Expertise in Online Markets
Online Appendix [1]

Stylianos Despotakis   Isa Hafalir   R. Ravi   Amin Sayedi

February 18, 2016

[1]Affiliations: Tepper School of Business, Carnegie Mellon University (Despotakis, Hafalir, Ravi), Foster School of Business, University of Washington (Sayedi). E-mails: steliosdes@cmu.edu (Despotakis), isaemin@cmu.edu (Hafalir), ravi@cmu.edu (Ravi), aminsa@uw.edu (Sayedi).

B Online Appendix

In this Online Appendix, we first give a complete proof of the equilibrium characterization lemma 2 in section B.1. In Section B.2, we calculate the explicit upper bound $\bar{\delta}$ on the value of δ alluded to in the Appendix Section A.3. We then provide a robustness check on the choice of our tie-breaking rule in Section B.3 by showing that the analog of the main lemma 2 continues to hold even if we invert the tie-breaking rule to favor experts instead of non-experts. In the next Section B.4, we show an extrapolation of our main result for the case when the private value distribution has a support of more than two values, thus lending support for our main observations in the limiting continuous case. Finally, in Section B.5, we consider equilibria when sellers sell in an auction but can choose between hard-close versus soft-close formats.

B.1 Proof of Lemma 2

Proof. We will group the nine equilibria into four cases. These are $v < \min\{M_1, M_2\}$ (equilibria 1, 2, 3 and 4), $M_2 \leq v < M_1$ (equilibria 5 and 6), $\max\{M_2, M_1\} \leq v$ (equilibrium 9), and $M_1 \leq v < M_2$ (equilibria 7 and 8).

Case 1. First, assume that $v < \min\{M_1, M_2\}$. This means that $cq + v < c$ and $v < cq$. Consider the following general set of strategies:

- High Expert: If $C = 0$, bids v in the first stage and does nothing in the second stage. If $C = c$, bids $cq + v$ in the first stage and bids $c + v$ in the second stage.

- Low Expert: If $C = 0$, does nothing. If $C = c$, he bids $cq + v$ in the first stage and c in the second stage.

- High Non-Expert: Bids $cq + v$ in the first stage. If he sees a bid other than $0, v, cq$, or $cq + v$ in the first stage, he bids $c + v$ in the second stage. Otherwise, he bids c in the second stage with probability $1 - a$.

- Low Non-Expert: Bids cq in the first stage with probability b and v with probability $1 - b$. If his bid was v, he bids cq in the second stage with probability $1 - g$.

The probabilities a, b, g are as yet undetermined . For now, we just assume that $a > 0$. We will examine if anyone has an incentive to change his strategy and at the same time try to determine the probabilities and the conditions for which the above is an equilibrium. These conditions will give us the proof that equilibria 1 and 3 are correct. Later, we will relax the assumption on a and examine what happens when $a = 0$; this will lead us to conditions that equilibria 2 and 4 are correct, and will conclude the proof of the four equilibria in the first case.

— High Expert with $C = 0$: His valuation is v and now he bids v in the first stage. If he does nothing in the first stage and he bids v in the second stage, then there is some probability that his bid will not go through with a payoff of 0, and in the case it goes through, his payoff would be the same in all cases as if he had bid v in the first stage (against a low non-expert, his payoff is 0 in both cases). Therefore, it is optimal for him to follow this strategy.

— High Expert with $C = c$: His valuation is $c + v$ and now he bids $cq + v$ in the first stage and bids $c + v$ in the second stage. We consider three alternative strategies which dominate all the rest, and we prove that he doesn't have any incentive to deviate to any of them.

One strategy is to bid v in the first stage and $c + v$ in the second stage. This strategy has a different result for him only if his bid in the second stage doesn't go through. In that case, by having a bid of v instead of a bid of $cq + v$ can only decrease his payoff.

Another strategy is to bid 0 in the first stage and $c + v$ in the second. The behavior of the rest of the bidders will not change, but his payoff will decrease because he can lose in some cases whereas the bid of $cq + v$ would give him a positive payoff.

The last strategy is to bid $c + v$ (or c) in the first stage and nothing (or $c + v$) in the second.

However, if we assume that his bid will go through in the second stage, with the alternative strategy the result would be the same in all cases except in the case he faces a high non-expert, where his payoff strictly decreases. Therefore, since δ is sufficiently small[17], it is better to bid in the second stage.

— Low Expert with $C = 0$: His value is 0 and he does nothing, which is optimal for him.

— Low Expert with $C = c$: His value is c. The payoff if he bids c in the first stage is

$$\begin{aligned}
A(\delta) = \quad & \underbrace{pr((1-\delta)0 + \delta(c - (cq + v)))}_{\text{opponent is high expert}} \\
& + \underbrace{p(1-r)((1-\delta)0 + \delta(c - (cq + v)))}_{\text{opponent is low expert}} \\
& + \underbrace{(1-p)r((1-\delta)0 + \delta(c - (cq + v)))}_{\text{opponent is high non-expert}} \\
& + \underbrace{(1-p)(1-r)(b(c - cq) + (1-b)(g(c-v) + (1-g)(1-\delta)(c - cq) + (1-g)\delta(c-v)))}_{\text{opponent is low non-expert}}.
\end{aligned}$$

The payoff with the current strategy is

$$\begin{aligned}
B(\delta) = \quad & \underbrace{(1-\delta)}_{\text{bid goes through}} \cdot \Bigg[\underbrace{pr((1-\delta)0 + \delta(c - (cq + v)))}_{\text{opponent is high expert}} \\
& + \underbrace{p(1-r)((1-\delta)0 + \delta(c - (cq + v)))}_{\text{opponent is low expert}} \\
& + \underbrace{(1-p)r(a(c - (cq + v)) + (1-a)(1-\delta)0 + (1-a)\delta(c - (cq + v)))}_{\text{opponent is high non-expert}} \\
& + \underbrace{(1-p)(1-r)(b(c - cq) + (1-b)(g(c-v) + (1-g)(1-\delta)(c - cq) + (1-g)\delta(c-v)))}_{\text{opponent is low non-expert}} \Bigg] \\
& + \underbrace{\delta}_{\text{bid doesn't go through}} \cdot \Bigg[\underbrace{pr((1-\delta)0 + \delta 0)}_{\text{opponent is high expert}} \\
& + \underbrace{p(1-r)\left((1-\delta)0 + \delta \frac{c - (cq + v)}{2}\right)}_{\text{opponent is low expert}} \\
& + \underbrace{(1-p)r(0)}_{\text{opponent is high non-expert}} \\
& + \underbrace{(1-p)(1-r)(b(c - cq) + (1-b)(g(c-v) + (1-g)(1-\delta)(c - cq) + (1-g)\delta(c-v)))}_{\text{opponent is low non-expert}} \Bigg].
\end{aligned}$$

[17] Formally, this condition means $\delta < \bar{\delta}$, which we discuss in Section B.2.

It holds that $B(0) - A(0) = (1-p)ra(c-(cq+v)) > 0$ (for $a > 0$), which means that for sufficiently small δ, $B(\delta) > A(\delta)$, i.e. the current strategy is better.

The alternative is to bid something else in the first stage other than $cq + v$ or c, and c in the second, but this doesn't increase the payoff.

— High Non-Expert: His expected valuation is $cq + v$. Bidding something else other than $cq + v$ in the first stage will not change the bidding behavior of the opponent to something better for him, therefore he prefers to bid $cq + v$ in the first stage rather than wait.

In the second stage, it doesn't matter what they do if they see a bid of 0 or v or cq, since the result cannot change. If they see a bid other than $0, v, cq$, or $cq + v$ (like c or $c + v$), something that doesn't happen in the equilibrium, they assume that the common value is high which means that their valuation is $c+v$, so they bid $c+v$. The reason is that the only one who might have incentive to deviate from the current strategies is an expert with $C = c$ who tries to bluff in some way to hide the common value.

If they see a bid of $cq + v$, then they know that their opponent is a high expert with $C = c$, or a low expert with $C = c$, or a high non-expert. Their payoff by doing nothing in the second stage is

$$\begin{aligned} A_2 = \quad & \underbrace{\frac{prq}{prq + p(1-r)q + (1-p)r}((1-\delta)0 + \delta(c + v - (cq + v)))}_{\text{opponent is high expert and } C=c} \\ &+ \underbrace{\frac{p(1-r)q}{prq + p(1-r)q + (1-p)r}((1-\delta)0 + \delta(c + v - (cq + v)))}_{\text{opponent is low expert and } C=c} \\ &+ \underbrace{\frac{(1-p)r}{prq + p(1-r)q + (1-p)r}(a0 + (1-a)(1-\delta)0 + (1-a)\delta 0)}_{\text{opponent is high non-expert}}, \end{aligned}$$

while their payoff by bidding c is

$$B_2 = \quad \underbrace{(1-\delta)}_{\text{bid goes through}} \cdot \Bigg[\underbrace{\frac{prq}{prq + p(1-r)q + (1-p)r}((1-\delta)0 + \delta(c + v - (cq + v)))}_{\text{opponent is high expert and } C=c}$$

$$+\underbrace{\frac{p(1-r)q}{prq+p(1-r)q+(1-p)r}((1-\delta)v+\delta(c+v-(cq+v)))}_{\text{opponent is low expert and } C=c}$$

$$\left. + \underbrace{\frac{(1-p)r}{prq+p(1-r)q+(1-p)r}(a0+(1-a)(1-\delta)\frac{cq+v-c}{2}+(1-a)\delta 0)}_{\text{opponent is high non-expert}}\right]$$

$$+\underbrace{\delta}_{\text{bid doesn't go through}}\cdot A_2.$$

By bidding $c+\epsilon$ for some small $\epsilon > 0$, his payoff can only decrease. By bidding $c-\epsilon$, the payoff is the same as if they stay with the bid of $cq+v$ (according to the tie-breaking rule, if two bidders are both high, the non-expert wins). It holds that

$$B_2 - A_2 = (1-\delta)\left[\frac{p(1-r)q}{prq+p(1-r)q+(1-p)r}(1-\delta)v + \frac{(1-p)r}{prq+p(1-r)q+(1-p)r}(1-a)(1-\delta)\frac{cq+v-c}{2}\right],$$

and we want this to be equal to 0 to permit mixing these strategies, which will give us an expression for the mixing probability a. This is

$$a = 1 - \frac{2p(1-r)qv}{(1-p)r(c-(cq+v))}.$$

This is always ≤ 1. We assumed also that $a > 0$, which is equivalent to $v < \frac{c(1-p)r(1-q)}{2p(1-r)q+(1-p)r} = m_1$. Therefore, we need this condition to have an equilibrium in this case.

If $1 - \frac{2p(1-r)qv}{(1-p)r(c-(cq+v))} \leq 0$, which is equivalent to $v \geq m_1$ and corresponds to $a = 0$, we need a different set of strategies and we consider this case later.

— Low Non-Expert: His expected valuation is cq. His payoff if he bids cq in the first stage is

$$A_3 = \underbrace{pr(q0+(1-q)(-v))}_{\text{opponent is high expert}}$$

$$+\underbrace{p(1-r)(q0+(1-q)0)}_{\text{opponent is low expert}}$$

$$+\underbrace{(1-p)r(0)}_{\text{opponent is high non-expert}}$$

$$+\underbrace{(1-p)(1-r)(b0+(1-b)(g(cq-v)+(1-g)(1-\delta)0+(1-g)\delta(cq-v)))}_{\text{opponent is low non-expert}}$$

His payoff if he bids v in the first stage and follows the current strategy in the second stage is

$$\begin{aligned}
B_3 = \quad & \underbrace{pr(q0+(1-q)(g0+(1-g)(1-\delta)(-v)+(1-g)\delta 0))}_{\text{opponent is high expert}} \\
& + \underbrace{p(1-r)(q0+(1-q)0)}_{\text{opponent is low expert}} \\
& + \underbrace{(1-p)r(0)}_{\text{opponent is high non-expert}} \\
& + (1-p)(1-r)\Bigg[\underbrace{b0+(1-b)(g^2(\frac{cq-v}{2})+(1-g)g((1-\delta)(cq-v)+\delta\frac{cq-v}{2})}_{\text{opponent is low non-expert}} \\
& + g(1-g)((1-\delta)0+\delta\frac{cq-v}{2})+(1-g)^2((1-\delta)^2 0+(1-\delta)\delta(cq-v)+\delta(1-\delta)0+\delta^2\frac{cq-v}{2}))\Bigg].
\end{aligned}$$

Now, in the second stage, if a low non-expert with a bid of v sees any bid other than v from the opponent, bidding cq or nothing in the second stage doesn't affect his payoff. If he sees a bid of v, then he knows that the opponent is either a high expert with $C=0$ or a low non-expert. If he does nothing in the second stage, his payoff is

$$\begin{aligned}
A_4 = \quad & \underbrace{\frac{pr(1-q)}{pr(1-q)+(1-p)(1-r)(1-b)}(0)}_{\text{opponent is high expert and } C=0} \\
& + \underbrace{\frac{(1-p)(1-r)(1-b)}{pr(1-q)+(1-p)(1-r)(1-b)}(g\frac{cq-v}{2}+(1-g)(1-\delta)0+(1-g)\delta\frac{cq-v}{2})}_{\text{opponent is low non-expert}},
\end{aligned}$$

while if he bids cq, the payoff is

$$\begin{aligned}
B_4 = \quad & \underbrace{(1-\delta)}_{\text{bid goes through}}\Bigg[\underbrace{\frac{pr(1-q)}{pr(1-q)+(1-p)(1-r)(1-b)}(-v)}_{\text{opponent is high expert and } C=0} \\
& + \underbrace{\frac{(1-p)(1-r)(1-b)}{pr(1-q)+(1-p)(1-r)(1-b)}(g(cq-v)+(1-g)(1-\delta)0+(1-g)\delta(cq-v))}_{\text{opponent is low non-expert}}\Bigg] \\
& + \underbrace{\delta A_4}_{\text{bid doesn't go through}}.
\end{aligned}$$

It must hold that $A_4 = B_4$ to permit mixing these strategies, from which we get an expression for the mixing probability g which is

$$g = \frac{\frac{2pr(1-q)v}{(1-p)(1-r)(1-b)(cq-v)} - \delta}{1-\delta}.$$

This expression is non-negative for sufficiently small δ and it is < 1 iff

$$v < \frac{c(1-p)(1-r)(1-b)q}{2pr(1-q) + (1-p)(1-r)(1-b)}.$$

For $b = 0$ and the corresponding g, we get $A_3 \leq B_3$ (for $g < 1$), therefore the current strategy of the low expert is optimal and we get an equilibrium. For this reason, we set $b = 0$. The above condition then becomes

$$v < \frac{c(1-p)(1-r)q}{2pr(1-q) + (1-p)(1-r)} = m_2.$$

If $v \geq m_2$, then we set $g = 1$ (which corresponds to strategy u^{LNE}).

This ends the proof for equilibria 1 and 3.

When $a = 0$, the strategy for the low expert we considered above is not always optimal. This happens when $v \geq m_1$. More specifically, since he knows that the high non-expert will bid c in the second stage for sure, he has no reason to wait until the second stage to bid, and bids c from the first stage. With the same logic, since a high non-expert knows for sure that he will bid c in the second stage, it is even better to bid c from the first stage. Moreover, when a high non-expert sees a bid of c in the first stage, he doesn't know for sure what the opponent is, so he doesn't increase his bid. This will change also the strategy for the high expert with $C = c$. In the first stage, he prefers to bid c instead of $cq + v$, because a bid of $cq + v$ would reveal that he is a high expert and $C = c$. So, the equilibrium when $v \geq \frac{c(1-p)r(1-q)}{2p(1-r)q+(1-p)r} = m_1$ is as follows.

- High Expert: If $C = 0$, bids v in the first stage and does nothing in the second stage. If $C = c$, bids c in the first stage and bids $c + v$ in the second stage.

- Low Expert: If $C = 0$, does nothing. If $C = c$, he bids c in the first stage and nothing in the second stage.

- High Non-Expert: Bids c in the first stage. If he sees a bid other than $0, v, cq$ or c in the first stage, he bids $c + v$ in the second stage. Otherwise, he does nothing in the second stage.

- Low Non-Expert: Bids v in the first stage. He bids cq in the second stage with probability $1 - g$.

The proofs for the high expert, the low expert and the low non-expert are the same. We need to check if the high non-expert has any reason to change strategy. An alternative strategy for him would be the one he had before, i.e. to bid $cq + v$ in the first stage and c in the second with some probability. So, suppose that he had bidden $cq + v$ in the first stage and he sees a bid of c. His payoff by doing nothing in the second is 0, while the payoff to bid c in the second stage is

$$
\begin{aligned}
B' = \quad & \underbrace{(1-\delta)}_{\text{bid goes through}} \cdot \Bigg[\underbrace{\frac{prq}{prq + p(1-r)q + (1-p)r}}_{\text{opponent is high expert and } C=c} ((1-\delta)0 + \delta(c + v - (c))) \\
& + \underbrace{\frac{p(1-r)q}{prq + p(1-r)q + (1-p)r}}_{\text{opponent is low expert and } C=c} (c + v - c) \\
& + \underbrace{\frac{(1-p)r}{prq + p(1-r)q + (1-p)r}}_{\text{opponent is high non-expert}} \left(\frac{cq + v - c}{2}\right) \Bigg] \\
& + \underbrace{\delta}_{\text{bid doesn't go through}} \cdot 0.
\end{aligned}
$$

This is ≥ 0 for $v \geq \frac{c(1-p)r(1-q)}{2prq\delta+2p(1-r)q+(1-p)r}$, which is true since

$$v > \frac{c(1-p)r(1-q)}{2p(1-r)q+(1-p)r} \geq \frac{c(1-p)r(1-q)}{2prq\delta+2p(1-r)q+(1-p)r}.$$

Therefore, he is better off by bidding c rather than 0 in the second stage. This means that by bidding in the first stage he can increase his payoff. All other possible strategies are trivially dominated by those we considered above.

This ends the proof for equilibria 2 and 4.

Summarizing the first case, when $a > 0$ (i.e. $v < m_1$) and $g < 1$ (i.e. $v < m_2$), we get the equilibrium $(s_1^{HE}, s^{LE}, x^{HNE}, x^{LNE})$, when $a = 0$ (i.e. $v \geq m_1$) and $g < 1$ (i.e. $v < m_2$), we get the equilibrium $(s_2^{HE}, t^{LE}, o^{HNE}, x^{LNE})$, when $a > 0$ (i.e. $v < m_1$) and $g = 1$ (i.e. $v \geq m_2$), we get the equilibrium $(s_1^{HE}, s^{LE}, x^{HNE}, u^{LNE})$, and when $a = 0$ (i.e. $v \geq m_1$) and $g = 1$ (i.e. $v \geq m_2$), we get the equilibrium $(s_2^{HE}, t^{LE}, o^{HNE}, u^{LNE})$.

Case 2. Assume now that $M_2 \leq v < M_1$. This means that $cq \leq v$ and $cq + v < c$. Consider the following set of strategies:

- High Expert: If $C = 0$, bids v in the first stage and does nothing in the second stage. If $C = c$, bids $cq + v$ in the first stage and bids $c + v$ in the second stage.

- Low Expert: If $C = 0$, does nothing. If $C = c$, he bids $cq + v$ in the first stage and c in the second stage.

- High Non-Expert: Bids $cq + v$ in the first stage. If he sees a bid other than $0, v, cq$, or $cq + v$ in the first stage, he bids $c + v$ in the second stage. Otherwise, he bids c in the second stage with probability $1 - a$.

- Low Non-Expert: Bids cq in the first stage and nothing in the second.

We now investigate if anyone has incentive to change strategy. For $a > 0$, the arguments for all types of bidders are the same as in the previous case except for the low non-expert.

The expected valuation of a low non-expert is cq. Now he bids cq in the first stage and his expected payoff is 0. The only way to get the item is only if he faces another low non-expert, in which case they both bid cq and there is a tie. But even in this case he has to pay cq, so his payoff is 0. He cannot achieve a better payoff, since it is never optimal to bid something above his expected valuation.

This ends the proof for equilibrium 5.

Similarly as in the previous case, the equilibrium when $v \geq \frac{c(1-p)r(1-q)}{2p(1-r)q+(1-p)r} = m_1$ (which means $a = 0$) is as follows.

- High Expert: If $C = 0$, bids v in the first stage and does nothing in the second stage. If $C = c$, bids c in the first stage and bids $c + v$ in the second stage.
- Low Expert: If $C = 0$, does nothing. If $C = c$, he bids c in the first stage and nothing in the second stage.
- High Non-Expert: Bids c in the first stage. If he sees a bid other than $0, v, cq$, or c in the first stage, he bids $c + v$ in the second stage. Otherwise, he does nothing in the second stage.
- Low Non-Expert: Bids cq in the first stage and nothing in the second.

This ends the proof for equilibrium 6.

Summarising the second case, when $a > 0$ (i.e. $v < m_1$), we get the equilibrium $(s_1^{HE}, s^{LE}, x^{HNE}, t^{LNE})$, and when $a = 0$ (i.e. $v \geq m_1$), we get the equilibrium $(s_2^{HE}, t^{LE}, o^{HNE}, t^{LNE})$.

Case 3. Next, suppose that $\max\{M_2, M_1\} \leq v$. This means that $\max\{cq, c(1-q)\} \leq v$. We consider the following set of strategies:

- High Expert: If $C = 0$, bids v in the first stage and does nothing in the second stage. If $C = c$, bids $cq + v$ in the first stage and bids $c + v$ in the second stage.
- Low Expert: If $C = 0$, does nothing. If $C = c$, he bids c in the first stage and nothing

in the second stage.

- High Non-Expert: Bids $cq+v$ in the first stage. If he sees a bid other than $0, v, cq, c,$, or $cq+v$ in the first stage, he bids $c+v$ in the second stage.

- Low Non-Expert: Bids cq in the first stage and nothing in the second.

This is the simplest case. Both high and low non-experts have nothing to lose by bidding their expected valuation, therefore they do so from the first stage. The low non-expert has no reason to hide his identity, therefore he bids his valuation from the first stage. The same is true for a high expert with $C = 0$. Finally, the high expert with $C = c$ bids the highest possible he can in the first stage without revealing that he is a high expert, which is a bid of $cq+v$, and then he bids $c+v$ in the second stage. If he bids $c+v$ from the first stage, then his payoff strictly decreases because of the possibility that the opponent is a high non-expert.

This ends the proof for equilibrium 9.

Summarizing the third case, we get the equilibrium $(s_1^{HE}, t^{LE}, t^{HNE}, t^{LNE})$.

Case 4. Finally, suppose that $M_1 \leq v < M_2$. This means that $c(1-q) \leq v < cq$. We consider two cases:

- If $v < \frac{c(1-p)(1-r)q}{2pr(1-q)+(1-p)(1-r)} = m_2$, the following is an equilibrium.
  - High Expert: If $C = 0$, bids v in the first stage and does nothing in the second stage. If $C = c$, bids $cq+v$ in the first stage and bids $c+v$ in the second stage.
  - Low Expert: If $C = 0$, does nothing. If $C = c$, he bids c in the first stage and nothing in the second stage.
  - High Non-Expert: Bids $cq+v$ in the first stage. If he sees a bid other than $0, v, cq, c,$, or $cq+v$ in the first stage, he bids $c+v$ in the second stage.
  - Low Non-Expert: Bids v in the first stage. He bids cq in the second stage with probability $1-g$, where $g = \frac{\frac{2pr(1-q)v}{(1-p)(1-r)(cq-v)} - \delta}{1-\delta}$.

- If $v \geq \frac{c(1-p)(1-r)q}{2pr(1-q)+(1-p)(1-r)} = m_2$, the following is an equilibrium.
  - High Expert: If $C = 0$, bids v in the first stage and does nothing in the second stage. If $C = c$, bids $cq + v$ in the first stage and bids $c + v$ in the second stage.
  - Low Expert: If $C = 0$, does nothing. If $C = c$, he bids c in the first stage and nothing in the second stage.
  - High Non-Expert: Bids $cq + v$ in the first stage. If he sees a bid other than $0, v, cq, c,$, or $cq + v$ in the first stage, he bids $c + v$ in the second stage.
  - Low Non-Expert: Bids v in the first stage and nothing in the second.

For the experts and the high non-expert, the proofs are similar to the previous case. For the low non-expert, the proof is similar to the second case.

This ends the proof for equilibria 7 and 8.

Summarizing the fourth case, when $g < 1$ (i.e. $v < m_2$), we get the equilibrium $(s_1^{HE}, t^{LE}, t^{HNE}, x^{LNE})$, and when $g = 1$ (i.e. $v \geq m_2$), we get the equilibrium $(s_1^{HE}, t^{LE}, t^{HNE}, u^{LNE})$.

□

B.2 More on δ

Recall that δ is the probability that a bid in the second stage does not go through (due to network or other technical difficulties). For the result of Lemma 2 to hold, in the model section, we assumed that $\delta \leq \bar{\delta}$. In this section, we elaborate on how to calculate the value of $\bar{\delta}$. We also briefly discuss how the equilibrium structure changes when $\delta > \bar{\delta}$.

We start with the first case of Lemma 2, i.e. when $v \leq \min\{m_1, m_2\}$. In that case, the set of strategies $(s_1^{HE}, s_1^{LE}, x^{HNE}, x^{LNE})$ is an equilibrium for sufficiently small δ.

More specifically, there are three threshold values τ_1, τ_2, τ_3, and the case 1 of Lemma 2 holds if $\delta \leq \min\{\tau_1, \tau_2, \tau_3\}$. The first threshold, τ_1, corresponds to the strategy of the high expert.

When δ exceeds this threshold, a high expert with $C = c$ prefers to bid $c + v$ in the first stage instead of waiting to bid in the second stage (i.e. instead of following strategy s_1^{HE}).

The second threshold, τ_2, corresponds to the strategy of the low expert. When δ exceeds this threshold, a low expert with $C = c$ prefers to bid c in the first stage instead of following the strategy s_1^{LE}. To compute τ_2, we have to find the minimum δ for which $B(\delta) \geq A(\delta)$ in the proof of Lemma 2, or equivalently solve the equation $B(\delta) = A(\delta)$ for δ.

The third threshold, τ_3, corresponds to the strategy of the low non-expert. When δ exceeds this threshold, a low non-expert prefers to bid cq in the first round, i.e. prefers to follow the strategy t^{LNE} instead of the strategy x^{LNE}. To compute τ_3, we have to find the minimum δ for which the probability g in the proof of Lemma 2 is non-negative, or equivalently solve the equation $g(\delta) = 0$ for δ.

The closed-form expressions for the three thresholds are given below.

$$\tau_1 = \frac{-\sqrt{2c^2(p-2)(p-1)(q-1)^2r^2 + 4c(p-2)p(q-1)q(r-1)rv + p^2(r-1)^2v^2} + 2c(p-1)(q-1)r + p(4q-1)(r-1)v}{cp(q-1)r + 2p(2q-1)(r-1)v},$$

$$\begin{aligned}\tau_2 = \max\left\{\frac{1}{\frac{(p(r-1)-2r)(c(q-1)+v)}{\sqrt{2}\sqrt{(p(r-1)-2r)(c(q-1)+v)(c(p-1)(q-1)r+v(p(2q(r-1)+r)-r))}}+1},\right.\\ \left.\frac{1}{-\frac{(p(r-1)-2r)(c(q-1)+v)}{\sqrt{2}\sqrt{(p(r-1)-2r)(c(q-1)+v)(c(p-1)(q-1)r+v(p(2q(r-1)+r)-r))}}+1}\right\} =\\ = \frac{1}{\frac{(p(r-1)-2r)(c(q-1)+v)}{\sqrt{2}\sqrt{(p(r-1)-2r)(c(q-1)+v)(c(p-1)(q-1)r+v(p(2q(r-1)+r)-r))}}+1},\end{aligned}$$

$$\tau_3 = \frac{2pr(1-q)v}{(1-p)(1-r)(cq+v)}.$$

The plot in Figure 9 shows how platform's revenue changes as δ increases in the interval

$[0, \min\{\tau_1, \tau_2, \tau_3\}]$.

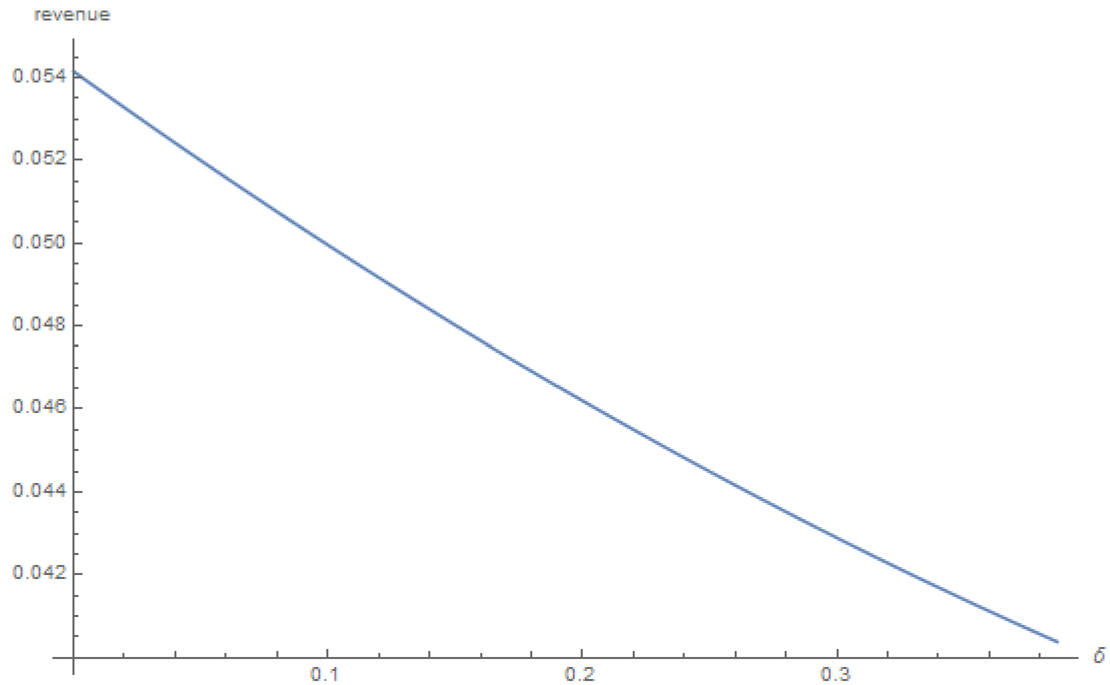

Figure 9: Platform's revenue as δ increases, for $v = 0.03$, $c = 1$, $p = 0.5$, $r = 0.5$, and $q = 0.1$. It is $\tau_1 = 0.445287$, $\tau_2 = 0.386605$, and $\tau_3 = 0.415385$.

We can see that as δ increases, platform's revenue decreases. The reason for this is that the equilibrium remains the same, i.e. the bids of the buyers are the same, but the probability that some bids don't go through increases, therefore the expected final price of the item is lower.

We continue with the second case of Lemma 2, where the equilibrium is $(s_2^{HE}, t^{LE}, o^{HNE}, x^{LNE})$. Here we need only two bounds for δ, one for the high expert and one for the low non-expert, since the low expert bids only in the first round independently of the value of δ.

The threshold for the low non-expert remains the same as in the previous case, i.e. it is τ_3. However, the bound for the high expert will change due to the change in the strategies of the other bidders.

Consider the following strategy for the high expert:

- t^{HE}: If $C = 0$, he bids v in the first stage and does nothing in the second stage. If $C = c$, he bids $c + v$ in the first stage and nothing in the second stage (truthful strategy).

To find the new threshold for the high expert, we need to compare his payoff when he uses the strategy s_2^{HE}, his payoff when he uses the strategy t^{HE}, and see when the first is larger than the second, which will make $(s_2^{HE}, t^{LE}, o^{HNE}, x^{LNE})$ an equilibrium. The new threshold is

$$\begin{aligned}\sigma_1 &= \max\left\{\frac{2p + \sqrt{2}\sqrt{(p-2)(p-1)} - 2}{p}, \frac{2p - \sqrt{2}\sqrt{(p-2)(p-1)} - 2}{p}\right\} = \\ &= \frac{2p + \sqrt{2}\sqrt{(p-2)(p-1)} - 2}{p}.\end{aligned}$$

Therefore, the necessary and sufficient condition for the second case of Lemma 2 is $\delta \leq \min\{\sigma_1, \tau_3\}$. Similarly for every case of the lemma, we can find the bound for δ. The following result summarizes all the cases.

Lemma 4. *The necessary and sufficient condition for δ in Lemma 2 is $\delta \leq \bar{\delta}$, where*

$$\bar{\delta} = \begin{cases} \min\{\tau_1, \tau_2, \tau_3\}, & \text{if } v \in [0, \min\{m_1, m_2\}) \text{ (case 1)}, \\ \min\{\sigma_1, \tau_3\}, & \text{if } v \in [m_1, m_2) \text{ (cases 2, 7)}, \\ \min\{\tau_1, \tau_2\}, & \text{if } v \in [m_2, m_1) \text{ (cases 3,5)}, \\ \sigma_1, & \text{if } v \in [\max\{m_1, m_2\}, +\infty) \text{ (cases 4,6,8,9)}. \end{cases}$$

An example of equilibrium for $\delta > \bar{\delta}$.

Since, according to the industry numbers, the probability that the sniping bid does not go through is less than 1%, in the main model we only consider the case in which δ is relatively small. However, it is theoretically interesting to know what happens for larger values of δ.

Depending on the value of δ and other parameters in the model, the full analysis leads to too many cases the discussion of which is beyond the scope of this paper. However, to gain some intuition, in the following we discuss one example of the equilibrium structure for $\delta > \bar{\delta}$. Interestingly, we see that the platform's revenue could be non-monotone in δ.

Consider the following parameter values, $c = 1$, $q = 0.1$, $r = 0.5$, $p = 0.5$, $v = 0.7$, and $\delta = 0.44$. We have $v > M_2 = 0.1$ and $v < m_1 = 0.75$; therefore, we are in case 5 of Lemma 2. However, it holds that $\tau_1 = 0.348355$ and $\tau_2 = 0.257284$, i.e., δ exceeds the necessary and sufficient threshold given in Lemma 4 for $(s_1^{HE}, s^{LE}, x^{HNE}, t^{LNE})$ to be an equilibrium.

In particular, the low expert does not want to snipe since δ is larger than τ_2, so he moves his bid to the first stage. In other words, he prefers to follow the strategy t^{LE} instead of the strategy s^{LE}. This will cause a change in the strategies of the other bidders. The new equilibrium will be $(s_2^{HE}, t^{LE}, o^{HNE}, t^{LNE})$, which happens to be the same as case 6 of Lemma 2.

This is because if we cosnider the proof of case 6, the requirements for the equilibrium are satisfied, even though $v < m_1$. First, it holds that $\delta < \sigma_1 = 0.44949$, so the high expert's best response is s_2^{HE}. Moreover, it is the case that $v > \frac{c(1-p)r(1-q)}{2prq\delta+2p(1-r)q+(1-p)r} = 0.698758$, which makes o^{HNE} the best response for the high non-expert. The value of v for the low expert and the low non-expert doesn't matter as long as the others follow the aforementioned strategies. Therefore, $(s_2^{HE}, t^{LE}, o^{HNE}, t^{LNE})$ is an equilibrium.

This causes the following interesting phenomenon. Even though in the interval $[0, 0.257284]$, platform's revenue is a decreasing function of δ, as depicted in Figure 9; as δ increases more, outside this interval, platform's revenue increases. In this example, for $\delta = 0.257284$, platform's revenue is 0.251106, while for $\delta = 0.44$, it is 0.264497.

This is mainly because the low expert has changed his strategy by moving his bid of c from the second stage to the first stage, i.e., from the point where his bid was not going through with probability $\delta = 0.257284$ to the point where his bid always goes through (when

$\delta = 0.44$); this change has a positive effect on the expected price of the item.

As δ increases even more (above σ_1), we see similar patterns: intervals in which platform's revenue is a decreasing function of δ, and some 'jumps' of the revenue in between due to the change of strategies by bidders. In the limit, when $\delta \approx 1$, no-one will bid in the second stage and the auction will be like a sealed-bid second price auction where everyone bids in the first stage.

B.3 Choice of Tie-breaking Rule

In this section, first we elaborate on the choice of our tie-breaking rule. We argue that this rule always favors the bidder who is willing to bid slightly higher than the current bid (i.e. his payoff continues to remain positive if he slightly raises his bid) which the other bidder is not able to match. Then, we show that our results are robust to the choice of the tie-breaking rule. In particular, we show that the equilibrium strategies remain almost unchanged, and our main results continue to hold, under a very different tie-breaking rule.

Recall that we use the following tie-breaking rule: If there is a tie between a low-type bidder and a high-type bidder, then the item goes to the high-type. If the two bidders are of the same type but of different expertise levels, then the item goes to the non-expert. Finally, if the two bidders are of the same type and of the same expertise level, then the winner is determined by a fair coin toss.

There are two interesting cases for which the tie-breaking rule has an effect in the equilibria described in the main lemma. The first is when a high non-expert faces a low expert who knows that $C = c$, and they both bid c. In this case, the high non-expert is willing to bid above c to win the tie and take the item, because his valuation is higher than c, but the low expert cannot do the same since his valuation is c. Therefore, the tie-breaking rule favors the bidder who would be willing to pay a slightly higher price.

The second case is when a high expert who knows that $C = 0$ faces a low non-expert, and they both bid v. In this case, the low non-expert does not want to win the item, because his valuation is below v. Therefore, he has an incentive to bid a bit below v, whereas the high expert does not want to do the same since his valuation is v. Thus, again the tie-breaking rule favors the bidder who has higher willingness to pay.

Intuitively, if we break the tie in favor of the other bidder in any of the above cases, one of the bidders would want to increase or decrease his bid by the smallest possible amount $\epsilon > 0$. Since in our model the strategy space is continuous and not discrete, such ϵ does not exist. We use this tie-breaking rule to avoid such complications. However, to further demonstrate the robustness of our results, in the following, we show that equilibrium strategies, and therefore all of our main results, continue to hold if we change the rule in the opposite direction and favor the experts over the non-experts.

Changing the tie-breaking rule.

Consider the following alternate tie-breaking rule: If there is a tie between two bidders of different expertise levels, then the item goes to the expert. Otherwise, the winner is determined by a fair coin toss. To reduce the number of cases in the analysis, we assume that $\delta = 0$, and only focus on the bids of the second stage.

As explained above, this game does not have a pure strategy Nash equilibrium unless we discretize the bidding space. We show that as the size of the discretization step converges to zero (i.e., the bidding space converges to continuous), the equilibrium outcome of the new tie-breaking rule converges to that of the old tie-breaking rule.

To discretize the strategy space, we assume that bidders can bid c or $c + \epsilon$, for some very small $\epsilon > 0$, but they cannot bid anything in between. This assumption will come into play when there is a tie between a low expert and a high non-expert who both bid c (the first case discussed earlier in this section). Note that without this discretization, high non-experts sometimes want to bid the smallest number strictly larger than c; this is because high non-experts want to win against experts but lose against other high non-experts. We assume that the rest of the strategy space remains unchanged.

We start by defining the strategies for the different types of bidders.

- For a high expert, consider the following strategy:
  - t^{HE}: If $C = 0$, he bids v. If $C = c$, he bids $c + v$.
- For a low expert, consider the following strategy:
  - t^{LE}: If $C = 0$, he does nothing. If $C = c$, he bids c.
- For a high non-expert, consider the following strategies:
  - x^{HNE}: He bids $cq + v$ with probability a and $c + \epsilon$ with probability $1 - a$, where $a := a(\epsilon) = 1 - \frac{2p(1-r)qv}{(1-p)r(c+\epsilon-(cq+v))}$.
  - o^{HNE}: He bids $c + \epsilon$.
  - t^{HNE}: He bids $cq + v$.
- For a low non-expert, consider the following strategies:
  - x^{LNE}: He bids v with probability g and cq with probability $1 - g$, where $g = \frac{2pr(1-q)v}{(1-p)(1-r)(cq-v)}$.
  - u^{LNE}: He bids v.
  - t^{LNE}: He bids cq.

We define also a new threshold for v, the analogous of the old m_1, that now depends also on ϵ. We have that $m_1 := m_1(\epsilon) = \frac{(c+\epsilon-cq)(1-p)r}{2p(1-r)q+(1-p)r}$.

Intuitively, high non-experts who want to over-bid now bid $c + \epsilon$ instead of c. This allows them to win against experts, even though ties are broken in favor of experts. We now describe the equilibrium bidding strategies for buyers in nine cases in the following lemma.

Lemma 5. *For the auction model described above with the alternative tie-breaking rule, the buyers' equilibrium bidding strategies are given below.*

1. *If $v \in [0, \min\{m_1(\epsilon), m_2\})$, the set of strategies $(t^{HE}, t^{LE}, x^{HNE}, x^{LNE})$ forms an equilibrium.*

2. *If $v \in [m_1, \min\{m_2, M_1\})$, the set of strategies $(t^{HE}, t^{LE}, o^{HNE}, x^{LNE})$ forms an equilibrium.*

3. *If $v \in [m_2, \min\{m_1(\epsilon), M_2\})$, the set of strategies $(t^{HE}, t^{LE}, x^{HNE}, u^{LNE})$ forms an equilibrium.*

4. *If $v \in [\max\{m_1(\epsilon), m_2\}, \min\{M_1, M_2\})$, the set of strategies $(t^{HE}, t^{LE}, o^{HNE}, u^{LNE})$ forms an equilibrium.*

5. *If $v \in [M_2, m_1(\epsilon))$, the set of strategies $(t^{HE}, t^{LE}, x^{HNE}, t^{LNE})$ forms an equilibrium.*

6. *If $v \in [\max\{m_1(\epsilon), M_2\}, M_1)$, the set of strategies $(t^{HE}, t^{LE}, o^{HNE}, t^{LNE})$ forms an equilibrium.*

7. *If $v \in [M_1, m_2)$, the set of strategies $(t^{HE}, t^{LE}, t^{HNE}, x^{LNE})$ forms an equilibrium.*

8. *If $v \in [\max\{m_2, M_1\}, M_2)$, the set of strategies $(t^{HE}, t^{LE}, t^{HNE}, u^{LNE})$ forms an equilibrium.*

9. *If $v \in [\max\{M_1, M_2\}, +\infty)$, the set of strategies $(t^{HE}, t^{LE}, t^{HNE}, t^{LNE})$ forms an equilibrium.*

Proof. We will prove the first four equilibria, i.e. when $v < M_1$ and $v < M_2$, which are the most general. The rest of the cases are similar to the proof of the Lemma 2.

We have that $v < \min\{M_1, M_2\}$. This means that $cq + v < c$ and $v < cq$. Consider the following general set of strategies:

- High Expert: If $C = 0$, he bids v. If $C = c$, he bids $c + v$.

- Low Expert: If $C = 0$, he does nothing. If $C = c$, he bids c.
- High Non-Expert: He bids $cq + v$ with probability a and $c + \epsilon$ with probability $1 - a$.
- Low Non-Expert: He bids v with probability g and cq with probability $1 - g$.

The probabilities a, g are as yet undetermined. We will examine if anyone has incentive to change strategy and at the same time try to determine the probabilities and the conditions for which the above is an equilibrium. These conditions will give us the proof that equilibria 1 and 3 are correct.

Both the high and the low experts bid truthfully and this is optimal for them. This is because they bid in the second stage, therefore they don't have any fear to reveal the common value to non-experts. They also know their true valuation, and since we have a second-price auction, it is optimal for them to bid their true values.

Now, we consider a high non-expert. His expected valuation is $cq + v$. Their payoff by bidding $cq + v$ is

$$\begin{aligned} A_2 = & \underbrace{prq(0)}_{\text{opponent is high expert and } C=c} \\ & + \underbrace{pr(1-q)(0)}_{\text{opponent is high expert and } C=0} \\ & + \underbrace{p(1-r)q(0)}_{\text{opponent is low expert and } C=c} \\ & + \underbrace{p(1-r)(1-q)(v)}_{\text{opponent is low expert and } C=0} \\ & + \underbrace{(1-p)r(0)}_{\text{opponent is high non-expert}} \\ & + \underbrace{(1-p)(1-r)(g(cq)+(1-g)(v))}_{\text{opponent is low non-expert}}, \end{aligned}$$

while their payoff by bidding $c + \epsilon$ is

$$B_2 = \underbrace{prq(0)}_{\text{opponent is high expert and } C=c}$$

$$
\begin{aligned}
&+ \underbrace{pr(1-q)(0)}_{\text{opponent is high expert and } C=0} \\
&+ \underbrace{p(1-r)q(v)}_{\text{opponent is low expert and } C=c} \\
&+ \underbrace{p(1-r)(1-q)(v)}_{\text{opponent is low expert and } C=0} \\
&+ \underbrace{(1-p)r\left((1-a)\frac{cq+v-(c+\epsilon)}{2}\right)}_{\text{opponent is high non-expert}} \\
&+ \underbrace{(1-p)(1-r)(g(cq)+(1-g)(v)).}_{\text{opponent is low non-expert}}
\end{aligned}
$$

By bidding $c+\epsilon+\zeta$ for some small $\zeta > 0$, his payoff can only decrease. By bidding c, the payoff is the same as with the bid of $cq+v$. It holds that

$$
B_2 - A_2 = p(1-r)q(v) + (1-p)r\left((1-a)\frac{cq+v-(c+\epsilon)}{2}\right),
$$

and we want this to be equal to 0 to permit mixing these strategies, which will give us an expression for the mixing probability a. This is

$$
a = 1 - \frac{2p(1-r)qv}{(1-p)r(c+\epsilon-(cq+v))}.
$$

This is always ≤ 1. The inequality $a > 0$ is equivalent to $v < \frac{(c+\epsilon-cq)(1-p)r}{2p(1-r)q+(1-p)r} = m_1$. So, we need this condition for equilibria 1 and 3. If $1 - \frac{2p(1-r)qv}{(1-p)r(c+\epsilon-(cq+v))} \leq 0$, then it is always better for the high non-expert to bid $c+\epsilon$, so we set $a = 0$ (equilibria 2 and 4).

Next, we consider a low non-expert. His expected valuation is cq. His payoff if he bids cq is

$$
\begin{aligned}
A_3 = \;& \underbrace{pr(q0+(1-q)(-v))}_{\text{opponent is high expert}} \\
&+ \underbrace{p(1-r)(q0+(1-q)0)}_{\text{opponent is low expert}} \\
&+ \underbrace{(1-p)r(0)}_{\text{opponent is high non-expert}}
\end{aligned}
$$

$$+ \underbrace{(1-p)(1-r)(g(cq-v)+(1-g)0)}_{\text{opponent is low non-expert}}.$$

His payoff if he bids v is

$$\begin{aligned} B_3 = & \underbrace{pr(q0+(1-q)0)}_{\text{opponent is high expert}} \\ & + \underbrace{p(1-r)(q0+(1-q)0)}_{\text{opponent is low expert}} \\ & + \underbrace{(1-p)r(0)}_{\text{opponent is high non-expert}} \\ & + \underbrace{(1-p)(1-r)\left(g\frac{cq-v}{2}\right)}_{\text{opponent is low non-expert}}. \end{aligned}$$

It must hold that $A_3 = B_3$ to permit mixing these strategies, from which we get an expression for the mixing probability g which is

$$g = \frac{2pr(1-q)v}{(1-p)(1-r)(cq-v)}.$$

This expression is always non-negative, and it is < 1 iff

$$v < \frac{c(1-p)(1-r)q}{2pr(1-q)+(1-p)(1-r)} = m_2.$$

If $v \geq m_2$, then we set $g = 1$ (which corresponds to strategy u^{LNE}).

This ends the proof. $\square$

Notice that as ϵ goes to 0, the bidding strategies of Lemma 5 approach the strategies of the main Lemma 2. This means that the analogues of Proposition 1 and Proposition 2 will continue to hold with the alternative tie-breaking rule.

B.4 Distribution of Bidders' Private Value

In the main model, we assumed that V has binary distribution with support $\{0, v\}$. In this section we relax that assumption and show that our main result, that non-experts sometimes bid more than their expected value, still holds. More specifically, for $k \geq 2$, we assume that the private value of each bidder is $V = \frac{i \cdot v}{k-1}$ with probability r_i, where $i \in \{0, 1, \ldots, k-1\}$ and $\sum_{i=0}^{k-1} r_i = 1$. In the main model, we had $k = 2$, $r_2 = r$, and $r_1 = 1 - r$.

The tie-breaking rule is a generalization of what we had in the main model. We assume that in case of a tie the bidder with the highest private value wins. If the two bidders have the same private value, then the non-expert wins. If both bidders have the same private value and the same expertise level, then the winner is determined with a fair coin toss.

To simplify the analysis, we assume that $\delta = 0$ and only focus on the bids in the second stage. It is easy to see that it is weakly dominant for all the experts to bid their true valuation. In other words, if an expert has private value $\frac{i \cdot v}{k-1}$, he will bid $\frac{i \cdot v}{k-1}$ when the common value is low ($C = 0$), and $c + \frac{i \cdot v}{k-1}$, when the common value is high ($C = c$). We also assume that $v \leq c \cdot \min\{q, 1 - q\}$ (which corresponds to the condition $v \leq \min\{M_1, M_2\}$ of our main model).

We show that, in equilibrium, non-experts will mix between at most three different bids. More specifically, if a non-expert is of type i, meaning that his private value is $\frac{i \cdot v}{k-1}$, he will mix between $\frac{(i+1) \cdot v}{k-1}$, $c \cdot q + \frac{i \cdot v}{k-1}$, and $c + \frac{(i-1) \cdot v}{k-1}$. The bid $\frac{(i-1) \cdot v}{k-1}$ is employed because he wants to lose against the experts of higher type when $C = 0$. The bid $c \cdot q + \frac{i \cdot v}{k-1}$ is used because this is his expected valuation and this is the bid he wants to have against a non-expert. The bid $c + \frac{(i-1) \cdot v}{k-1}$ is used because he wants to win against an expert of lower type when $C = c$.

The reason that bidders use only these three bids in equilibrium is that, assuming that the opponent also uses the same strategy in equilibrium, every other bid is dominated by at least one of these three. The intuition is as follows. A bid $y \in [0, cq)$ will lose against all the bids in $[cq, +\infty)$ and will win only against some bids in $[0, v]$. But a bid in $[0, v]$ means that either the opponent is an expert and $C = 0$, in which case we want to win against bids in $[0, v_i]$ and lose against bids in $[v_{i+1}, v]$, or the opponent is a non-expert who happened to underbid, in which case we want to win all the time, something achieved by the bid of $cq + v_i$. So, depending on the parameters of the model, y is dominated by either v_{i+1} or $cq + v_i$. When these two give the same payoff, i.e. when the non-expert is mixing between the two, y is dominated by both.

Similarly, a bid $y \in [cq, c)$ will lose against all the bids in $[c, +\infty)$, will win against all bids in $[0, v]$, and will win against some bids in $[cq, cq + v]$. But a bid in $[cq, cq + v]$ means that the opponent is a non-expert, in which case we want to win against all bids in $[cq, cq + v_{i-1}]$ and lose against bids in $[cq + v_{i+1}, cq + v]$. So, y is dominated by the bid $cq + v_i$.

Finally, a bid $y \in [c, +\infty)$ will win against all bids in $[0, cq + v]$, and will win against some bids in $[c, +\infty)$. But a bid in $[c, +\infty)$ means that either the opponent is an expert and $C = c$, in which case we want to win against bids in $[c, c + v_{i-1}]$ and lose against bids in $[c + v_i, v]$, or the opponent is a non-expert who happened to overbid, in which case we want to lose all the time, something achieved by the bid of $cq + v_i$. So, depending on the parameters of the model, y is dominated by either $c + v_{i-1}$ or $cq + v_i$. When these two give the same payoff, i.e. when the non-expert is mixing between the two, y is dominated by both.

Suppose that the non-experts of type i will bid $\frac{(i+1)\cdot v}{k-1}$ with probability $\theta_{i,1}$, $c \cdot q + \frac{i \cdot v}{k-1}$ with probability $\theta_{i,2}$, and $c + \frac{(i-1)\cdot v}{k-1}$ with probability $\theta_{i,3}$, where $\theta_{i,1} + \theta_{i,2} + \theta_{i,3} = 1$. It holds that $\theta_{0,3} = 0$ and $\theta_{k-1,1} = 0$.

The expected payoff of a non-expert of type $i < k - 1$ when he bids $\frac{(i+1)\cdot v}{k-1}$ is

$$\begin{aligned}\Phi_i =&(1-p)\left(\sum_{j=0}^{i-1} r_j \theta_{j,1}\left(cq + \frac{iv}{k-1} - \frac{(j+1)v}{k-1}\right) + \frac{1}{2} r_i \theta_{i,1}\left(cq + \frac{iv}{k-1} - \frac{(i+1)v}{k-1}\right)\right) \\ &+ p(1-q)\sum_{j=0}^{i} \frac{v(i-j)r_j}{k-1}.\end{aligned}$$

The expected payoff of a non-expert of type i when he bids $c \cdot q + \frac{i \cdot v}{k-1}$ is

$$\begin{aligned}\Psi_i =&(1-p)\left(\sum_{j=0}^{k-1} r_j\theta_{j,1}\left(cq+\frac{iv}{k-1}-\frac{(j+1)v}{k-1}\right)+\sum_{j=0}^{i-1} r_j\theta_{j,2}\left(\frac{iv}{k-1}-\frac{jv}{k-1}\right)\right)\\ &+p(1-q)\sum_{j=0}^{k-1}\frac{v(i-j)r_j}{k-1}.\end{aligned}$$

The expected payoff of a non-expert of type $i > 0$ when he bids $c + \frac{(i-1)\cdot v}{k-1}$ is

$$\begin{aligned}\Omega_i =&(1-p)\left(\sum_{j=0}^{k-1} r_j\theta_{j,1}\left(cq+\frac{iv}{k-1}-\frac{(j+1)v}{k-1}\right)+\sum_{j=0}^{i-1} r_j\theta_{j,3}\left(cq-c+\frac{iv}{k-1}-\frac{(j-1)v}{k-1}\right)\right.\\ &\left.+\sum_{j=0}^{k-1} r_j\theta_{j,2}\left(\frac{iv}{k-1}-\frac{jv}{k-1}\right)+\frac{1}{2}r_i\theta_{i,3}\left(cq-c+\frac{iv}{k-1}-\frac{(i-1)v}{k-1}\right)\right)\\ &+p\left((1-q)\sum_{j=0}^{k-1}\frac{v(i-j)r_j}{k-1}+q\sum_{j=0}^{i-1} r_j\left(\frac{iv}{k-1}-\frac{jv}{k-1}\right)\right).\end{aligned}$$

Consider the case where $\theta_{i,1} > 0$ for every $i < k-1$, $\theta_{i,2} > 0$ for every i, and $\theta_{i,3} > 0$ for every $i > 0$. In other words, all types of non-experts are mixing between all their potential bids. This case corresponds to the equilibrium in case 1 of Lemma 2. To find all the probabilities t, we need to solve the system

$$\begin{aligned}&\Phi_i = \Psi_i, \quad \text{for } i \in \{0,1,\ldots,k-2\}\\ &\Psi_i = \Omega_i, \quad \text{for } i \in \{1,2,\ldots,k-1\}\\ &\theta_{0,3} = 0\\ &\theta_{k-1,1} = 0\\ &\sum_{m=1}^{3}\theta_{i,m} = 1, \quad \text{for } i \in \{0,1,\ldots,k-1\}.\end{aligned}$$

The solution to this system for $k = 2$ is

$$\theta_{0,1} = \frac{2p(q-1)r_1v}{(p-1)r_0(cq-v)}$$

$$\theta_{0,2} = 1 - \frac{2p(q-1)r_1 v}{(p-1)r_0(cq-v)}$$

$$\theta_{0,3} = 0$$

$$\theta_{1,1} = 0$$

$$\theta_{1,2} = 1 - \frac{2pqr_0 v}{(p-1)r_1(c(q-1)+v)}$$

$$\theta_{1,3} = \frac{2pqr_0 v}{(p-1)r_1(c(q-1)+v)},$$

which is the same as our solution in the main body of the paper ($\theta_{1,2} = a$ and $\theta_{0,1} = g$).

The solution to the system for $k = 3$ is

$$\theta_{0,1} = \frac{2v(2cq(p(q-1)r_1 + 2(p-1)r_0) + v(-p(q-1)(r_1 - 2r_2) - 4(p-1)r_0))}{(p-1)r_0\left(4c^2q^2 + 4cqv - 7v^2\right)}$$

$$\theta_{0,2} = \frac{(2cq-v)((p-1)r_0(2cq-v) - 2p(q-1)r_1 v) - 4p(q-1)r_2 v^2}{(p-1)r_0\left(4c^2q^2 + 4cqv - 7v^2\right)}$$

$$\theta_{0,3} = 0$$

$$\theta_{1,1} = \frac{2v(2cq(p(q-1)r_2 - (p-1)r_0) + v(p(q-1)(2r_1 + 3r_2) + (p-1)r_0))}{(p-1)r_1\left(4c^2q^2 + 4cqv - 7v^2\right)}$$

$$\begin{aligned}
\theta_{1,2} = \Big(&16c^4(p-1)(q-1)^2q^2r_1 - 16c^3(q-1)qv\Big(r_2(pq^2 - (p+1)q + p) \\
&+ r_0(p((q-1)q+1) + q - 1) + (p-1)r_1\Big) \\
&+ 4c^2v^2\Big(2r_0(p(q((q-1)q+4) - 1) + 3q^2 - 4q + 1) \\
&+ r_1(p(-6(q-1)q - 3) + 18(q-1)q + 7) - 2r_2(p(q((q-2)q+5) - 3) + q(2-3q))\Big) \\
&+ 4cv^3\Big(r_0(p(q(13q-12) - 2) + 5q + 2) + r_1(p(8(q-1)q - 3) + 7) \\
&+ r_2(p(q(13q-14) - 1) - 5q + 7)\Big) + 7v^4\Big(2r_0(-3pq + p - 1) \\
&+ 2r_2(3pq - 2p - 1) + (3p-7)r_1\Big)\Big) \\
&\Big/\Big((p-1)r_1\left(4c^2(q-1)^2 - 4c(q-1)v - 7v^2\right)\left(4c^2q^2 + 4cqv - 7v^2\right)\Big)
\end{aligned}$$

$$\theta_{1,3} = \frac{2(p-1)r_2 v(2c(q-1)+v) - 2pqv((3r_0 + 2r_1)v - 2c(q-1)r_0)}{(p-1)r_1\left(4c^2(q-1)^2 - 4c(q-1)v - 7v^2\right)}$$

$$\theta_{2,1} = 0$$
$$\theta_{2,2} = \frac{(p-1)r_2(2c(q-1)+v)^2 + 2pqv(2r_0v - r_1(2c(q-1)+v))}{(p-1)r_2\left(4c^2(q-1)^2 - 4c(q-1)v - 7v^2\right)}$$
$$\theta_{2,3} = \frac{2pqv(r_1(2c(q-1)+v) - 2r_0v) - 8(p-1)r_2v(c(q-1)+v)}{(p-1)r_2\left(4c^2(q-1)^2 - 4c(q-1)v - 7v^2\right)}.$$

Even though we can analytically solve the system for larger values of k, the closed-form solution does not have any meaningful pattern. In the following example, we numerically solve the system for the case of $k = 20$, which would be an approximation of uniform continuous distribution of v.

Figure 10 shows a plot of the three mixing probabilities as functions of the private value of a non-expert for $k = 20$ and small v. We can see that as the private value increases, the probability of underbidding decreases and the probability of overbidding increases.

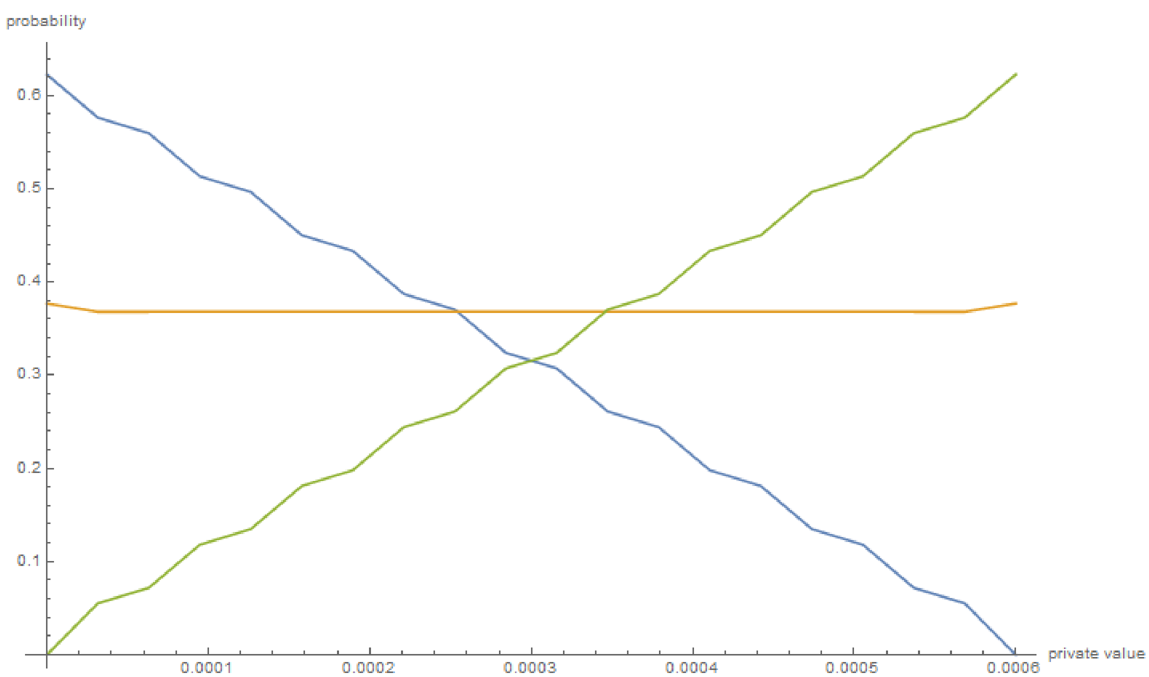

Figure 10: Mixing probabilities as a function of the private value. Blue is for underbidding, orange for bidding the expected valuation, and green for overbidding. The plot is for $k = 20$, $r_i = 1/k$ for $i \in \{0, \ldots, k-1\}$, $c = 1$, $q = 0.5$, $p = 0.999$, and $v = 0.0006$.

As v increases, we will get different equilibria where some types of non-experts don't mix between all their potential bids, i.e., some $\theta_{i,1}$'s become 0. A complete analysis of the equilibrium is beyond the scope of this paper as the number of cases in equilibrium analysis grows exponentially in k as v increases; however, the general pattern is that as v increases, non-experts bid more aggressively. This is consistent with our findings in the main body of the paper.

B.5 Signaling Using Closing Format: Soft versus Hard Close

In this section, we consider a situation in which the platform lets the sellers to decide whether to sell in an auction with hard-close or soft-close format. We call the seller of an item with high common value a high-type seller, and the seller of an item with low common value a low-type seller. A seller is high-type with probability q where q is common knowledge. A seller naturally knows his own type; experts also know the seller's type (since they know the common value of items being offered). But non-experts do not know the seller's type. We investigate whether a seller can signal his type using the closing format (soft versus hard). In particular, we derive conditions for existence of a separating equilibrium.

A seller sets his closing format F (soft or hard). For a format F, we assume that all non-experts have the same belief about a seller who uses F. In general, non-experts' belief about a format is the probability that they think a seller using that format is high-type. However, since we only consider pure strategy Nash equilibria of the game, non-experts' belief about a format is limited to three possibilities: Low (L), High (H), and Unknown (X). In belief L, non-experts believe that a seller using format F is always a low-type seller. In belief H, non-experts believe that a seller using format F is always a high-type seller. Finally, in belief X, non-experts cannot infer anything about the seller's type and believe that the seller is high-type with probability q.

Non-experts have beliefs about each format F. In equilibrium, the beliefs must be consistent with sellers' strategies. In particular, if both types of sellers use the same format in (a pooling) equilibrium, non-experts' belief for that format must be X. If the two types of sellers use different formats in (a separating) equilibrium, non-experts' belief for the format used by the low-type seller must be L and for the format used by the high-type seller must be H. Furthermore, in an equilibrium, given the non-experts' beliefs, sellers should not be able to benefit from changing their strategies.

We use the following notation to explain the results of this section: Let $\pi_T^B(F)$, where $T \in \{L, H\}$ and $B \in \{L, H, X\}$ denote the expected profit of a seller who uses mechanism $F \in \{soft, hard\}$ and has type T, and non-experts believe has type B.

Lemma 6. *For any $B \in \{L, H\}$ and any $T \in \{L, H\}$ we have $\pi_T^B(soft) = \pi_T^B(hard)$. In other words, if non-experts have no uncertainty about the type of the seller ($B \neq X$), soft-close and hard-close formats both lead to the same revenue for the seller (no matter what type the seller is).*

Proof. Note that when non-experts have no uncertainty about the type of the seller, they do not infer anything from other bidders' bids, and do not update their expected value. The auction reduces to a full-information second price auction in this case. □

The seller's revenue, in each case, is given by

$$\pi_H^H(soft) = \pi_H^H(hard) = c + r^2 v$$

$$\pi_L^L(soft) = \pi_L^L(hard) = r^2 v$$

$$\pi_H^L(soft) = \pi_H^L(hard) = \begin{cases} cp^2 + rv(2(1-p)p(1-r) + r) & \text{if } v \leq c \\ cp\left(2(1-p)r + p - 2(1-p)r^2\right) + r^2 v & \text{if } v > c. \end{cases}$$

$$\pi_L^H(soft) = \pi_L^H(hard) = \begin{cases} c(1-p)^2 + rv(2(1-p)p(1-r)+r) & \text{if } v \leq c \\ c(1-p)(p(2(1-r)r-1)+1) + r^2 v & \text{if } v > c. \end{cases}$$

Lemma 7. *No separating equilibrium exists.*

Proof. Assume for sake of contradiction that there is a separating equilibrium in which the low-type uses format F and the high-type uses format F'. Note that, using the above expressions, we have $\pi_L^L(F) < \pi_L^H(F) = \pi_L^H(F')$ for any F and F'. Therefore, the low-type benefits from mimicking, contradicting the equilibrium condition. □

Depending on out-of-equilibrium beliefs, the game could have multiple pooling equilibria (both soft- and hard-close). We can show that soft-close is always a pooling equilibrium. Furthermore, for regions in which hard-close provides higher expected revenue for the high-type seller, as shown in Figure 6, hard-close is also a pooling equilibrium. The intuitive criterion is not sufficient for refining the equilibrium set to a unique equilibrium. However, we can show that only soft-close pooling equilibrium can survive the D1 criterion refinement. Intuitively, the high-type always gains more (loses less) than low-type by deviating to soft-close format in a hard-close pooling equilibrium. Therefore, out-of-equilibrium beliefs on soft-close auction, subject to D1 requirement, is high. This makes the deviation to soft-close always profitable (in a hypothetical hard-close equilibrium). Therefore, hard-close pooling equilibrium cannot survive D1 criterion refinement. This is formally proved in the following lemma.

Lemma 8. *A hard-close pooling equilibrium cannot survive D1 criterion refinement.*

Proof. Assume for sake of contradiction that there is a hard-close pooling equilibrium. Let z be the buyer's belief, the probability that the seller is high-type, on observing the soft-close format. We show that for any $z < 1$, if a low-type seller weakly benefits from deviating to soft-close, a high-type seller strictly benefits from deviating. Then, according to D1 criterion, this implies that out-of-equilibrium belief on soft-close has to be high. Therefore, hard-close cannot be an equilibrium.

Assume for sake of contradiction that there is a z for which a low-type seller weakly benefits from deviating to soft-close, but a high-type seller does not strictly benefit from deviating to soft-close. First, note that if both buyers are non-experts, the equilibrium outcome is not affected by the type of the seller. In other words, both types of sellers would have the same revenue in each closing format. Similarly, if both buyers are experts, the equilibrium outcome is not affected by the closing format. Therefore, to compare the benefit of deviation (for sellers), we can assume that the buyers have different levels of expertise: an expert and a non-expert.

If the expert is low-value, then the revenue is zero in both hard-close and soft-close formats for the low-type seller. The revenue for the high-type seller is always greater than or equal in soft-close (always c) than in hard-close (at most c, depending on the value of z and whether the non-expert is using an aggressive strategy or not).

Finally, consider the case that the expert is high-value. First, assume that the non-expert is also high-value. In this case, the revenue of soft-close for high-type seller is always c and for low-type seller is always v. The revenue of hard-close for the low-type seller is always v. Therefore, the low-type cannot prefer hard-close to soft-close in this sub-case. Next, assume that the non-expert is low-value. In this case, the revenue of the auction in both soft-close and hard-close cases is determined by the bid of the non-expert. A low-value non-expert has the exact same strategy in soft-close and hard-close formats. Furthermore, this strategy does not depend on whether the common value is high or low. Therefore, low-type and high-type sellers have the same revenue in the hard-close format and in the soft-close format. As shown, there is no case (for any z) in which a low-type seller benefits from deviating to soft-close while a high-type seller does not. Furthermore, it is easy to see that there are cases in which the high-type seller strictly benefits from this deviation. Therefore, according to

D1 criterion, buyers' out-of-equilibrium belief on soft-close auction has to be high.

Given that, in a hypothetical hard-close equilibrium, buyers' belief on soft-close is high, sellers always benefit from deviating to soft-close. Therefore, hard-close cannot be a pooling equilibrium. □

Finally, note that a soft-close pooling equilibrium always survives D1 criterion refinement. This is because whenever the high-type seller benefits from deviating to hard-close, the low-type seller also benefits from deviating to hard-close (the proof is very similar to the proof of Lemma 8). Therefore, a soft-close pooling equilibrium in which out-of-equilibrium belief on hard-close is low survives D1 criterion refinement.

Given that the only pure strategy equilibrium that survives D1 criterion refinement is a soft-close pooling equilibrium, we show that, compared to the case where the platform decides closing format, a low-type seller and the platform are both (weakly) worse off if the closing format decision is left to the sellers. A high-type seller may be worse off or better off, depending on other parameters, as shown in Figure 6.