Optimal Communication Design

James Best Dan Quigley Maryam Saeedi Ali Shourideh

Carnegie Mellon and Cambridge
SAET 2023

July 17, 2023

Aggregation in the Age of Information

- Online platforms often rely on information aggregation
  - Review aggregators: Yelp, IMDb, Goodreads, etc.
  - Promotion of content on social media
  - ...
- Common issue:
  - Strategic manipulation

This Paper

- Question: Is there a way for an aggregator to overcome this strategic manipulation while being informative?
- Model:
  - 2 senders + 1 receiver
  - Senders are biased relative to receiver
  - Each sender observes a signal of the state privately
  - Sends a message independently
  - Aggregator: what to show to the receiver

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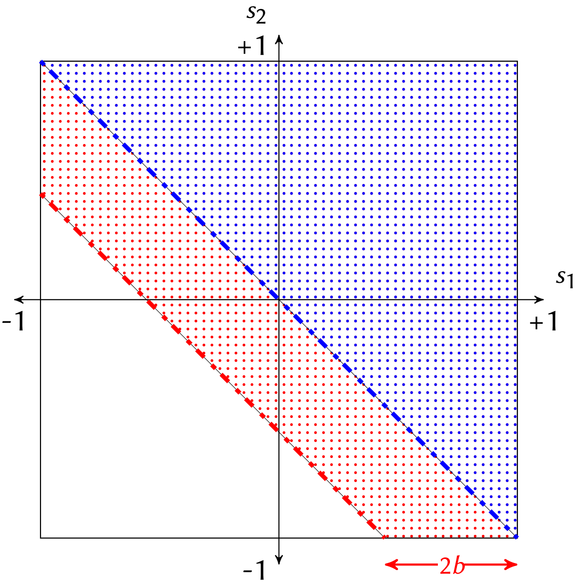
- State:

$$\omega = \frac{s_1 + s_2}{2}$$

Example



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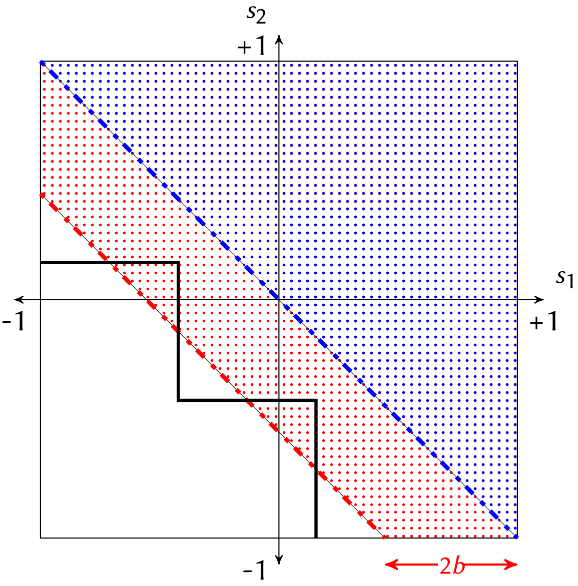

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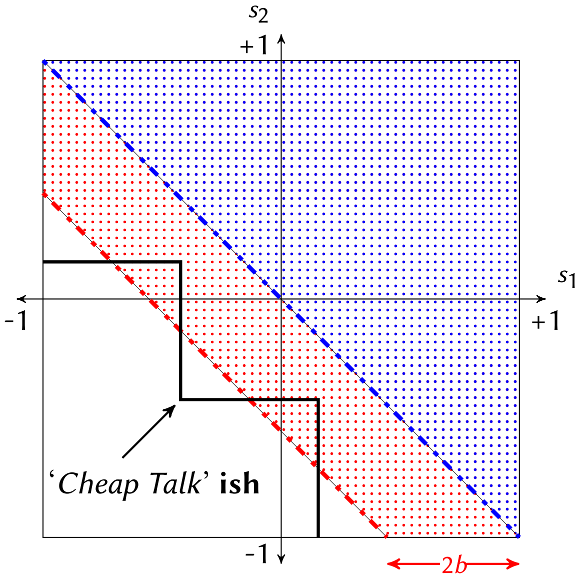

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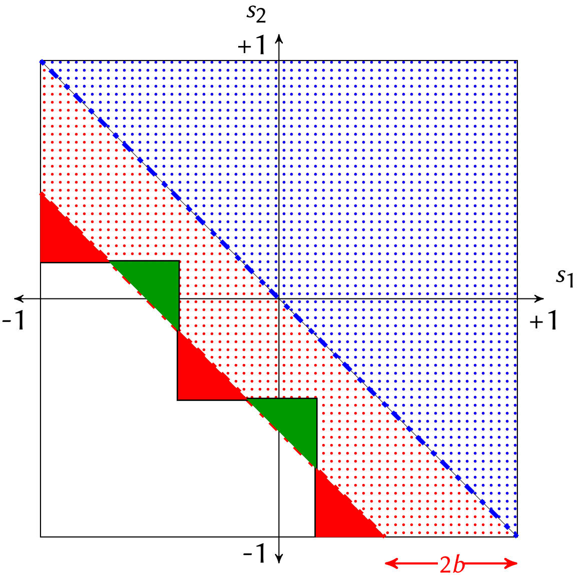

Example

- Notation:
  - *Collusion:* senders' best allocation
- In the example:
  - Value of collusion for R is higher than value of cheap talk.
  - Red Areas $>$ Green Areas
  - (Cond.) Avg. value of ω in R $<$ Avg. value of ω in G
- Later: we will show that there is some generality to this insight

Literature

- Wolinsky (GEB, 2002): verifiable disclosure model (limited state space)
- Goltsman, Hörner, Pavlov, and Squintani (JET, 2009): comparing different benchmarks for communication (mediation vs. negotiation)
- Multiple sender cheap talk games: Krishna and Morgan (QJE, ..), Battagalini (Ecta, 2004), Meyer et al (?) and many more; our focus is on what can be achieved with commitment.
- Incentives in information design: Onuchic and Ray (2022), Boleslavski and Kim (2023), Saeedi and Shourideh (2023)
- **This paper:** Optimal mechanisms (for now partially)

Model

- Payoffs are the same as before:
  - $u^S(a, \omega) = a \times (\omega + b), b > 0$
  - $u^R(a, \omega) = a \times \omega$
  - $\omega = \frac{s_1 + s_2}{2}$
  - $s_i \sim F(s_i), \mathrm{Supp}(F) = [-1, 1]$
- For now, only one assumption on F

Assumption. *Mean of s is non-positive, i.e., information is valuable for R.*

Mechanisms

- Information/mechanism design under commitment:
  - R commits to ignore information
  - A review aggregator
- Myerson (1984): WLOG, direct mechanisms:
  - Sender i reveals s_i
  - Mechanism recommends $a = 1$, with $\Pr = \sigma(s_1, s_2) \in [0, 1]$
- IC:

$$\mathbb{E}_{-i}\left[\left(s_i + s_{-i} + 2b\right)\sigma(s)\right] \geq \mathbb{E}_{-i}\left[\left(s_i + s_{-i} + 2b\right)\sigma\left(s_{-i}; \hat{s}\right)\right], \forall \hat{s}$$

- Obedience:

$$\int (s_1 + s_2)\,\sigma(s) \prod_{i=1,2} f(s_i)\,ds_i \geq 0$$

Mechanisms: Examples

- (Independent) Cheap talk with partition: $[-1,1] = \bigcup_i \left[\underline{s}_i, \underline{s}_{i+1}\right]$
    - IC: $\mathbb{E}\left[\mathbf{1}\left[s \geq \underline{s}_{j^*(i)}\right]\left(\underline{s}_i + s + 2b\right)\right] = \mathbb{E}\left[\mathbf{1}\left[s \geq \underline{s}_{j^*(i)+1}\right]\left(\underline{s}_i + s + 2b\right)\right]$
    - Ob.: $\mathbb{E}\left[\mathbf{1}\left[\underline{s}_{j+1} \geq s \geq \underline{s}_j\right]\mathbf{1}\left[\underline{s}_{i+1} \geq s' \geq \underline{s}_i\right](s + s')\right] \geq 0 \iff j \geq j^*(i)$
- Cheap talk-ish – not an equilibrium:
    - IC the same as before
    - Ob.: $\mathbb{E}\left[(s_1 + s_2)\,\sigma(s)\right] \geq 0$
- Collusion: $\sigma(s) = \mathbf{1}\left[s_1 + s_2 \geq -2b\right]$
    - Obviously IC
    - Ob.: $\int_{s_1+s_2 \geq -2b} (s_1 + s_2) f(s_1) f(s_2)\, ds \geq 0$

Mechanism Design

- Focus on R optimal mechanisms

$$\max \mathbb{E}\left[(s_1 + s_2)\,\sigma(s)\right]$$

subject to

$$(\text{IC}), \sigma \in [0, 1]$$

- WLOG, focus on σ: symmetric; if not, just use $\hat{\sigma}(s) = \frac{\sigma(s)+\sigma\left(s^T\right)}{2}$.
- Need to only impose one sender's IC

Small Bias: Collusion is optimal

Theorem. Suppose that $\alpha \geq \frac{b}{1+b}$ exists such that

$$1 - 3\alpha + 2\alpha\left(1 - s\right)\frac{f'\left(s\right)}{f\left(s\right)} \geq 0$$

$$1 - 3\alpha + 2\left(b - \alpha\left(s + b\right)\right)\frac{f'\left(s\right)}{f\left(s\right)} \geq 0$$

Then collusive allocation is optimal. Moreover, there exists a distribution for which these bounds are tight.

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Then collusive allocation is optimal. Moreover, there exists a distribution for which these bounds are tight.

- Roughly speaking: we cannot have $0 \gg f'(s)/f(s)$ and $b \gg 0$.
- Tightness: If $s \sim U[-1, 1]$, $b \leq 1/2$, collusion is optimal. If $b > 1/2$, collusion is not optimal.
  - More on this later.
- Proof: by constructing lagrange multipliers for the 2D linear programming problem

Small Bias: Examples

- Linear density: $f(s) = 1 - As, A \leq 0, b \leq \frac{1}{2}\frac{1+A}{1+3A}$.
- Single-peaked density

$$b \leq \frac{1}{2} \frac{1}{1 - \min_s \frac{(1-s)f'(s)}{f(s)}}$$

- In general, the bounds imply that $b \leq 1/2$
- something more positive: distributions that do not blow up give us a non-zero bound:

Corollary. If $|f'(s)/f(s)| < M$ for some $M \in \mathbb{R}_+$, then there exists $\overline{b}_C > 0$ such that collusion is optimal when $0 \leq b \leq \overline{b}_C$.

Higher bias: What to do

- In general, we do not know the answer yet but we have hopefull conjectures!!
- Wolinsky (2002): monotone vs. non-monotone mechanisms

Definition. A mechanism is monotone if and only if $\sigma(s) \geq \sigma(s'), \forall s \geq s'$.

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Definition. A mechanism is monotone if and only if $\sigma(s) \geq \sigma(s')$, $\forall s \geq s'$.

- High bias: monotone does not exist.

Lemma. There is no informative monotone mechanism if $b \geq 1$.

Non-monotone mechanisms

- Non-monotone mechanisms often exist!!

Lemma. Existence

1. There exists $\overline{b}_N > 1$ such that if $1/2 \leq b \leq \overline{b}_N$ then an informative non-monotone mechanism exists.
2. If $\mathbb{E}s = 0$, then $\overline{b}_N = \infty$, i.e., non-montone informative mechanism always exists.

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- Simple non-monotone mechanism:

$$\sigma(s) = \begin{cases} 0 & s_1, s_2 \geq \hat{s} \\ 1 & s_1 \leq \hat{s}, s_2 \geq \hat{s} \\ 1 & s_1 \geq \hat{s}, s_2 \leq \hat{s} \\ 0 & s_1, s_2 \leq \hat{s} \end{cases}$$

Non-monotone mechanisms

- Conjecture:

Conjecture. For values of b high enough, simple non-monotone is almost optimal.

Optimal Non-monotone mechanisms look crazy

$$b = 1.2, s \sim U[-1, 1]$$