E-appendix for
Integrated Optimization of Customer and Supplier Logistics at Robert Bosch LLC

Hakan Yildiz∗
Eli Broad College of Business, Michigan State University, East Lansing, MI 48824 USA

R. Ravi
Tepper School of Business, Carnegie Mellon University, Pittsburgh, PA 15213 USA

Wayne Fairey
Customer Service, Logistics & Planning (ChP/CLP), Robert Bosch LLC, Charleston, SC 29418 USA

Abstract
This appendix presents the optimization model developed and solved.

Optimization Model

Data and Parameters

pl = the plant.
xd = the crossdock.
C = set of all customers.
S = set of all suppliers.
N = set of all customers and suppliers: \( N = C \cup S \).
\( N' = \) set of all nodes including the crossdock and the plant: \( N' = N \cup \{ pl, xd \} \).
\( C^{LTL} = \) set of customers currently on LTL routes.
\( S^{TL} = \) set of suppliers currently on milkruns.
\( R^S = \) set of current supplier milkruns.
\( S^{TL}_j = \) set of suppliers currently on milkrun \( j \in R^S \).
dist_{i,j} = \) distance between nodes \( i, j \in N' \).

∗Corresponding author. Tel: +1 517 4326439; Fax: +1 517 4321112
Email address: yildiz@msu.edu (Hakan Yildiz)

Preprint submitted to Elsevier March 30, 2010
\( d_i = \) average demand for node \( i \in N \) in units of a truck load.

\( s_i = \) average supply for node \( i \in N \) in units of a truck load.

\( d_i^{\text{NPack}} = \) average demand for node \( i \in N \) in units of packages.

\( s_i^{\text{NPack}} = \) average supply for node \( i \in N \) in units of packages.

\( d_i^{\text{weight}} = \) average weight of node \( i \in N \)’s demand in pounds.

\( s_i^{\text{weight}} = \) average weight of node \( i \in N \)’s supply in pounds.

\( \text{maxstops} = \) maximum number of stops allowed on a route excluding the origin (pl or xd).

\( \text{maxvol} = \) maximum percentage volume capacity allowed on a TL in units of a truck.

\( \text{maxweight} = \) maximum weight capacity of a truck in pounds.

\( m = \) an arbitrary large number.

**Costs**

\( p^{\text{TL}} = \) per mile cost for round trip TL trucks.

\( p^{\text{OneWay}} = \) per mile cost for oneway TL trucks.

\( p^{\text{TL-Stop}} = \) per stop charge for a TL truck.

\( p_{i,pl}^{\text{LTL}} = \) LTL cost between \( i \in S \) and the Plant.

\( p_{i,xd}^{\text{LTL}} = \) LTL cost between \( i \in N \) and the crossdock.

\( p_{xd}^{h} = \) per container handling cost at the crossdock.

\( p_i^{\text{TL}}(C) = \) current milkrun cost for \( i \in C \).

\( p_j^{\text{TL}}(S) = \) current milkrun cost for current supplier route \( j \in R^S \).
**Decision Variables**

We used three types of variables in this model: Binary, General Integer and Continuous.

**Binary Variables**

\[ Z_r = 1 \text{ if the current supplier milkrun } r \in R^S \text{ is used, } 0 \text{ otherwise.} \]

\[ V_i = 1 \text{ if customer } i \text{ uses its current way of transportation or if supplier } i \text{ uses its current milkrun,} \]

\[ Y_i = 1 \text{ if node } i \text{ uses LTL to and from the Plant, } 0 \text{ otherwise, } \forall i \in N. \]

\[ W_i = 1 \text{ if node } i \text{ is on a new milkrun, } 0 \text{ otherwise, } \forall i \in N. \]

\[ Dock_i^d = 1 \text{ if node } i \text{'s demand goes through the crossdock and } 0 \text{ otherwise, } \forall i \in N. \]

\[ Dock_i^s = 1 \text{ if node } i \text{'s supply goes through the crossdock and } 0 \text{ otherwise, } \forall i \in N. \]

\[ Visit_j^i = 1 \text{ if } i \text{'s demand or supply goes through node } j, \text{and } 0 \text{ otherwise } \forall i \in N, \forall j \in N'. \]

As mentioned in Section 7, in addition to the current ways of transportation, two new means of transportation for each customer and supplier are allowed. In this formulation, the two new means are represented by \( X_i \) and \( W_i \). The decision of keeping the current mode is represented by \( V_i \) for suppliers that currently use TL carriers and customers, and represented by \( Y_i \) is for suppliers that currently use LTL carriers.

**General Integer Variables**

\[ RT = \text{number of round trip trucks between the crossdock and the Plant.}\]

\[ A_{u,v} = \text{number of times the milkrun arc } (u, v) \text{ is used, } \forall u, v \in N'. \]

\[ OW_{pl,xd} = \text{Number of times the one way arc from the Plant to the crossdock is used.}\]

\[ OW_{xd,pl} = \text{Number of times the one way arc from the crossdock to the Plant is used.}\]

**Continuous (Flow) Variables**

\[ F^{TL}_{u,v}(i) = \text{flow of node } i \text{'s demand on milkrun arc } (u, v), \forall u, v \in N', \forall i \in N. \]

\[ G^{TL}_{u,v}(i) = \text{flow of node } i \text{'s supply on milkrun arc } (u, v), \forall u, v \in N', \forall i \in N. \]

\[ F^{RT}_{pl,xd}(i) = \text{flow of } i \text{'s demand from the Plant to the crossdock on RT trucks, } \forall i \in N. \]

\[ G^{RT}_{xd,pl}(i) = \text{flow of } i \text{'s supply from the crossdock to the Plant on RT trucks, } \forall i \in N. \]

\[ F^{OW}_{pl,xd}(i) = \text{flow of } i \text{'s demand from the Plant to the crossdock on one way trucks, } \forall i \in N. \]

\[ G^{OW}_{xd,pl}(i) = \text{flow of node } i \text{'s supply from the crossdock to the Plant on one way trucks, } \forall i \in N. \]
Objective Function

\[
\text{minimize}(\text{MilkrunCost}_{\text{new}} + \text{StopCost}_{\text{new}} + \text{MilkrunCost}_{\text{customer}}^\text{old} + \text{MilkrunCost}_{\text{supplier}}^\text{old} + \text{LTLcost}_{\text{xd}} + \text{LTLcost}_{\text{pl}} + \text{RTcost} + \text{OneWayCost} + \text{CrossdockCost})
\]

where,

\[
\text{MilkrunCost}_{\text{new}} = p^{TL} \ast (\sum_{u,v \in N} \text{dist}_{u,v} \ast A_{u,v}) : \text{the total cost of all new milkruns.}
\]

\[
\text{StopCost}_{\text{new}} = p^{TL\text{-Stop}} \ast (\sum_{i \in N} W_i + A_{xd,pl} + A_{pl,xd} + RT) : \text{the total stopping cost incurred in the new milkruns.}
\]

\[
\text{MilkrunCost}_{\text{customer}}^\text{old} = \sum_{i \in C} V_i \ast p_i^{TL}(C) : \text{the total cost of current milkruns used by the customers in the new solution.}
\]

\[
\text{MilkrunCost}_{\text{supplier}}^\text{old} = \sum_{i \in S} Z_i \ast p_i^{TL}(S) : \text{the total cost of current milkruns used by the suppliers in the new solution.}
\]

\[
\text{LTLcost}_{\text{xd}} = \sum_{i \in N} X_i \ast p_{i,xd}^{LTL} : \text{the total cost of LTL trucks between nodes and crossdock.}
\]

\[
\text{LTLcost}_{\text{pl}} = \sum_{i \in S} Y_i \ast p_{i,pl}^{LTL} : \text{the total cost of LTL trucks between the nodes and the plant.}
\]

\[
\text{RTcost} = RT \ast p^{TL} \ast (\text{dist}_{pl,xd} + \text{dist}_{xd,pl}) : \text{the total cost of round trip TL trucks between the plant and the crossdock.}
\]

\[
\text{OneWayCost} = p^{OneWay} \ast (\text{OW}_{pl,xd} \ast \text{dist}_{pl,xd} + \text{OW}_{xd,pl} \ast \text{dist}_{xd,pl}) : \text{the total cost of one way TL trucks between the plant and the crossdock.}
\]

\[
\text{CrossdockCost} = p^h \ast \sum_{i \in N} (\text{Dock}_i^d \ast N_{Pack} + \text{Dock}_i^s \ast N_{Pack}) : \text{total crossdock usage cost.}
\]
Constraints

For ease of reading, we grouped similar constraints below.

\[ Y_i + V_i + W_i + X_i = 1, \quad \forall i \in N \quad (1) \]

\[ V_i = 0, \quad \forall i \in S \setminus S^{TL}. \quad (2) \]

\[ Y_i = 0, \quad \forall i \in C \cup S^{TL}. \quad (3) \]

\[ |S_r^{TL}| \cdot Z_r \geq \sum_{i \in S_r^{TL}} V_i, \quad \forall r \in R. \quad (4) \]

\[ \sum_{u \in N'} F_{u,i}^{TL}(i) = d_i \cdot W_i, \quad \forall i \in N. \quad (5) \]

\[ \sum_{u \in N'} G_{u,i}^{TL}(i) = s_i \cdot W_i, \quad \forall i \in N. \quad (6) \]

\[ \sum_{u \in N \cup \{pl\}} (F_{u,i}^{TL}(i) + F_{x,i}^{TL}(i)) + F_{pl,i}^{RT}(i) + F_{pl,i}^{OW}(i) \leq m \cdot Dock^d_i, \quad \forall i \in N \quad (7) \]

\[ \sum_{u \in N \cup \{pl\}} (G_{u,i}^{TL}(i) + G_{x,i}^{TL}(i)) + G_{pl,i}^{RT}(i) + G_{pl,i}^{OW}(i) \leq m \cdot Dock^s_i, \quad \forall i \in N \quad (8) \]

\[ F_{i,u}^{TL}(i) + G_{u,i}^{TL}(i) + F_{u,i}^{TL}(i) + G_{u,i}^{TL}(i) + G_{pl,i}^{TL}(i) + F_{pl,i}^{TL}(i) = 0 \quad \forall i \in N, \forall u \in N' \quad (9) \]

Short Description: (1): only one mode of transportation must be used by each customer and supplier; (2): suppliers that currently use LTL service, naturally, do not have old milkruns to be assigned in the new solution; (3): for suppliers, only the ones that currently use LTL service can use an LTL service to and from the plant; (4): if a supplier \( i \) that is on the current milkrun \( r \) uses that milkrun in the new solution, \( Z_r \) becomes 1; (5 & 6): if node \( i \) is on a new milkrun, than the amount of \( i \)'s demand flow into node \( i \) should be equal to its demand and the amount of \( i \)'s supply flow out of node \( i \) should be equal to its supply; (7 & 8): If node \( i \)'s flow passes through the crossdock, then the crossdock is used by node \( i \); (9): unallowed flows are set to zero;
\[
\sum_{v \in N} A_{u,v} \leq 1 \quad \forall u \in N
\]  
(10)

\[
A_{u,v} + A_{v,u} \leq 1 \quad \forall u, v \in N \setminus \{pl\}
\]  
(11)

\[
\sum_{v \in N} A_{v,u} = \sum_{v \in N} A_{u,v} \quad \forall u \in N
\]  
(12)

\[
\sum_{u \in N} A_{x_d,u} = \sum_{u \in N} A_{u,x_d}
\]  
(13)

Short Description: (10): at most one milkrun truck should enter each customer or supplier; (11): only one of the two arcs between two nodes can be used; (12, 13): number of milkrun trucks entering and leaving a node should be equal;

Flow Balance Type Constraints:

\[
V_i \ast d_i + Y_i \ast d_i + F_{pl,x_d}(i) + F_{pl,x_d}(i) + \sum_{u \in N} F_{pl,u}(i) = d_i, \quad \forall i \in N
\]  
(14)

\[
V_i \ast s_i + Y_i \ast s_i + G_{pl,pl}(i) + G_{pl,pl}(i) + \sum_{u \in N} G_{u,pl}(i) = s_i, \quad \forall i \in N
\]  
(15)

\[
\sum_{u \in N} F_{u,j}(i) - \sum_{u \in N} F_{j,u}(i) = 0, \quad \forall i, j \in N : i \neq j
\]  
(16)

\[
\sum_{u \in N} G_{u,j}(i) - \sum_{u \in N} G_{j,u}(i) = 0, \quad \forall i, j \in N : i \neq j
\]  
(17)

\[
\sum_{u \in N} F_{pl,x_d}(i) + F_{pl,x_d}(i) + F_{pl,x_d}(i) - \sum_{u \in N} F_{pl,u}(i) - X_i \ast d_i = 0, \quad \forall i \in N.
\]  
(18)

\[
\sum_{u \in N} G_{pl,x_d}(i) + G_{pl,pl}(i) + G_{pl,pl}(i) - \sum_{u \in N} G_{u,x_d}(i) - X_i \ast s_i = 0, \quad \forall i \in N.
\]  
(19)

Short Description: (14 & 15): supply and demand satisfaction at the plant; (16 & 17): flow balance at nodes except the crossdock and the plant; (18 & 19) demand and supply flow balance at the crossdock;
Volume and Weight Capacity Constraints:

\[
\sum_{i \in N} (F_{u,v}^{TL}(i) + G_{u,v}^{TL}(i)) \leq A_{u,v} \ast \maxvol \quad \forall u, v \in N'. \quad (20)
\]

\[
\sum_{i \in N} F_{pl,xd}^{RT}(i) \leq RT \ast \maxvol \quad (21)
\]

\[
\sum_{i \in N} G_{xd,pl}^{RT}(i) \leq RT \ast \maxvol \quad (22)
\]

\[
\sum_{i \in N} F_{pl,xd}^{OW}(i) \leq OW_{pl,xd} \ast \maxvol \quad (23)
\]

\[
\sum_{i \in N} G_{xd,pl}^{OW}(i) \leq OW_{xd,pl} \ast \maxvol \quad (24)
\]

\[
\sum_{i \in N} (d_i^{weight} F_{u,v}^{TL}(i) + s_i^{weight} G_{u,v}^{TL}(i)) \leq \maxweight \ast A_{u,v}, \quad \forall u, v \in N' \quad (25)
\]

\[
\sum_{i \in N} d_i^{weight} F_{pl,xd}^{RT}(i) \leq \maxweight \ast RT \quad (26)
\]

\[
\sum_{i \in N} s_i^{weight} G_{xd,pl}^{RT}(i) \leq \maxweight \ast RT \quad (27)
\]

\[
\sum_{i \in N} d_i^{weight} F_{pl,xd}^{OW}(i) \leq \maxweight \ast OW_{pl,xd} \quad (28)
\]

\[
\sum_{i \in N} s_i^{weight} G_{xd,pl}^{OW}(i) \leq \maxweight \ast OW_{xd,pl} \quad (29)
\]

Short Description: (20, 21, 22, 23, 24): the total flow between two nodes can not exceed the total volume capacity on the TL trucks operating between those two nodes; (25, 26, 27, 28, 29): the total flow between two nodes can not exceed the total weight capacity on the TL trucks operating between those two nodes;
Customer Sensitive Constraints:

\[
\sum_{k \in N'} G_{ik}^{TL}(j) \leq \text{Visit}_j^i \quad \forall i \in N, \forall j \in S \cup \{pl, xd\} : i \neq j
\]  
(30)

\[
\sum_{k \in N'} F_{jk}^{TL}(i) \leq \text{Visit}_j^i \quad \forall i \in N, j \in N' : i \neq j
\]  
(31)

\[
\sum_{j \in N', j \neq i} \text{Visit}_j^i \leq \text{maxstops} \quad \forall i \in N.
\]  
(32)

\[
X_i = 0 \quad \forall i \in C \setminus C^{LTL}
\]  
(33)

\[
A_{i,j} = 0 \quad \forall i, j \in C
\]  
(34)

\[
F_{vl}^{TL}(i) + G_{li}^{TL}(i) = 0 \quad \forall v \in N', \forall i \in C, \forall j \in C \setminus C^{LTL} : i \neq j
\]  
(35)

\[
A_{i,j} = 0 \quad \forall i \in S, \forall j \in C
\]  
(36)

\[
\sum_{k \in N', j \in C} G_{ik}^{TL}(j) \leq 0 \quad \forall i \in C
\]  
(37)

\[
\sum_{k \in N', j \in C} F_{ik}^{TL}(j) \leq 0 \quad \forall i \in S
\]  
(38)

\[
\sum_{k \in N, j \in N} G_{jk}^{TL}(j) \leq 0
\]  
(39)

Short Description: (30, 31): if node \( i \)'s supply or demand flows through a node \( j \), then node \( i \)'s flow visits node \( j \); (32): the number of nodes that node \( i \)'s flow goes through is at most \( \text{maxstops} \); (33): customers should not ship with LTL unless they currently use LTL; (34): no arc between two customers can be used; (35): no customer's demand or supply should go through another customer; (36): no arc from a supplier to a customer can be used; (37): no supply flow should go through a customer; (38): no customer demand flow should go through a supplier; (39): the crossdock should only be the last stop on a milkrun (no supply flow should flow from crossdock to other customers or suppliers).