List of Notation

$\circ$	entry-wise multiplication of vectors
∇	the gradient: $\nabla f(x) = (\mathrm{D}_1 f(x), \ldots, \mathrm{D}_n f(x))$
$\ni$	$S \ni i$ is equivalent to $i \in S$
$\oplus$	logical XOR (exclusive-or)
$\hat{\lVert} f \hat{\rVert}_p$	$(\sum_{\gamma \in \widehat{\mathbb{F}_2^n}} \lvert\widehat{f}(\gamma)\rvert^p)^{1/p}$
$\triangle$	symmetric difference of sets; i.e., $S \triangle T = \{i : i$ is in exactly one of $S, T\}$
$\vee$	logical OR
$\wedge$	logical AND
$*$	the convolution operator
$[z^k]F(z)$	coefficient on z^k in the power series $F(z)$
1_A	0-1 indicator function for A
$\mathbf{1}_B$	0-1 indicator random variable for event B
2^A	the set of all subsets of A
AND_n	the logical AND function on n bits: False unless all inputs are True
$\chi(b)$	when $b \in \mathbb{F}_2^n$, denotes $(-1)^b \in \mathbb{R}$
$\chi_S(x)$	when $x \in \mathbb{R}^n$, denotes $\prod_{i \in S} x_i$, where $S \subseteq [n]$; when $x \in \mathbb{F}_2^n$, denotes $(-1)^{\sum_{i \in S} x_i}$ if $x \in \mathbb{F}_2^n$
$\mathrm{codim}\, H$	for subspace $H \le \mathbb{F}^n$, denotes $n - \dim H$
$\mathbf{Cov}[f,g]$	the covariance of f and g, $\mathbf{Cov}[f] = \mathbf{E}[fg] - \mathbf{E}[f]\mathbf{E}[g]$
$d_{\chi^2}(\varphi, 1)$	chi-squared distance of the distribution with density φ from the uniform distribution

$\deg(f)$	the degree of f; the least k such that f is a real linear combination of k-juntas
D_i	the ith discrete derivative: $\mathrm{D}_i f(x) = \frac{f(x^{(i\mapsto 1)}) - f(x^{(i\mapsto -1)})}{2}$
$\mathrm{dist}(g,h)$	the relative Hamming distance; i.e., the fraction of places where g and h disagree
$\mathrm{Dist}(x,y)$	the Hamming distance, $\#\{i : x_i \neq y_i\}$
$\mathrm{DNF}_{\mathrm{size}}(f)$	least possible size of a DNF formula computing f
$\mathrm{DNF}_{\mathrm{width}}(f)$	least possible width of a DNF formula computing f
$\mathrm{DT}(f)$	least possible depth of a decision tree computing f
$\mathrm{DT}_{\mathrm{size}}(f)$	least possible size of a decision tree computing f
$d_{\mathrm{TV}}(\varphi,\psi)$	total variation distance between the distributions with densities φ, ψ
E_i	the ith expectation operator: $\mathrm{E}_i f(x) = \mathbf{E}_{\boldsymbol{x}_i}[f(x_1,\dots,x_{i-1},\boldsymbol{x}_i,x_{i+1},\dots,x_n))]$
$f \otimes g$	if $f : \{-1,1\}^m \to \{-1,1\}$ and $g : \{-1,1\}^n \to \{-1,1\}$, denotes the function $h : \{-1,1\}^{mn} \to \{-1,1\}$ defined by $h(x^{(1)},\dots,x^{(m)}) = f(g(x^{(1)}),\dots,g(x^{(m)}))$
$f^{\otimes d}$	if $f : \{-1,1\}^n \to \{-1,1\}$ then $f^{\otimes d} : \{-1,1\}^{n^d} \to \{-1,1\}$ is defined inductively by $f^{\otimes 1} = f$, $f^{\otimes(d+1)} = f \otimes f^{\otimes d}$
$f^{\dagger}$	the boolean dual defined by $f^{\dagger}(x) = -f(-x)$
$\mathbb{F}_2$	the finite field of size 2
$\widehat{\mathbb{F}_2^n}$	the group (vector space) indexing the Fourier characters of functions $f : \mathbb{F}_2^n \to \mathbb{R}$
f^{even}	the even part of f, $(f(x)+f(-x))/2$
$\langle f,g\rangle$	$\mathbf{E}_{\boldsymbol{x}}[f(\boldsymbol{x})g(\boldsymbol{x})]$
$\widehat{f}(i)$	shorthand for $\widehat{f}(\{i\})$ when $i \in \mathbb{N}$
$f_{H\mid z}$	if $f : \mathbb{F}_2^n \to \mathbb{R}$, $H \leq \mathbb{F}_2^n$, and $z \in H^{\perp}$, denotes the function $H \to \mathbb{R}$ defined by $f_{H\mid z}(y) = f(y+z)$
$f_{\mid z}$	if $f : \{-1,1\}^n \to \mathbb{R}$, $J \subseteq [n]$, and $z \in \{-1,1\}^{\overline{J}}$, denotes the restriction of f given by fixing the coordinates in $\overline{J}$ to z
$f_{J\mid z}$	if $f : \mathbb{F}_2^n \to \mathbb{R}$, $J \subseteq [n]$, and $z \in \mathbb{F}_2^{\overline{J}}$, denotes the restriction of f given by fixing the coordinates in $\overline{J}$ to z
$f^{=k}$	$\sum_{\lvert S\rvert = k} \widehat{f}(S)\,\chi_S$
$f^{\leq k}$	$\sum_{\lvert S\rvert \leq k} \widehat{f}(S)\,\chi_S$
f^{odd}	the odd part of f, $(f(x)-f(-x))/2$
$\widehat{f}(S)$	the Fourier coefficient of f on character χ_S
$\mathrm{F}_{\beta\mid H^{\perp}} f(z)$	for $\beta \in H \leq \mathbb{F}_2^n$, denotes $\widehat{f_{H\mid z}}(\beta)$

$\mathrm{F}_{S\mid\overline{J}}f(z)$	for $S \subseteq J \subseteq [n]$, denotes $\widehat{f_{J\mid z}}(S)$
$\mathbf{Inf}_i[f]$	the influence of coordinate i on f
$\mathbf{Inf}_i^{(\rho)}[f]$	the noisy influence, $\mathbf{Stab}_\rho[\mathrm{D}_i f]$
$\overline{J}$	if $J \subseteq [n]$, denotes $[n] \setminus J$
L	the Laplacian operator: $\mathrm{L}f = \sum_{i=1}^n \mathrm{L}_i f$
$\Lambda_\rho(\mu)$	$\mathbf{Pr}[\boldsymbol{z}_1 > t, \boldsymbol{z}_2 > t]$, where $\boldsymbol{z}_1, \boldsymbol{z}_2$ are standard Gaussians with correlation $\mathbf{E}[\boldsymbol{z}_1\boldsymbol{z}_2] = \rho$ and $t = \overline{\Phi}^{-1}(\mu)$.
L_i	the ith directional Laplacian operator: $\mathrm{L}_i f = f - \mathrm{E}_i f$
$\ln x$	$\log_e x$
$\log x$	$\log_2 x$
Maj_n	the majority function on n bits
$[n]$	$\{1, 2, 3, \dots, n\}$
$\mathbb{N}$	$\{0, 1, 2, 3, \dots\}$
$\mathrm{N}(0,1)$	the standard Gaussian distribution
$\mathbb{N}^+$	$\{1, 2, 3, \dots\}$
$N_\rho(x)$	the probability distribution generating strings ρ-correlated to x
$\mathbf{NS}_\delta[f]$	the noise sensitivity of f at δ; i.e., $\frac{1}{2} - \frac{1}{2}\mathbf{Stab}_{1-2\delta}[f]$
OR_n	the logical OR function on n bits: True unless all inputs are False
Φ	the standard Gaussian cdf, $\Phi(t) = \int_{-\infty}^t \phi(z)\,dz$
ϕ	the standard Gaussian pdf, $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$
φ_A	the density function for the uniform probability distribution on A; i.e., $1_A/\mathbf{E}[1_A]$
$\overline{\Phi}$	the standard Gaussian complementary cdf, $\overline{\Phi}(t) = \int_t^\infty \phi(z)\,dz$
$\mathbb{R}$	the real numbers
$\mathbb{R}^{\geq 0}$	the nonnegative real numbers
$\mathrm{sens}_f(x)$	the number of pivotal coordinates for f at x
$\mathrm{sgn}(t)$	$+1$ if $t \geq 0$, -1 if $t < 0$
S_n	the symmetric group on $[n]$
$\mathrm{sparsity}(f)$	$\mathbf{Pr}_{\boldsymbol{x}}[f(\boldsymbol{x}) \neq 0]$
$\mathrm{sparsity}(\widehat{f})$	$\lvert\mathrm{supp}(\widehat{f})\rvert$
$\mathbf{Stab}_\rho[f]$	the noise stability of f at ρ: $\mathbf{E}[f(\boldsymbol{x})f(\boldsymbol{y})]$ where $\boldsymbol{x}, \boldsymbol{y}$ are a ρ-correlated pair
$\mathrm{supp}(f)$	the set of inputs where f is nonzero

T_ρ	the noise operator: $\mathrm{T}_\rho f(x) = \mathbf{E}_{\boldsymbol{y} \sim N_\rho(x)}[f(\boldsymbol{y})]$
$\mathscr{U}$	the Gaussian isoperimetric function, $\mathscr{U} = \phi \circ \Phi^{-1}$
$\mathbf{Var}[f]$	the variance of f, $\mathbf{Var}[f] = \mathbf{E}[f^2] - \mathbf{E}[f]^2$
Var_i	defined by $\mathrm{Var}_i f(x) = \mathbf{Var}_{\boldsymbol{x}_i}[f(x_1, \dots, x_{i-1}, \boldsymbol{x}_i, x_{i+1}, \dots, x_n))]$
$\mathbf{W}^k[f]$	the Fourier weight of f at degree k
$\mathbf{W}^{>k}[f]$	the Fourier weight of f at degrees above k
$\boldsymbol{x} \sim \{-1,1\}^n$	the random variable $\boldsymbol{x}$ is chosen uniformly from $\{-1,1\}^n$
$\boldsymbol{x} \sim A$	the random variable $\boldsymbol{x}$ is chosen uniformly from the set A
$x^{(i \mapsto b)}$	the string $(x_1, \dots, x_{i-1}, b, x_{i+1}, \dots, x_n)$
$x^{\oplus i}$	$(x_1, \dots, x_{i-1}, -x_i, x_{i+1}, \dots, x_n)$
$\boldsymbol{x} \sim \varphi$	the random variable $\boldsymbol{x}$ is chosen from the probability distribution with density φ
x^S	$\prod_{i \in S} x_i$, with the convention $x^{\emptyset} = 1$
(y, z)	if $J \subseteq [n]$, $y \in \{-1,1\}^J$, $z \in \{-1,1\}^{\overline{J}}$, denotes the natural composite string in $\{-1,1\}^n$