95-771 Data Structures and Algorithms
Spring 2026

Midterm Exam
Carnegie Mellon University

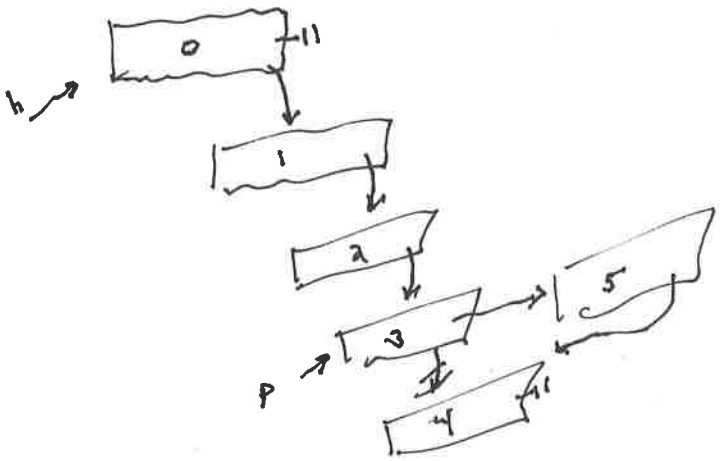
Name Key **Andrew ID** Key

Total points = 131. Score will be a percentage of 131.

Tracing Lists and Trees (28 Points) ✓

1. You will be asked to show the exact output of the following program.

```
public class Node {

   public int data;
   public Node next;
   public Node(int value, Node ptr) {
      data = value;
      next = ptr;
   }
   public String toString() {          /* 1.a */
      String temp = Integer.toString(data);
      if(next == null) {
         temp = temp + "--| ";
         return temp;
      }
      else {  // make a recursive call
         return temp + "," + next.toString()  ;
      }
   }
   public static void main(String args[]) {

      Node head = new Node(0,null);
      System.out.println("Line 1: " + head);    /* 1.b */
      Node p = head;
      for (int i = 1; i <5; i++) {
         Node t = new Node(i, null);
         p.next = t;
         p = p.next;
      }
      System.out.println("Line 2: " + head);    /* 1.c */
      p = head;
      while(p.data < 3) {
         p = p.next;
      }
      p.next = new Node(5,p.next);
      System.out.println("Line 3: " + head);   /* 1.d */
   }
}
```



For question 1, assume n is the number of nodes on the list.

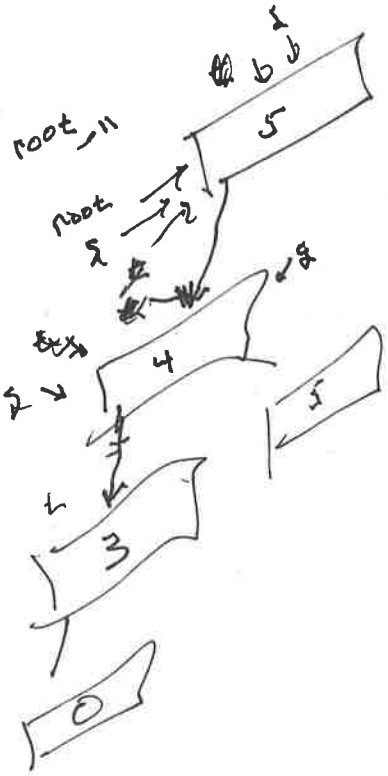
1.(a) What is the Big Theta for the toString() method marked with (1.a)? $\Theta(N)$ (2 pts)
1.(b) Show the output of the code marked (1.b). 0 --| (2 pts)
1.(c) Show the output of the code marked (1.c). 0,1,2,3,4 --| (2 pts)
1.(d) Show the output of the code marked (1.d). 0,1,2,3,5,4 --| (6 pts)
1.(e) Is it correct to say that the method toString() of the Node class runs in $O(2^n)$?
Circle (True) or False (3 pt.)

1.(f) Is it correct to say that the method toString of the Node class runs in O(Log n) ?
Circle True or False (3 pt.)

key

2. Study the execution of the following program. This is not the same program that appeared on a previous exam. Five questions appear below. Note that there are println() statements in various places.

```
class Node {
   public int data;
   public Node lc;
   public Node rc;
   public Node(Node lc, int x, Node rc) {
      this.lc = lc;
      this.data = x;
      this.rc = rc;
   }
}

public class SimpleTree {

   public Node root;

   public SimpleTree() {
      root = null;
   }

   // This tree allows duplicate keys to be added.
   public void add(int x){

      if (root == null) {
         System.out.println("Adding at root");
         root = new Node(null,x,null);
      }
      else {
         Node t = root;
         Node q = t;
         while(t != null) {
            if(x <= t.data ) {
               q = t;
               t = t.lc;
            }
            else {
               q = t;
               t = t.rc;
            }
         } // end while
         if(x <= q.data) {
            System.out.println("Going left");
            q.lc = new Node(null, x, null);
         }
         else {
            System.out.println("Going right");
            q.rc = new Node(null, x, null);
         }
```



Key

```
    }
  }
  public void traversal(Node r) {
    if(r == null) return;
    if(r.lc != null)traversal(r.lc);
    System.out.print(r.data + " ");
    if(r.rc != null)traversal(r.rc);

  }
  public void traversal() {
    traversal(root);
    System.out.println();
  }

  public static void main(String[] args) {
    SimpleTree st = new SimpleTree();
    st.add(5);
    st.add(4);
    st.add(3);
    System.out.println("Three values have been added.");
    st.traversal();

    st.add(5);

    System.out.println("An additional value has been added");
    st.traversal();

    st.add(0);
    System.out.println("The final value has been added");
    st.traversal();

  }
}
```

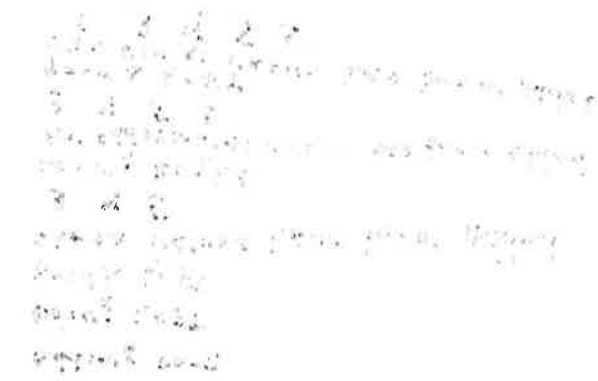
What is the exact output of this program? (6 Points)

```
Adding root
Going left
Going left
three values have been added
3 4 5
Going right
An additional value has been added
3 4 5 5
going left
The final value has been added
0 3 4 5 5
```

2.(d) What is the Big Theta value for traversal(). Our only assumption is that the tree holds n nodes in total. $\Theta(N)$ (1 point)

2.(e) Suppose we built a CLR complete tree and it was perfectly balanced (unlike the tree created above). Suppose too that the tree held n nodes in total. Provide a Big Theta value for traversal(). $\Theta(N)$ (1 point)

2.(f) Does it make sense to describe different "cases" when analyzing the runtime of traversal()? Circle YES or (NO) (1 Point)



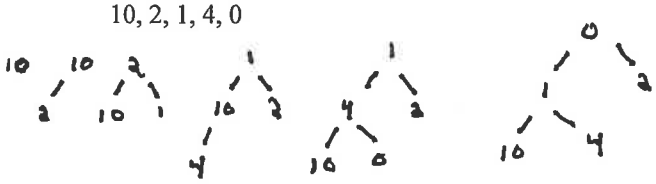
Key

2.(g) Suppose we built a CLR complete tree and it was perfectly balanced (unlike the tree created above). Suppose too that the tree held n nodes in total. Is it correct to say that traversal() is Ω (Log n)? Circle TRUE or FALSE. (1 Point)

Heaps (12 points)

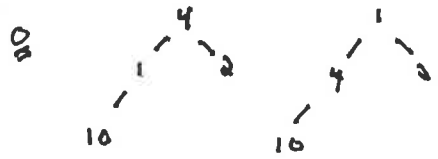
3) Insert the following 5 numbers into a min heap. Draw a new tree for each heap insertion. (4 Points)

10, 2, 1, 4, 0



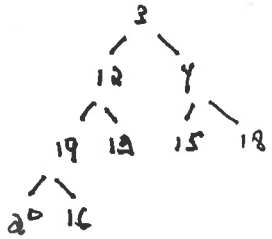
4) What is the height of the tree that you drew in question 3? (A single node in a tree gives a height of 0.) (2 Points) 2

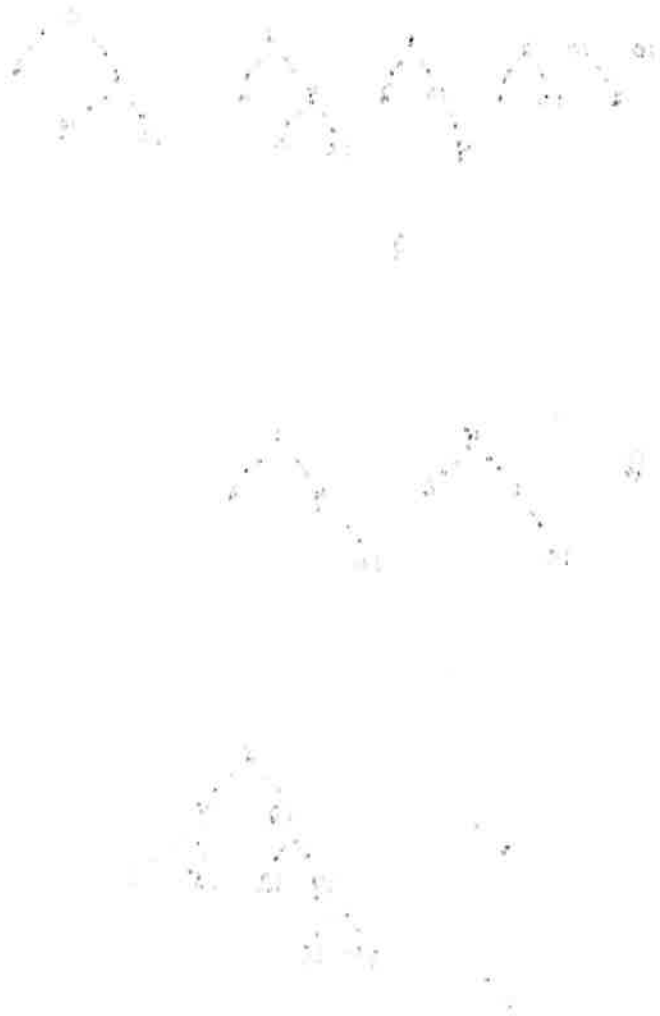
5) Perform exactly one deleteMin() operation on the heap that you drew in question 3. Draw the resulting tree. With a careful drawing, make it clear to the reader what is going on. (3 Points)



6) Consider the following min heap implemented in an array. It is not quite correct. To make it a proper min heap exactly one swap must occur. What two numbers (child and parent) need to be swapped in order to make this a min heap? (3 points). PLACE CHECK MARKS NEXT TO THE TWO NUMBERS THAT NEED TO BE SWAPPED.

3
12
4
19 ✓
13
15
18
20
16 ✓

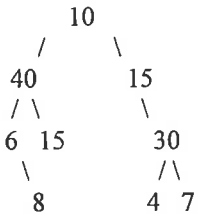




Binary Trees (16 points)

7. Parts (a), (b), (c) refer to the following binary tree:

Key



(a) List the data that would be accessed by a pre-order traversal on the given tree by writing out the values in the nodes as they would be accessed, separated by commas. (3 points)

10 40 6 8 15 15 30 4 7

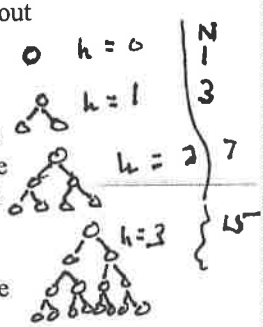
(b) List the data that would be accessed by an in-order traversal on the given tree by writing out the values in the nodes as they would be accessed, separated by commas. (2 points)

6, 8, 40, 15, 10, 15, 4, 30, 7

(c) List the data that would be accessed by a level-order traversal on the given tree by writing out the values in the nodes as they would be accessed, separated by commas. (2 points)

10, 40, 15, 6, 15, 30, 8, 4, 7

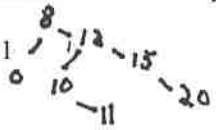
(d) In general, if a binary (at most two children per node) tree is perfectly balanced (unlike the tree pictured here) and complete with height h, how many nodes, in terms of h, will the tree have? (2 points) $2^{h+1}-1$ Note, this tree has a perfectly flat bottom. We need the total number of nodes in terms of h. This is an exact answer, not Big O.



(e) In general, if a binary tree is perfectly balanced (unlike the tree pictured here) and complete with exactly k leaves. What is the number of nodes (in terms of k) in this tree? (2 points)

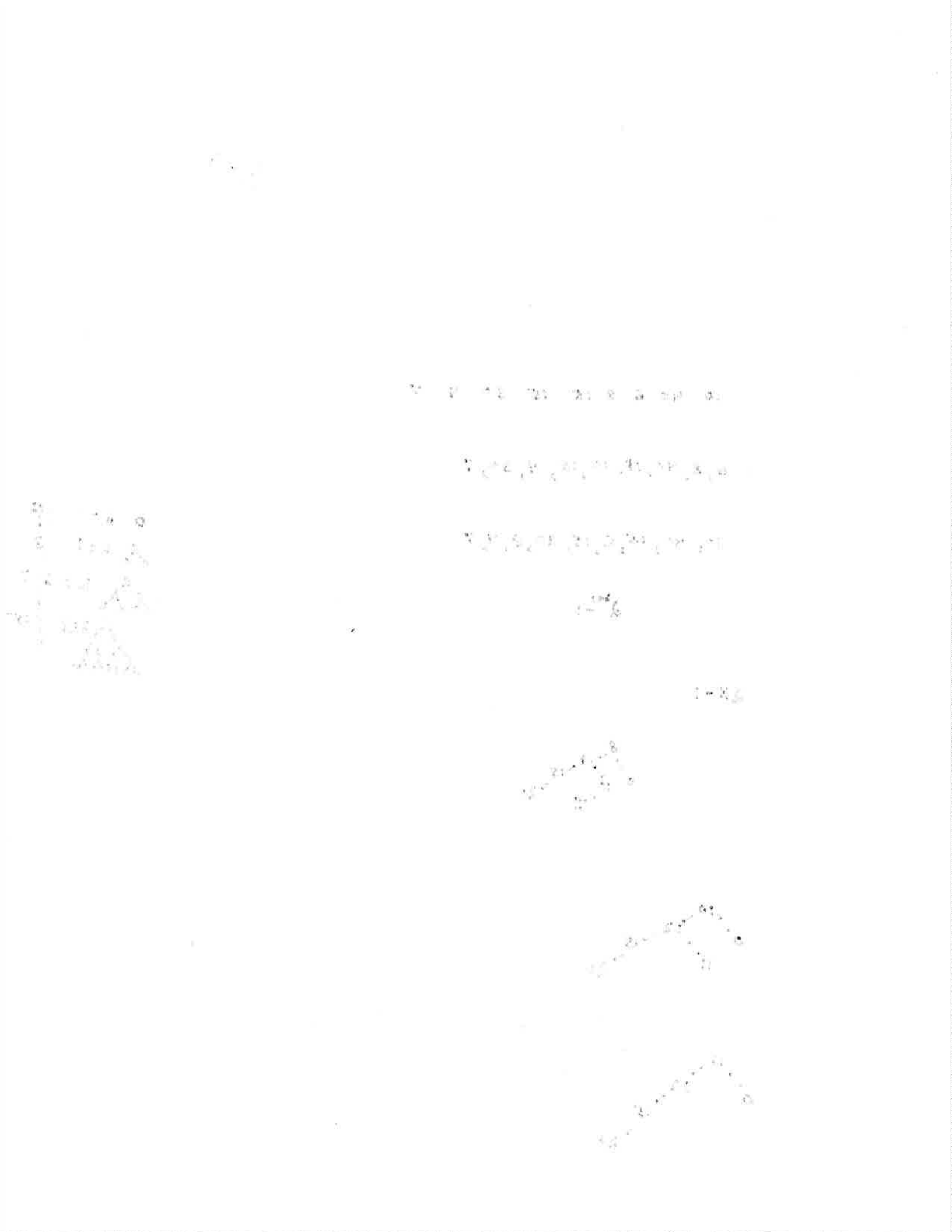
$2K-1$ Note, this tree has a perfectly flat bottom. This is an exact answer, not Big O.

8. (a) Insert he following numbers into a Binary Search Tree. Draw the tree after all insertions are complete. (1 Point)
8, 12, 15, 20, 0, 10, 11



(b) Delete 8 from the final tree that you drew in 8 (a). We are following the "Go Right, Left Hard" rule. Draw this final tree. (2 Point)

(c) Delete 10 from the final tree that you drew in 8 (b). This is the tree that resulted from the deletion of 8 in question 8 (b). We are following the "Go Right, Left Hard" rule. Draw this final tree. (2 Point)

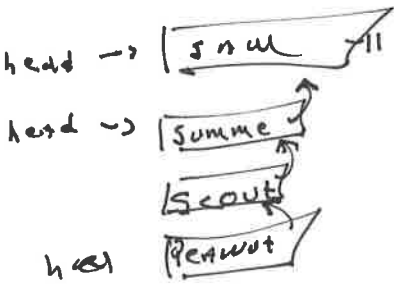


Project Questions (20 points) ✓

(9) Recall your work with linked lists from Project 1, 2-D trees from Project 2 and Red Black trees from Project 3.

(a) Given the following LinkedList program, complete the method named display(). The method named display() will print every data value in the list and then halt. The first value on the list is printed first, then the second, and so on. Place your Java programming logic in the method named display(). (4 Points)

```
class Node {
        String data;
        Node link;
        public Node(String value, Node next) {
                data = value;
                link = next;
        }
}
public class LinkedList {

    public static void display(Node ptr) {
         while (ptr != null) {
              System.out.println(ptr.data);
              ptr = ptr.next;
                                  link
         }

    }
    public static void main(String[] args) {
        Node head = new Node("Sam",null);
        head = new Node("Summer",head);
        head = new Node("Scout", head);
        head = new Node("Peanut", head);
        display(head);
    }
}
```



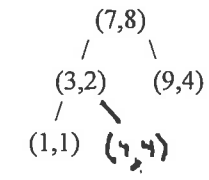
(b) What is an appropriate pre-condition for the method named display() in question 9 (a) ? (1 Point)

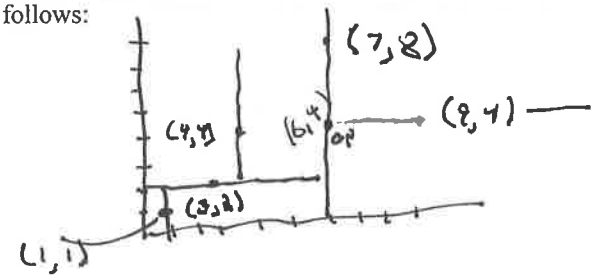
// Pre: ptr points to a null terminated list.

(c) What will be the output of the completed program in 9 (a)? This is the program with your completed display() method. (2 Points)

Peanut Scout Summer Sam

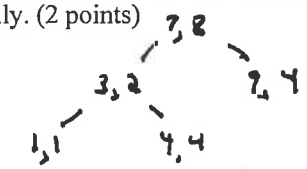
(d) The following points, in a standard (x,y) coordinate plane, have been added to a 2-d tree. (7,8), (3,2), (9,4), (1,1). The 2-d tree appears as follows:





KEY

Add the point (4,4) to this 2-d tree. Redraw the tree with this new point added. The first point, (7,8), breaks the plane vertically. (2 points)



(e) Consider the 2-d tree that you created, with the addition of (4,4), in (9.d). Suppose that we performed a nearest neighbor search for the query point (6,4). Would we need to compute the distance between (6,4) and (9,4)? Circle YES or NO. (2 Points)

(f) In Project 3 we wrote a Red Black binary search tree. Suppose we are doing an insert of a course name into a Red Black Tree. Let T(n) be the number of operations required to do the insert. In the worst case, which of the following are true about T(n)? Circle all of those that are true. (You may or may not have more than one answer.) (5 Points)

$T(n) \in \Theta(\log N)$

-1 for each error, max off 5

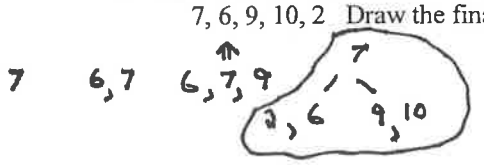
1. $T(n) \in O(1)$
2. $T(n) \in \Omega(1)$
3. $T(n) \in \Omega(n^2)$
4. $T(n) \in \Theta(\log n)$
5. $T(n) \in O(2^n)$
6. $T(n) \in \Theta(n)$
7. $T(n) \in O(n!)$
8. $T(n) \in O(\log(n))$
9. $T(n) \in O(n)$ Hist

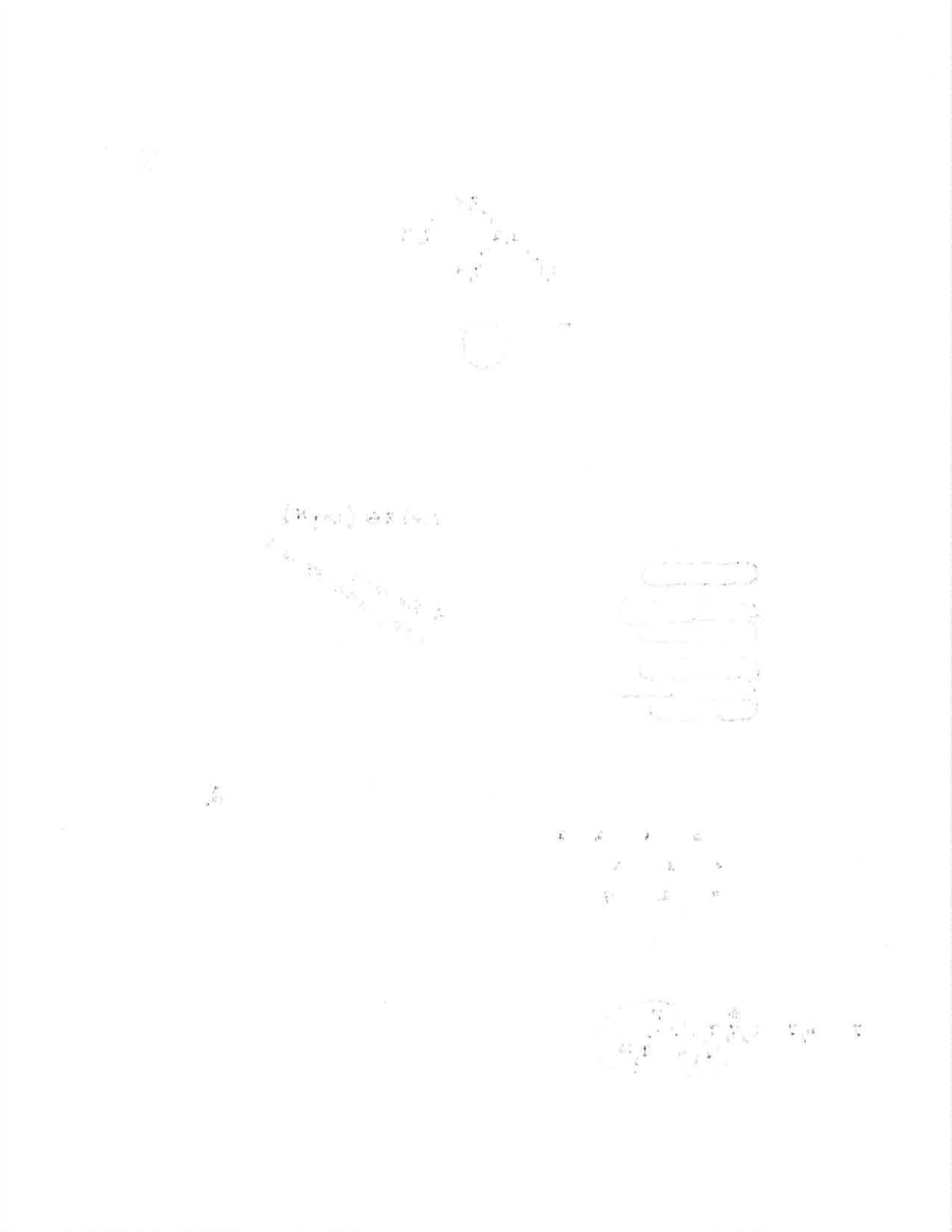
(g)The following is a data file for Project 3. Note the course Hist3 that is taken by Sue. The purpose of the Red-black tree was to maintain an integer with each course name. What integer will be assigned to Sue's Hist3 in the Red-Black tree of Project 3? The course numbers begin at 0. (4 Points) 2

	0	1	2	3
Amy	Calc1	Span1	Hist3	Philo2
Bill	Calc1	Philo2	Hist3	
Sue	Calc1		Hist3	Chem1
	0		2	4

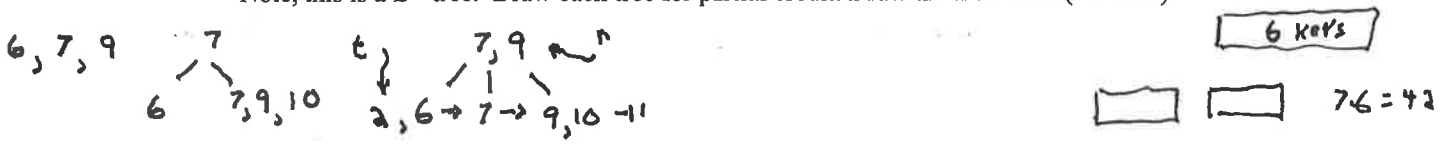
B Trees (21 points) ✓

10. (a) Insert the following numbers into a B-Tree with a minimum of 1.
7, 6, 9, 10, 2 Draw the final tree. (7 Points)





(b) Insert the following numbers into a B+ Tree with a minimum of 1.
7, 6, 9, 10, 2
Note, this is a B+ tree. Draw each tree for partial credit. Draw the final tree. (7 Points)



(c) Consider a B-Tree with a minimum of 3. What is the exact maximum number of keys such a tree could hold if the tree were of height 1? 48 (7 Points)

42 + 6 = 48

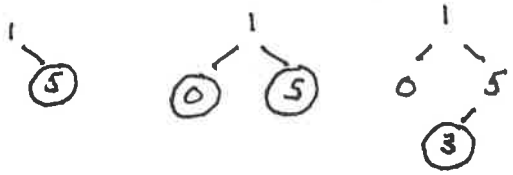
Red Black Trees (8 points) ✓

-3 IF NOT connect

11. Red Black Trees

(a) Insert the following numbers, one by one, into a Red-Black Tree. Show the tree after each insertion. Draw RED nodes with a circle or a label 'R'. (8 points)

1, 5, 0, 3



-3 IF WRONG

Graph Algorithms (26 points)

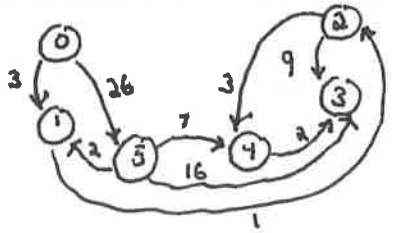
Consider the weighted, directed graph G_1. The graph is represented by an adjacency matrix m. If there is an edge from i to j with weight k then m[i,j] = k.

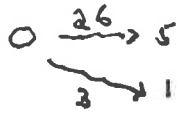
Matrix m

vertex	0	1	2	3	4	5
0		3				26
1			1			
2				9	3	
3						
4				2		
5		2		16	7	

G_1

12. (a) Draw the graph G_1 with circles and edges. (2 Points)





12. (b) What is the shortest path from the start node 0 to node 3 in the graph G_1? Your path must be a list of ordered pairs. (2 points) (0,1), (1,2), (2,4), (4,3)

key

Key

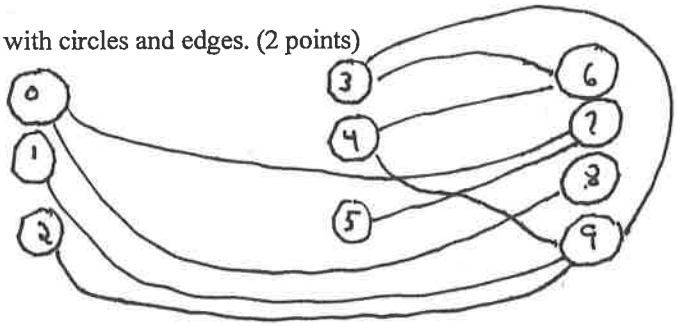
Consider the undirected graph G_2. The graph is represented by an adjacency matrix n. If vertex i shares an edge with vertex j then n[i,j] = T.

Matrix n

Vertex	0	1	2	3	4	5	6	7	8	9
0								T	T	
1										T
2										T
3							T			T
4							T			T
5								T		
6				T	T					
7	T					T				
8	T									
9		T	T	T	T					

G_2

12. (c) Draw the G_2 graph with circles and edges. (2 points)



12. (d) Show the list of nodes that would be visited by a breadth first search in the graph G_2. We are starting from vertex 0. (3 points) 0, {1, 8}, 5,

↑ any order

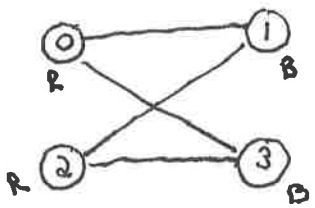
Consider the undirected graph G_3. The graph is represented by an adjacency matrix o. If vertex i shares an edge with vertex j then o[i,j] = T.

Martrix o

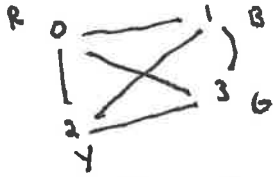
vertex	0	1	2	3
0		T		T
1	T		T	
2		T		T
3	T		T	

G_3

12. (e) What is the minimum number of colors that we could color G_3 with? (3 Points) 2



Key



Key

Consider the undirected graph G_4. The graph is represented by an adjacency matrix p. If vertex i shares an edge with vertex j then p[i,j] = T.

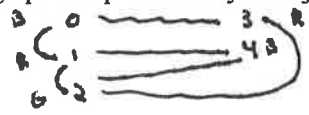
Martrix p

vertex	0	1	2	3
0		T	T	T
1	T		T	T
2	T	T		T
3	T	T	T	

G_4

12. (f) What is the minimum number of colors that we could color G_4 with ? (3 Points) 4

Consider the undirected graph G_5. The graph is represented by an adjacency matrix q. If vertex i shares an edge with vertex j then q[i,j] = T.



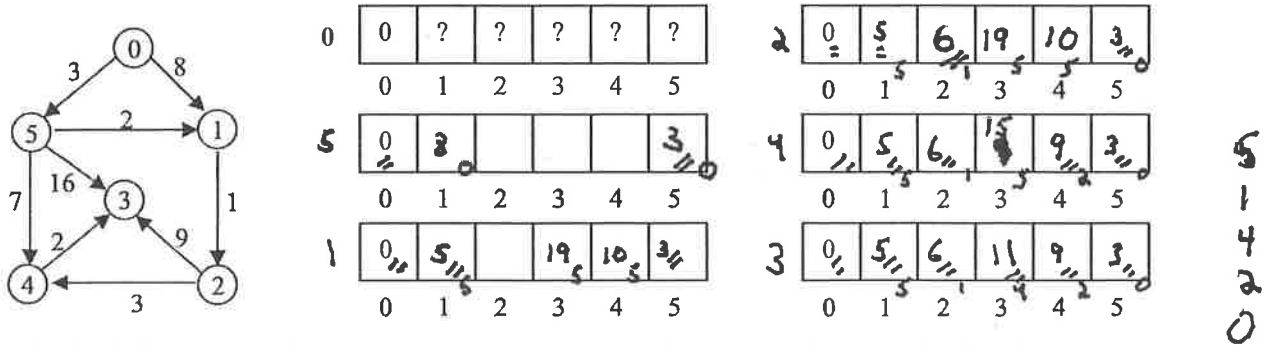
Martrix q

Vertex	0	1	2	3	4
0		T		T	
1	T		T		T
2		T		T	T
3	T		T		
4		T	T		

G_5

12. (g) What is the minimum number of colors that we could color G_5 with ? (3 Points) 3

12. (h) Draw the contents of the distance array for each iteration of Dijkstra's Algorithm as it works on this graph. The initial state is given. **Mark the node to be selected next to the left of the array (note how 0 is marked to the left of the first array.)** Fill in each array cell working downward. That is, complete the first column of arrays before the second column of arrays. (6 Points)



5 1 4 2 0

(d) After Dijkstra is complete it also collects parent pointers for each node in the graph. Complete the chart below showing the parent of each node as computed by Dijkstra. (2 points)

Node	Parent
0	Nil
1	5
2	1
3	4
4	2
5	0

k=2