Please print single-sided with each problem on its own pages. List any collaborators or sources (including yourself) at the end of your submission.

0 (optional ungraded practice)

(a) Prove the following languages are not regular:

i. $\{a\text{+}b\text{=}c \mid a, b, c \in \{0,1\}^*$ and interpreted as binary representations of integers, $a + b = c\}$

ii. $\{1^p \mid p$ prime $\}$

(b) Give regular expressions for the following languages:

i. $\{x \in \{c, a, b\}^* \mid x$ contains the substring "cab" $\}$

ii. $\{x \in \{a, b, c\}^* \mid x$ contains an even number of b's $\}$

iii. $\{x \in \{a, b, c\}^* \mid$ every a in x is followed by a b, and x ends with a c $\}$

1 Deterministic Infinite Automata

We define a DIA $= (Q, \Sigma, q_0, \delta, F)$ identically as for DFAs, except the set Q of states must be **infinite**.

What set of languages do DIAs recognize? Prove your claim.

2 Blow-Up

Using the Rabin-Scott powerset construction from lecture, an NFA with n states can be made into an equivalent DFA with 2^n states, but this is usually much larger than the minimal equivalent DFA.

Prove that for all $n > 0$, there is an n-state NFA whose minimal equivalent DFA has at least 2^{n-1} states.

3 Converse Pumping Lemma

Consider the language $A = \{a^i b^j c^k \mid i, j, k \geq 0$ and if $i = 1$ then $j = k.\}$

(a) Show that A is not regular.

(b) Show that the pumping lemma is insufficient to prove A is not regular, that is, we have some P where if $w \in A$ and $|w| \geq P$, we can write $w = xyz$ with $|y| > 0$, $|xy| \leq P$, and for every $i \geq 0$, $xy^iz \in A$.

(c) Do (a) and (b) contradict the pumping lemma? Explain.