Information Disclosure in Matching Markets

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SAET- July, 2023

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Motivation

- Matching environment (consumer, products)
- Vertical differentiation
- Allocation (possibly random) of products to consumers
- Implementation: Information and prices
- Example: rating systems and allocation
- Sorting (information) and consumer welfare

Setting

- Matching Environment - Vertical Differentiation
- Product qualities (given): $y \sim \bar{G}(y)$, inelastically supplied - measure one, mean $\bar{y}$
- Consumer heterogeneity: $x \sim F(x)$, measure $m > 1$. Continuous strictly increasing.
  - Define $\tilde{F}$ to be F restricted to the set $x \geq x_0$, where $m(1 - F(x_0)) = 1$
- Utility $u(x, y) = xy - t(x)$

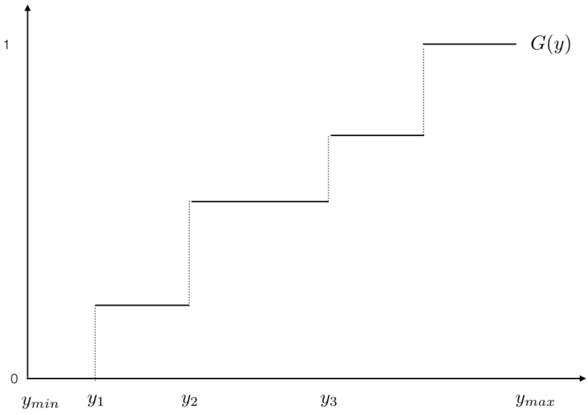
Monotone Allocation

- Any G such that $\bar{G}$ is a mean preserving spread of G
  - suppressing information or pooling
  - Example: Split $y's$ into two pooling groups: $\{[y_{min}, y_1), [y_1, y_{max}]\}$ and G point mass on conditional means y_L, y_H
  - Extremes: $\underline{G}$ (point mass 1 at $\bar{y}$) and $\bar{G}$.
- Monotone allocation y(x)
  - If G is continuous,

$$G(y(x)) = \tilde{F}(x) \text{ for } x \geq x_0$$

  - If G has points of discontinuity (e.g. pooling regions):

Discontinuous G



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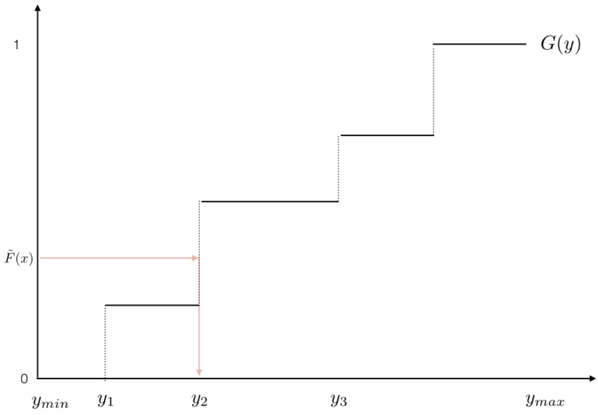

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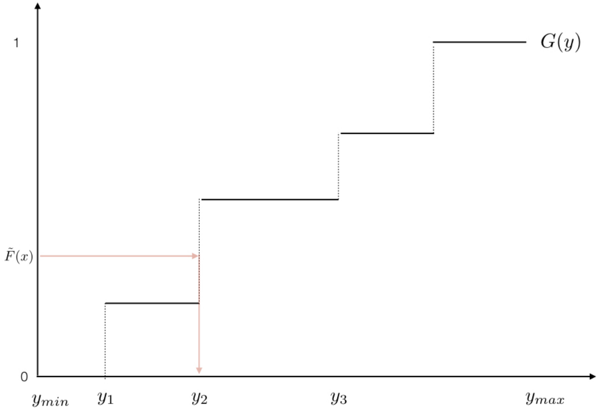

$$y(x) = G^{-1}(F(x))$$

Incentive Feasible Allocation

- $G \lesssim_{SOSD} \bar{G}$
- monotone allocation $y(x) = G^{-1}(F(x))$ and transfers $t(x)$

$$u(x) = xy(x) - t(x)$$

- Individual Rationality: x_0 must be indifferent in participating, so $u(x_0) = 0$
- Incentive compatibility:

$$u(x) = \int_{x_0}^{x} y(u)\, du$$

Pure Random Matching (no information)

Distribution $\underline{G}$ point mass on $\bar{y}$

$$y(x) = \bar{y} \text{ for all } x \geq x_0$$

$$\begin{aligned} u(x) &= \int_{x_0}^{x} y(u)\,du + u(x_0) \\ &= (x - x_0)\,\bar{y} \end{aligned}$$

Using $\bar{y} = \int G^{-1}(u)\,du$

$$u(x) = (x - x_0) \int G^{-1}(u)\,du$$

Sorting and Consumer Welfare

Proposition

For any distribution G such that $\underline{G} \prec_{SOSD} G \precsim_{SOSD} \bar{G}$ there exists $x_1 > x_0$ such that for all $x_0 < x \leq x_1$, $u(x) < (x - x_0)\, \bar{y}$

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Proof.

Take x_1 such that $y(x_1) < \bar{y}$. Then for all $x \in (0, x_1]$,

$$u(x) = \int_{x_0}^{x} y(u)\,du \leq (x - x_0)\,y(x_1) < (x - x_0)\,\bar{y}$$

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- In particular, the random allocation Pareto dominates G iff this holds for x_{max}.

Sorting and Consumer Welfare

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- **Sufficiency**
  - Without loss of generality take $x_0 = 1$ and $x_{max} = 1$.
  - Suppose $\tilde{F}(u) \leq u$ for all $u \in [0, 1]$. Then:

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$$u(1) = \int_0^1 G^{-1}\left(\tilde{F}(u)\right) du \leq \int_0^1 G^{-1}(u)\, du = \bar{y}$$

Sorting and Welfare

- **Necessity (need continuous $\bar{G}$)**
  - Take some x_1 such that $x_1 < \tilde{F}(x_1)$.
  - Take y_1 with $\bar{G}(y_1) = \tilde{F}(x_1)$
  - Split $y's$ into two pooling groups: $\{[y_{min}, y_1), [y_1, y_{max}]\}$ and G point mass on conditional means y_L, y_H

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$$\begin{aligned} u(1) &= x_1 y_L + (1 - x_1) y_H \\ &> \tilde{F}(x_1) y_L + \left(1 - \tilde{F}(x_1)\right) y_H = \bar{y} \end{aligned}$$

Consumer Pareto Frontier

- Characterize consumer's Pareto frontier allocations

$$\int_{x_0}^{1} u(x)\, dH(x)$$

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Theorem

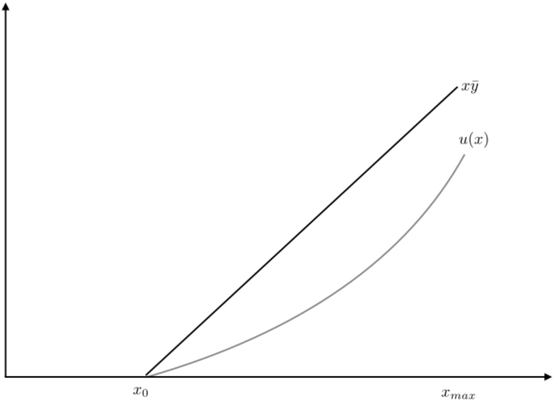
(Myerson (1981), Kleiner et. al. (2021), Saeedi-Shourideh (2023))
Optimal matching satisfies the following:

1. It partitions $[x_0, 1]$ into finite intervals and alternates between fully assortative and fully random matching over the intervals
2. If $\frac{d}{dx}\frac{1-H(x)}{\tilde{f}(x)} < 0$, then x belongs to a random matching interval
3. The highest consumer $x = 1$ is always randomly matched.

Endogenous Entry

- Cost of entry k. Random draw of quality from G_0
- Equilibrium condition $\int p(y)\, G(y) = k$
- $p(y)$ decrease in k
- Total consumer surplus increases
- Increases at the lower end
- Might not increase everywhere

Endogenous Entry



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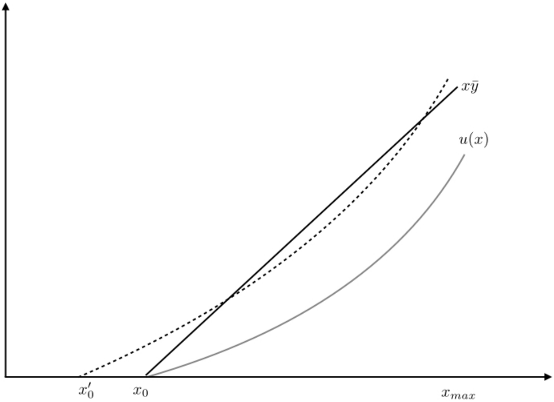

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