Please print single-sided with each problem on its own pages and your name on every page.

List any collaborators or sources (including yourself) at the end of your submission.

1 Using the Pumping Lemma for CFLs

Use the pumping lemma for context-free languages to prove the following are not context-free:

(a) $\{w \in \{0,1\} \mid w \text{ is a palindrome with equal numbers of 0s and 1s}\}$

(b) $\{0^n 1^n 0^n 1^n \mid n \in \mathbb{N}\}$

(c) $\{0^i 1^j \mid j \text{ divides } i\}$

2 Strengthening the Pumping Lemma for CFLs

Prove a stronger form of the pumping lemma for CFLs, where v and y are both non-empty. That is:

If L is a context-free language, there exists a number k such that any string $s \in L$, $|s| \geq k$, can be divided into five pieces $s = uvxyz$ where

1. for each $i \geq 0$, $uv^i xy^i z \in L$,
2. $v \neq \varepsilon$ and $y \neq \varepsilon$, and
3. $|vxy| \leq k$.

3 Stronger Machines, Weaker Closures

For any language L, we define $\textsc{Swap}(L) = \{bac \mid a, b, c \in \Sigma^*, abc \in L\}$.

Prove that the context-free languages are not closed under $\textsc{Swap}$, i.e. there exists a context-free language L such that $\textsc{Swap}(L)$ is not context-free.

Optional: The regular languages *are* closed under $\textsc{Swap}$, by a construction similar to one you've seen before. Intuitively, why can a class of languages be closed under an operation when a strict superset of the class is not?

4 I Miss Intersection

Prove the language $\{a^i b^j \mid i \neq j \text{ and } 2i \neq j\}$ is context-free.