







































is backboneed.

Consider a problem  $\Pi$ , and let the optimal solution to the given instance on  $G$  with edge costs  $c(\cdot)$  be a subgraph  $H \subseteq G$ . Then, from the low-stretch property of the random embedding, the expected cost of  $H$  under the cost function  $\hat{c}_T$  is  $\mathbb{E}_T[\sum_{e \in H} \hat{c}_T(e)] \leq \tilde{O}(\log n) \cdot \sum_{e \in H} c(e)$ . Therefore the expected cost of any optimal solution under edge costs  $\hat{c}_T$  is at most  $\tilde{O}(\log n) \cdot \sum_{e \in H} c(e)$ . Consequently, any  $\beta$ -approximation algorithm for the problem  $\Pi$  on backboneed graphs would return a subgraph  $H' \subseteq G$ , with expected cost (with respect to  $\hat{c}_T$ ) at most  $(\beta \times \tilde{O}(\log n)) \cdot \sum_{e \in H} c(e)$ . Since  $\hat{c}(e) \geq c(e)$  for any edge  $e \in E(G)$ , the expected cost of the subgraph  $H'$  with respect to edge costs  $c(\cdot)$  is also at most  $(\beta \times \tilde{O}(\log n)) \cdot \sum_{e \in H} c(e) \leq (\beta \times \tilde{O}(\log n))c(H)$ . Therefore, the solution  $H'$  is a randomized  $\beta \times \tilde{O}(\log n)$ -approximate solution on the original edge costs  $c(\cdot)$ . ■

## B Proofs from Section 4

**B.1 2-CFL on Non-Metric Instances** We now consider instances where the connection cost for the clients is given by some distance function  $d(\cdot, \cdot)$  which may itself not satisfy triangle inequality, and the edge costs for building the 2-connected core is  $c(\cdot)$ .

We show how we can get poly-logarithmic approximations for the above “non-metric” 2-CFL problem using essentially the same techniques we used for 2-ECGS. We first guess one facility which the optimal solution opens and call it  $r$ . The LP is almost identical to the one given for 2-CFL on general graphs, except for the client-facility connection cost being some arbitrary function  $d(\cdot, \cdot)$  instead of the tree distances  $c(\cdot)$ . Here is a brief overview of the rounding algorithm for 2-CFL. We skip the details of the proofs as they are very similar to the ones given in the earlier sections.

- (i) Solve the LP relaxation optimally. Then filter the client connection costs: If we let  $D_u^* = \sum_{v \in V} d(u, v)z_{uv}^*$ , it must be that  $\sum_{v \in V \mid d(u, v) \leq 2D_i^*} z_{uv}^* \geq \frac{1}{2}$ . Set  $z_{uv}^* \leftarrow 0$  if  $d(u, v) > 2D_i^*$  and scale the solution by factor 2.
- (ii) For each  $u \in \mathcal{D}$ , create a group  $g_u = \{v \in V \mid z_{uv}^* \neq 0\}$  of facilities associated with this client. It is easy to check that the solution  $(x^*, y^*)$  is a feasible solution for the 2-ECGS LP with these groups.
- (iii) Perform Stage I and Stage II of the 2-ECGS algorithm once; if a group  $g_u$  is 2-connected to the root, open a facility at the representative vertex  $v_{g_i}$ . Because the 2-ECGS algorithm ensures that  $\Omega(\frac{1}{\log n})$  groups are 2-connected to the root, and we open facilities for these groups, we know that  $\Omega(\frac{1}{\log n})$  clients have a facility opened near them. A similar

analysis as the one for the 2-ECGS problem can be used to see that the total cost spent in this step is at most  $O(\log n)\text{LPOpt}$ .

- (iv) We can then repeat this process  $O(\log^2 n)$  times and output the union of all previous partial solutions to guarantee with high probability a feasible solution to the 2-CFL problem.

**THEOREM B.1.** *Non-metric 2-CFL admits an  $O(\log^3 n)$  approximation algorithm on backboneed graphs, and an  $O(\log^4 n)$  approximation algorithm for general graphs.*