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0 (ungraded practice)

Draw DFAs for the following languages. Briefly justify why your DFAs recognize the correct language.

1. The language of strings in $\{a, b, c\}$ containing the string cab at least twice.
2. The language of strings in $\{a, b, c\}$ containing the string cab at most twice.
3. The language of strings in $\{a, b, c\}$ whose length is either even or divisible by 5 (or both!)
4. The language of strings in $\{a, b, c\}$ containing at least one a and an even number of b's.

1

Given a string $w \in \{0, 1, 2\}^*$, let $\textsc{Int}(w)$ be w interpreted as a base-3, most-significant-digit first integer, so:
$\textsc{Int}(002) = \textsc{Int}(2) = 2$
$\textsc{Int}(012) = \textsc{Int}(12) = 5$
$\textsc{Int}(20) = 6$
$\textsc{Int}(21) = 7$
$\textsc{Int}(120) = 15$
and so on.

Let $L = \{w \in \{0, 1, 2\}^* \mid \textsc{Int}(w) \equiv 0 \bmod (5)\}$.
Prove that L is regular by providing a DFA that recognizes it.

2

For any language A, we define

$$\textsc{Del}(A) = \{wx \in \Sigma^* \mid \exists\, \sigma \in \Sigma \text{ such that } w\sigma x \in A\}$$

In other words, the set of strings from A with one symbol removed.

Prove that if A is regular, $\textsc{Del}(a)$ is regular.

Hint: Assume you have a DFA that recognizes A. How would you modify it to recognize $\textsc{Del}(A)$?

3

For any languages L_1, L_2, we define

$$\textsc{Cut}(L_1, L_2) = \{xyz \in \Sigma^* \mid xz \in L_1, y \in L_2\}$$

Prove that if L_1 and L_2 are regular, $\textsc{Cut}(L_1, L_2)$ is regular.