

Please print single-sided with each problem on its own pages and your name on every page.

List any collaborators or sources (including yourself) at the end of your submission.

1 NP-completeness of the student body

Consider the following problem:

Let U be a set of students (integers). Let S be a set of clubs (sets of integers, not necessarily disjoint). Let a congress C be defined as a set of students (subset of U) that represents all clubs (for every $s \in S$, there is some r such that $r \in s$ and $r \in C$). Let k be a provided natural number. The problem is to determine if there is a congress of size k . Formally:

$$\text{CONGRESS} = \{\langle U, S, k \rangle \mid \text{There is a congress of size } k \text{ from } U \text{ representing } S\}$$

Equivalently:

$$\text{CONGRESS} = \{\langle U, S, k \rangle \mid \exists C \text{ such that } |C| = k \text{ and } C \subseteq U \text{ and no } s \in S \text{ is disjoint from } C\}$$

Prove that CONGRESS is NP-complete.

For this problem, you may assume a problem is NP-complete if and only if it is covered in the lecture notes.

Hint: Prove the problem is in NP and then reduce from an NP-complete problem. You should have at most one sentence justifying NP and at most one sentence justifying that the reduction is polynomial time. Expressing the reduction precisely and proving it correct (both directions) are the important parts of this problem.

2 Can't mash these potatoes

Informally, prove that you can't concatenate DFAs quickly. Formally, prove that there is no polynomial-space-computable function that takes encodings of pairs of DFAs to an encoding of a DFA that recognizes the concatenation of the languages of the input DFAs. (*Hint: Look at previous homework results. Remember that a TM must leave the output on the tape in order to compute a function.*)