Key

Coding Lists and Trees (18 points)

1. The following program contains two println() statements. You will be asked to describe the output of the program when each of the println() statements is executed. You will also be asked to describe the run time complexity Big Θ of two methods. See below. (8 points):

```
class Node {
   private int data;
   private Node next;

   public Node(int data, Node next) {
      this.data = data;
      this.next = next;
   }

   public int getData() {
      return data;
   }

   public void setData(int data) {
      this.data = data;
   }

   public Node getNext() {
      return next;
   }

   public void setNext(Node next) {
      this.next = next;
   }
}
public class DSAMidtermProject {

  private Node list = null;

  public void push(int x) {
     list = new Node(x,list);
  }

  public int pop() {
     int x = list.getData();
     list = list.getNext();
     return x;
  }

  public boolean isEmpty() {
     return list == null;
  }
```

```
  public String toString() {
     String v = "";
     Node c = list;
     while (c != null) {
        v = v + c.getData();
        if(c.getNext()!= null) v = v + "->";
        else v = v + "--||";
        c = c.getNext();
     }
     return v;
  }

  public int retieveTheValue() {
     DSAMidtermProject temp = new DSAMidtermProject();
     int x = 1000000;
     while(!this.isEmpty()) {
        int y = this.pop();
        if(y < x) x = y;
        temp.push(y);
     }
     while(!temp.isEmpty()) {
        this.push(temp.pop());
     }
     return x;
  }

  public static void main(String args[]) {

     DSAMidtermProject dsa = new DSAMidtermProject();

     for (int j = 10; j < 15; j++) {
        dsa.push(j);
     }
     dsa.push(2);
     dsa.push(10);
     dsa.push(100);

     // 1.a
     System.out.println(dsa.retieveTheValue());

     // 1.b
     System.out.println(dsa);

  }
}
```

1.a What will the program display at the println marked 1.a? 2

1.b What will the program display at the println marked 1.b? 100 -> 10 -> 2 -> 14 -> 13 -> 12 -> 11 -> 10 -11

1.c What is the run time complexity (Big Θ) of retieveTheValue()? Θ(n)

1.d Is it correct to say that the toString() method runs in $O(2^n)$? Circle True or False

2. Study the execution of the following program. Questions appear below. (10 points):

```
class Node {
   public int data;
   public Node lc;
   public Node rc;
   public Node(Node lc, int x, Node rc) {
      this.lc = lc;
      this.data = x;
      this.rc = rc;
   }
}
public class SimpleTree {

   public Node root;
   public int ctr;

   public SimpleTree() {
      root = null;
      ctr = 0;
   }

   private Node add(Node t){

      if (t == null) {
         ctr = ctr + 1;
         return new Node(null,ctr,null);
      }
      t.lc = add(t.lc);
      t.rc = add(t.rc);
      return t;
   }

   public void add() {
      if (root == null) {
         ctr = ctr + 1;
         root = new Node(null,ctr,null);
      }
      else {
         add(root);
      }
   }
```

```
  public void traversal(Node t) {

    if(t != null) {
      traversal(t.lc);
      System.out.print(t.data + ":");
      traversal(t.rc);
    }
  }
  public void traversal() {
    traversal(root);
  }

  public static void main(String[] args) {
    SimpleTree st = new SimpleTree();
    st.add();
    st.add();

    // Question 2.a
    st.traversal();
    System.out.println();

    SimpleTree coolTree = new SimpleTree();
    coolTree.add();
    coolTree.add();
    coolTree.add();

    // Question 2.b
    coolTree.traversal();

  }
}
```

2.a What will the program display at the traversal marked Question 2.a?

2: 1: 3:

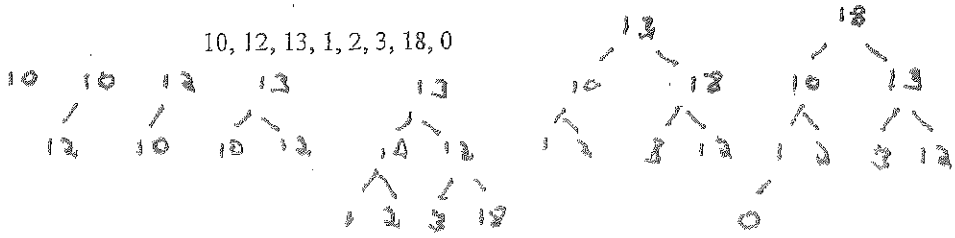
2.b What will the program display at the traversal marked Question 2.b?

4: 2: 5: 1: 6: 3: 7:

Heaps (12 points)

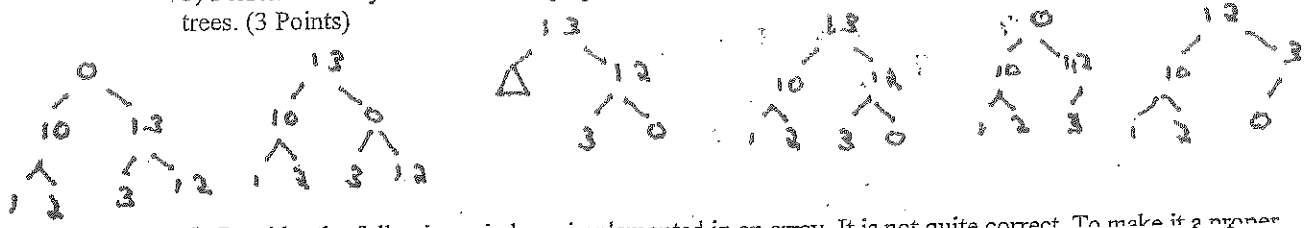
3) Insert the following 8 numbers into a max heap. Draw a new tree for each heap insertion. (4 Points)

10, 12, 13, 1, 2, 3, 18, 0



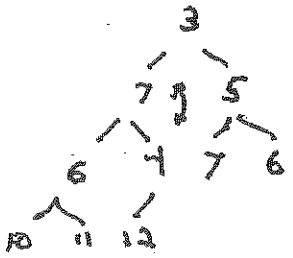
4) What is the height of the tree that you drew in question 3? (2 Points) 3

5) Perform exactly two deleteMax() operations on the heap that you drew in question 3. Draw the resulting trees. (3 Points)



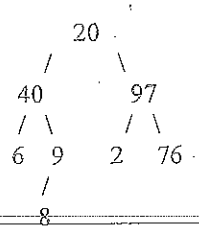
6) Consider the following min heap implemented in an array. It is not quite correct. To make it a proper min heap exactly one swap must occur. What two numbers (child and parent) need to be swapped in order to make this a min heap? (3 points). PLACE CHECK MARKS NEXT TO THE TWO NUMBERS THAT NEED SWAPPED.

3
7 ✓
5
6
4 ✓
7
6
10
11
12



Binary Trees (16 points)

7. Parts (a), (b), (c) refer to the following binary tree:



(a) List the data that would be accessed by a pre-order traversal on the given tree by writing out the values in the nodes as they would be accessed, separated by commas. (3 points)

20, 40, 6, 9, 8, 97, 2, 76

(b) List the data that would be accessed by an in-order traversal on the given tree by writing out the values in the nodes as they would be accessed, separated by commas. (2 points)

6, 40, 8, 9, 20, 2, 97, 76

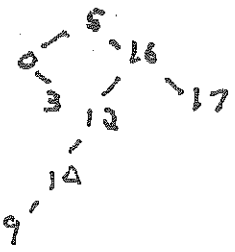
(c) List the data that would be accessed by a level-order traversal on the given tree by writing out the values in the nodes as they would be accessed, separated by commas. (2 points)

20, 40, 97, 6, 9, 2, 76, 8

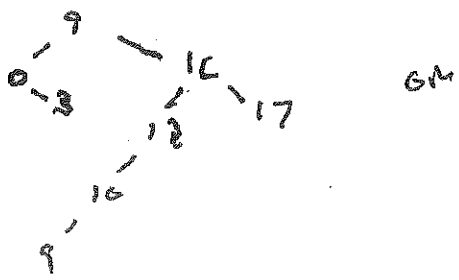
(d) In general, if a ternary (at most three children per node) tree is perfectly balanced (unlike the tree pictured here) and complete with height h, how many leaves, in terms of h, will the tree have? (2 points) 3^h Note, this tree has a perfectly flat bottom.

(e) In general, if a binary tree is perfectly balanced (unlike the tree pictured here) and complete with exactly k leaves. What is the height (in terms of k) of this tree? (2 points) $\log_2 K$ Note, this tree has a perfectly flat bottom.

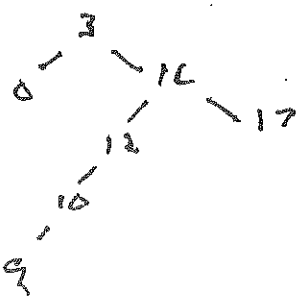
8. (a) Insert he following numbers into a Binary Search Tree. Draw the tree after all insertions are complete. (3 Points)
5,16,17,12,10,9,0,3



(b) Delete 5 from the final tree that you drew in 8 (a). Draw this final tree. (2 Points)
There are two possible answers. Either one is fine.



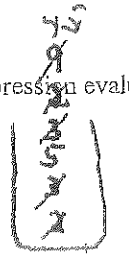
GM

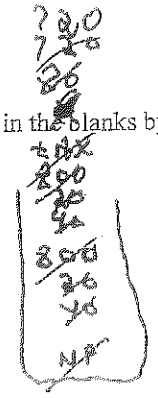


Project Questions (20 points)

(9) (a) In Project 1, we wrote an RPN expression evaluator. Fill in the blanks by evaluating each expression (8 points):

2 3 + 3 2 ^ *
35
tax .10 =
.10
netPay 40 20 * 40 20 * tax * - =
720





(b) In Project 2 we wrote an implementation of Shunting Yard. Show the content of the stack and the output as Shunting Yard processes the following expression. Draw pictures so that it is clear to the reader that you understand the algorithm. (8 points)

3 + 4 * 2 / (1 - 5) ^ 2 ^ 3

3 4 2 * 15- 2 3 ^ ^ / +

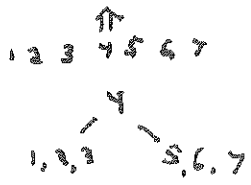
(c) In Project 3 we wrote a binary search tree containing lists of crime records. It was organized with the crime address being the key in each tree node.

Circle all of those that are true. (You may or may not have more than one answer.) (4 Points)

In the best case, a lookup on an address is:

(1) O(LogN)
(2) O(1)
3. $\Omega(N^2)$
(4) O(N)
5. Θ(LogN)
(6) $O(2^n)$
(7) Ω(1)

B Trees (21 points)



10. a) Insert the following numbers into a B-Tree with a minimum of 3. 1,2,3,4,5,6,7. Draw the final tree. (5 Points)

b) Insert the following numbers into a B+ Tree with a minimum of 3. 1,2,3,4,5,6,7. Draw the final tree. (5 Points)

c) Consider a B-Tree with a minimum of 50. What is the exact maximum number of keys such a tree could hold if the tree were of height 1? 10,200 (5 Points)

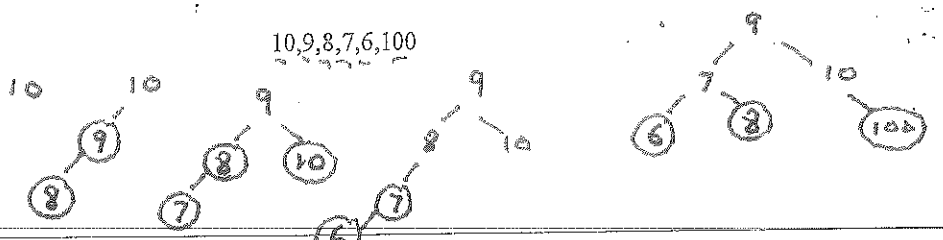
d) Suppose we have a B-Tree with a minimum of 50 stored on disk. Suppose too that we have 10,000 keys stored in the tree. How many disk accesses (in the worst case) would we need to find a given key? (Hint: solve 10 c. before doing 10 d.). 2 (6 Points)

Red Black Trees (8 points)

11. Red Black Trees

(a) Insert the following numbers, one by one, into a Red-Black Tree. Show the tree after each insertion. Draw RED nodes with a circle and BLACK nodes with a rectangle. (5 points)

10,9,8,7,6,100



(b) When trying to analyze the run time complexity of an inorder traversal of a Red Black tree, one should consider the worst case, average case and best case separately. Each case may have a different run time performance. Circle TRUE or FALSE (1 point)

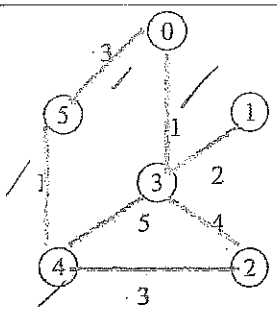
(c) What is the best-case runtime complexity of a Red Black Tree insert operation? Use Big Theta notation. (2 points) $\Theta(\log N)$ or $\Theta(\log_2 N)$

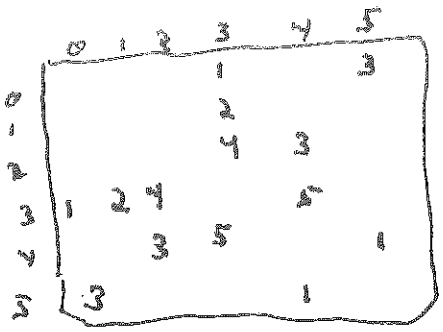
Graphs Representations (25 points)

13. (a) Consider the weighted and undirected graph below. When starting at vertex 0, breadth first search (BFS) could be used to enumerate (visit) each of the vertices. Show an ordering that BFS might produce. Starting from vertex 0, visit all the nodes. List the nodes as BFS would visit.
(5 points) 0, 5, 3, 1, 4, 2 — ANY ORDER
3, 5

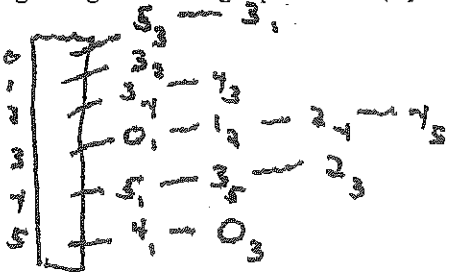
(b) If we want to learn if a path exists between vertex A and vertex B, we can simply run either BFS or DFS to find out. Circle True or False. (3 points)

(c) Show an adjacency matrix representation of the following graph (10 points)





(d) Draw an adjacency list representation of the graph shown immediately above. You need to think about how you will represent the distances associated with the edges. The picture that you draw will make it clear how the edge weights are being represented. (5 points)



(e) The graph shown above is a simple, undirected graph. It contains no loops or multiple edges. Suppose instead that we have a simple directed graph G with no loops or multiple edges. Suppose too that G has V vertices. What is the maximum number of edges that such a graph can contain (in terms of V)? Your answer should be an exact formula. Answers in Big Theta notation don't count. (2 point) V·(V-1)

V	edges
1	0
2	2
3	6
4	12
V	V·(V-1)