Approximation Algorithms for the Traveling Purchaser Problem and its Variants in Network Design

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Abstract. The traveling purchaser problem is a generalization of the traveling salesman problem with applications in a wide range of areas including network design and scheduling. The input consists of a set of markets and a set of products. Each market offers a price for each product and there is a cost associated with traveling from one market to another. The problem is to purchase all products by visiting a subset of the markets in a tour such that the total travel and purchase costs are minimized. This problem includes many well-known NP-hard problems such as uncapacitated facility location, set cover and group Steiner tree problems as its special cases.
We give an approximation algorithm with a poly-logarithmic worst-case ratio for the traveling purchaser problem with metric travel costs. For a special case of the problem that models the ring-star network design problem, we give a constant-factor approximation algorithm. Our algorithms are based on rounding LP relaxation solutions.

1 Introduction

Problem. The traveling purchaser problem (TPP), originally proposed by Ramesh [Ram 81], is a generalization of the traveling salesman problem (TSP). The problem can be stated as follows. We are given a set $M = \{1, \ldots, m\}$ of markets and a set $N = \{1, \ldots, n\}$ of products. Also, we are given c_{ij}, cost of travel from market city i to city j, and nonnegative d_{ij}, the cost of product i at market j. A purchaser starts from his home city, say city 1, and travels to a subset of the m cities and purchases each of the n products in one of the cities he visits, and returns back to his home city. The problem is to find a tour for the purchaser such that the sum of the travel and purchase costs is minimized. It is assumed that each product is available in at least one market city. If a product i is not available at market j, then d_{ij} is set to a high value.

Applications. The traveling purchaser problem has applications in many areas including parts procurement in manufacturing facilities, warehousing, transportation, telecommunication network design and scheduling. An interesting scheduling application involves sequencing n jobs on a machine that has m states [Ong 82]. There is a set-up cost of c_{ij} to change the state of the machine from i to j. A cost d_{ij} is specified to process job i at state j. The objective is to minimize the sum of machine set-up and job processing costs.

The traveling purchaser problem contains the TSP, the prize collecting TSP, uncapacitated facility location problem, group Steiner tree problem and the set cover problem as its immediate special cases. The TSP is the case when each market city has a product available only at that city. In the uncapacitated facility location problem, let the fixed cost for opening facility j be f_j and the cost of servicing client i by facility j be d_{ij}. Then the problem is equivalent to a TPP with a market for each facility and a product for each client, where the travel cost between markets i and j is $c_{ij} = (f_i + f_j)/2$ and the purchase cost of product i at market j is d_{ij}. In the set cover problem, we are given a set S and subsets $S_1, \ldots, S_n \subset S$. The problem is to find a minimum size collection of subsets whose union gives S. This corresponds to a TPP where S is the set of products and there is a market j for each subset S_j. The cost of purchasing product i at market j (of S_j) is zero if $i \in S_j$ and is a large number otherwise. There is a unit cost of travel between each market. Then, there is a set cover of size k if and only if there is a TPP solution of cost k.

Hardness. Note that since there is no polynomial time approximation algorithm for the general TSP, TPP with no assumptions on the costs cannot be approximated in polynomial time unless $P = NP$ [GJ 79]. The TPP instance into which we reduce the set cover problem has metric travel costs. Therefore, from the above approximation-preserving reduction and current hardness results for set cover [F 96,RS 97,AS 97] it follows that there is no polynomial time approximation algorithm for the traveling purchaser problem even with metric travel costs whose performance ratio is better than $(1-o(1)) \ln n$ unless $P = NP$.

Related Work. Due to the hardness of the problem, many researchers have focused on developing heuristics. Most of these algorithms are local search heuristics (Golden, Levy and Dahl [GLD 81], Ong [Ong 82], Pearn and Chien [PC 98]). Voss [V 96] generated solutions by tabu search. The exact solution methods are limited to the branch-and-bound algorithm of Singh and van Oudheusden [SvO 97], which solves relaxations in the form of the uncapacitated facility location problem.

Our Results. We give the first approximation results for the traveling purchaser problem. We give an approximation algorithm with a poly-logarithmic worst-case ratio for the TPP problem with metric travel costs (Corollary 1). In fact, this algorithm approximates a more general bicriteria version of the problem (Theorem 1). For a special case of the TPP problem that models the ring-star network design problem with proportional costs, we give a constant-factor approximation algorithm (Theorem 4 and Corollary 2).

2 Bicriteria Traveling Purchaser Problem

We consider a bicriteria version of the traveling purchaser problem, where minimizing the purchase costs and the travel costs are two separate objectives. The bicriteria problem is a generalization of the TPP, whose solutions provide the decision-maker insight into the tradeoffs between the two objectives.

We use the framework due to Marathe et al. [MRS+ 95] for approximating a bicriteria problem. We choose one of the criteria as the objective and bound the value of the other by a budget constraint. Suppose we want to minimize objectives A and B. We consider the problem

$$P: \ \min B \ s.t. \ A \leq a$$

Definition 1. *An (α, β)-approximation algorithm for the problem P outputs a solution with A-cost at most α times the budget a and B-cost at most β times the optimum value of P, where α, $\beta \geq 1$.*

Our approximation algorithm rounds an LP relaxation solution. It uses the "filtering" technique of Lin and Vitter [LV 92] to obtain a solution feasible to the LP relaxation of a closely related Group Steiner Tree (GST) problem. Then, the LP rounding algorithm of Garg, Konjevod and Ravi [GKR 97] is utilized to obtain a feasible solution.

2.1 Formulation:

We represent the bicriteria TPP as the problem of minimizing the travel costs subject to a budget D on the purchasing costs. The following IP formulation is a relaxation of the TPP problem, where the market cities that the purchaser visits are connected by a 2-edge-connected subgraph instead of a tour. In the formulation, the variable x_{ij} indicates whether product i is purchased at market j, and variable z_{jk} indicates whether markets j and k are connected by an edge of the 2-connected subgraph.

$$\begin{aligned}
&\min \sum_{j,k \in M} c_{jk}\, z_{jk} \\
&\text{st} \\
&\quad \sum_{i=1}^{n} \sum_{j=1}^{m} d_{ij} x_{ij} \leq D && && (1)\\
&\quad \sum_{j=1}^{m} x_{ij} = 1 && i \in N && (2)\\
&\quad \sum_{j \notin S} x_{ij} + \tfrac{1}{2} \sum_{j \in S,\, k \notin S} z_{jk} \geq 1 && i \in N,\ S \subset M,\, 1 \notin S && (3)\\
&\quad x_{ij} \in \{0,1\} && i \in N,\ j \in M && (4)\\
&\quad z_{jk} \in \{0,1\} && j,k \in M && (5)
\end{aligned}$$

Constraint (1) is the budget constraint on purchase cost. Constraints (2) enforce that each product is purchased. Constraint set (3) is intended to capture the requirement of crossing certain cuts in the graph by edges in the subgraph that connect the visited markets. Consider a set of markets S not including the traveler's start node 1, and a particular product i: Either i is purchased at a market not in S or the 2-edge-connected subgraph containing 1 must contain at least one market in S from where i is purchased, thus crossing at least two of the edges in the cut around S. This disjunction is expressed by constraints (3).

The LP relaxation relaxes the integrality of x_{ij} and z_{jk} variables. Although the LP has an exponential number of constraints, it can be solved in polynomial time using a separation oracle [GLS 88] based on a minimum cut procedure. To separate a given solution (z, x) over constraints (3) for a particular product i, we set up a capacitated undirected graph as follows: For every edge (i, j) of the complete graph on the market nodes, we assign an edge-capacity $z_{ij}/2$. We add a new node p_i and assign the capacity of the undirected edge between p_i and market node j to be x_{ij}. A polynomial-time procedure to determine the minimum cut separating 1 and p_i [AMO 93] can now be used to test violation of all constraints of type (3) for product i. Repeating this for every product i provides a polynomial-time separation oracle for constraints (3).

2.2 Filtering

Let $\hat{x}, \hat{z}$ be an optimal solution to the LP relaxation defined above. By filtering, we limit the set of markets a product can be purchased at. For each product, we filter out markets that offer a price substantially over the average purchase cost of the product in the LP solution.

Let D_i denote the purchase cost of product i in the solution $\hat{x}, \hat{z}$, i.e. $D_i = \sum_{j=1}^{m} d_{ij}\hat{x}_{ij}$. For a given $\epsilon > 0$, define a group of markets for product i: $G_i = \{j \in M : d_{ij} \leq (1+\epsilon)D_i\}$. Every group G_i gets at least a certain amount of fractional assignment of product i to its markets in the LP solution as shown by the next lemma.

Lemma 1. *For every product $i \in N$ and $\epsilon > 0$, $\sum_{j \in G_i} \hat{x}_{ij} \geq \frac{\epsilon}{1+\epsilon}$.*

Proof. Suppose for a contradiction that $\sum_{j \in G_i} \hat{x}_{ij} < \frac{\epsilon}{1+\epsilon}$. Then, $\sum_{j \notin G_i} \hat{x}_{ij} \geq \frac{1}{1+\epsilon}$. Note that $D_i = \sum_{j \in M} d_{ij}\hat{x}_{ij} \geq \sum_{j \notin G_i} d_{ij}\hat{x}_{ij} > (1+\epsilon)D_i \sum_{j \notin G_i} \hat{x}_{ij}$ by the definition of G_i. Since $\sum_{j \notin G_i} \hat{x}_{ij} \geq \frac{1}{1+\epsilon}$, we get the contradiction $D_i > D_i$.

2.3 Transformation to Group Steiner Tree Problem

For each product we identified a group of markets to purchase the product. We now need to select at least one market from each group and connect them by a tour. For this, we take advantage of the *Group Steiner Tree* (GST) problem which can be stated as follows. Given an edge-weighted graph with some subsets of vertices specified as groups, the problem is to find a minimum weight subtree which contains at least one vertex from each group. We assume without loss of generality that node 1 is required to be included in the tree. We define the following GST instance. Let G be a complete graph on vertex set equal to the market set M. The weight of edge (i, j) is set to c_{ij} (note that we assume c_{ij} is metric). Let the G_i defined as above for each product i be the groups.

Consider the LP relaxation of this GST problem, which we denote by LP-GST. The variables z_{jk} denote whether the edge between j and k is included in the tree.

$$\min \sum_{j,k \in M} c_{jk}\, z_{jk}$$

st

$$\sum_{j \in S,\, k \notin S} z_{jk} \geq 1 \qquad S \subset M,\ 1 \notin S \text{ and } G_i \subseteq S \text{ for some } i \quad (6)$$

$$0 \leq z_{jk} \leq 1 \qquad j, k \in M \quad (7)$$

The nontrivial constraints (6) enforce that there is a path from node 1 to some node in group G_i, for every i, in the solution.

Lemma 2. *Let $\bar{z}_{jk} = (\frac{1+\epsilon}{2\epsilon})\hat{z}_{jk}$. Then, $\bar{z}$ is feasible to LP-GST.*

Proof. Consider $S \subset M$ containing G_i but not city 1. By constraint (2), $\sum_{j \notin S} \hat{x}_{ij}$ $+\frac{1}{2}\sum_{j \in S,\, k \notin S} \hat{z}_{jk} \geq 1$. Also, $\sum_{j \notin S} \hat{x}_{ij} \leq \sum_{j \notin G_i} \hat{x}_{ij} \leq \frac{1}{1+\epsilon}$ by Lemma 1. Then, $\frac{1}{2}\sum_{j \in S,\, k \notin S} \hat{z}_{jk} \geq 1 - \frac{1}{1+\epsilon} = \frac{\epsilon}{1+\epsilon}$. So, we have $\sum_{j \in S,\, k \notin S} \bar{z}_{jk} \geq 1$.

Garg, Konjevod and Ravi [GKR 97] gave a randomized approximation algorithm that rounds a solution to LP-GST. A de-randomized version can be found in [CCGG 98]. Using any of these algorithms to round the solution $\bar{z}$ provides a tree that includes at least one vertex from each group and has cost $O(\log^3 m \log\log m)$ times $\sum_{j,k \in M} c_{jk}\, \bar{z}_{jk}$.

We obtain a solution to the TPP as follows. Let T be the tree output by the GST rounding algorithm. Let v_i be a market in G_i included in T. We purchase product i at market v_i. We duplicate each edge in T and find an Eulerian tour. We obtain a Hamiltonian tour on the markets in T by short-cutting the Eulerian tour. That is, while traversing the Eulerian tour, when a node that has already been visited is next, we skip to the next unvisited node, say u, and include an edge that connects the current node to u.

The following lemmas are now immediate.

Lemma 3. *The TPP rounding algorithm outputs a solution with total purchase cost at most $(1+\epsilon)\sum_{i=1}^{n}\sum_{j=1}^{m} d_{ij}\hat{x}_{ij}$, which is at most $(1+\epsilon)$ times the budget D, for any chosen $\epsilon > 0$.*

Lemma 4. *The TPP rounding algorithm outputs a solution with total travel cost at most $O((1+\frac{1}{\epsilon})(\log^3 m \log\log m))\sum_{j,k \in M} c_{jk}\, \hat{z}_{jk}$, which is at most $O((1+\frac{1}{\epsilon})(\log^3 m \log\log m))$ times the optimal TPP cost, for any chosen $\epsilon > 0$.*

From Lemmas 3 and 4 we get the following theorem.

Theorem 1. *The TPP rounding algorithm outputs a $((1+\epsilon), (1+\frac{1}{\epsilon})O(\log^3 m \log\log m))$-approximate solution for the bicriteria TPP problem with metric travel costs in polynomial time, for any $\epsilon > 0$.*

The same analysis gives a poly-logarithmic approximation for the TPP as well, where we relax the budget constraint on total purchase cost and add the cost to the objective function.

Corollary 1. *For any* $\epsilon > 0$*, the TPP rounding algorithm finds a solution for the TPP with metric travel costs, whose cost is* $\max\{(1+\epsilon), (1+\frac{1}{\epsilon})\ O(\log^3 m \log\log m)\}$ *times the optimal TPP cost in polynomial time.*

We note that the TPP with metric costs can be directly transformed to a group Steiner *tour* problem[1] on a metric with $m + nm$ nodes, i.e., one of finding a tour that visits at least one node from each group. To construct this metric, we begin with the original metric c on the market nodes. To each market node, we attach n new nodes via "leaf" edges, one for each product - such an edge from market node j to its product node i is assigned cost $d_{ij}/2$. All other edges incident on the new nodes are given costs implied by the triangle inequality. All the nodes corresponding to a product i specify a group - Thus, there are n groups, each with m nodes. It is now straightforward to verify that any group Steiner tour can be transformed to a solution to the original traveling purchaser instance with the same cost. Applying the rounding algorithms for group Steiner trees and short-cutting the tree obtained to a tour gives a direct $O(\log^3(m+nm)\log\log(m+nm))$ approximation to the metric TPP.

3 Network Design with Proportional Cost Metrics

In this section we consider a special case of the traveling purchaser problem, which models a telecommunication network design problem. A communication network consists of several local access network (LANs) that collect traffic of user nodes at the switching centers, and a backbone network that routes high-volume traffic among switching centers. We model this problem by requiring a ring architecture for the backbone network and a star architecture for the LANs. The ring structure is preferred for its reliability. Because of the "self-healing" properties associated with SONET rings, ring structures promise to be of increasing importance in future telecommunication networks ([Kli 98]). The formal model follows.

We are given a graph $G = (V, E)$, with length l_e on edge e. Without loss of generality, we use the metric completion of the given graph. That is, length of an edge e is replaced by the shortest-path length d_e between its endpoints. The problem is to pick a tour (ring backbone) on a subset of the nodes and connect the remaining nodes to the tour such that sum of the *tour cost* and the *access cost* is minimized. The access cost of connecting a non-tour node i to a tour node j is d_{ij}, i.e. the shortest-path length between i and j. The access cost includes the cost of connecting all non-tour nodes to the tour. On the other hand, the cost of including an edge e in the tour is $\rho\, d_e$, where the constant $\rho \geq 1$ reflects the more expensive cost of higher bandwidth connections in the backbone network.

[1] This is also called the generalized TSP in the literature; see [FGT 97].

This problem is a special case of TPP where the vertices of the graph correspond to both the set of markets and the set of products [V 90]. With the TPP terminology, the purchase cost of a product of node i at the market of node j is the shortest path length between nodes i and j. Thus, if node i is included in the tour, its product is purchased at its own market at zero cost. We consider a bicriteria version of this problem with the two objectives of minimizing the tour cost and minimizing the access cost. We use the following notation to denote the problems considered.

(A, T, ρ): Minimize tour cost T subject to a budget on the access cost A, where a tour edge costs ρ times the edge length.

$(A + T, \rho)$: Minimize sum of the tour and access costs, where a tour edge costs ρ times the edge length.

3.1 Hardness

The bicriteria problem (A, T, ρ) is NP-hard even when $\rho = 1$. When the budget on the access cost A is set to zero, the problem reduces to the TSP since every node must be included in the tour. We show that it is NP-hard to approximate this problem with a sub-logarithmic performance ratio without violating the budget constraint. This result does not follow from the inapproximability of TSP since we assume that the distances d_{ij} are metric.

Theorem 2. *There exists no $(1, \alpha)$-approximation algorithm, for any $\alpha = o(\log n)$, for the $(A, T, 1)$ problem unless $P = NP$. Here n is the number of nodes in the $(A, T, 1)$ instance.*

The proof (omitted) is by an approximation preserving reduction from the connected dominating set problem. Note that since $(A, T, 1)$ is a special case of (A, T, ρ), the same hardness result holds for (A, T, ρ).

Theorem 3. *The single criteria problem $(A + T, 1)$ is NP-hard.*

The proof (omitted) is by a reduction from the Hamiltonian tour problem in an unweighted graph which is known to be NP-hard [GJ 79]. Again, since $(A, T, 1)$ is a special case of (A, T, ρ), NP-hardness of the latter follows as well.

3.2 Approximation

There exists a simple 2-approximation algorithm for the $(A + T, 1)$ problem. Find a minimum spanning tree of G, say MST, duplicate the edges of MST and shortcut this to a tour. Note that every node is included in the tour so that the access cost is zero. The cost of the tour is at most 2 times the cost of MST, which is a lower bound on the optimal cost.

Note that this heuristic is a 2ρ-approximation algorithm for $(A + T, \rho)$. However, we obtain a stronger constant factor approximation for both the bicriteria and single objective problems for arbitrary ρ by LP rounding. The LP rounding algorithm uses filtering to limit the set of tour nodes a node can be connected to, as in the TPP rounding algorithm. However, the construction of the tour differs from the TPP rounding algorithm. Tour nodes are chosen based on the access costs and the tour is built by shortcutting an MST on a graph obtained by contracting balls around the tour nodes.

We assume that a root node r is required to be included to the tour (this is similar to including the home city in the TPP). If no such node is specified, we can run the algorithm n times, each time with a different root node, and pick the best solution. We use the following relaxation of (A, T, ρ), which is very similar to the relaxation that we used in the TPP rounding algorithm.

$$\min \rho \sum_{e \in E} d_e z_e$$

st

$$\sum_{i \in V} \sum_{j \in V} d_{ij} x_{ij} \leq D \qquad (1)$$

$$\sum_{j \in V} x_{ij} = 1 \qquad i \in V \qquad (2)$$

$$\sum_{j \notin S} x_{ij} + \frac{1}{2} \sum_{e \in \delta(S)} z_e \geq 1 \qquad i \in V,\ S \subset V, r \notin S \qquad (3)$$

$$x_{ij} \in \{0, 1\} \qquad i \in N,\ j \in M \qquad (4)$$

$$z_{jk} \in \{0, 1\} \qquad j, k \in M \qquad (5)$$

Variable x_{ij} indicates whether node i is connected to the tour at node j, and variable z_e indicates whether edge e is included in the tour. Constraint (1) is the budget constraint on access cost. Here, d_{ij} denotes the shortest path length between nodes i and j. Constraint (2) ensures that every node has access to the tour. For a node set S excluding r, constraint (3) ensures that at least two edges of the cut around S, denoted by $\delta(S)$, is included in the tour, if some node has been assigned to access the tour at a node in S. We obtain the LP relaxation (LPR) by relaxing the integrality in constraints (4) and (5).

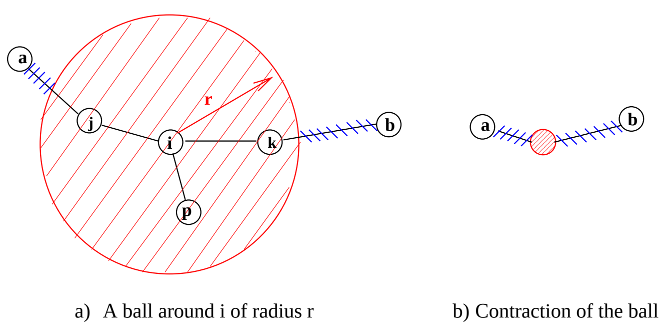

Fig. 1. The definition and contraction of a ball

We need a few definitions before we describe the algorithm. A *ball* of radius r around a node i is the set of all points in G that are within distance r from i under the length function d_e on the edges. The ball may include nodes, edges and partial edges as illustrated in Figure 1a. When we *contract* a ball around a node into a single node, (i) we delete edges with both ends in the ball; (ii) we connect the edges with exactly one endpoint in the ball to the new node and shorten their length by the length remaining in the ball (Figure 1b). Let $\epsilon > 0$ and $\alpha > 1$ be input parameters. The algorithm is as follows:

(1) Solve LPR, let $\hat{x}, \hat{z}$ be an optimal solution.
(2) Let D_i denote the access cost of node i in this solution, i.e. $D_i = \sum_{j \in V} d_{ij} \hat{x}_{ij}$.
(3) Let $\widehat{D}_i = (1+\epsilon) D_i$ and define a ball B_i around every node i of radius $\alpha \widehat{D}_i$.
(4) Preprocessing step: remove all balls containing r and connect their centers to r in the access network.
(5) While unprocessed balls remain:
  (5.1) Pick a ball with minimum radius, say B_k, and mark it as a "tour ball".
  (5.2) Remove all balls intersecting B_k and mark them as "connected via B_k".
(6) Contract each *tour ball* to a node. Let G' be a complete graph on the contracted nodes and r, with edge weights equal to shortest path lengths in G (after contractions).
(7) Find an MST of G' and construct H by replacing edges of the MST by shortest paths in G.
(8) Duplicate edges of H and shortcut them to a tour PT.
(9) Uncontract the balls. Construct tour T by connecting the center node i of each ball B_i to PT.
(10) Connect the center of every ball marked "connected via B_k" directly to k in the access network.

Before we analyze the worst-case performance of the algorithm, let us clarify how we process ball B_i in Step (9). Let b_i be the contracted node corresponding to B_i. Let e_1 and e_2 be the edges incident on b_i in PT. Let v_1 and v_2 be the endpoints of e_1 and e_2 in B_i. Connect the center node i to the tour by adding edges (i, v_1) and (i, v_2) (see Figure 2). Extend e_1 and e_2 to include the portions in the ball.

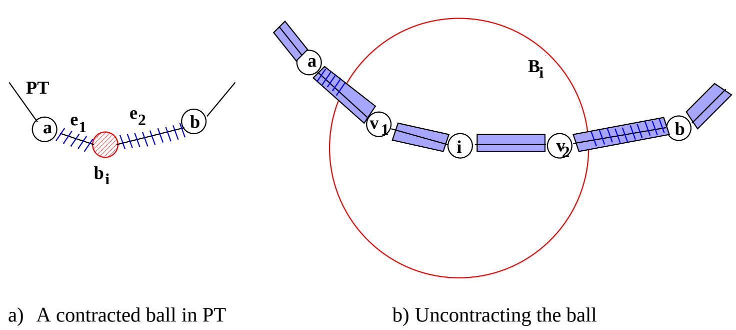

Fig. 2. Uncontracting a ball B_i to include in PT

Lemma 5. *The rounding algorithm outputs a solution with access cost at most $2\alpha(1+\epsilon)$ times the budget D.*

Proof. Each nontour node i is connected to a tour node k such that $B_i \cap B_k$ is nonempty and $\widehat{D}_k \leq \widehat{D}_i$ by the choice of the tour balls in the algorithm. Then, the access cost of i is at most $\alpha\widehat{D}_i + \alpha\widehat{D}_k \leq 2\alpha\widehat{D}_i = 2\alpha(1+\epsilon)\sum_{j\in V} d_{ij}\hat{x}_{ij}$. Since $\hat{x}$ is a solution to the relaxation LPR, it satisfies the budget constraint $\sum\limits_{i\in V}\sum\limits_{j\in m} d_{ij}\hat{x}_{ij} \leq D$. Thus, the access cost is at most $2\alpha(1+\epsilon)D$.

Remark 1. The argument in the above proof is also valid for a problem where an access cost budget is specified separately for each node instead of a single budget constraint on the total access cost.

Lemma 6. *The rounding algorithm outputs a solution with tour cost at most $\max\{2, \frac{\alpha}{\alpha-1}\}(1+\frac{1}{\epsilon})$ times the optimal cost.*

Proof. We use the following definitions. For an edge set M, let $c(M) = \sum_{e\in M}\rho d_e$. Let P be the set of nodes included in the tour T output by the algorithm. Let G_i be a ball around i of radius $\widehat{D}_i$. Let E_C denote the edge set of the contracted graph. That is, E_C excludes from E all edges with both ends in a tour ball as well as portions of the edges with one end point strictly inside a tour ball.

The proof follows from the following claims.

Claim 1: $c(T) \leq c(PT) + 2\alpha(1+\epsilon)\rho\sum\limits_{i\in P} D_i$.

Claim 2: $2(\alpha-1)\epsilon\rho\sum\limits_{i\in P} D_i \leq \rho\sum\limits_{i\in P}\sum\limits_{e\in(B_i-G_i)} d_e\hat{z}_e$.

Claim 3: $c(PT) \leq 2c(MST) \leq 2(1+\frac{1}{\epsilon})\rho\sum\limits_{e\in E_C} d_e\hat{z}_e$.

Proof of Claim 1: The cost of the tour T equals the cost of PT, the tour on the contracted nodes, plus the cost of the edges in the tour balls that connect the tour nodes to PT. For a tour ball B_i, suppose PT touches B_i at points k_1 and k_2. The path in B_i connecting k_1 to the center node i and i to k_2 has cost at most $2\alpha(1+\epsilon)\rho D_i$ since B_i has radius $\alpha(1+\epsilon)D_i$.

Proof of Claim 2: By an argument similar to the proof Lemma 1 it can be shown that for any $i\in V$, $\sum_{j\in G_i}\hat{x}_{ij} \geq \frac{\epsilon}{1+\epsilon}$. Then, by constraint (3) of LPR, it follows that $\sum_{e\in\delta(G_i)}\hat{z}_e \geq \frac{2\epsilon}{(1+\epsilon)}$ for any $i\in V$, and G_i excluding r. Note that a fractional $\hat{z}$ value of at least $\frac{2\epsilon}{1+\epsilon}$ must go a distance of at least $(\alpha-1)\widehat{D}_i$ to get out of the ball B_i. We can consider this distance as a moat around G_i of width $(\alpha-1)\widehat{D}_i$. So, we get $\rho\sum_{e\in(B_i-G_i)} d_e\hat{z}_e \geq \rho\frac{2\epsilon}{(1+\epsilon)}(\alpha-1)\widehat{D}_i = 2\rho\epsilon(\alpha-1)D_i$ for any $i\in P$, since $\widehat{D}_i = (1+\epsilon)D_i$.

Proof of Claim 3: The first inequality easily follows since we obtain PT by shortcutting MST. To show the second inequality, we show that $\bar{z} = \frac{(1+\epsilon)}{2\epsilon}\hat{z}$ is a feasible solution to an LP relaxation of a Steiner tree problem on the contracted graph $G_C = (V_C, E_C)$, with terminal nodes being the contracted balls and r.

Consider S containing B_i but not r. By constraint (3) of LPR, $\sum_{j\notin S}\hat{x}_{ij} + \frac{1}{2}\sum_{e\in\delta(S)}\hat{z}_e \geq 1$. By the definition of B_i, we also have $\sum_{j\notin S}\hat{x}_{ij} \leq \sum_{j\notin B_i}\hat{x}_{ij} \leq$

$\frac{1}{1+\epsilon}$. Then, $\frac{1}{2}\sum_{e\in\delta(S)}\hat{z}_e \geq 1-\frac{1}{1+\epsilon} = \frac{\epsilon}{1+\epsilon}$. So, $\sum_{e\in\delta(S)}\bar{z}_e \geq 1$. Thus, $\bar{z}$ is a feasible solution to the LP relaxation of the Steiner tree problem on G_C. Let $c(ST)$ be the cost of the LP relaxation of the Steiner tree problem on G_C with terminal set the contracted nodes plus r and edge costs ρd_e. Then, $c(MST) \leq 2c(ST)$ (see, e.g. [AKR 95]). Since $\bar{z}$ is a feasible solution, $c(ST) \leq \sum_{e\in E_C}\rho d_e \bar{z}_e = \frac{(1+\epsilon)}{2\epsilon}\sum_{e\in E_C}\rho d_e \hat{z}_e$. Thus, the claim follows.

From Claims 1, 2 and 3 we get,

$$C(T) \leq 2(1+\frac{1}{\epsilon})(\rho\sum_{e\in E_C} d_e\hat{z}_e) + \frac{\alpha}{\alpha-1}(1+\frac{1}{\epsilon})\rho\sum_{i\in P}\sum_{e\in(B_i-G_i)} d_e\hat{z}_e.$$

Since E_C excludes edges in B_i for any $i \in P$, $C(T) \leq \max\{2, \frac{\alpha}{\alpha-1}\}(1+\frac{1}{\epsilon})$ $\rho\sum\limits_{e\notin G_i} d_e\hat{z}_e \leq \max\{2, \frac{\alpha}{\alpha-1}\}(1+\frac{1}{\epsilon})OPT$, where OPT is the optimal cost to (A, T, ρ) problem.

From Lemmas 5 and 6, the next result follows immediately.

Theorem 4. *For any $\epsilon > 0$, $\alpha > 1$ and any ρ, the rounding algorithm outputs a $(2\alpha(1+\epsilon), \max\{2, \frac{\alpha}{\alpha-1}\}\ (1+\frac{1}{\epsilon}))$ approximate solution for the bicriteria problem (A, T, ρ) in polynomial time.*

For minimizing the sum of the two objectives, the performance ratio of the algorithm is the maximum of the two ratios for the separate objectives. The best ratio is obtained by setting $\epsilon = 1/\sqrt{2}$ and $\alpha = 1+1/\sqrt{2}$, yielding a performance ratio of $3+2\sqrt{2}$.

Corollary 2. *The rounding algorithm is a $(3+2\sqrt{2})$-approximation algorithm for $(A+T, \rho)$ problem.*

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