Carnegie Mellon University in Qatar
15415 - Spring 2018

Recitation 8

1 Practicing B^+ Tree Insertions

1. Consider a B^+ tree of order **2**.
   (a) How many maximum number of keys can we have in a single node? ________________
   (b) What is the least number of keys we can have in a root node? ________________
   (c) What is the least number of keys we can have in a non-root node? ________________
   (d) What is the maximum number of pointers for a non-leaf page? ________________
   (e) Starting from an empty B^+ , insert the following keys in the same order as shown (*we need not show the tree at each step; just the final one*):

   `15, 21, 13, 30, 42, 50`

(f) For each of the following sub-questions, we will be doing some more insertions **on the resultant tree from question 1**.

i. Insert key `60`.
ii. Insert key `70`.
iii. Insert key `28`.
iv. Insert key `80`.

We will be doing each of these insertions in two ways (show the tree after each step):

1. Use **only splitting** of nodes (if necessary).

2. **Try to redistribute** with neighbors if possible, otherwise, split (if necessary).

   Sub-questions repeated below for reference:

   i. Insert key `60`.
   ii. Insert key `70`.
   iii. Insert key `28`.
   iv. Insert key `80`.

2. What is the **minimum number of insertions** that we must perform on the *resultant tree from question 1* to increase the height of the tree by **1**?

2 B^+ Tree Deletions

We have seen how to look-up and insert keys into a B^+ tree. We will now look at how to perform **deletions**.

To delete key ***K*** from a B^+ tree:

- Start at the root, and find the leaf ***L*** where entry K belongs to.
- Remove the entry.
- If ***L*** is at least half-full, we are done!
- If ***L*** underflows:
  - Try to **re-distribute** (i.e., borrow from a "rich sibling" and "copy up" its *lowest key*).
  - If **re-distribution** fails, ***merge L*** and a "poor sibling."
    - Update parent.
    - And possibly, merge *recursively*.

2.1 Examples

Suppose we have the following B^+ tree with order 2.

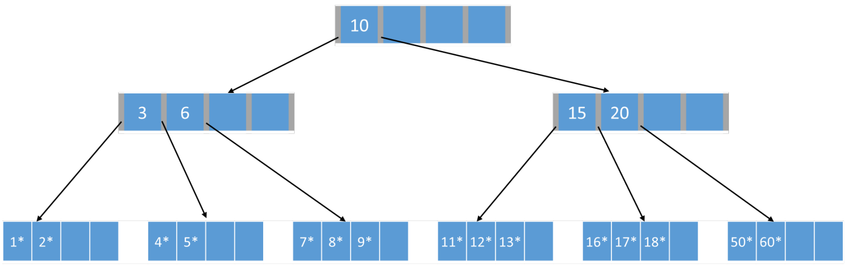


Figure 1: Our initial B^+ tree

1. Deleting key `18` from the original tree (Figure 1) will result in the following:

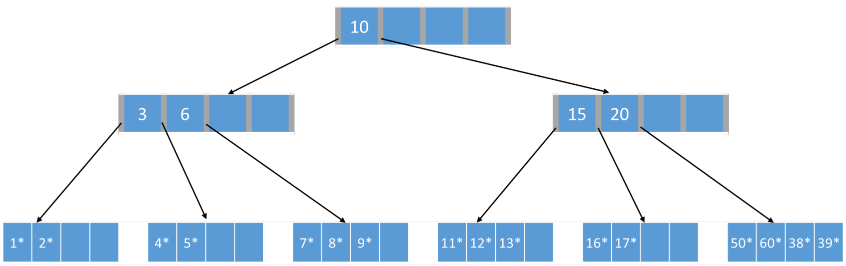


Figure 2: Our resulting B^+ tree after deleting `18` from Figure 1

We simply found our way to the correct leaf, and removed the key.

2. Deleting key 5 from the resultant tree (Figure 2) will result in the following:

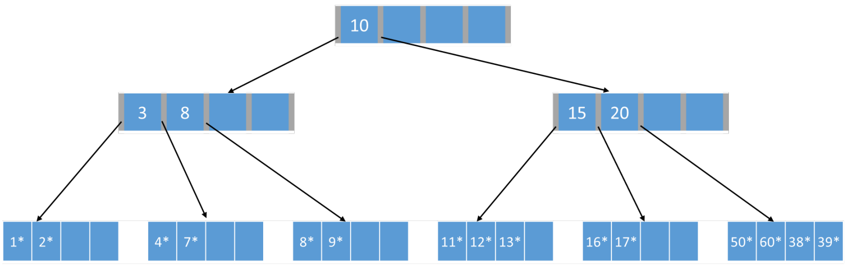


Figure 3: Our resulting B^+ tree after deleting 5 from Figure 2

This was a bit more involved than in the previous example. Here's what happened:

- We found are leaf where **k** resides (leaf with keys 4 and 5).
- We deleted key 5.
- This resulted in an underflow! Our leaf now has less than the **d** keys. We fix it by:
  - Redistribution: checking if we have a 'rich neighbor' we can borrow from. Indeed, if we check our right neighbor, we can borrow an entry. Therefore, we move 7 to our leaf.
  - Last step: we need to 'copy up' the lowest value in the leaf from which we borrowed from. In this case, this value is 8.

3. Deleting key 9 from the resultant tree (Figure 3) will result in the following:

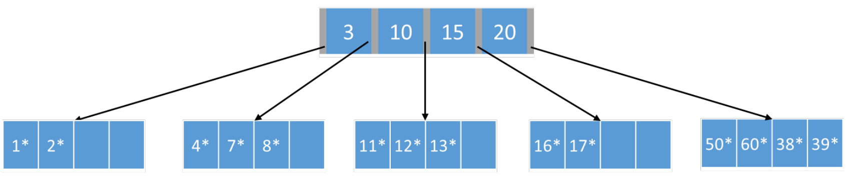


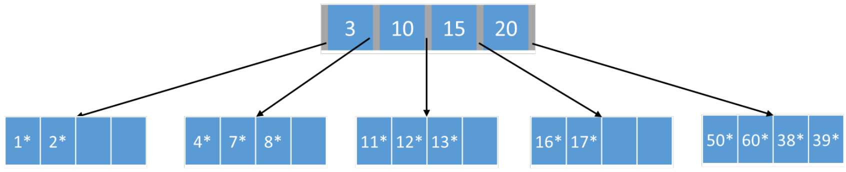
Figure 4: Our resulting B^+ tree after deleting 9 from Figure 3

This was a bit further involved than in the previous example. Here's what happened:

- We found are leaf where **k** resides (leaf with keys 7, 8 and 9).
- We deleted key 9.
- This resulted in an underflow! Our leaf now has less than the **d** keys. We fix it by:
  - Redistribution: checking if we have a 'rich neighbor' we can borrow from. However, we don't have any neighbors, so we must 'merge'!
  - We merge this leaf with the previous one:
    * 8 is merged with the the leaf containing 4 and 7.
    * The 8 from the parent is 'tossed' as the page it points to doesn't exist anymore.
    * Now, we have a new problem; tossing 8 from the parent resulted in another underflow! We fix this by doing exactly the steps we did when we removed 9:
      · Try to redistribute (doesn't work here! The only other neighbor is the node with 15 and 20, which is 'poor').
      · Merge again!

2.2 Practicing B^+ Tree deletions

Suppose we have the following B^+ tree with order 2.



1. Delete the entry with key `2`.

2. Delete the entry with key `1`.