# The Palladian grammar

G Stiny
Centre for Configurational Studies, The Open University, Milton Keynes MK7 6AA, England
W J Mitchell
The Martin Centre for Architectural and Urban Studies, University of Cambridge, Cambridge, England
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**Abstract.** A parametric shape grammar that generates the ground plans of Palladio's villas is developed as a definition of the Palladian style. The grammar is applied to generate the plan for the Villa Malcontenta.

> "And altho' variety and things new may please every one, yet they ought not to be done contrary to the precepts of art, and contrary to that which reason dictates; whence one sees, that altho' the ancients did vary, yet they never departed from the universal and necessary rules of art, as shall be seen in my *book of antiquities*" (Palladio, 1965, page 26).

Palladio's *I Quattro Libri dell'Architettura* sets out rules of classical architectural usage in much the same way as a traditional grammar sets out rules of Latin usage. The conventions of composition and construction governing correct building practice are established by prescription and example. An architectural language is thus defined that owes its power and richness to the coherence and clarity of these "precepts of art ... which reason dictates".

In this paper a first attempt is made to recast parts of Palladio's architectural grammar in a modern, generative form. The rules of a *parametric shape grammar* (Stiny, 1977) that generates villa ground plans are specified. In many cases these rules are direct translations of Palladio's explicit canons of design; in others they are based on examples of villa plans in the *Quattro Libri*. The grammar so defined generates the main features of most of Palladio's villa plans as drawn in the *Quattro Libri*. The complete generation of the ground plan of the Villa Malcontenta (figure 1) is given.

The geometry of Palladio's villa plans is studied instead of other aspects of his architectural system, for example, the use of decorative motifs, because it characterizes the Palladian style in a fundamental sense. As Wittkower (1952) observes, the distinguishing feature of Palladio's villas is "the systematization of the groundplan". Ackerman (1966) further points out that "Often the porch is the only antique reference in the design; all the rest of the detail is simple geometry, which is consistent with the concept of a hierarchy of elements".

The quotations appearing in the following sections are taken from the Dover reprint edition (1965) of Isaac Ware's 1738 English translation of the *Quattro Libri*; references to particular villa ground plans are made via plate numbers from the *Second Book*.
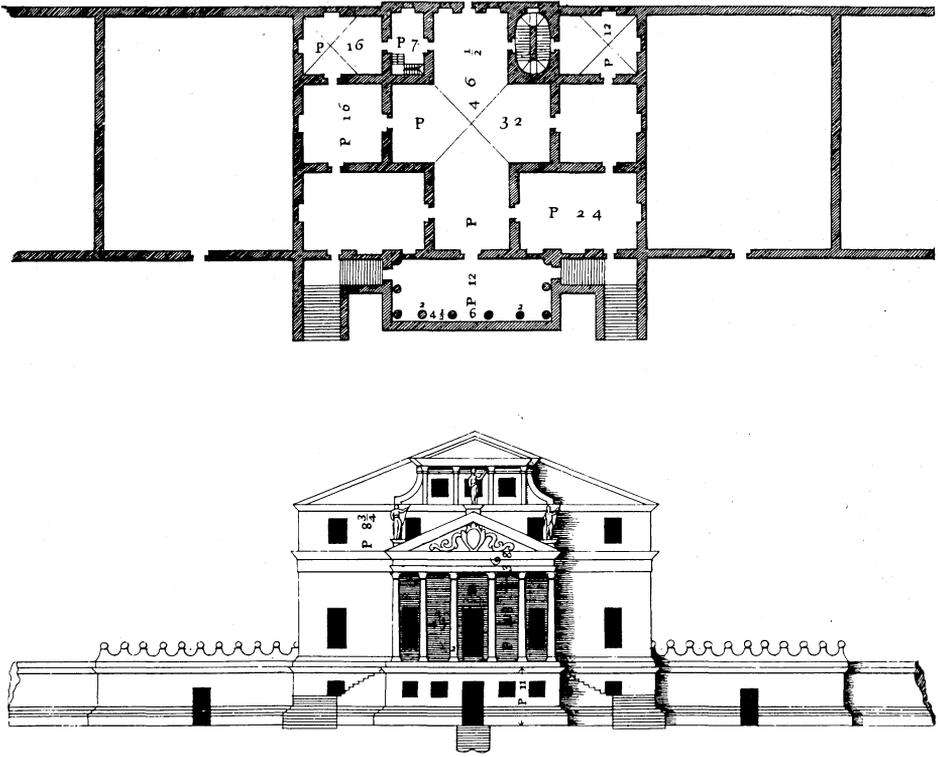
**Figure 1.** The Villa Malcontenta as drawn by Palladio in the *Quattro Libri*. The generation of villa facades is not considered in this paper. A parametric shape grammar that generates villa facades could be defined by use of the techniques developed here.

### Uniaxial villa plans

The plans of all but two of Palladio's villas (plates XIII and XLIII) in the *Quattro Libri* are laid out with respect to a single axis. These plans may be generated in eight stages which correspond more or less to a natural and intuitive design process. The stages are applied in this sequence:

(1) grid definition;
(2) exterior-wall definition;
(3) room layout;
(4) interior-wall realignment;
(5) principal entrances—porticos and exterior-wall inflections;
(6) exterior ornamentation—columns;
(7) windows and doors;
(8) termination.

*Preliminaries*

"This line is half the *Vicentine* foot, with which the following fabrics have been measured" (page 39).

The unit of measure employed by Palladio was the ancient foot of Vicenza. After allowances are made for inaccuracies and minor variations in drawing and construction, it appears that the standard ground-floor wall thickness employed in the villas was two of these feet. Rooms almost invariably have dimensions that are small integer multiples of the unit foot.

Based on these observations, uniaxial villa plans are defined in a two-dimensional Cartesian coordinate system with unit measure equal to one Vicentine foot, that is, equal to one half a standard base wall thickness. Plans are generated with respect to the north–south axis of this coordinate system. (The convention is followed that walls and dimensions parallel to the $x$-axis of the coordinate system are 'east–west'; walls and dimensions parallel to the $y$-axis are 'north–south'.) For easy reference, the axis is indicated in the initial shape, in the rules, and in the steps in the generation of villa plans by a broken straight line.

The initial shape from which all plans are generated by the sequential and recursive application of the rules specified below is the labelled shape $\langle s_\phi, \{(0, 0): A\}\rangle$ shown in figure 2. The initial shape requires that plans be rooted at the origin of the coordinate system.

(0, 0) ┆ $A$

**Figure 2.** The initial shape from which all villa plans are generated.

*Stage 1: grid definition*

"The rooms ought to be distributed on each side of the entry and hall; and it is to be observed, that those on the right correspond to those on the left, that so the fabrick may be the same in one place as in the other ..." (page 27).

The most conspicuous feature of Palladio's villa plans is their bilateral symmetry. These plans are constructed in terms of labelled rectangular 'tartan' grids with bilateral symmetry relative to the north–south axis of the coordinate system. Tartan grids are generated using the rules specified in figure 3. The grid required for the layout of the ground plan of the Villa Malcontenta is shown in figure 4.
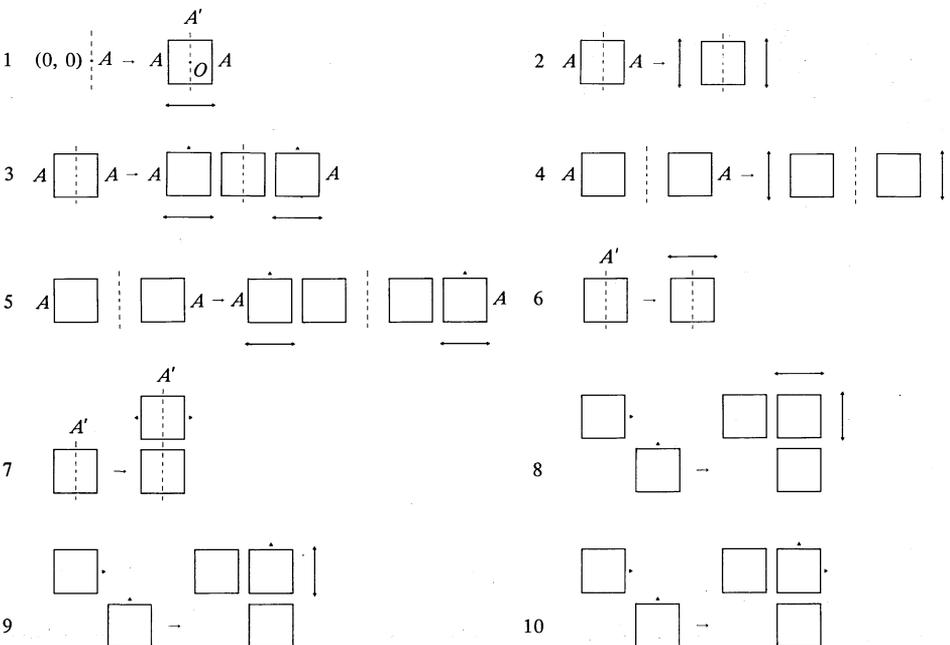


**Figure 3.** Rules for the generation of 'tartan' grids with bilateral symmetry. Grids generated by these rules are used to fix the underlying structure of villa plans.

Every grid generated by the rules in figure 3 consists of a $(2m+1) \times n$ array of variously dimensioned rectangles, where $m$ and $n$ are integers such that $m \geqslant 0$ and $n > 0$. Most of Palladio's villa plans are based on $5 \times 3$ grids. These grids correspond to Wittkower's (1952) "geometrical pattern" for Palladian villas. Some villa plans in the *Quattro Libri* require smaller grids (for example, plate XLVI); others require larger ones (for example, plate XLI).

In each grid, the central column of rectangles is bisected by the north–south axis of the coordinate system. Each rectangle to the left of the axis has a corresponding reflection to the right of the axis. Adjacent rectangles in the grid are separated by a fixed distance of one wall thickness; their parallel edges are the same length. The dimensions of these rectangles are determined by the assignments used to apply the rules that generate the grid. Usually these dimensions are small integer multiples of the unit measure. The line segments, with arrowheads at their end points, surrounding the grid are used to record the original dimensions of its component rectangles. These dimensions are used mainly from stage 5 onwards.

In order to ensure bilateral symmetry, to fix wall thickness at a constant value, and to incorporate proportioning rules simultaneously, it is necessary to associate parameters and parametric expressions with the grid generation rules and with the rules specified in the following stages. The parameterization of rules in this first stage is the most critical. Once a grid has been generated, it constrains all subsequent applications of rules in such a way that their associated variables are forced to assume the correct values.

Any consistent well-defined set of dimensioning and proportioning rules can be incorporated into this grammar by parameterizing its rules. The actual parameterization of the rules is straightforward but is not given here, as a complete discussion of Palladio's system of proportion (see Wittkower, 1952) would take us outside the constructive theme of this paper. The reader may wish to attempt the simple exercise of parameterizing the rules in accordance with the following precepts set forth by Palladio.

"The most beautiful and proportionable manner of rooms, and which succeed best, are seven, because they are either made round (tho' but seldom) or square, or their length will be the diagonal line of the square, or of a square and a third, or of one square and a half, or of one square and two thirds, or of two squares" (page 27).
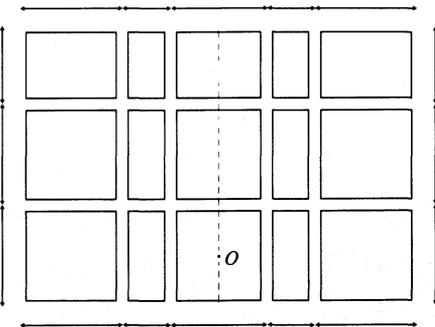


**Figure 4.** The underlying grid generated for the Villa Malcontenta.

*Stage 2: exterior-wall definition*
Once a grid is generated, it is circumscribed by a rectangle to form an exterior wall. This operation is performed by the rule specified in figure 5. The underlying wall pattern generated for the Villa Malcontenta by applying this rule to the grid in figure 4 is shown in figure 6. The walls in this drawing are hatched.
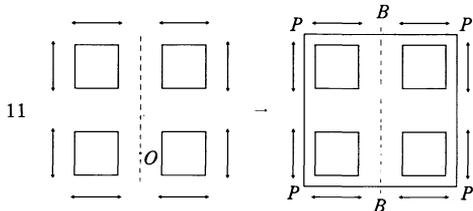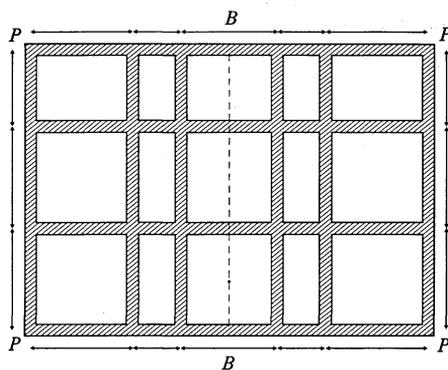
Figure 5. Rule for the generation of exterior walls.

Figure 6. The underlying wall pattern for the Villa Malcontenta.

*Stage 3: room layout*
The interior spaces in Palladio's uniaxial villa plans may be rectangular, I-shaped, T-shaped, or +-shaped. Rooms with these shapes are formed by recursively concatenating the spaces in the wall pattern generated for a villa, in accordance with the rules specified in figure 7. Applications of these rules preserve the symmetry of the plan. Notice that a plan can have at most one nonrectangular space and that this space must be bisected by the north–south axis of the coordinate system.



19    $\langle s_\phi, \{(0,0): B\}\rangle \rightarrow \langle s_\phi, \{(0,0): C\}\rangle$
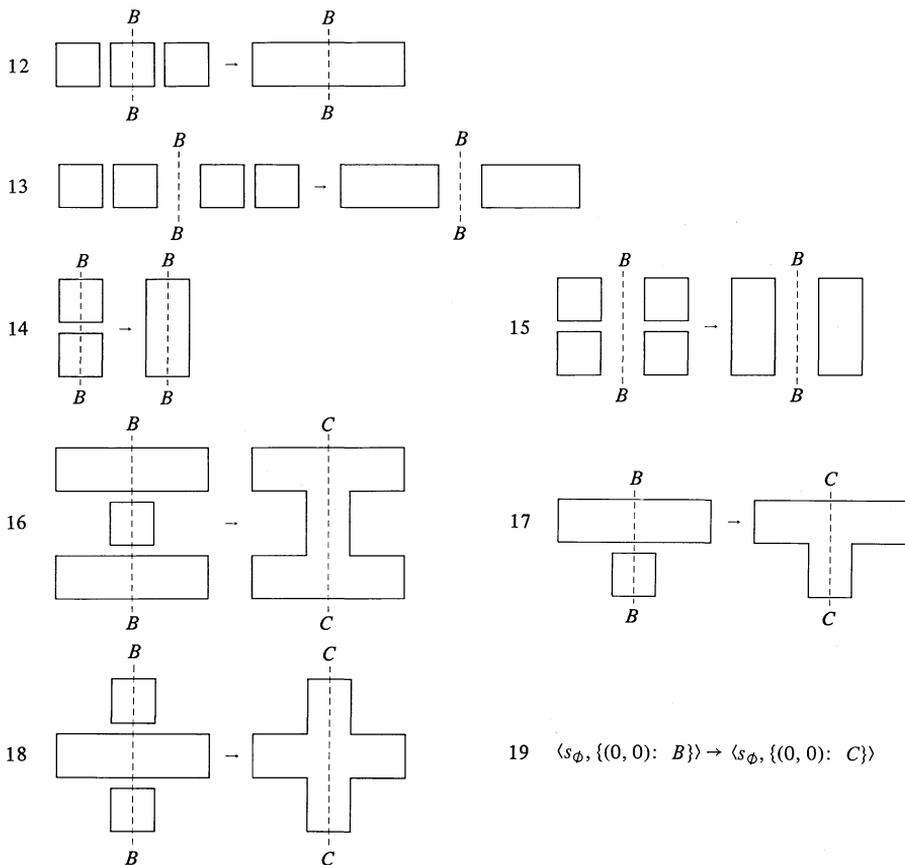
Figure 7. Rules for room layout.

The generation of the room layout for the Villa Malcontenta is given in figure 8. The first drawing in this figure is produced by applying rule 13 to the wall pattern in figure 6.
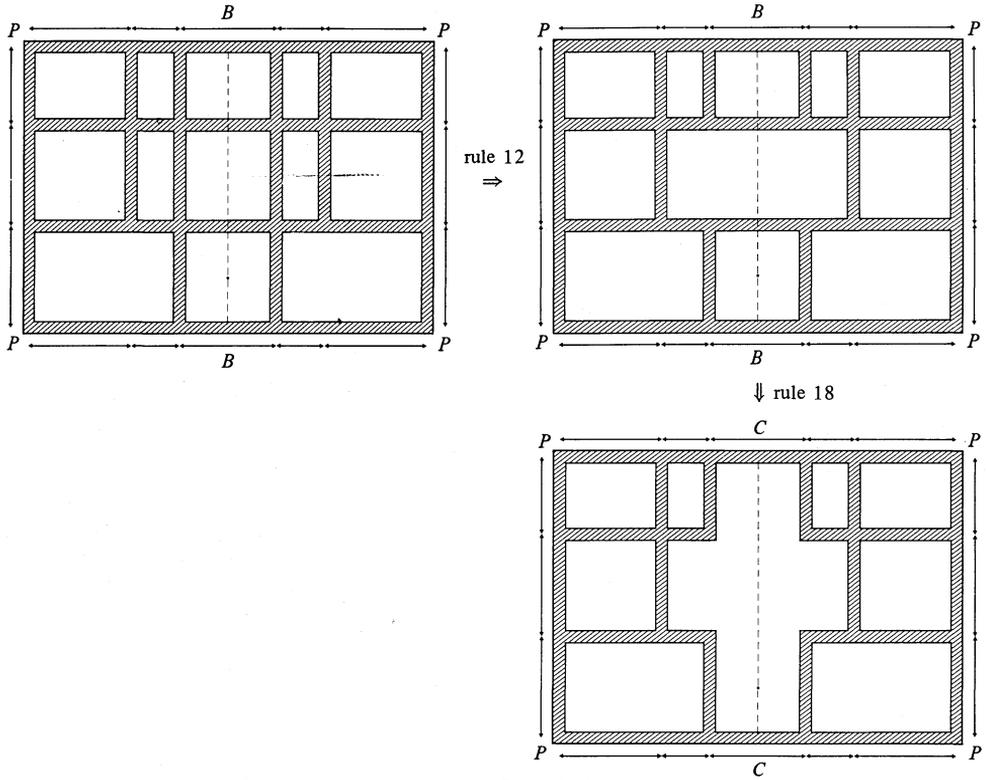


Figure 8. Generation of the room layout for the Villa Malcontenta.

*Stage 4: interior-wall realignment*
North–south and east–west shifts of internal walls away from the underlying grid are sometimes used by Palladio to make minor adjustments to room layouts (plates XLII and XLV). The rules specified in figure 9 produce these wall realignments in all
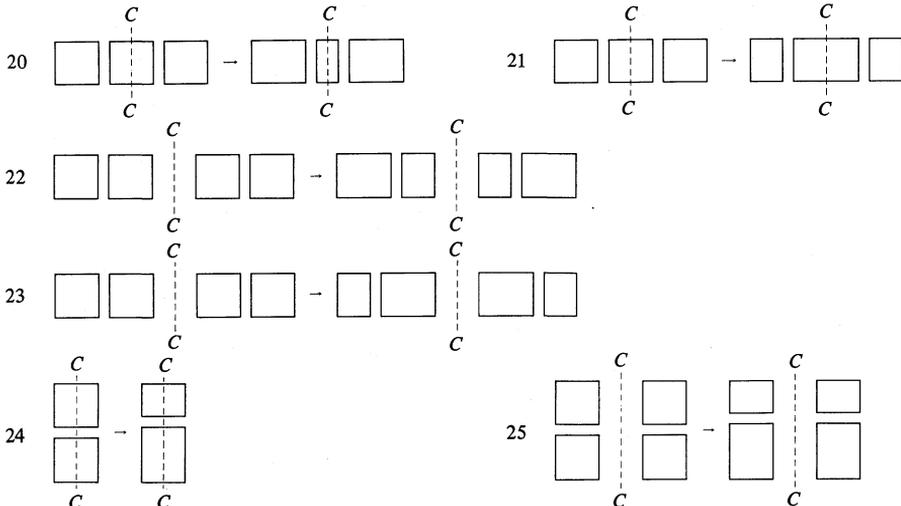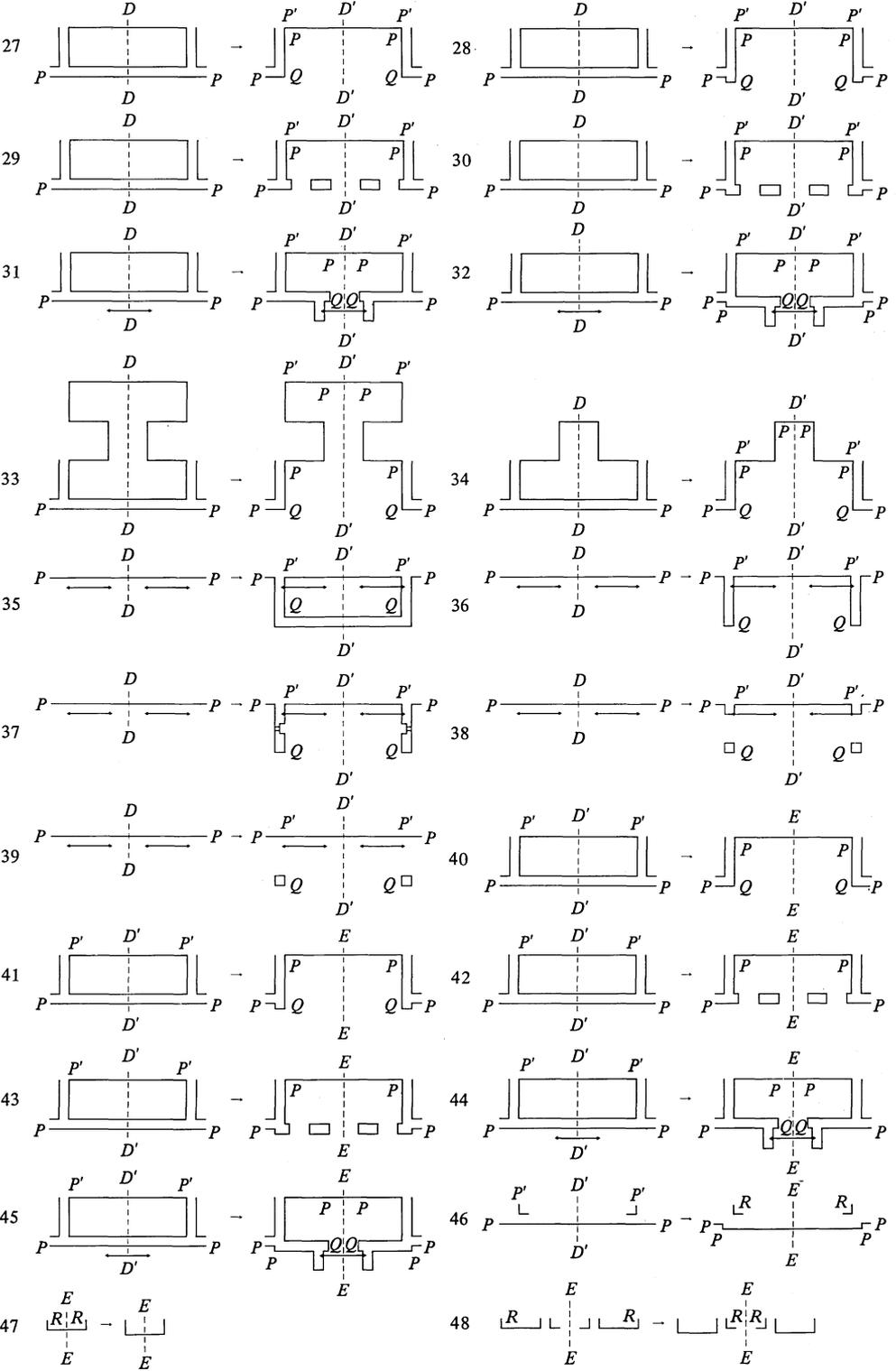


Figure 9. Rules for realigning interior walls.

26    $\langle s_\phi, \{(0, 0): \ C\}\rangle \rightarrow \langle s_\phi, \{(0, 0): \ D\}\rangle$



**Figure 10.** Rules for adding porticos and wall inflections to the principal entrances of a villa.

cases; they apply to room layouts generated in stage 3. The application of these rules preserves the symmetry of the plan. Wall realignment is an optional operation; it is used very infrequently.

*Stage 5: principal entrances*
The rules given in figure 10 specify the handling of the principal entrances in a Palladian villa. These entrances are located on the plan's axis of symmetry. One is
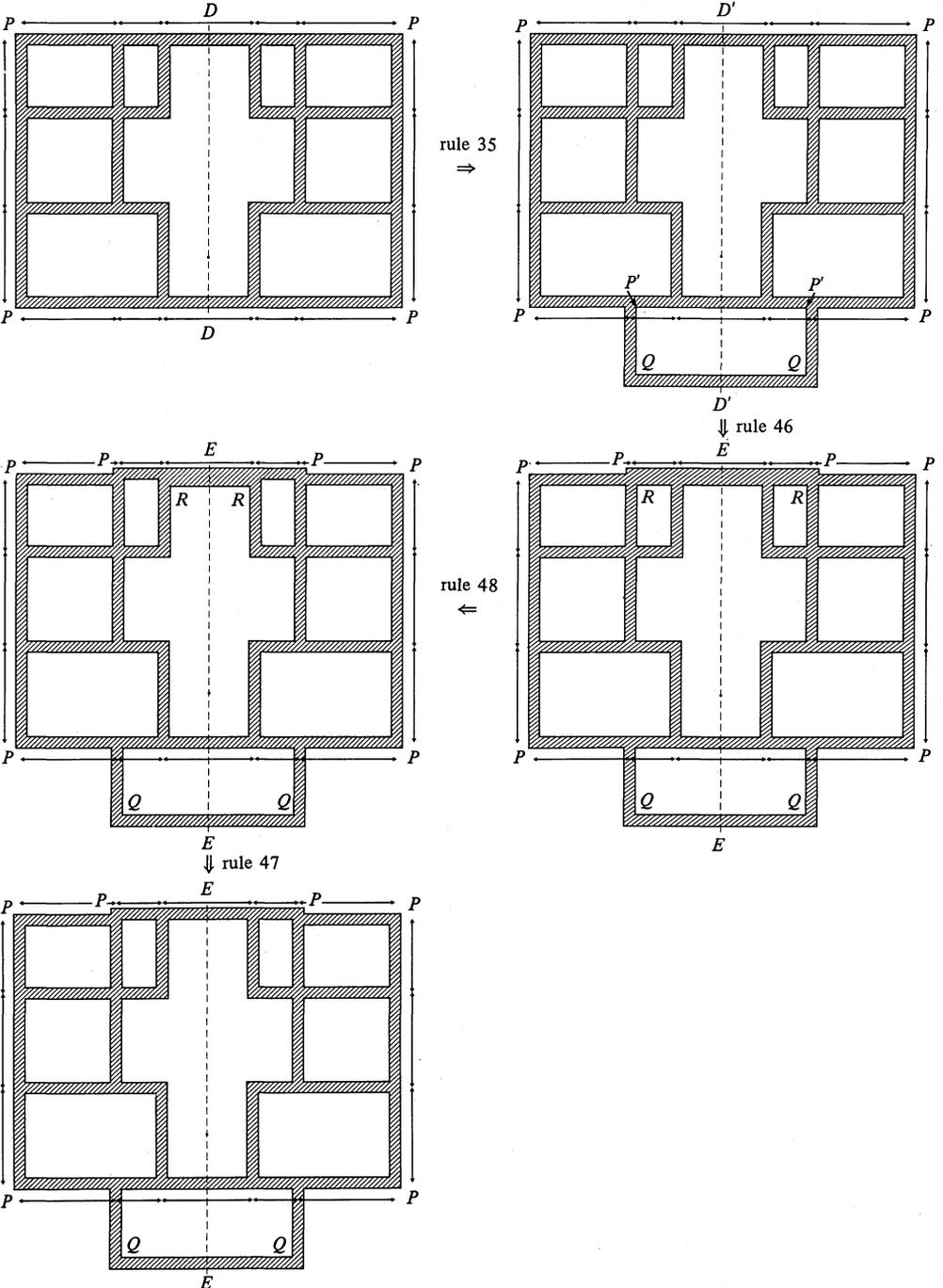


**Figure 11.** Generation of the portico and wall inflection for the Villa Malcontenta.

distinguished by a portico, *in antis* (rules 27–34) or *prostyle* (rules 35–39); the other is distinguished by a portico *in antis* (rules 40–45), an exterior-wall inflection of one half a wall thickness (rule 46), or not at all (rules 49 and 50). For a portico *in antis*, those parts of the interior east–west wall directly open to the outside are considered exterior walls. Rooms adjacent to a wall inflection are extended correspondingly (rules 47 and 48). When a villa plan contains two porticos or a portico and a wall inflection, their east–west dimensions are the same. Notice that the line segments with arrowheads at their end points determine the east–west dimensions of some of these additions in terms of one or more of the original rectangular elements in the underlying grid for the plan (rules 31 and 32, 35–39, and 44 and 45). The addition of a portico and a wall inflection to the plan of the Villa Malcontenta is shown in figure 11. The first drawing in figure 11 is produced by applying rule 26 to the last drawing in figure 8.

*Stage 6: exterior ornamentation*

"Having thus treated of plain walls, we shall next consider their ornaments; among which none are more considerable than columns, when they are properly placed, and in a just proportion to the whole edifice" (page 11).

The rules given in figure 12 specify the different ways columns can be added to finish porticos. Each of these rules may be considered a column schema which applies under certain 'lexical' conditions. The use of different arrangements of columns is similar to the use of different words in a fixed syntactic context. Column insertion for the Villa Malcontenta is shown in figure 13. This drawing is produced from the last drawing in figure 11 by applying rule 56.
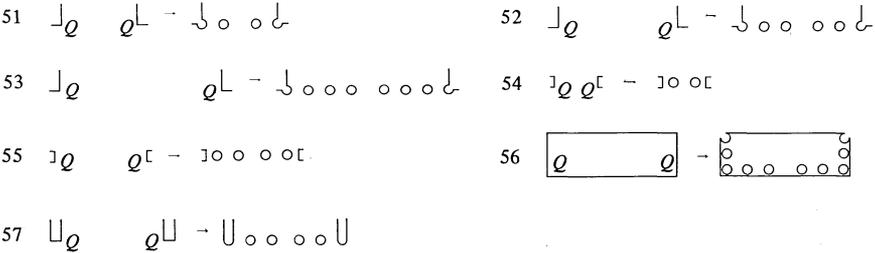


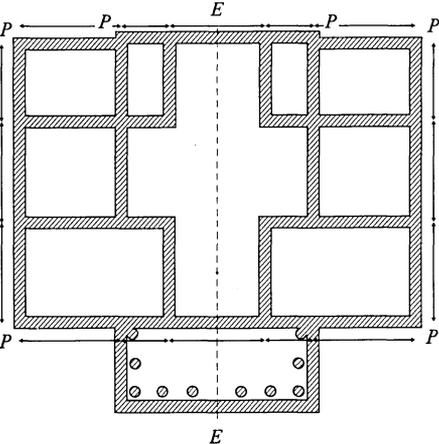Figure 12. Rules for finishing porticos by the addition of columns.



Figure 13. Addition of columns for the Villa Malcontenta.

## Stage 7: windows and doors

"The windows on the right hand ought to correspond to those on the left, and those above directly over them that are below; and the doors likewise ought to be directly over one another, that the void may be over the void, and the solid upon the solid, and all face one another, so that standing at one end of the house one may see to the other, which affords both beauty and cool air in summer, besides other conveniences" (page 31).

Rules 58 and 59 specified in figure 14 fix the conditions under which windows can be inserted in a villa plan. These windows occur on both sides of but not on the plan's axis of symmetry. One such opening is inserted in each nonaxial exterior-wall segment corresponding to a north–south or east–west dimension of one of the original rectangles in the underlying grid for the plan. Exterior walls are distinguished by occurrences of the label $P$; their component parts are determined by the line segments, with arrowheads at their end points, surrounding the plan. Typically windows are centered in the wall segments to which they are added. Windows in parallel wall segments are inserted together and linked via a line segment distinguished by arrowheads at its end points. This line segment is used to locate nonaxial interior doors as will be indicated presently. The application of rules for window insertion preserves the symmetry of the plan. Window insertion is obligatory for each nonaxial external wall segment in the plan and once performed is indicated by locating the symbol ● at the midpoint of the line segment, with arrowheads at its end points, which determines this element.

Rules 60 and 61 in figure 14 allow for nonaxial interior doors to be located in a plan symmetrically so that they line up with previously inserted windows and with each other. A door can be placed in a wall just in the case when it is cut by a line segment linking windows. In this way the characteristic *enfilade* is produced.

Rules 62–65 in figure 14 provide for exterior doors to be located on the plan's axis of symmetry. These doors may have ancillary windows associated with them. The remaining rule in figure 14 determines the location of interior doors on this axis.
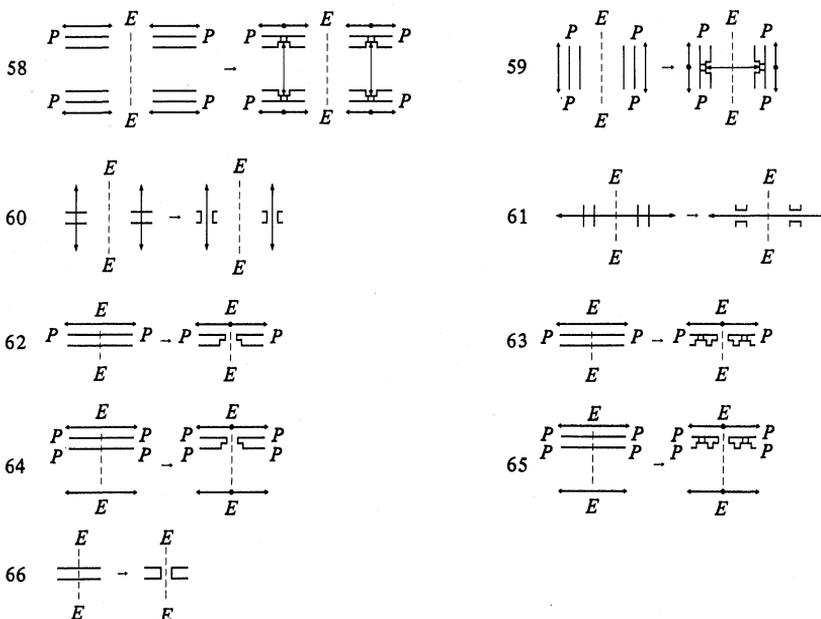


Figure 14. Rules for locating windows and doors in villa plans.

Because of the bilateral symmetry of a villa plan, the application of these rules produces an odd number of openings in each of its east–west elevations. This result would have pleased Alberti who declares in his *De Re Aedificatoria* (originally published in 1485; for a modern translation see Alberti et al, 1955) that the supports of ancient buildings were generally even in number, as animals support themselves on
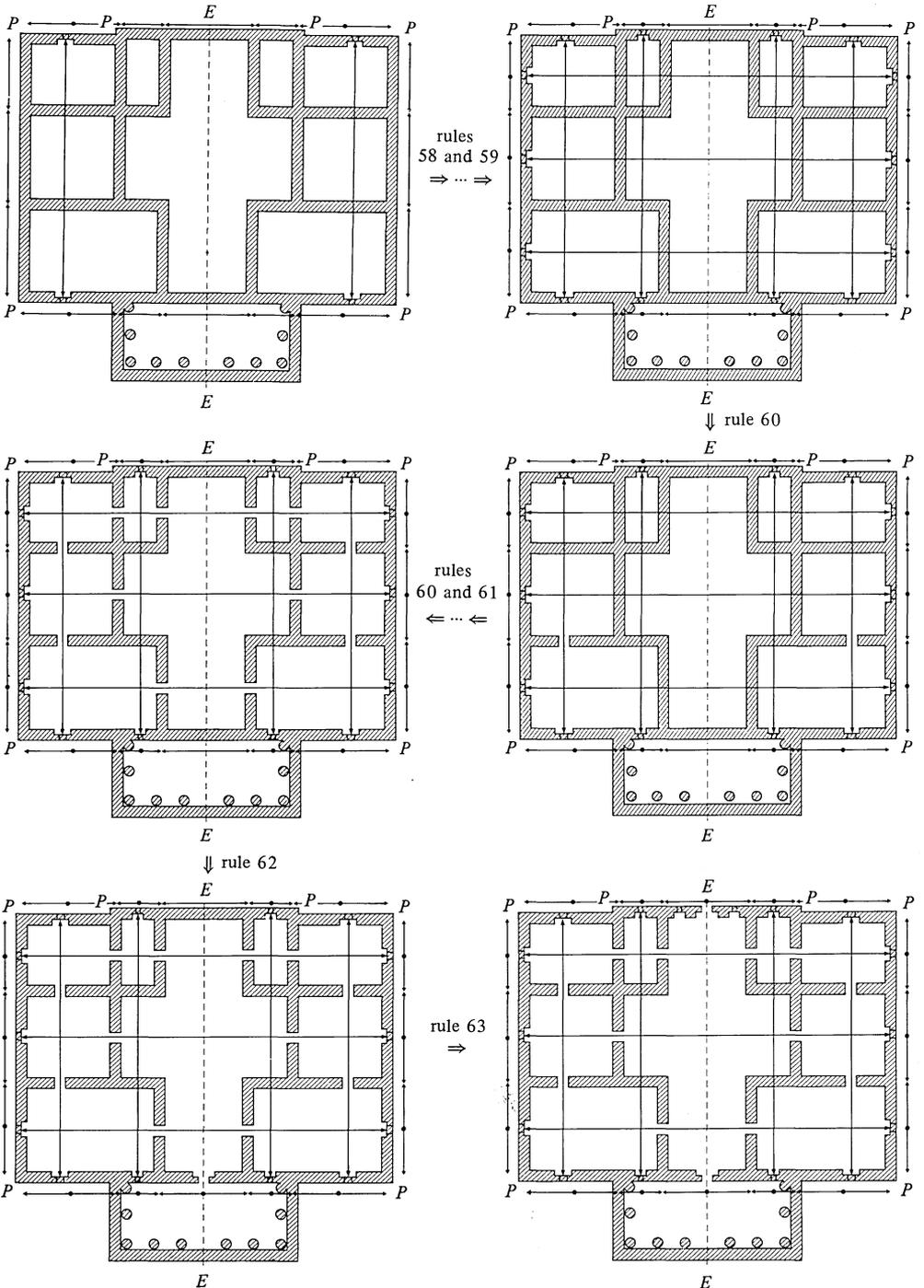


**Figure 15.** Generation of the windows and doors in the Villa Malcontenta.

an even number of feet; but the apertures were generally odd in number, in the same way that the mouth is a single opening in the face with eyes disposed on either side.

Window and door insertion in the plan for the Villa Malcontenta is shown in figure 15. The first drawing in this figure is obtained from the drawing in figure 13 by applying rule 58.

*Stage 8: termination*

Termination rules are specified in figure 16. For the most part, these rules provide for the erasing of the labels and labelled line segments (line segments with arrowheads at their end points and possibly the symbol ● at their midpoints) used to guide the plan generation process. Notice that the labelled line segments, with arrowheads at their end points, which surround a plan, can be erased if and only if there is an interior-wall segment associated with them (rules 68 and 69). These rules ensure that some part of each wall in the underlying wall pattern generated for a villa (stages 1 and 2) remains in the final plan. For this condition to be satisfied, the grid for the plan must be minimal. Only plans which have been correctly generated in the previous seven stages can have all of their labels erased. These unlabelled plans are in the language of villa plans defined by the parametric shape grammar given here. The final plan generated for the Villa Malcontenta is shown in figure 17.
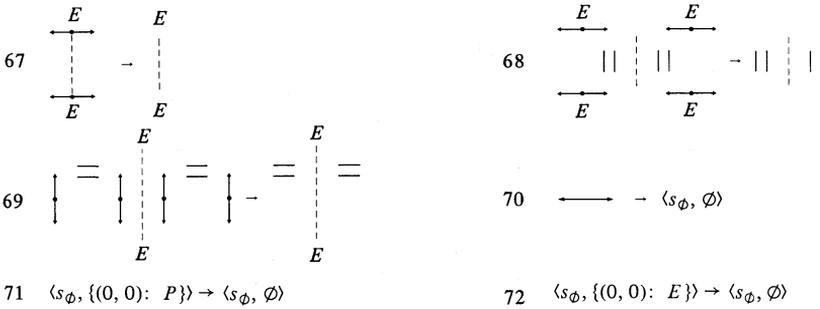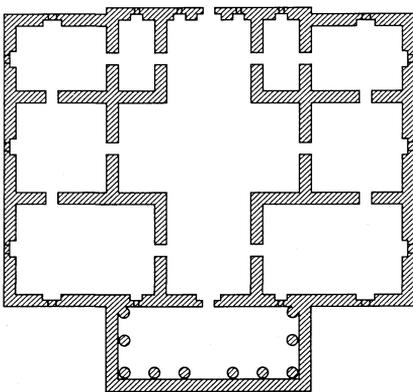


Figure 16. Termination rules.



Figure 17. The final plan generated for the Villa Malcontenta.

## Biaxial villa plans

A parametric shape grammar that generates Palladio's biaxial villa plans, of which the Villa Rotonda (plate XIII) is the best known, can be defined by extending the rules given for the uniaxial case to two axes of symmetry. These extensions are formed by locating an east–west axis, in each uniaxial rule, orthogonal to the original north–south one and then reflecting the rule with respect to this new axis. By following

this procedure for all distinct east–west axis locations, the main biaxial rules are specified. In addition to these rules, rules for central circular spaces (stage 3) are also required. The definition of these rules is straightforward: square spaces with their centres at the intersection of the two axes of symmetry are replaced by circular ones with their centres also at this point and their radii determined by the dimensions of the squares.

## Discussion

When several buildings each create a similar impression, they are said to exemplify a particular architectural *style*. Given a finite corpus of buildings that are perceived to be alike in some sense, the problem of style consists of characterizing the basis for this likeness. Ideally this characterization has three main purposes: (1) it should clarify the underlying commonality of structure and appearance manifest for the buildings in the corpus; (2) it should supply the conventions and criteria necessary to determine whether any other building not in the original corpus is an instance of the style; and (3) it should provide the compositional machinery needed to design new buildings that are instances of the style. If the characterization of a particular architectural style is to have any explanatory or predictive value, it must satisfy these descriptive, analytic, and synthetic tests of adequacy.

The parametric shape grammar given for the generation of Palladio's villa ground plans may be considered a partial definition of the Palladian style. (A more complete definition would require a grammar for facade generation and more detailed treatment of Palladio's system of proportion.) Clearly this generative characterization satisfies the preceding criteria: the rules of the grammar elucidate the structure and appearance of villa plans; any villa can be determined to be an instance of the style by simply determining whether there is a sequence of rule applications that generates its plan; and plans of villas not already existing can be generated by the grammar.

It should be noted that the grammar given here is arbitrary in certain respects. The corpus of buildings to be described might have been selected in a different way. We might, for example, have restricted it to Palladio's executed villa designs, or we might have attempted to broaden it to include all of the villa projects which are extant in Palladio's drawings. A change in definition of the corpus might result in changes in the grammar. But it seems clear that Palladio intended the corpus given in the *Quattro Libri* as a canonical presentation of the essential features of his villa architecture, so we feel justified in adopting it.

The drawings in the *Quattro Libri* contain numerous small errors and inconsistencies, and the constructed buildings often deviate in minor ways from the drawings. Thus there is often a certain arbitrariness in what we take to be the intended design. Furthermore the definition of what constitutes a trivial or accidental variation in form, which need not be accounted for by the grammar, is also arbitrary to some extent. We have simply exercised our best judgement on these matters. Different decisions would not affect the grammar in any fundamental way.

The definition of the Palladian style by use of the parametric shape grammar specified here allows other issues and questions of aesthetic and historical interest to be investigated. For example, the grammar provides the basis for classifying villas in terms of the properties of the sequences of rules applied to generate their plans and even to count the number of possible villas of a certain type. Indeed a simple combinatorial analysis of possible sequences of rule applications would allow for the computation of the number of all possible villas that have an underlying grid size of say $5 \times 3$. The grammar could also be used to distinguish those stylistic features, in neo-Palladian movements, that are canonical in the Palladian sense from those that diverge from Palladio's standard architectural usage. Using the grammar explicated

here, architectural historians could ask and find answers to questions such as: how deep (or superficial) was Lord Burlington's appreciation of his mentor's system of design? Finally, the parametric shape grammar for Palladio's villa ground plans provides the necessary generative machinery for the definition of an aesthetic system based on the constructive mode of understanding (Stiny and Gips, 1978). By use of this aesthetic system, a detailed critical account of villas could be given in terms of algorithmically based interpretative conventions and evaluative criteria corresponding to Palladio's own.

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