	logical NOT
$\ni$	$S \ni i$ is equivalent to $i \in S$
$\oplus$	logical XOR (exclusive-or)
$\hat{\|} f \hat{\|}_p$	$(\sum_{\gamma \in \widehat{\mathbb{F}_2^n}} \|\widehat{f}(\gamma)\|^p)^{1/p}$
$\triangle$	symmetric difference of sets; i.e., $S \triangle T = \{i : i \text{ is in exactly one of } S, T\}$
$\vee$	logical OR
$\wedge$	logical AND
$*$	the convolution operator
$[z^k]F(z)$	coefficient on z^k in the power series $F(z)$
1_A	0-1 indicator function for A
$\mathbf{1}_B$	0-1 indicator random variable for event B
1_x	the 0-1 indicator function for x
2^A	the set of all subsets of A
AND_n	the logical AND function on n bits: False unless all inputs are True
$\mathrm{Aut}(f)$	the group of automorphisms of Boolean function f
$\mathrm{BitsToGaussians}_M^d$	on input the bit matrix $x \in \{-1,1\}^{d \times M}$, has output $z \in \mathbb{R}^d$ equal to $\frac{1}{\sqrt{M}}$ times the column-wise sum x; if d is omitted it's taken to be 1
$\mathbb{C}$	the complex numbers

$\chi(b)$	when $b \in \mathbb{F}_2^n$, denotes $(-1)^b \in \mathbb{R}$
$\chi_S(x)$	when $x \in \mathbb{R}^n$, denotes $\prod_{i \in S} x_i$, where $S \subseteq [n]$; when $x \in \mathbb{F}_2^n$, denotes $(-1)^{\sum_{i \in S} x_i}$ if $x \in \mathbb{F}_2^n$
$\operatorname{codim} H$	for subspace $H \leq \mathbb{F}^n$, denotes $n - \dim H$
$\mathbf{Cov}[f,g]$	the covariance of f and g, $\mathbf{Cov}[f] = \mathbf{E}[fg] - \mathbf{E}[f]\mathbf{E}[g]$
$d_{\chi^2}(\varphi, 1)$	chi-squared distance of the distribution with density φ from the uniform distribution
$\deg(f)$	the degree of f; the least k such that f is a real linear combination of k-juntas
$\deg_{\mathbb{F}_2}(f)$	for Boolean-valued f, the degree of its $\mathbb{F}_2$-polynomial representation
$\Delta^{(\pi)}(f)$	the expected number of queries made by the best decision tree computing f when the input bits are chosen from the distribution π
$\delta^{(\pi)}(f)$	the revealment of f; i.e., $\min\{\max_i \delta_i^{(\pi)}(\mathcal{T}) : \mathcal{T}$ computes $f\}$
$\Delta^{(\pi)}(\mathcal{T})$	the expected number of queries made by randomized decision tree $\mathcal{T}$ when the input bits are chosen from the distribution π
$\delta_i^{(\pi)}(\mathcal{T})$	the probability randomized decision tree $\mathcal{T}$ queries coordinate i when the input bits are chosen from the distribution π
$\Delta_y f$	for $f : \mathbb{F}_2^n \to \mathbb{F}_2$, the function $\mathbb{F}_2^n \to \mathbb{F}_2$ defined by $\Delta_y f(x) = f(x+y) - f(x)$
D_i	the ith discrete derivative: $\mathrm{D}_i f(x) = \frac{f(x^{(i \mapsto 1)}) - f(x^{(i \mapsto -1)})}{2}$
$\operatorname{dist}(g,h)$	the relative Hamming distance; i.e., the fraction of places where g and h disagree
$\Delta(x,y)$	the Hamming distance, $\#\{i : x_i \neq y_i\}$
$\mathrm{DNF}_{\mathrm{size}}(f)$	least possible size of a DNF formula computing f
$\mathrm{DNF}_{\mathrm{width}}(f)$	least possible width of a DNF formula computing f
$\mathrm{DT}(f)$	least possible depth of a decision tree computing f
$\mathrm{DT}_{\mathrm{size}}(f)$	least possible size of a decision tree computing f
$d_{\mathrm{TV}}(\varphi, \psi)$	total variation distance between the distributions with densities φ, ψ
E_i	the ith expectation operator: $\mathrm{E}_i f(x) = \mathbf{E}_{\boldsymbol{x}_i}[f(x_1, \dots, x_{i-1}, \boldsymbol{x}_i, x_{i+1}, \dots, x_n))]$
E_I	the expectation over coordinates I operator
$\mathbf{Ent}[f]$	for a nonnegative function on a probability space, denotes $\mathbf{E}[f \ln f] - \mathbf{E}[f] \ln \mathbf{E}[f]$

$\mathbf{E}_{\pi_p}[\cdot]$	an abbreviation for $\mathbf{E}_{\boldsymbol{x}\sim\pi_p^{\otimes n}}[\cdot]$
$f \oplus g$	if $f : \{-1,1\}^m \to \{-1,1\}$ and $g : \{-1,1\}^n \to \{-1,1\}$, denotes the function $h : \{-1,1\}^{m+n} \to \{-1,1\}$ defined by $h(x,y) = f(x)g(y)$
$f \otimes g$	if $f : \{-1,1\}^m \to \{-1,1\}$ and $g : \{-1,1\}^n \to \{-1,1\}$, denotes the function $h : \{-1,1\}^{mn} \to \{-1,1\}$ defined by $h(x^{(1)},\dots,x^{(m)}) = f(g(x^{(1)}),\dots,g(x^{(m)}))$
f^{*n}	the n-fold convolution, $f * f * \cdots * f$
$f^{\dagger}$	the Boolean dual defined by $f^{\dagger}(x) = -f(-x)$
f^{+z}	if $f : \mathbb{F}_2^n \to \mathbb{R}$, $z \in \mathbb{F}_2^n$, denotes the function $f^{+z}(x) = f(x+z)$
f_H^{+z}	denotes $(f^{+z})_H$
$\mathbb{F}_2$	the finite field of size 2
$\widehat{\mathbb{F}_2^n}$	the group (vector space) indexing the Fourier characters of functions $f : \mathbb{F}_2^n \to \mathbb{R}$
f^{even}	the even part of f, $(f(x)+f(-x))/2$
$\langle f,g\rangle$	$\mathbf{E}_{\boldsymbol{x}}[f(\boldsymbol{x})g(\boldsymbol{x})]$
f_H	if $f : \mathbb{F}_2^n \to \mathbb{R}$, $H \le \mathbb{F}_2^n$, denotes the restriction of f to H
$\widehat{f}(i)$	shorthand for $\widehat{f}(\{i\})$ when $i \in \mathbb{N}$
$f^{\subseteq J}$	$\sum_{S\subseteq J} \widehat{f}(S)\chi_S$
$f_{\mid z}$	if $f : \{-1,1\}^n \to \mathbb{R}$, $J \subseteq [n]$, and $z \in \{-1,1\}^{\overline{J}}$, denotes the restriction of f given by fixing the coordinates in $\overline{J}$ to z
$f_{J\mid z}$	if $f : \{-1,1\}^n \to \mathbb{R}$, $J \subseteq [n]$, and $z \in \{-1,1\}^{\overline{J}}$, denotes the restriction of f given by fixing the coordinates in $\overline{J}$ to z
$f^{=k}$	$\sum_{\lvert S\rvert=k} \widehat{f}(S)\chi_S$
$f^{\le k}$	$\sum_{\lvert S\rvert\le k} \widehat{f}(S)\chi_S$
f^{odd}	the odd part of f, $(f(x)-f(-x))/2$
$\mathbb{F}_{p^\ell}$	for p prime and $\ell \in \mathbb{N}^+$, denotes the finite field of p^ℓ elements
$\widehat{f}(S)$	the Fourier coefficient of f on character χ_S
$\mathrm{F}_{S\mid\overline{J}}f(z)$	for $S \subseteq J \subseteq [n]$, denotes $\widehat{f_{J\mid z}}(S)$
$f^{\otimes d}$	if $f : \{-1,1\}^n \to \{-1,1\}$ then $f^{\otimes d} : \{-1,1\}^{n^d} \to \{-1,1\}$ is defined inductively by $f^{\otimes 1} = f$, $f^{\otimes(d+1)} = f \otimes f^{\otimes d}$
$\widetilde{f}$	the randomization/symmetrization of f, defined by $\widetilde{f}(r,x) = \sum_S \boldsymbol{r}^S f^{=S}(\boldsymbol{x})$
$\gamma^+(\partial A)$	the Gaussian Minkowski content of ∂A

$\mathcal{G}(v,p)$	the Erdős–Rényi random graph distribution, $\pi_p^{\otimes\binom{v}{2}}$
h_i	the ith (normalized) Hermite polynomial, $h_i = \frac{1}{\sqrt{i!}}H_i$
h_α	for $\alpha \in \mathbb{N}^n$ a multi-index, the n-variate (normalized) Hermite polynomial $h_\alpha(z) = \prod_{i=1}^n h_{\alpha_i}(z_i)$
H_i	the ith probabilists' Hermite polynomial, defined by $\exp(tz - \frac{1}{2}t^2) = \sum_{i=0}^\infty \frac{1}{i!}H_i(z)t^i$
$A^\perp$	$\{\gamma : \gamma \cdot x = 0 \text{ for all } x \in A\}$
$\mathbf{Inf}_i[f]$	the influence of coordinate i on f
$\mathbf{Inf}_i^{(\rho)}[f]$	the ρ-stable influence, $\mathbf{Stab}_\rho[\mathrm{D}_i f]$
$\widetilde{\mathbf{Inf}}_J[f]$	the coalitional influence of $J \subseteq [n]$ on $f : \{-1,1\}^n \to \{-1,1\}$, namely $\mathbf{Pr}_{\boldsymbol{z}\sim\{-1,1\}^{\overline{J}}}[f_{J\mid\boldsymbol{z}} \text{ is not constant}]$
$\widetilde{\mathbf{Inf}}_J^b[f]$	for $b \in \{-1,1\}$, equals $\mathbf{Pr}_{\boldsymbol{z}\sim\{-1,1\}^{\overline{J}}}[f_{J\mid\boldsymbol{z}} \not\equiv -b] - \mathbf{Pr}[f = b]$
$\overline{J}$	if $J \subseteq [n]$, denotes $[n] \setminus J$
$L^2(\{-1,1\}^n)$	denotes $L^2(\{-1,1\}^n, \pi_{1/2}^{\otimes n})$
$L^2(G^n)$	if G is a finite abelian group, denotes the complex inner product space of functions $G^n \to \mathbb{R}$ with inner product $\langle f,g\rangle = \mathbf{E}_{\boldsymbol{x}\sim G^n}[f(\boldsymbol{x})\overline{g(\boldsymbol{x})}]$
$L^2(\Omega,\pi)$	the inner product space of (square-integrable) functions $\Omega \to \mathbb{R}$ with inner product $\langle f,g\rangle = \mathbf{E}_{\boldsymbol{x}\sim\pi}[f(\boldsymbol{x})g(\boldsymbol{x})]$
$\Lambda_\rho(\alpha)$	$\mathbf{Pr}[\boldsymbol{z}_1 \le t, \boldsymbol{z}_2 \le t]$, where $\boldsymbol{z}_1, \boldsymbol{z}_2$ are standard Gaussians with correlation $\mathbf{E}[\boldsymbol{z}_1\boldsymbol{z}_2] = \rho$ and $t = \Phi^{-1}(\alpha)$.
$\mathrm{L}f$	the Laplacian operator applied to the Boolean function f, defined by $\mathrm{L}f = \sum_{i=1}^n \mathrm{L}_i f$ (or, the Ornstein–Uhlenbeck operator if f is a function on Gaussian space)
L_i	the ith coordinate Laplacian operator: $\mathrm{L}_i f = f - \mathrm{E}_i f$
$\ln x$	$\log_e x$
$\log x$	$\log_2 x$
Maj_n	the majority function on n bits
$\mathbf{MaxInf}[f]$	$\max_i\{\mathbf{Inf}_i[f]\}$
$\mathbf{MaxInf}[f]$	$\max_i\{\mathbf{Inf}_i[f]\}$
$[n]$	$\{1,2,3,\dots,n\}$
$\mathbb{N}$	$\{0,1,2,3,\dots\}$
$\mathrm{N}(0,1)$	the standard Gaussian distribution
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$\mathrm{N}(0,1)^d$	the distribution of d independent standard Gaussians; i.e., $\mathrm{N}(0, I_{d\times d})$

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$\mathrm{N}(\mu,\Sigma)$	for $\mu \in \mathbb{R}^d$ and $\Sigma \in \mathbb{R}^{d\times d}$ positive semidefinite, the d-variate Gaussian distribution with mean μ and covariance matrix Σ

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$\mathbb{N}^+$	$\{1,2,3,\dots\}$
$\mathbb{N}_{<m}$	$\{0,1,\dots,m-1\}$
$N_\rho(x)$	when $x \in \{-1,1\}^n$, denotes the probability distribution generating a string ρ-correlated to x
$N_\rho(z)$	when $z \in \mathbb{R}^n$, denotes the probability distribution of $\rho z + \sqrt{1-\rho^2}\boldsymbol{g}$ where $\boldsymbol{g} \sim \mathrm{N}(0,1)^n$
$\mathbf{NS}_\delta[f]$	the noise sensitivity of f at δ; i.e., $\frac{1}{2} - \frac{1}{2}\mathbf{Stab}_{1-2\delta}[f]$
OR_n	the logical OR function on n bits: True unless all inputs are False
Φ	the standard Gaussian cdf, $\Phi(t) = \int_{-\infty}^{t} \phi(z)\,dz$
ϕ	the standard Gaussian pdf, $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$
φ_A	the density function for the uniform probability distribution on A; i.e., $1_A/\mathbf{E}[1_A]$
ϕ_α	given functions $\phi_0,\dots,\phi_{m-1}$ and a multi-index α, denotes $\prod_{i=1}^n \phi_{\alpha_i}$
$\overline{\Phi}$	the standard Gaussian complementary cdf, $\overline{\Phi}(t) = \int_t^\infty \phi(z)\,dz$
$\pi^{\otimes n}$	if π is a probability distribution on Ω, denotes the associated product probability distribution on Ω^n
$\pi_{1/2}$	the uniform distribution on $\{-1,1\}$
π_p	the "p-biased" distribution on bits: $\pi_p(-1) = p$, $\pi_p(1) = 1-p$
$\mathbf{Pr}_{\pi_p}[\cdot]$	an abbreviation for $\mathbf{Pr}_{\boldsymbol{x}\sim\pi_p^{\otimes n}}[\cdot]$
$\mathbb{R}$	the real numbers
$\mathbb{R}^{\geq 0}$	the nonnegative real numbers
$\mathrm{RDT}(f)$	zero-error randomized decision tree complexity of f
$\mathbf{RS}_A(\delta)$	the rotation sensitivity of A at δ; i.e., $\mathbf{Pr}[1_A(\boldsymbol{z}) \neq 1_A(\boldsymbol{z}')]$ for a $\cos\delta$-correlated pair $(\boldsymbol{z},\boldsymbol{z}')$

$\mathrm{sens}_f(x)$	the number of pivotal coordinates for f at x
$\mathrm{sgn}(t)$	$+1$ if $t \geq 0$, -1 if $t < 0$
S_n	the symmetric group on $[n]$
$\mathrm{sparsity}(f)$	$\mathbf{Pr}_{\boldsymbol{x}}[f(\boldsymbol{x}) \neq 0]$
$\mathrm{sparsity}(\widehat{f})$	$\lvert\mathrm{supp}(\widehat{f})\rvert$
$\mathbf{Stab}_\rho[f]$	the noise stability of f at ρ: $\mathbf{E}[f(\boldsymbol{x})f(\boldsymbol{y})]$ where $\boldsymbol{x}, \boldsymbol{y}$ are a ρ-correlated pair
$\mathrm{supp}(\alpha)$	if α is a multi-index, denotes $\{i : \alpha_i \neq 0\}$
$\mathrm{supp}(f)$	if f is a function, denotes the set of inputs where f is nonzero
T_ρ	the noise operator: $\mathrm{T}_\rho f(x) = \mathbf{E}_{\boldsymbol{y} \sim N_\rho(x)}[f(\boldsymbol{y})]$
T^i_ρ	the operator defined by $\mathrm{T}^i_\rho f(x) = \rho f + (1-\rho)\mathrm{E}_i f$
T_r	for $r \in \mathbb{R}^n$, denotes the operator defined by $\mathrm{T}^1_{r_1}\mathrm{T}^2_{r_2}\cdots\mathrm{T}^n_{r_n}$
$\mathscr{U}$	the Gaussian isoperimetric function, $\mathscr{U} = \phi \circ \Phi^{-1}$
U_ρ	the Gaussian noise operator: $\mathrm{U}_\rho f(z) = \mathbf{E}_{\boldsymbol{z}' \sim N_\rho(z)}[f(\boldsymbol{z}')]$
$\mathbf{Var}[f]$	the variance of f, $\mathbf{Var}[f] = \mathbf{E}[f^2] - \mathbf{E}[f]^2$
Var_i	defined by $\mathrm{Var}_i f(x) = \mathbf{Var}_{\boldsymbol{x}_i}[f(x_1,\dots,x_{i-1},\boldsymbol{x}_i,x_{i+1},\dots,x_n))]$
$\mathrm{vol}_\gamma(A)$	$\mathbf{Pr}_{\boldsymbol{z} \sim \mathrm{N}(0,1)^n}[\boldsymbol{z} \in A]$, the Gaussian volume of A
$\mathbf{W}^k[f]$	the Fourier weight of f at degree k
$\mathbf{W}^{>k}[f]$	the Fourier weight of f at degrees above k
$\boldsymbol{x} \sim A$	the random variable $\boldsymbol{x}$ is chosen uniformly from the set A
$x^{(i \mapsto b)}$	the string $(x_1,\dots,x_{i-1},b,x_{i+1},\dots,x_n)$
$x^{\oplus i}$	$(x_1,\dots,x_{i-1},-x_i,x_{i+1},\dots,x_n)$
$\boldsymbol{x} \sim \varphi$	the random variable $\boldsymbol{x}$ is chosen from the probability distribution with density φ
x^S	$\prod_{i \in S} x_i$, with the convention $x^\emptyset = 1$
$\boldsymbol{x} \sim \{-1,1\}^n$	the random variable $\boldsymbol{x}$ is chosen uniformly from $\{-1,1\}^n$
(y,z)	if $J \subseteq [n]$, $y \in \{-1,1\}^J$, $z \in \{-1,1\}^{\overline{J}}$, denotes the natural composite string in $\{-1,1\}^n$
$\mathbb{Z}$	the additive group of integers modulo m
$\widehat{\mathbb{Z}^n_m}$	the group indexing the Fourier characters of functions $f : \mathbb{Z}^n_m \to \mathbb{C}$