1. (1p) State the pumping lemma for regular languages.

2. (1p) Draw a picture giving the intuition behind the proof of the pumping lemma for regular languages.

3. (1p) Name and describe the three regular operations.

4. (2p) Give a brief, English characterization of the language of each of the following regular expressions. For extra fun, draw NFAs! a. $(0 \cup 1)^* 01(0 \cup 1)^*$

b. $0((0 \cup 1)^* 0)^* \cup 1((0 \cup 1)^* 1)^*$

c. $(1^*01^*01^*)^*$

4. (1p) The regular languages are the languages recognizable by computation with what resource constraint?

- **5.** (2p) Circle the regular languages among the following languages. The alphabet is $\{0, 1\}$.
 - a. The set of strings with length at most 453.
 - b. The set of palindromes (strings w such that $w^R = w$).
 - c. The set of strings that are accepted by any minimal DFA with at least 453 states.
 - d. Strings whose middle character (rounded down) and final character are equal.
 - e. The set of png encodings of 1080p pictures of cats.

6. (2p) Circle the models of computations that are equivalent to DFAs among the following.

a. An NFA where transitions can be labelled with arbitrary regular expressions.

b. A DFA whose transition function has an extra output indicating whether it sees the previous or next character of the string next. The string is given special delimiting characters at each end. (For those of you who have seen Turing machines before, this is a read-only Turing machine.)

c. A DFA equipped with a counter. The transition function takes an extra input indicating whether the counter is zero and has an extra output indicating whether to increment it or decrement it (not below zero).

d. A DFA that gets a record of the set of states it has ever visited as an extra input to its transition function.

e. A DFA that on input x receives as input to its transition function the previous $\lg x$ letters of the string instead of the previous 1.

f. An actual, real, physical computer.

7. (1p) Let L be a language and M a DFA with n states accepting L such that no DFA with fewer states accepts L. Circle the true statement(s) below.

a. Every DFA accepting L has n states.

a. Every DFA accepting L is isomorphic to M.

a. Every DFA with n states accepting L is isomorphic to M.

a. Every DFA with n states accepting L is equal to M. (*Hint*: This is false.)

a. Every DFA with no inaccessible states and no pairs of distinct, indistinguishable states is isomorphic to M. (*Hint*: This is true.)