

Please print single-sided with each problem on its own pages and your name on every page.

List any collaborators or sources (including yourself) at the end of your submission.

## 1 Cartography

Let  $L = \{\langle M \rangle \mid M \text{ is a Turing machine and on some input } w, M \text{ changes the input portion of its tape}\}$ . Prove that  $L$  is semidecidable. Prove that it is undecidable by giving a *mapping reduction* (see Sipser p. 235) from some undecidable language to  $L$ .

## 2 Shortlexly Enumerable

Prove that a language is decidable iff some enumerator enumerates it in shortlex order.

## 3 Pushup Automata

A *pushup automaton* is similar to a deterministic pushdown automaton, except:

1. a pop operation returns the least (instead of most) recently pushed element that has not yet been popped,
2. the machine accepts by entering a special accept state that has immediate effect (a la Turing machines), and
3. the input is followed by a special delimiter character.

Prove that the class of languages recognized by pushup automata is the set of Turing-recognizable languages. (To do this, you should prove that Turing machines and pushup automata are mutually simulable.)