

Please print single-sided with each problem on its own pages and your name on every page.

List any collaborators or sources (including yourself) at the end of your submission.

1 $\lambda f.(\lambda x.fxx)(\lambda x.fxx)$

Rudich proved the fixed point theorem using the recursion theorem and mentioned that it is easy to go the other way, too. Prove the recursion theorem using the following stronger version of the fixed point theorem, which you may assume henceforth:

Let $f: \Sigma^* \rightarrow \Sigma^*$. Then there is a Turing machine M such that $f(\langle M \rangle) = \langle N \rangle$ where for every $w \in \Sigma^*$, if M on w hangs, then N on w hangs, and if M on w accepts (resp. rejects) with s left on its tape, then N on w accepts (resp. rejects) with s left on its tape.

(*Hint:* See http://en.wikipedia.org/wiki/Lambda_calculus#Recursion_and_fixed_points for inspiration.)

2 The Cardinal Incompressibility Theorem

- Recall that $\text{COMPRESS} = \{\langle w, n \rangle \mid K(w) \leq n\}$ is undecidable. Prove that $\text{COMPRESS} \in \Sigma_1^0$.
- Prove that the function $f: \Sigma^* \rightarrow \Sigma^*$ given by $w \mapsto 0^{\lfloor s|K(s) \leq |w| \rfloor}$ is uncomputable (informally, a Turing machine cannot determine how many strings can be described with at most a given number of bits).

3 NP Is Closed Under Kleene Star

Prove that NP is closed under Kleene star.