

Complete this checkpoint on your own, and email your solution to okahn and afrank@andrew.cmu.edu by 11:59 PM on February 24th with “[FLAC checkpoint 1]” in the subject.

Your work should be legible and preferably typed, but you don’t have to use TeX.

1 Decidability Is a Formal Notion

At http://upload.wikimedia.org/wikipedia/commons/a/ab/Kaninchen_und_Ent_e.png is a picture of a rabbit or a duck (not both). w is a string: 0 if that’s a rabbit; 1 if it’s a duck. Explain why $\{w\}$ is decidable.

2 Strengthening? Turing Machines

Show that the following modifications do not add any power to Turing machines by explaining at a high level how normal Turing machines can simulate them.

- (a) A bi-infinite tape (with infinitely many blanks before as well as after the input).
- (b) The ability to “mark” cells on the tape with annotations from an arbitrary “marking” alphabet Π . (It might help to begin by formalizing this.)

3 Sloppy Pseudocode

In mathematics, we can consider structures or processes that are not physically realizable. In specifying Turing machines, we cannot. For each of the following specifications, explain why it does not properly specify a Turing machine and correct it so that it specifies the intended Turing machine.

- (a) (Part of an attempt to prove that E_{TM} is co-semidecidable.) List all the strings in Σ^* and run M on each one.
- (b) (Part of an attempt to reduce $HALTS$ to ALL_{TM} .) Run M on w . If it halts, accept.
- (c) (Part of an attempt to prove that, for fixed $w \in \Sigma^*$, $\{\langle M \rangle \mid M \text{ is a Turing machine that halts on } w\}$ is recursively enumerable. Here and in general, imagine that Turing machines are canonicalized somehow, so there are countable many of them.) Run every Turing machine on w in parallel.

From now on, you will frequently have to describe Turing machines and prove them correct. Remember to keep your mathematical reasoning separate from your algorithmic specifications.

4 Carpentry

Prove that $\{\langle M \rangle \mid M \text{ is a Turing machine that halts on at most 453 inputs}\}$ is co-semidecidable. (*Hint:* Your answer should include the word *dovetail(ing)*.)