In the U.S., the growth of the organ transplantation waiting list significantly outpaces the supply of donated organs. Among the many initiatives aiming to boost the supply, the donor priority rule, under which a registered organ donor—in case of needing a transplant—is given priority to receive an organ over a non-donor, has been considered by policy makers in the U.S. In this paper, we model the U.S. organ donation and allocation system using the strategic queueing theoretic framework. To capture the tradeoff behind individuals’ donating behavior, we use quality-adjusted life expectancy (QALE) to measure their benefits from potential organ transplants, and use the cost of donating to measure their loss from becoming potential cadaveric organ donors. Then we characterize the equilibrium before and after introducing the donor priority rule. When individuals are homogeneous in their health status, introducing donor priority rule always leads to improved social welfare. When individuals possess heterogeneous health status, in contrast to the extant literature, we show that the social welfare can be worse off after introducing the donor priority rule due to the unbalanced incentive structure. Finally, we propose a simple freeze-period mechanism, and prove that in conjunction with the donor priority rule, it can increase the share of organ donors without distorting the average quality of the donated organs.

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We assume some of the most peculiar and temporary of our late advantages as natural, permanent, and to be depended on, and we lay our plans accordingly... On this sandy and false foundation we scheme for social improvement and dress our political platforms...

John Maynard Keynes: *The Economic Consequences of The Peace*

1. Introduction

U.S. is experiencing an ongoing organ shortage crisis, with about 18 people die in the U.S. while waiting for transplants each day, and a new candidate is added to the waiting list every 10 minutes (Organdonor.gov 2012). Furthermore, the waiting list of transplant candidates outgrows the registry of potential organ donors: between 1989 and 2009, the number of people wait-listed for an organ increased by 4.89 times, but the number of cadaveric organ donors (i.e., those who agree to donate their organs in case of premature brain death) grew by 1.47 times. Because cadaveric organs remain a major source of organs for transplantation, one major cause of the current organ crisis is a low share of registered organ donors, currently at around 40%, although the public has an 85% approval rate of organ donation (Gallup 2005).

A myriad of initiatives have been suggested to encourage more people to add their names to the organ donor registry. The contemporary public discourse mostly focuses on educational and promotional measures to enhance the public’s awareness of the benefits of organ donation. On April 2, 2012, President Barack Obama proclaimed April 2012 as the National Donate Life Month to “call upon health care professionals, volunteers, educators, government agencies, faith-based and community groups, and private organizations to join forces to boost the number of organ and tissue donors.” On May 1, 2012, Facebook announced a new sharing function that enabled its users to advertise their donor status on their timelines, in the hope that this move will exert peer pressure to people who have not registered as organ donors.
Health economists, on the other hand, have long been at the forefront to propose to provide monetary incentives (in cash or non-cash forms) to individuals for registering as potential cadaveric organ donors. While few would contest that doing so would lead to an immediate jump in the number of organ donors, this market-based approach been widely criticized as it allegedly “fosters class distinctions (and exploitation), infringes on the inalienable values of life and liberty, and is therefore ethically unacceptable” (Delmonico et al. 2002).

In addition to enhancing public awareness and providing monetary incentives, there are two popular initiatives that have been weighed by Federal and state governments as well as non-government organizations: (i) Donor priority rule, which provides a priority status to individuals registering to become potential organ donors. Under the rule, in case that registered donors need organ transplants, they are given higher priorities in receiving cadaveric organs than non-donors. (ii) Presumed consent (a.k.a, “opt-out”) policy, which, in contrary to the current practice in the U.S., automatically registers people as potential organ donors (e.g., when they apply for a driver license) unless they follow required procedures to opt out of the organ donation program. The legislation of the presumed consent has been endorsed by various studies (e.g., Abadie and Gay 2006) but faces many hurdles, including the public’s fear of misrepresentation of individuals’ willingness to donate.

Our paper focuses on analyzing the donor priority rule. We develop a theoretic model of the system of donor registration and organ allocation. As we focus on the broad implication of adopting the rule, we do not restrict ourselves to a specific type of organ (e.g., kidney, liver, tissue).

In modeling the tradeoffs behind each individual’s decision of registering as a potential organ donor, we follow Kessler and Roth (2012) to assume that each individual has a cost
of donating; different from Kessler and Roth (2012), however, we capture each individual’s utility from organ transplantation using the expected total quality-adjusted life expectancy (QALE) by applying the approximation results from the queueing literature (e.g., Zenios 1999). Our analysis shows that when all individuals are homogeneous in their health status, the introduction of the donor priority rule will expand the size of the donor registry, increase the overall availability of obtaining an organ, and result in increased social welfare. This result is consistent with the findings by Kessler and Roth (2012). When the individuals are heterogeneous, however, we show that, different from what Kessler and Roth (2012) predict, the introduction of the donor priority rule can indeed reduce the social welfare. This is because under the rule, even for individuals with the same cost of donating, as they can have different probabilities of needing organ transplants in the future, they will respond differently to rule in their decisions as to whether to register to be a potential organ donor or not. Specifically, we show that ceteris paribus, the donor priority rule provides a higher incentives to high-risk individuals than to low-risk individuals, and essentially leads to a pool of donated organs with an average quality lower than the that of general population. When this disproportionality of incentives becomes significant enough, we show that the resultant social welfare loss can outweigh the social welfare gain from the expanded organ donor registry.

We proceed to show that a simple freeze-period remedy, under which an individual may not enjoy a higher queueing priority until after having registered as an organ donor for a period of time (i.e., the freeze period), can overcome the aforementioned quality distorting effect. When the freeze-period remedy is implemented in conjunction with the donor priority rule, the average quality of the donated organs can be restored to the population average level. The underlying reason for this improvement is that the freeze
period remedy essentially dis incentives individuals from becoming organ donors, but the
level of disincentive differs for individuals with different risk levels such that high-risk
individuals are more discouraged than low-risk ones. We prove that the remedy, if designed
properly, can (i) help fix the biased incentive structure due to the donor priority rule, and
(ii) expand the donor base compared to the case without the donor priority rule. Thus
the freeze-period remedy, in conjunction with the donor priority rule, will lead to a better
social outcome compared to the current organ donation system.

The rest of the paper is organized as follows. Section 2 reviews the relevant literature.
In §3, we describe our analytical framework. In §4, we consider the case where all the
individuals have homogeneous health status. Section 5 models population heterogeneity in
health status. Section 6 proposes a freeze-period remedy. Section 7 concludes the paper
with a summary of our key insights and future research directions.

2. Literature

We add to a thin but growing body of Operations Management (OM) literature on organ
transplantation services, most of which focuses on organ allocation (e.g., Su and Zenios
2004, 2006; Akan et al. 2012; Ata et al. 2012) and surgical decisions (e.g., Howard 2002;
Alagoz et al. 2004). To the best of our knowledge, we are the first to develop an OM
model to examine the organ donation policy. Here we briefly review several papers on
organ allocation that are most relevant to our work. Su and Zenios (2004) develop and
analyze a queueing model to examine the role of patient choice in the kidney transplant
waiting system, and highlight the conflict between equity and efficiency in kidney allo-
cation. They show that under the LCFS priority discipline, the competitive equilibrium
is socially optimal. Our paper, with a specific focus on organ donation, also considers a
priority queueing discipline, but the priority is tied to each individual’s organ donor status.
rather than the sequence of arrivals. Su and Zenios (2006) propose an organ allocation method where heterogeneous patients have to declare which types of kidneys they would be willing to accept at the time they join the waiting list (rather than at the time they are offered the kidney). By doing so, a lengthy search at time of transportation is eliminated, and each candidate’s private information is reflected in the allocation process. Our paper highlights the impact of individual heterogeneity as well and shows that individuals’ heterogeneous health status can influence their organ donation behavior and, in turn, the average quality of the donated organs.

In the vast health economics literature, there is a paucity of rigorous treatment of organ donation through modeling the donor priority rule. The only paper that analytically examines the same issue is Kessler and Roth (2012) but there are two important differences that separate their work from ours. First, Kessler and Roth (2012) address the impact of the donor priority rule mainly through behavioral experiments. Their analytic model, built to illustrate their experimental findings, only considers how the donor priority rule affects an individual’s probability of receiving an organ. Our paper, by contrast, builds a queueing model of the organ allocation process, and uses an individual’s QALE, rather than the probability of receiving organs, as the measure of an individual’s utility. This different problem setup gives rich and interesting insights into the social welfare consequences of the donor priority rule.

3. Modeling Framework

Each individual can be at one of the three states: healthy (in a condition not requiring an organ transplant), sick and in need of an organ transplant, or dead from brain death. Two types of stochastic events are central to our modeling of the organ allocation system: some healthy individuals become sick and are in need of an organ, while other healthy
individuals suffer from brain death and can potentially provide a number of organs. We model them as two separate, independent stochastic processes. We denote by $\theta$ the arrival rate of individuals in need of organs, and $\phi$ the rate of all brain deaths; both processes are assumed to be Poisson processes without loss of generality. Letting $n$ denote the number of organs each potential donor can provide, the maximum possible arrival rate of organs is $\phi n$. We assume that $\phi n > \theta$, meaning that the organs from all the brain deaths—whether organ donor registry or not—are adequate to support those in need of organs; this is a realistic assumption given the real statistics: in 2007, for example, there are 2.5 million brain deaths in the U.S., which is more than twenty times of the number of patients on the waiting list for cadaveric organs (Organdonor.gov 2012).

Each individual incurs a cost to register as an organ donor. The cost is denoted by $c$ and has a support of $(-\infty, \infty)$ and a cumulative density function $F(\cdot)$. When an individual has a cost $c > 0$, it means there are certain burdens to be overcome for the individual to register to be a potential organ donor. For example, some individuals fear that physicians might not try their best to save registered organ donors’ lives (Teresi 2012). As another example, certain religious beliefs disfavor the practice of organ donation (Bruzzone 2008). When an individual has a cost $c < 0$, it means that the individual earns a positive non-monetary gain (e.g., social recognition, self-fulfilment) from registering to be an organ donor (Prottas 1983).

Following the organ transplantation literature (e.g., Su and Zenios 2006), we use QALE to measure the utility of an individual that needs an organ transplant. An individual’s QALE is written as

$$u = \alpha D + \beta \pi T,$$

where $\alpha$ is the quality-of-life score while on the waiting list; $D$ is the individual’s life expectancy from the time he is put on the waiting list to the time he dies or receives an
organ, whichever comes earlier; $\beta$ is the quality-of-life score after transplantation; $\pi$ is the probability of receiving an organ; $T$ is the individual’s post-transplantation life expectancy.

For a healthy individual, the tradeoff behind the decision of becoming organ donor involves the cost of donating versus the potential benefits from organ donation (if there are any). It is evident that ceteris paribus, an individual with a higher cost will have a lower incentive to become an organ donor. In other words, given any organ donation policy, there must exist a threshold $x \in (-\infty, \infty)$ such that all the individuals with $c \leq x$ will become organ donors, and all the individuals with $c > x$ will not. Given $x$, the corresponding donation rate (i.e., the proportion of the population who are donors) is $F(x)$. As $F(x)$ increases in $x$, a larger $x$ means that there is a higher share of organ donors. We define a constant $\hat{x}$ at which $F(\hat{x})\phi n = \theta$, that is, $\hat{x} = F^{-1} \left( \frac{\theta \phi n}{\phi m} \right)$. In other words, $F(\hat{x})$ is the share of organ donors at which the supply rate of organs is equal to the demand rate.

Let $d$ denote a sick individual’s life expectancy without organ transplantation. When the threshold is $x$, we can approximate the individual’s pre-transplantation life expectancy as

$$D(x) = \left( \frac{\theta - F(x)\phi n}{\theta} \right)^+ \cdot d$$

$$= \begin{cases} \frac{\theta - F(x)\phi n}{\theta} \cdot d & \text{if } x \leq \hat{x} \\ 0 & \text{otherwise.} \end{cases}$$

The probability for each individual to receive an organ $\pi$ can be approximated as

$$\pi(x) = \min \left\{ 1, \frac{F(x)\phi n}{\theta} \right\}$$

$$= \begin{cases} \frac{F(x)\phi n}{\theta} & \text{if } x \leq \hat{x} \\ 1 & \text{otherwise.} \end{cases}$$

The above approximations were developed in Zenios (1999).
3.1. Benchmark: Social Optimum

Before proceeding to investigate the effect of a specific policy, we first characterize the social optimum. Since all candidates are homogeneous in their ex-ante expected utility of receiving an organ in our model, they have the same probability to receive an organ. In order to achieve social optimum, the donors, as chosen by the social planner, must be the ones with low costs of donating. Therefore, the social optimum is dictated by a threshold, denoted by $x^{SO}$, such that all individuals with costs lower than $x^{SO}$ will register to become donors, and those with costs higher than $x^{SO}$ will not.

The social welfare as a function of $x$ can thus be written as the expected aggregate QALE of all the listed individuals less the aggregated costs of donating of all the registered organ donors (i.e., whose cost of donating is lower than $x$):

$$W_s(x) = \theta (\alpha D(x) + \beta \pi(x)T) - \mathbb{E}[(c|c \leq x)]F(x)$$

$$= \begin{cases} 
\alpha d(\theta - F(x)\phi n) + \beta T F(x)\phi n - \int_{-\infty}^{x} cf(c)dc & \text{if } x < \hat{x}, \\
\theta \beta T - \int_{-\infty}^{x} cf(c)dc & \text{otherwise.} 
\end{cases}$$

(3)

Lemma 1. The socially efficient cost threshold is:

$$x^{SO} = \min\{(\beta T - \alpha d)\phi n, \hat{x}\}.$$ 

Proof. When $x < \hat{x}$, we have from (3) that

$$\frac{dW_s(x)}{dx} = ((\beta T - \alpha d)\phi n - x) f(x).$$

This implies the regional maximum for $x \in (-\infty, \hat{x})$ is achieved at $(\beta T - \alpha d)\phi n$ or $\hat{x}$, whichever is lower. If $x > \hat{x}$, we observe from (3) that $W_s(x)$ decreases in $x$, indicating that the regional maximum for $x \in (\hat{x}, \infty)$ is achieved at $\hat{x}$. Taken together, the socially optimal threshold $x^{SO}$ is $(\beta T - \alpha d)\phi n$ or $\hat{x}$, whichever is lower. 

Q.E.D.
4. Preliminary: Homogeneous Health Status

As a benchmark, we consider in this section the case individuals with heterogeneous costs of donating but homogeneous health status. We first characterize the equilibrium without the donor priority rule. Then we characterize the equilibrium after the introduction of the donor priority rule. We then compare the social welfare and show that introducing donor priority rule leads to increased social welfare.

4.1. Without Priority Rule

As a benchmark, we consider the case where the allocation of organs follows a first-come-first-serve (FCFS) queueing discipline, and registered organ donors do not enjoy any priority if they are in need of organs in the future. Because each individual gains no benefits from becoming an organ donor, only those with \( c < 0 \) have the incentive to do so. Therefore, in equilibrium, the threshold cost, denoted by \( x_{np}^* \), is zero; the share of organ donors (i.e., the proportion of the population registering as donors) is \( F(0) \). The resultant arrival rate for the supply process of organs is therefore \( n \phi F(0) \). We assume that \( n \phi F(0) < \theta \), meaning that the supply rate of organs is lower than the demand rate when there is no priority rule, which is in line with the reality of the U.S. organ transplantation system (Organdonor.gov 2012).

Each sick individual’s pre-transplantation life-expectancy can be approximated as

\[
D(x_{np}^*) = \left( \frac{\theta - F(0) \phi n}{\theta} \right)^+ \cdot d = \frac{\theta - F(0) \phi n}{\theta} \cdot d.
\]

The probability for each sick individual to receive an organ can be approximated as

\[
\pi(x_{np}^*) = \frac{F(0) \phi n}{\theta} < 1.
\]
The social welfare with no donor priority rule can thus be written as the expected aggregate QALE of all the sick individuals less the costs of donating of all those who choose to register as organ donors:

\[
W_{np} = \theta (\alpha D + \beta \pi T) - \mathbb{E}[(c|c \leq 0)] F(0)
\]

\[
= \theta \left( \alpha d \cdot \frac{\theta - F(0) \phi n}{\theta} + \beta T \cdot \frac{F(0) \phi n}{\theta} \right) - \mathbb{E}[(c|c \leq 0)] F(0)
\]

\[
= \alpha d (\theta - F(0) \phi n) + \beta F(0) \phi n T - \mathbb{E}[(c|c \leq 0)] F(0).
\]

4.1.1. Comparison with Social Optimum
In the social optimum characterized in §3.1, we have \(x^{SO} = \min \{ (\theta \beta T - \alpha d) \phi n, \hat{x} \}\). Since \(n \phi F(0) < \theta\), we have \(\hat{x} = F^{-1} \left( \frac{\theta}{\phi n} \right) > 0\). In addition, the condition \(\beta T > \alpha d\) gives \((\beta T - \alpha d) \phi n > 0 = x^*_{np}\). Therefore, \(x^{SO} = \min \{ (\beta T - \alpha d) \phi n, \hat{x} \} > 0\). In other words, without the priority rule, the donation rate is strictly below the socially optimal level.

4.2. With Priority Rule
Now we consider the case where all the registered donors have the priority over non-donors to receive organs if and when they need cadaveric organs. Due to the introduction of this rule, there are two types of queues waiting for donated organs: the priority queue with organ donors, and the regular queue with non-donors. The implication is that there are two competing effects behind the equilibrium: first, when an individual decides to be an organ donor, he or she is in essence purchasing an option to join a priority queue in the future when he needs an organ. Therefore, the individual would benefit from a larger organ pool provided by more organ donors. Second, the value of becoming a donor is diminishing as more people become donors because each organ donor, if in need of an organ, would face a longer waiting time in the priority queue.

To seek the point at which the equilibrium is reached, we first need to derive an organ donor’s utility. We use \(x^*_{p}\) to denote the cut-off cost at which an individual is indifferent
as to whether to elect to be a donor or not. As a result, the supply rate of organs is now \( \phi n F(x_p^*) \), and the demand rate of organs remains \( \theta \), of which \( F(x_p^*) \theta \) is the arrival rate of donors, and the remaining \([1 - F(x_p^*)]\) is the arrival rate of non-donors. In consequence, there are two waiting lists—donor list and non-donor list—for organs. Because a donor has a higher priority than a non-donor, they have different utility values.

**Donor Utility.** A donor’s pre-transplantation life cannot be computed using (1)–(2) because the arrival rate of donors is \( \lambda_d = F(x_p^*) \theta \) is always lower than the supply rate of organs \( \mu_d = F(x_p^*) \phi n \) because \( \theta \ll \phi n \). Rather, a donor has a probability of 1 of receiving an organ, and his pre-transplantation life expectancy is \( E[\min\{d,W\}] \), where \( W \) is the donor’s expected waiting time for an organ, which, by the classic queueing theory (c.f. Kleinrock 1975), is exponentially distributed with an mean of

\[
\frac{\lambda_d}{\mu_d - \lambda_d} = \frac{F(x_p^*) \theta}{F(x_p^*) \phi n - F(x_p^*) \theta} = \frac{\theta}{\phi n} \approx 0,
\]

since \( \theta \ll \phi n \). Therefore, a donor’s net utility is his QALE in case that he will need a organ less his cost, that is,

\[
u_d = \beta \theta T - c.
\]

**Non-Donor Utility.** A non-donor has a lower priority than the donor, and faces scarce supply. The total arrival rate of sick non-donors is \( \lambda_n = (1 - F(x_p^*)) \theta \), and the supply rate of organs for non-donors is \( \mu_n = F(x_p^*) (\phi n - \theta) \). His pre-transplantation life expectancy and probability of receiving an organ can be computed using (1)–(2):

\[
D_n = \frac{\lambda_n - \mu_n}{\lambda_n} \cdot d,
= \frac{(1 - F(x_p^*)) \theta - F(x_p^*) (\phi n - \theta)}{(1 - F(x_p^*)) \theta} \cdot d,
= \frac{\theta - \phi n F(x_p^*)}{(1 - F(x_p^*)) \theta} \cdot d,
\]
and his probability of receiving an organ is
\[ \pi_n = \frac{\mu_n}{\lambda_n} = \frac{F(x_p^*)(\phi_n - \theta)}{(1 - F(x_p^*))\theta}. \]

The following proposition characterizes the equilibrium.

**Proposition 1.** In equilibrium,
\[ x_p^* = \frac{\theta - \phi_n F(x_p^*)}{1 - F(x_p^*)} \cdot (\beta T - \alpha d), \tag{4} \]
and only those individuals with donation costs \( c \) below the cutoff cost \( x^* \) will elect to become organ donors.

**Proof.** An individual with the cutoff cost \( x_p^* \) is indifferent as to whether to become an organ donor or not, that is,
\[ \theta \beta T - x_p^* = \theta \left( \alpha d \cdot \frac{\lambda_n - \mu_n}{\lambda_n} + \beta T \cdot \frac{\mu_n}{\lambda_n} \right), \]
\[ \Rightarrow \theta \beta T - x_p^* = \theta \left( \alpha d \cdot \frac{\theta - F(x_p^*) \phi_n}{\theta (1 - F(x_p^*))} + \beta T \cdot \frac{F(x_p^*) (\phi_n - \theta)}{\theta (1 - F(x_p^*))} \right), \]
which is equivalent to (4).

Based on Proposition 1, we can write the social welfare under the donor priority rule as
\[
W_p = \int_{-\infty}^{x_p^*} (\theta \beta T - c) f(c) dc + \theta \int_{x_p^*}^{\infty} \left( \frac{\lambda_n - \mu_n}{\lambda_n} + \frac{\mu_n}{\lambda_n} \right) f(c) dc
\]
\[
= \int_{-\infty}^{x_p^*} (\theta \beta T - c) f(c) dc + \theta \int_{x_p^*}^{\infty} \left( \frac{\lambda_n - \mu_n}{\lambda_n} + \frac{\mu_n}{\lambda_n} \right) f(c) dc
\]
\[
= \theta \beta T - E(c | c \leq x_p^*) F(x_p^*) - x_p^* \left( 1 - F(x_p^*) \right).
\]

Proposition 1 gives the following corollary:

**Corollary 1.** \( x_p^* > x_{np}^* = 0 \).

**Corollary 2.** \( x_p^* \) increases in \( \theta \), and decreases in \( \phi \).
Proof. We have from (4) that

\[ 1 - \frac{\theta}{\phi n} = \left(1 - \frac{x_p^*}{\phi n(\beta T - \alpha d)}\right) \left(1 - F(x_p^*)\right). \] (5)

As \( \theta \) increases, the left-hand side of (5) decreases, requiring a higher \( x_p^* \) to decreases the right-hand side of (5) and balance the equation. Similarly, we can show that \( x_p^* \) decreases in \( \phi \).

Q.E.D.

The above corollary suggests that an individual is more likely to join the organ donor registry if his chance of becoming sick and needing an organ increases, and vice versa. In addition, an individual is less likely to become an organ donor when brain deaths occur more frequently, providing a higher supply rate of organs.

**Comparison with Social Optimum.** Corollary 1 implies that introducing the donor priority rule will lead to a higher donation rate. However, we can verify that the equilibrium donation rate is still below the social optimal donation rate, i.e. \( x_p^* < x^{SO} \). To see this, recall that \( x^{SO} = \min \{ (\beta T - \alpha d) \phi n, \hat{x} \} \). On one hand, as \( \theta - n\phi F(x_p^*) > 0 \) (from (4)), we have \( x_p^* < \hat{x} = F^{-1} \left( \frac{\theta}{\phi n} \right) \). On the other hand, we have

\[
x_p^* = \frac{\theta - \phi n F(x_p^*)}{1 - F(x_p^*)} (\beta T - \alpha d)
= \frac{\theta - \phi n}{\phi n} F(x_p^*) (\beta T - \alpha d) \phi n
< (\beta T - \alpha d) \phi n \text{ (since } \theta < \phi n).\]

Hence we have \( x_p^* < \min \{ \hat{x}, (\beta T - \alpha d) \phi n \} = x^{SO} \).

**4.3. Comparison of Social Welfare**

We now compare the social welfare before and after the introduction of the donor priority rule. The result is summarized in the following proposition:

**Proposition 2.** Under homogeneous population, the introduction of the donor priority rule always increases the social welfare.
Proof. We examine the difference of the social welfare before and after the introduction of the donor priority rule:

\[ W_p - W_{np} = (\theta - F(0) \phi n) (\beta T - \alpha d) + E(c|c \leq 0) F(0) - E(c|c \leq x^*_p) F(x^*_p) - x^*_p (1 - F(x^*_p)) \]

which, by Proposition 1, can be rewritten as

\[ W_p - W_{np} = (\theta - F(0) \phi n) (\beta T - \alpha d) - \int_0^{x^*_p} cf(c) dc - x^*_p (1 - F(x^*_p)) , \]

Now, since \( \theta < \phi n \), we have

\[ \frac{1 - F(x^*_p)}{\theta / (\phi n) - F(x^*_p)} > 1, \]

which gives

\[ W_p - W_{np} > x^*_p (F(x^*_p) - F(0)) - \int_0^{x^*_p} cf(c) dc > 0. \]

That is, the social welfare improves after the introduction of the donor priority rule. Q.E.D.

Giving donors the priority to receive organs has two immediate effects: it increases the total costs of donating because a proportion of donors with positive costs are incentivized to register as donors; it increases the supply of donors. Proposition 2 suggests that the second effect outweighs the first, leading to increased social welfare.

The following corollary further refines Proposition 2.

**Corollary 3.** The social welfare difference \( W_p - W_{np} \) increases in \( \theta \), and decreases in \( \phi \).
Proof. The social welfare difference, by (6), can be rewritten as

\[ W_p - W_{np} = x_p^* \left( F(x_p^*) - F(0) \right) \cdot \left( 1 + \frac{1 - \theta/(\phi n)}{\theta/(\phi n) - F(x_p^*)} \right) - \int_{0}^{x_p^*} cf(c)dc, \]

which is increasing in \( x_p^* \). As \( \theta \) increases, we have from Corollary 2 that \( x_p^* \) increases, and so does the social welfare improvement. Similarly, we can show that the social welfare improvement decreases in \( \phi \).

Q.E.D.

As \( \theta \) increases or \( \phi \) decreases, it becomes increasingly challenging for a candidate to be matched to an organ. The donor priority provides a stronger incentive for individuals to become registered donors and leads to a higher social welfare increase.

5. The Unexpected Welfare Consequences of Donor Priority Rule

In the preceding section, we show that the introduction of donor priority rule increases the social welfare. The baseline model allows individuals to be heterogeneous in their costs of donating. In this section, we extend our preliminary model to incorporate the second dimension of the heterogeneity among these individuals: their chance of becoming sick and needing an organ.

We categorize the whole population into two groups: high- and low-risk population. A high-risk individual is more likely to become sick and need an organ; in addition, a high-risk individual is more likely to have low-quality organs. To be specific, with probability \( p_H \), an individual is high-risk; with probability \( p_L \), an individual is low-risk. We have \( p_H + p_L = 1 \). Furthermore, We now break the demand process into two separate, independent subprocesses such that the arrival rate for high-risk individuals to become sick and need organs is \( \theta_H \), and the arrival rate for low-risk individuals to become sick and need organs is \( \theta_L \). The total arrival rate of individuals needing organ is now \( p_H \theta_H + p_L \theta_L \). As the two groups of population are different in their potential needs for organs due to their different
health status, it is reasonable to assume that they also differ in the quality of their organs. The quality of an individual’s organs is measured in terms of the post-transplantation life expectancy of another individual who receives one of these organs. We use $G_H(t)$ and $G_L(t)$ to denote the respective c.d.f.’s of the post-transplantation life expectancy associated with organs receiving from a high-risk and low-risk individual. We make the following assumptions:

**Assumption 1.** The life expectancy of a person that receives an organ from a high-risk individual is lower than that from a low-risk individual, that is, $E[G_H(t)] = T_H < E[G_L(t)] = T_L$.

**Assumption 2.** The total demand rate for donated organs is lower than the maximum possible organ supply rate, that is, $P_H\theta_H + \pi_L\theta_L < \phi n$.

**Assumption 3.** The arrival rate for a high-risk individual to become sick and need an organ is higher than that for a low-risk individual, that is, $\theta_H > \theta_L$.

5.1. **Without Priority Rule**

We first consider the case where the priority rule is not enforced. Similar to the preliminary model, only those with negative costs of donating have the incentive to register as organ donors. In other words, the cut-off cost in equilibrium is $x_i^* = 0$. Hence the aggregated post-transplantation life expectancy is

$$T_a = \sum_{i=L,H} p_i T_i.$$

The social welfare is thus

$$W_{np}^h = \sum_{i=L,H} p_i \theta_i \left( \alpha_d \frac{\sum_{i=L,H} p_i \theta_i - F(0) \phi n}{\sum_{i=L,H} p_i \theta_i} + \beta T_a \frac{F(0) \phi n}{\sum_{i=L,H} p_i \theta_i} \right) - \mathbb{E}[(c|c \leq 0)] F(0)$$

$$= \alpha d \left( \sum_{i=L,H} p_i \theta_i - F(0) \phi n \right) + F(0) \phi n \beta T_a - \mathbb{E}[(c|c \leq 0)] F(0).$$
5.2. With Priority Rule

We now introduce the priority rule to the organ allocation system. One key aspect different from our analysis in the preliminary model is that now since there are two types of individuals with different risk levels, they would respond to priority rule differently by setting different cut-off costs, denoted by $x_H^*$ and $x_L^*$, respectively, for high- and low-risk individuals. As a result, the arrival rate of type $i$ donor patients is

$$\lambda_d^i = p_i F(x_i^*) \theta_i, \text{ for } i = H, L,$$

and the arrival rate of type $i$ non-donor patients is

$$\lambda_n^i = p_i (1 - F(x_i^*)) \theta_i, \text{ for } i = H, L.$$

In addition, the total arrival rate of organs from both high- and low-risk organ donors is

$$\mu = \sum_{i=L,H} p_i F(x_i^*) \phi_n.$$

**Donor Utility.** The total arrival rate of organ donors is

$$\lambda_d = \sum_{i=L,H} \lambda_d^i = \sum_{i=L,H} p_i F(x_i^*) \theta_i.$$

The donors are endowed with the priority of receiving all the available organs, i.e., total supply rate of organs available to donors is $\mu_d = \mu$. Hence a type $i$ organ donor with a cost $c$ has a net utility of

$$u_d^i(c) = \theta_i \beta T_p(x_H^*, x_L^*) - c,$$

where $T_p(x_H^*, x_L^*)$ is the aggregated post-transplantation life expectancy such that

$$T_p(x_H^*, x_L^*) = \frac{\sum_{i=L,H} p_i F(x_i^*) T_i}{\sum_{i=L,H} p_i F(x_i^*)}.$$
Non-Donor Utility. The total arrival rate of non-donors is

\[ \lambda_n = \sum_{i=L,H} \lambda^i_n = \sum_{i=L,H} p_i (1 - F(x^*_i)) \theta_i, \]

and the total supply rate of donors available to non-donors is

\[ \mu_n = \mu - \lambda_d = \sum_{i=L,H} p_i F(x^*_i) (\phi_n - \theta_i). \]

Hence a type-\(i\) non-donor with a cost \(c\) has a net utility of

\[ u^i_n(c) = \theta_i \left( \frac{\lambda_n - \mu_n}{\lambda_n} d + \frac{\mu_n}{\lambda_n} T_p(x^*_H, x^*_L) \right). \]

We characterize the equilibrium in the following proposition.

**Proposition 3.** In equilibrium, the cut-off costs \(x^*_i\) satisfy:

\[ x^*_i = \theta_i \cdot (\beta T_p(x^*_H, x^*_L) - \alpha d) \cdot \frac{\sum_{j=L,H} p_j (\theta_j - F(c^*_j) \phi_n)}{\sum_{j=L,H} p_j (1 - F(c^*_j)) \theta_j} \]

for \(i = H, L\).

**Proof.** Consider a type-\(i\) individual with the cut-off cost \(x^*_i, i = H, L\). In equilibrium, the individual is indifferent as to whether to become an organ donor or not, that is,

\[ \theta_i \beta T_p(x^*_H, x^*_L) - x^*_i = \theta_i \left( \frac{\lambda_n - \mu_n}{\lambda_n} d + \frac{\mu_n}{\lambda_n} T_p(x^*_H, x^*_L) \right). \]

The above equation can be rewritten as

\[ x^*_i = \theta_i (\beta T_p(x^*_H, x^*_L) - \alpha d) \frac{\lambda_n - \mu_n}{\lambda_n}, \]

which can be further reorganized as (3). \(Q.E.D.\)

The following corollary immediately follows from Proposition 3:

**Corollary 4.** \(\frac{x^*_H}{x^*_L} = \frac{\theta_H}{\theta_L} > 1\).
Corollary 4, in turn, gives the following corollary:

**Corollary 5.** \( T_p(x^*_h, x^*_l) < \sum_{i=L,H} p_i T_i \).

**Proof.** By noticing that \( T_p(x^*_h, x^*_l) = \sum_{i=L,H} p_i F(x^*_i) T_i \) and \( x^*_h < x^*_l \).

Corollaries 4 and 5 reveal an unexpected consequence of introducing the donor priority rule: people with different health risks perceive disproportional level of attractiveness of becoming registered organ donors. More specifically, high-risk individuals are more likely to become organ donors. As a result, the average quality of the donated organs is lower than the average quality of organs from the overall population.

Next, we show that the introduction of the donor priority rule can lead to a lower social welfare.

**Proposition 4.** The social welfare decreases after the introduction of the donor priority rule if

\[
\beta \cdot T_a(x^*_h, x^*_l) - \alpha d > \frac{\sum_{i=L,H} p_i \theta_i}{F(0) \phi n} \sum_{i=L,H} p_i \theta_i
\]

**Proof.** We use \( \Delta U(c) \) to denote the utility change of a type-\( i \) individual with a cost of \( c \) due to the introduction of the donor priority rule.

\[
\Delta U(c) = \begin{cases} 
\beta T_p(x^*_h, x^*_l) - \alpha d \frac{\sum_{i=L,H} p_i \theta_i - F(0) \phi n}{\sum_{i=L,H} p_i \theta_i} - \beta T_a(x^*_h, x^*_l) \frac{F(0) \phi n}{\sum_{i=L,H} p_i \theta_i} & \text{if } c \leq 0 \\
\beta T_p(x^*_h, x^*_l) - \alpha d \frac{\sum_{i=L,H} p_i \theta_i - F(0) \phi n}{\sum_{i=L,H} p_i \theta_i} - \beta T_a(x^*_h, x^*_l) \frac{F(0) \phi n}{\sum_{i=L,H} p_i \theta_i} - c & \text{if } 0 < c \leq x^*_i \\
\beta T_p(x^*_h, x^*_l) - \alpha d \frac{\sum_{i=L,H} p_i \theta_i - F(0) \phi n}{\sum_{i=L,H} p_i \theta_i} - \beta T_a(x^*_h, x^*_l) \frac{F(0) \phi n}{\sum_{i=L,H} p_i \theta_i} - x^*_i & \text{otherwise,}
\end{cases}
\]

which implies that \( \Delta U(c) \) is decreasing in \( c \). This, in turn, gives one of the sufficient conditions for \( W_D^1 - W_N^1 < 0 \):

\[
\beta T_p(x^*_h, x^*_l) - \alpha d \frac{\sum_{i=L,H} p_i \theta_i - F(0) \phi n}{\sum_{i=L,H} p_i \theta_i} - \beta T_a(x^*_h, x^*_l) \frac{F(0) \phi n}{\sum_{i=L,H} p_i \theta_i} < 0,
\]

which can be rewritten as (7).  

\[Q.E.D.\]
6. A Freeze-Period Remedy

In the previous section, we have shown that the introduction of the donor priority rule can lead to the reduction in the social welfare due to the resultant imbalanced incentive structure to individuals with varying health status. In this section, we introduce a simple and easy-to-implement freeze-period remedy. We show that the remedy can offset the quality distortion effect as a result of the donor priority rule. Therefore, when used in conjunction with the donor priority rule, this remedy can improve the social welfare by expanding the size of the donor registry without reducing the average quality of donated organs.

By introducing the freeze-period mechanism, the donor priority is effectively a fixed period after the registration. Assume the fixed period is $S$, then as the time a person of type $i$ getting sick and needing an organ satisfies the exponential distribution with mean $1/\theta_i$, the probability that he/she gets sick after the fixed period $S$ is $e^{-\theta_i S}$. Therefore, assuming that people of type $i$ choose to register as an organ donor if the cost is no more that $x^\#_i$, the arrival rate of the patients under the donor priority rule is

$$\lambda_p = \sum_{i=L,H} p_i F\left(x^\#_i\right) \theta_i e^{-\theta_i S},$$

and the arrival rate of the patients without donor priority is

$$\lambda_n = \sum_{i=L,H} p_i \theta_i \left(1 - F\left(x^\#_i\right) e^{-\theta_i S}\right)$$

The total arrival rate of organs is

$$\mu = \sum_{i=L,H} p_i F(x^\#_i) \phi n.$$
The patients with priority are endowed with the priority of receiving all the available organs, i.e., total supply rate of organs available to donors is \( \mu_p = \mu \), while the total supply rate of donors available to the patients without donor priority is

\[
\mu_n = \mu - \lambda_p = \sum_{i=L,H} p_i F(x^#_i) (\phi n - \theta_i e^{-\theta_i S}).
\]

The aggregated post-transplantation life expectancy for the patients receiving organs is

\[
T_p(x^#_H, x^#_L) = \sum_{i=L,H} p_i T_i x^#_i F(x^#_i) T_p(x^#_H, x^#_L) \sum_{i=L,H} p_i F(x^#_i).
\]

Therefore, a type \( i \) organ donor with a cost \( c \) has a net utility of

\[
u^d_i(c) = \theta_i \left( e^{-\theta_i S} \beta T_p(x^#_H, x^#_L) + (1 - e^{-\theta_i S}) \left( \alpha \frac{\lambda_n - \mu_n}{\lambda_n} d + \beta \frac{\mu_n}{\lambda_n} T_p(x^#_H, x^#_L) \right) \right) - c,
\]

while a type-\( i \) non-donor with a cost \( c \) has a net utility of

\[
u^i_n(c) = \theta_i \left( \alpha \frac{\lambda_n - \mu_n}{\lambda_n} d + \beta \frac{\mu_n}{\lambda_n} T_p(x^#_H, x^#_L) \right).
\]

We characterize the equilibrium in the following proposition.

**Proposition 5.** In equilibrium, the cut-off costs \( x^#_i, i = H, L \) satisfy:

\[
x^#_i = \theta_i e^{-\theta_i S} \left( \beta T_p(x^#_H, x^#_L) - \alpha d \right) \frac{\sum_{i=L,H} \lambda_n - \mu_n}{\lambda_n} \frac{\theta_i \left( 1 - F(x^#_i) e^{-\theta_i S} \right)}{\sum_{i=L,H} p_i \theta_i \left( 1 - F(x^#_i) \right) e^{-\theta_i S}} > 0
\]

**Proof.** A type-\( i \) individual with the cut-off cost \( c_i, i = H, L \), is indifferent as to whether to become an organ donor or not. In other words,

\[
\theta_i \left( e^{-\theta_i S} \beta T_p(x^#_H, x^#_L) + (1 - e^{-\theta_i S}) \left( \alpha \frac{\lambda_n - \mu_n}{\lambda_n} d + \beta \frac{\mu_n}{\lambda_n} T_p(x^#_H, x^#_L) \right) \right) - x^#_i
\]

\[
= \theta_i \left( \alpha \frac{\lambda_n - \mu_n}{\lambda_n} d + \beta \frac{\mu_n}{\lambda_n} T_p(x^#_H, x^#_L) \right)
\]

which can be rearranged as

\[
x^#_i = \theta_i e^{-\theta_i S} \left( \beta T_p(x^#_H, x^#_L) - \alpha d \right) \frac{\lambda_n - \mu_n}{\lambda_n},
\]
or

\[ x_i^# = \theta_i e^{-\theta_i S} \left( \beta T_p(x_H^#, x_L^#) - \alpha d \right) \frac{\sum_{i=L,H} p_i \left[ \theta_i - F(x_i^#) \phi n \right]}{\sum_{i=L,H} p_i \theta_i \left( 1 - F(x_i^#) e^{-\theta_i S} \right)} . \]

Q.E.D.

The following corollary immediately follows from Proposition 3:

**Corollary 6.** \( \frac{x_H^#}{x_L^#} = \frac{\theta_H e^{-\theta_H S}}{\theta_L e^{-\theta_L S}} \).

**Corollary 7.** If \( S = \ln \left( \frac{\theta_H}{\theta_L} \right) \), then \( \frac{x_H^#}{x_L^#} = \frac{\theta_H e^{-\theta_H S}}{\theta_L e^{-\theta_L S}} = 1 \) and \( T_p(x_H^#, x_L^#) = \sum_{i=L,H} p_i T_i \).

**Proof.** By noticing that \( T_p(x_H^#, x_L^#) = \frac{\sum_{i=L,H} p_i F(x_i^#) T_i}{\sum_{i=L,H} p_i F(x_i^#)} \) and \( x_H^# = x_L^# \). Q.E.D.

The above two corollaries imply that there is a unique solution to the freeze period that eliminates the distorted incentives due to the donor priority rule.

### 7. Concluding Remarks

This research is motivated by the widening gap between the increasing demand for donated organs and the steady size of the donor registry. We focus on analyzing the donor priority rule under which registered organ donors have a higher priority over non-donors in receiving transplants. One would naturally expect that the society is better off since more people would sign up for organ donation. Whereas this is indeed the case when individuals are homogeneous (§4), our analysis in §5 reveals a hidden incentive issue associated with the donor priority rule. Specifically, the policy will exert an unbalanced incentive to the people with varying health conditions: less healthy individuals have a higher incentive than healthier ones to become registered organ donors. As a result, although the organ donation rate is higher in response to the policy, the average quality of the donated organs can be lower due to the distorted incentives.

Our proposed remedy is to introduce a freeze period, which can not only overcome the loophole that people might register as an organ donor last minute before they need one,
but can also restore the efficiency of the organ donation and allocation system. The reason is that the existence of a freeze period discourages both type of individuals to become organ donors, but it discourages high-risk individuals more than low-risk individuals. Thus, by appropriately choosing the length of the freeze period, the quality distorting effect introduced by the donor priority rule can be mitigated.

Opportunities for future research include extending the modeling of population heterogeneity as two group into multiple groups or as a continuous type. In either of these cases, we expect that our major insights remain unchanged, that is the donor priority rule can lead to a distorted pool of donated organs and hence reduce the social welfare. Nevertheless, we expect our proposed freeze-period scheme to evolve into a scheme where the length of the freeze depends on individual characteristics (e.g., age). Further research could also explore the policy design problems specific to the characteristics of each type of organ.
References


