Appendix A Proofs of Results

Proof of Theorem 1: To prove Theorem 1, we first prove the following statement: Let $f(x)$ and $g(x)$ be two unimodal continuous and differentiable functions on a closed interval $[a,b]$, and x_f^* and x_g^* be the x values that maximize $f(x)$ and $g(x)$, respectively. If $f'(x) \geq g'(x)$ for all x in the domain, then $x_f^* \geq x_g^*$.

Because $f'(x) \geq g'(x)$, we have $f'(x_g^*) \geq g'(x_g^*)$. It means that the value of $f(x)$ can increase by choosing the value of x above x_g^*. Thus, there exists a better feasible solution for optimizing $f(x)$, and that feasible solution is higher than x_g^*. Since $f(x)$ is a unimodal function, we have $x_f^* \geq x_g^*$.

We now proceed to proving the main result in Theorem 1. The derivative of the two profit functions are:

$$\pi'_{\mathrm{ET}}(s) = \frac{\alpha_p\gamma_r}{(1-\gamma_p s)^2} + \frac{\alpha_r\gamma_r}{(1-\gamma_r s)^2} + \frac{\beta_p\gamma_p}{(1-\gamma_p s)^2} - \frac{\beta_p\gamma_r}{(1-\gamma_p s)^2} - \tau'(s), \quad s \in [0, \frac{1}{2\gamma_r}];$$
$$\pi'_{\mathrm{EO}}(s) = \frac{\alpha_p\gamma_r}{(1-\gamma_r s)^2} + \frac{\alpha_r\gamma_r}{(1-\gamma_r s)^2} + \frac{\beta_p\gamma_p}{(1-\gamma_p s)^2} - \frac{\beta_p\gamma_r}{(1-\gamma_r s)^2} - \tau'(s), \quad s \in [0, \frac{1}{2\gamma_r}].$$

It is easy to verify that for $\tau(s) = \frac{ks}{1-\gamma_r s}$, $\pi_{\mathrm{ET}}(s)$ and $\pi_{\mathrm{EO}}(s)$ are both unimodal functions. Using above equations, $\forall s \in [0, \frac{1}{2\gamma_r}]$ we have

$$\begin{aligned}\pi'_{\mathrm{ET}}(s) - \pi'_{\mathrm{EO}}(s) &= \Big(\frac{\alpha_p\gamma_r}{(1-\gamma_p s)^2} - \frac{\beta_p\gamma_r}{(1-\gamma_p s)^2}\Big) - \Big(\frac{\alpha_p\gamma_r}{(1-\gamma_r s)^2} - \frac{\beta_p\gamma_r}{(1-\gamma_r s)^2}\Big),\\ &= \gamma_r(\beta_p - \alpha_p)\Big(\frac{1}{(1-\gamma_r s)^2} - \frac{1}{(1-\gamma_p s)^2}\Big) \geq 0.\end{aligned}$$

Using the statement proven in the beginning of this proof, we have $s^{\mathrm{ET}} \geq s^{\mathrm{EO}}$. ∎

Proof of Proposition 2: For any given $s \in [0, \frac{1}{2\gamma_r}]$, we have

$$\begin{aligned}\pi_{\mathrm{ET}}(s) - \pi_{\mathrm{EO}}(s) &= \Big(\frac{\alpha_p\gamma_r s}{1-\gamma_p s} - \frac{\beta_p\gamma_r s}{1-\gamma_p s}\Big) - \Big(\frac{\alpha_p\gamma_r s}{1-\gamma_r s} - \frac{\beta_p\gamma_r}{1-\gamma_r s}\Big)\\ &= \gamma_r(\beta_p - \alpha_p)\Big(\frac{s}{1-\gamma_r s} - \frac{s}{1-\gamma_p s}\Big) \geq 0.\end{aligned}$$

Therefore,

$$\pi_{\mathrm{ET}}(s^{\mathrm{EO}}) \geq \pi_{\mathrm{EO}}(s^{\mathrm{EO}}).$$

Since s^{ET} is the optimal amount of learning effort under ET,

$$\pi_{\mathrm{ET}}(s^{\mathrm{ET}}) \geq \pi_{\mathrm{ET}}(s^{\mathrm{EO}}).$$

By transitivity,

$$\pi_{\mathrm{ET}}(s^{\mathrm{ET}}) \geq \pi_{\mathrm{EO}}(s^{\mathrm{EO}}),$$

i.e., $\pi^*_{\mathrm{ET}} \geq \pi^*_{\mathrm{EO}}$. ∎

Proof of Proposition 3:

$$\phi_r^{ET} = \frac{1-c_r^{\mathrm{ET}}}{1-\gamma_r s^{\mathrm{ET}}} = \frac{\gamma_r s^{\mathrm{ET}}}{1-\gamma_r s^{\mathrm{ET}}},$$
$$\phi_r^{EO} = \frac{1-c_r^{\mathrm{EO}}}{1-\gamma_r s^{\mathrm{EO}}} = \frac{\gamma_r s^{\mathrm{EO}}}{1-\gamma_r s^{\mathrm{EO}}}.$$

Theorem 1 shows

$$s^{\mathrm{ET}} \geq s^{\mathrm{EO}},$$

therefore,

$$\frac{\gamma_r s^{\mathrm{ET}}}{1-\gamma_r s^{\mathrm{ET}}} \geq \frac{\gamma_r s^{\mathrm{EO}}}{1-\gamma_r s^{\mathrm{EO}}},$$

i.e, $\phi_r^{ET} \geq \phi_r^{EO}$. ■

Proof of Theorem 2: First Order Condition gives us:

$$\pi'_{\mathrm{ET}}(s^{\mathrm{ET}}) = \frac{\alpha_p\gamma_r}{(1-\gamma_p s^{\mathrm{ET}})^2} + \frac{\alpha_r\gamma_r}{(1-\gamma_r s^{\mathrm{ET}})^2} + \frac{\beta_p\gamma_p}{(1-\gamma_p s^{\mathrm{ET}})^2} - \frac{\beta_p\gamma_r}{(1-\gamma_p s^{\mathrm{ET}})^2} - \frac{k}{(1-\gamma_r s^{\mathrm{ET}})^2} = 0; \quad (47)$$
$$\pi'_{\mathrm{EO}}(s^{\mathrm{EO}}) = \frac{\alpha_p\gamma_r}{(1-\gamma_r s^{\mathrm{EO}})^2} + \frac{\alpha_r\gamma_r}{(1-\gamma_r s^{\mathrm{EO}})^2} + \frac{\beta_p\gamma_p}{(1-\gamma_p s^{\mathrm{EO}})^2} - \frac{\beta_p\gamma_r}{(1-\gamma_r s^{\mathrm{EO}})^2} - \frac{k}{(1-\gamma_r s^{\mathrm{EO}})^2} = 0. \quad (48)$$

Multiply (47) by $(1-\gamma_r s^{\mathrm{ET}})^2$ and arrange the equation:

$$(\frac{1-\gamma_r s^{\mathrm{ET}}}{1-\gamma_p s^{\mathrm{ET}}})^2 \cdot (\alpha_p\gamma_r + \beta_p\gamma_p - \beta_p\gamma_r) = k - \alpha_r\gamma_r \quad (49)$$

Note that

$$\frac{1-\gamma_r s^{\mathrm{ET}}}{1-\gamma_p s^{\mathrm{ET}}} = 1 - (\gamma_r - \gamma_p)\frac{s^{\mathrm{ET}}}{1-\gamma_p s^{\mathrm{ET}}}, \quad (50)$$

Substituting (50) into (49), with some algebra we have

$$\phi_p^{\mathrm{ET}} \equiv \frac{\gamma_r s^{\mathrm{ET}}}{1-\gamma_p s^{\mathrm{ET}}} = \frac{\gamma_r}{\gamma_r - \gamma_p}[1 - (\frac{k - \alpha_r\gamma_r}{\alpha_p\gamma_r + \beta_p\gamma_p - \beta_p\gamma_r})^{\frac{1}{2}}] \quad (51)$$

Similarly, from (48) we derive the expression for the coverage rate under equal opportunity:

$$\phi_p^{\mathrm{EO}} \equiv \frac{\gamma_r s^{\mathrm{EO}}}{1-\gamma_r s^{\mathrm{EO}}} = \frac{\gamma_r}{\gamma_r - \gamma_p}[(\frac{\beta_p\gamma_p}{k - \alpha_r\gamma_r + \beta_p\gamma_r - \alpha_p\gamma_r})^{\frac{1}{2}} - 1] \quad (52)$$

Therefore, we have:

$$\phi_p^{\mathrm{ET}} - \phi_p^{\mathrm{EO}} = \frac{\gamma_r}{\gamma_r - \gamma_p}[2 - (\frac{k - \alpha_r\gamma_r}{\alpha_p\gamma_r + \beta_p\gamma_p - \beta_p\gamma_r})^{\frac{1}{2}} - (\frac{\beta_p\gamma_p}{k - \alpha_r\gamma_r + \beta_p\gamma_r - \alpha_p\gamma_r})^{\frac{1}{2}}] \quad (53)$$
$$= \frac{1}{\gamma_r - \gamma_p}[2 - \sigma - (\frac{\beta_p\gamma_p}{\sigma^2\beta_p\gamma_p + (1-\sigma^2)(\beta_p\gamma_r - \alpha_p\gamma_r)})^{\frac{1}{2}}] \quad (54)$$

When $\frac{\alpha_p}{\beta_p} \le 1 - \frac{1-(2-\sigma)^2)\sigma^2}{(1-\sigma^2)(2-\sigma)^2} \cdot \frac{\gamma_p}{\gamma_r}$, we have

$$(\beta_p - \alpha_p)\gamma_r \ge \frac{1-(2-\sigma)^2)\sigma^2}{(1-\sigma^2)(2-\sigma)^2} \cdot \beta_p\gamma_p, \tag{55}$$

$$[1-(2-\sigma)^2\sigma^2]\beta_p\gamma_p \le (1-\sigma^2)(2-\sigma)^2(\beta_p\gamma_r - \alpha_p\gamma_r), \tag{56}$$

$$\beta_p\gamma_p \le (2-\sigma)^2\sigma^2\beta_p\gamma_p + (2-\sigma)^2(1-\sigma^2)(\beta_p\gamma_r - \alpha_p\gamma_r), \tag{57}$$

$$\frac{\beta_p\gamma_p}{\sigma^2\beta_p\gamma_p + (1-\sigma^2)(\beta_p\gamma_r - \alpha_p\gamma_r)} \le (2-\sigma)^2, \tag{58}$$

therefore,

$$(\frac{\beta_p\gamma_p}{\sigma^2\beta_p\gamma_p + (1-\sigma^2)(\beta_p\gamma_r - \alpha_p\gamma_r)})^{\frac{1}{2}} \le 2-\sigma, \tag{59}$$

which implies that $\phi_p^{\mathrm{ET}} > \phi_p^{\mathrm{EO}}$, since $\gamma_r - \gamma_p > 0$. ■

Proof of Theorem 3: Let NS_p^{ET} and NF_p^{ET} be the number of successful acceptance and the number of failed acceptance in the protected group under equal treatment, respectively. Then,

$$NS_p^{\mathrm{ET}} = \frac{1-c^{\mathrm{ET}}}{1-\gamma_p s^{\mathrm{ET}}}(1-d_p) = \frac{\gamma_r s^{\mathrm{ET}}}{1-\gamma_p s^{\mathrm{ET}}}(1-d_p);$$
$$NF_p^{\mathrm{ET}} = \frac{1-\gamma_p s^{\mathrm{ET}} - c^{\mathrm{ET}}}{1-\gamma_p s^{\mathrm{ET}}}d_p = \frac{(\gamma_r - \gamma_p)s^{\mathrm{ET}}}{1-\gamma_p s^{\mathrm{ET}}}d.$$

Therefore, we have

$$\delta_p^{\mathrm{ET}} = \frac{NS_p^{\mathrm{ET}}}{NS_p^{\mathrm{ET}} + NF_p^{\mathrm{ET}}} = \frac{(1-d_p)\gamma_r}{\gamma_r - \gamma_p d_p}.$$

Similarly, under equal opportunity, we have

$$NS_p^{\mathrm{EO}} = \frac{1-c_p^{\mathrm{EO}}}{1-\gamma_p s^{\mathrm{EO}}}(1-d_p) = \frac{\gamma_r s^{\mathrm{EO}}}{1-\gamma_r s^{\mathrm{EO}}}(1-d_p)$$
$$NF_p^{\mathrm{EO}} = \frac{1-\gamma_p s^{\mathrm{EO}} - c_p^{\mathrm{EO}}}{1-\gamma_p s^{\mathrm{EO}}}d_p = (\frac{\gamma_r s^{\mathrm{EO}}}{1-\gamma_r s^{\mathrm{EO}}} - \frac{\gamma_p s^{\mathrm{EO}}}{1-\gamma_p s^{\mathrm{EO}}}) \cdot d_p$$

Thus,

$$\delta_p^{\mathrm{EO}} = \frac{NS_p^{\mathrm{EO}}}{NS_p^{\mathrm{ET}} + NF_p^{\mathrm{ET}}} = \frac{(1-d_p)\gamma_r}{\gamma_r - \frac{1-\gamma_r s^{\mathrm{EO}}}{1-\gamma_p s^{\mathrm{EO}}} \cdot \gamma_p d_p} < \frac{(1-d_p)\gamma_r}{\gamma_r - \gamma_p d_p} = \delta_p^{\mathrm{ET}}.$$

■

Appendix B Proofs of the Results in the Two Other Cases

B.1 Medium Expected Loss

In this section, we show that our results hold for the case of medium expected loss, i.e.,

$$\alpha_p + \alpha_r < \beta_p \le \alpha_p + \frac{1-\gamma_p s}{1-\gamma_r s}\alpha_r.$$

From the analysis in section 3.3, we know that in this case

$$c_p^{\mathrm{ET}} = c_r^{\mathrm{ET}} = 1 - \gamma_r s, \qquad \pi_{\mathrm{ET}}(s) = \frac{\alpha_p \gamma_r s}{1-\gamma_p s} + \frac{\alpha_r \gamma_r s}{1-\gamma_r s} + \frac{\beta_p \gamma_p s}{1-\gamma_p s} - \frac{\beta_p \gamma_r s}{1-\gamma_p s} - \tau(s)$$
$$c_p^{\mathrm{EO}} = 1 - \gamma_p s, c_r^{\mathrm{EO}} = 1 - \frac{1-\gamma_r s}{1-\gamma_p s}\gamma_p s, \qquad \pi_{\mathrm{EO}}(s) = \frac{\alpha_p \gamma_p s}{1-\gamma_p s} + \frac{\alpha_r \gamma_p s}{1-\gamma_p s} - \tau(s);$$

Learning

The derivative of the two profit functions are:

$$\pi'_{\mathrm{ET}}(s) = \frac{\alpha_p \gamma_r}{(1-\gamma_p s)^2} + \frac{\alpha_r \gamma_r}{(1-\gamma_r s)^2} + \frac{\beta_p \gamma_p}{(1-\gamma_p s)^2} - \frac{\beta_p \gamma_r}{(1-\gamma_p s)^2} - \tau'(s)$$
$$\pi'_{\mathrm{EO}}(s) = \frac{\alpha_p \gamma_p}{(1-\gamma_p s)^2} + \frac{\alpha_r \gamma_p}{(1-\gamma_p s)^2} - \tau'(s);$$

Therefore,

$$\begin{aligned}\pi'_{\mathrm{ET}}(s) - \pi'_{\mathrm{EO}}(s) &= \frac{\alpha_p \gamma_r}{(1-\gamma_p s)^2} + \frac{\alpha_r \gamma_r}{(1-\gamma_r s)^2} + \frac{\beta_p \gamma_p}{(1-\gamma_p s)^2} - \frac{\beta_p \gamma_r}{(1-\gamma_p s)^2} - \frac{\alpha_p \gamma_p}{(1-\gamma_p s)^2} - \frac{\alpha_r \gamma_p}{(1-\gamma_p s)^2} \\ &= \frac{(\beta_p - \alpha_p - \alpha_r)\gamma_p}{(1-\gamma_p s)^2} + [\frac{\alpha_r}{(1-\gamma_r s)^2} - \frac{\beta_p}{(1-\gamma_p s)^2}]\gamma_r\end{aligned}$$

As

$$\beta_p \leq \alpha_p + \frac{1-\gamma_p s}{1-\gamma_r s}\alpha_r,$$

we have

$$\beta_p < \frac{1-\gamma_p s}{1-\gamma_r s}\alpha_r < (\frac{1-\gamma_p s}{1-\gamma_r s})^2 \alpha_r,$$

thus,

$$\frac{\beta_p}{(1-\gamma_p s)^2} < \frac{\alpha_r}{(1-\gamma_r s)^2}.$$

Also

$$\beta_p > \alpha_p + \alpha_r.$$

Therefore,

$$\pi'_{\mathrm{ET}}(s) - \pi'_{\mathrm{EO}}(s) > 0.$$

By the statement shown in the proof of Theorem 1, we have $s^{\mathrm{ET}} \geq s^{\mathrm{EO}}$. ■

Impact on the firm

For any given $s \in [0, \frac{1}{2\gamma_r}]$, we have

$$\pi_{\mathrm{ET}}(s) - \pi_{\mathrm{EO}}(s) = \frac{(\beta_p - \alpha_p - \alpha_r)\gamma_p s}{1-\gamma_p s} + [\frac{\alpha_r}{1-\gamma_r s} - \frac{\beta_p}{1-\gamma_p s}]\gamma_r s \geq 0$$

Therefore,

$$\pi_{\mathrm{ET}}(s^{\mathrm{EO}}) \geq \pi_{\mathrm{EO}}(s^{\mathrm{EO}}).$$

Since s^{ET} is the optimal amount of learning effort under ET,

$$\pi_{\mathrm{ET}}(s^{\mathrm{ET}}) \geq \pi_{\mathrm{ET}}(s^{\mathrm{EO}}).$$

By transitivity,

$$\pi_{\mathrm{ET}}(s^{\mathrm{ET}}) \geq \pi_{\mathrm{EO}}(s^{\mathrm{EO}}),$$

i.e., $\pi^*_{\mathrm{ET}} \geq \pi^*_{\mathrm{EO}}$. ■

Impact on the Regular Group

$$\phi_r^{ET} = \frac{1-c_r^{\mathrm{ET}}}{1-\gamma_r s^{\mathrm{ET}}} = \frac{\gamma_r s^{\mathrm{ET}}}{1-\gamma_r s^{\mathrm{ET}}},$$
$$\phi_r^{EO} = \frac{1-c_r^{\mathrm{EO}}}{1-\gamma_r s^{\mathrm{EO}}} = \frac{\gamma_p s^{\mathrm{EO}}}{1-\gamma_r s^{\mathrm{EO}}}.$$

Since

$$s^{\mathrm{ET}} \geq s^{\mathrm{EO}}, \gamma_r \geq \gamma_p$$

therefore,

$$\frac{\gamma_r s^{\mathrm{ET}}}{1-\gamma_r s^{\mathrm{ET}}} \geq \frac{\gamma_r s^{\mathrm{EO}}}{1-\gamma_r s^{\mathrm{EO}}} \geq \frac{\gamma_p s^{\mathrm{EO}}}{1-\gamma_r s^{\mathrm{EO}}},$$

i.e, $\phi_r^{ET} \geq \phi_r^{EO}$.

Since $c_p^{\mathrm{ET}}, c_p^{\mathrm{EO}} \geq 1-\gamma_r s$, we have $\delta_r^{\mathrm{ET}} = \delta_r^{\mathrm{EO}} = 1$. ■

Impact on the Protected Group

$$\phi_p^{ET} = \frac{1-c_p^{\mathrm{ET}}}{1-\gamma_p s^{\mathrm{ET}}} = \frac{\gamma_r s^{\mathrm{ET}}}{1-\gamma_p s^{\mathrm{ET}}},$$
$$\phi_r^{EO} = \frac{1-c_p^{\mathrm{EO}}}{1-\gamma_p s^{\mathrm{EO}}} = \frac{\gamma_p s^{\mathrm{EO}}}{1-\gamma_p s^{\mathrm{EO}}}.$$

Since

$$s^{\mathrm{ET}} \geq s^{\mathrm{EO}}, \gamma_r \geq \gamma_p,$$

we have $\phi_p^{ET} \geq \phi_r^{EO}$. ■

B.2 Large Expected Loss

In this section, we show that our results hold for the case of large expected loss, i.e.,

$$\beta_p > \alpha_p + \frac{1-\gamma_p s}{1-\gamma_r s}\alpha_r.$$

From the analysis in section 3.3, we know that in this case

$$c_p^{\mathrm{ET}} = c_r^{\mathrm{ET}} = 1-\gamma_p s, \qquad \pi_{\mathrm{ET}}(s) = \frac{\alpha_p\gamma_p s}{1-\gamma_p s} + \frac{\alpha_r\gamma_p s}{1-\gamma_r s} - \tau(s);$$

$$c_p^{\mathrm{EO}} = 1-\gamma_p s, c_r^{\mathrm{EO}} = 1 - \frac{1-\gamma_r s}{1-\gamma_p s}\gamma_p s, \qquad \pi_{\mathrm{EO}}(s) = \frac{\alpha_p\gamma_p s}{1-\gamma_p s} + \frac{\alpha_r\gamma_p s}{1-\gamma_p s} - \tau(s).$$

Learning

The derivative of the two profit functions are:

$$\pi'_{\mathrm{ET}}(s) = \frac{\alpha_p\gamma_p}{(1-\gamma_p s)^2} + \frac{\alpha_r\gamma_p}{(1-\gamma_r s)^2} - \tau'(s);$$
$$\pi'_{\mathrm{EO}}(s) = \frac{\alpha_p\gamma_p}{(1-\gamma_p s)^2} + \frac{\alpha_r\gamma_p}{(1-\gamma_p s)^2} - \tau'(s).$$

Therefore,

$$\pi'_{\mathrm{ET}}(s) - \pi'_{\mathrm{EO}}(s) = \frac{\alpha_r\gamma_p}{(1-\gamma_r s)^2} - \frac{\alpha_r\gamma_p}{(1-\gamma_p s)^2} \geq 0.$$

By the statement shown in the proof of Theorem 1, we have $s^{\mathrm{ET}} \geq s^{\mathrm{EO}}$. ■

Impact on the decision maker

For any given $s \in [0, \frac{1}{2\gamma_r}]$, we have

$$\pi_{\mathrm{ET}}(s) - \pi_{\mathrm{EO}}(s) = \frac{\alpha_r\gamma_p s}{1-\gamma_r s} - \frac{\alpha_r\gamma_p s}{1-\gamma_p s} \geq 0$$

Therefore,

$$\pi_{\mathrm{ET}}(s^{\mathrm{EO}}) \geq \pi_{\mathrm{EO}}(s^{\mathrm{EO}}).$$

Since s^{ET} is the optimal amount of learning effort under ET,

$$\pi_{\mathrm{ET}}(s^{\mathrm{ET}}) \geq \pi_{\mathrm{ET}}(s^{\mathrm{EO}}).$$

By transitivity,

$$\pi_{\mathrm{ET}}(s^{\mathrm{ET}}) \geq \pi_{\mathrm{EO}}(s^{\mathrm{EO}}),$$

i.e., $\pi^*_{\mathrm{ET}} \geq \pi^*_{\mathrm{EO}}$. ■

Impact on the Regular Group

$$\phi_r^{ET} = \frac{1 - c_r^{\text{ET}}}{1 - \gamma_r s^{\text{ET}}} = \frac{\gamma_p s^{\text{ET}}}{1 - \gamma_r s^{\text{ET}}},$$
$$\phi_r^{EO} = \frac{1 - c_r^{\text{EO}}}{1 - \gamma_r s^{\text{EO}}} = \frac{\gamma_p s^{\text{EO}}}{1 - \gamma_p s^{\text{EO}}}.$$

Since

$$s^{\text{ET}} \geq s^{\text{EO}}, \gamma_r \geq \gamma_p$$

therefore,

$$\frac{\gamma_p s^{\text{ET}}}{1 - \gamma_r s^{\text{ET}}} \geq \frac{\gamma_p s^{\text{ET}}}{1 - \gamma_p s^{\text{ET}}} \geq \frac{\gamma_p s^{\text{EO}}}{1 - \gamma_p s^{\text{EO}}},$$

i.e, $\phi_r^{ET} \geq \phi_r^{EO}$.

As $c_p^{\text{ET}}, c_p^{\text{EO}} \geq 1 - \gamma_r s$, we have $\delta_r^{\text{ET}} = \delta_r^{\text{EO}} = 1$. ■

Impact on the Protected Group

$$\phi_p^{ET} = \frac{1 - c_p^{\text{ET}}}{1 - \gamma_p s^{\text{ET}}} = \frac{\gamma_p s^{\text{ET}}}{1 - \gamma_p s^{\text{ET}}},$$
$$\phi_r^{EO} = \frac{1 - c_p^{\text{EO}}}{1 - \gamma_p s^{\text{EO}}} = \frac{\gamma_p s^{\text{EO}}}{1 - \gamma_p s^{\text{EO}}}.$$

Since

$$s^{\text{ET}} \geq s^{\text{EO}}$$

we have $\phi_p^{ET} \geq \phi_r^{EO}$. ■