

# 15-122: Principles of Imperative Computation, Spring 2024

## Written Homework 6

Due on [Gradescope](#): Monday 19<sup>th</sup> February, 2024 by 9pm

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This written homework covers doubly-linked lists.

**Preparing your Submission** You can prepare your submission with any PDF editor that you like. Here are a few that prior-semester students recommended:

- [Kami](#), [Adobe Acrobat Online](#), or [DocHub](#), some web-based PDF editors that work from anywhere.
- *Acrobat Pro*, installed on all [non-CS cluster machines](#), works on many platforms.
- *iAnnotate* works on any iOS and Android mobile device.

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**Please do not add, remove or reorder pages.**

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Question:	1	2	3	Total
Points:	4	5	6	15
Score:				

## 1. Multiple Return Values

One of the difficulties that both structs and pointers can solve in different ways is the problem of returning more than one piece of information from a function. For instance, a function that tries to parse a string as an integer needs to return both the successfully-parsed integer and information about whether that parse succeeded. Because *any* number could be the result of a successful parse, if the function only returns an **int**, there's no way to distinguish a failed parse from a successful one.

```
int parse_int(string str) {  
    int k;  
    bool parse_successful;  
    ...  
    if (parse_successful) return k;  
    return ???; /* What do we do now? */  
}
```

In each of the following exercises, a main function wants to print the value that `parse_int` is storing in the variable `k`, but *only* when the boolean value stored in `parse_successful` is true; otherwise we want to print out *"Parse error"*.

You don't have to use all the blank lines we have provided, but you shouldn't use any extra lines. **Double-check your syntax; we will be picky about syntax errors for this question.**

2pts

1.1 Finish this program so that the code will parse the first command-line argument as an **int** if possible. Make sure all your pointer dereferences are provably safe.

```
int* parse_int(string str) {
    int k;
    bool parse_successful;
    // Omitted code that tries to parse the string. It puts
    // the result in the local variable k and sets
    // parse_successful to true if it can, otherwise sets
    // parse_successful to false.

    if (parse_successful) {
        _____;
        _____;

        return _____;
    }
    return _____;
}

int main() {
    args_t A = args_parse();
    if (A->argc != 1) error("Wrong number of arguments");
    int* k_ptr = parse_int(A->argv[0]);

    if ( _____ ) printf("%d\n", _____ );
    else error("Parse error");
    return 0;
}
```

2pts

- 1.2 Complete another program that works the same way, but that calls `parse_int` with an additional argument. This argument should have pointer type. Make sure all your pointer dereferences are provably safe.

```
bool parse_int(string str, _____)
//@requires _____;
{
    int k;
    bool parse_successful;
    // Same omitted code...

    if (parse_successful) {
        _____;

        return _____;
    }
    return _____;
}

int main() {
    args_t A = args_parse();
    if (A->argc != 1) error("Wrong number of arguments");

    _____;

    bool res = parse_int(A->argv[0], _____);

    if ( _____ ) printf("%d\n", _____);
    else error("Parse error");
    return 0;
}
```

## 2. Reasoning with Linked Lists

You are given the following C0 type definitions for a linked list of integers.

```
typedef struct list_node list;
struct list_node {
    int data;
    list* next;
};

struct segment_header {
    list* start;
    list* end;
};
typedef struct segment_header list_segment;
```

An empty list consists of one dummy `list_node`. All lists have one additional node (the dummy) at the end that does not contain any relevant data, as discussed in class.

In this task, we ask you to analyze a list function and reason that each pointer access is safe. You will do this by indicating the line(s) in the code that you can use to conclude that the access is safe. Your analysis must be precise and minimal: mention only the line(s) upon which the safety of a pointer dereference depends. If a line does not include a pointer dereference, indicate this by writing `NONE` after the line in the space provided. As an example, we show the analysis for an `is_segment` function below.

```
1 bool is_segment(list* s, list* e) {
2     if (s == NULL) return false;           // NONE
3     if (e == NULL) return false;         // NONE
4     if (!is_acyclic(s)) return false;    // NONE
5     if (s->next == e) return true;       // 2
6     list* c = s;                         // NONE
7     while (c != e && c != NULL) {        // NONE
8         c = c->next;                     // 7
9     }                                     // NONE
10    if (c == NULL)                        // NONE
11        return false;                    // NONE
12    return true;                           // NONE
13 }
```

When we reason that a pointer dereference is safe, the argument applies *only* to that dereference. So, in the example below, we have to use line 31 to prove both line 32 and line 33 safe.

```
31 // @assert is_segment(a, b); // ASSUME VALID
32 a->next = b;
33 list* l = a->next;
```

We don't allow you to say that, because line 32 didn't raise an error, `a` must not be `NULL` and therefore line 33 must be safe.

Here's a mystery function:

```

67 void mystery(list_segment* a, list_segment* b)
68 //@requires a != NULL;           // NONE
69 //@requires b != NULL;           // NONE
70 //@requires is_segment(a->start, a->end); // _____
71 //@requires is_segment(b->start, b->end); // _____
72 {
73     list* t1 = a->start;           // _____
74     list* t2 = b->start;           // _____
75     while (t1 != a->end && t2 != b->end) // _____
76         //@loop_invariant is_segment(t1, a->end); // _____
77         //@loop_invariant is_segment(t2, b->end); // _____
78     {
79         list* t = t2;             // _____
80         t2 = t2->next;            // _____
81         t->next = t1->next;        // _____
82         t1->next = t;             // _____
83         t1 = t1->next->next;       // _____
84     }
85     b->start = t2;                // _____
86 }

```

You can use the blanks on the right for scratchwork — they are not graded.

1pt

2.1 Explain when line 75 is safe: first, clearly state what the conditions for the safety of line 75 are, and then write down the line numbers that support each of them — *only cite the necessary line numbers*.

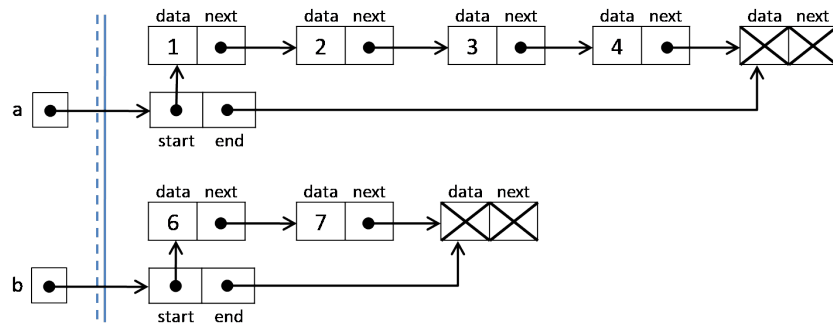
1pt

2.2 Why can we not use the combination of lines 68 (which tells us that a is not NULL), 70 (which tells us that a->start is not NULL) and 73 (which tells us that t1 is a->start) to reason that t1 is not NULL and therefore that line 82 is safe?

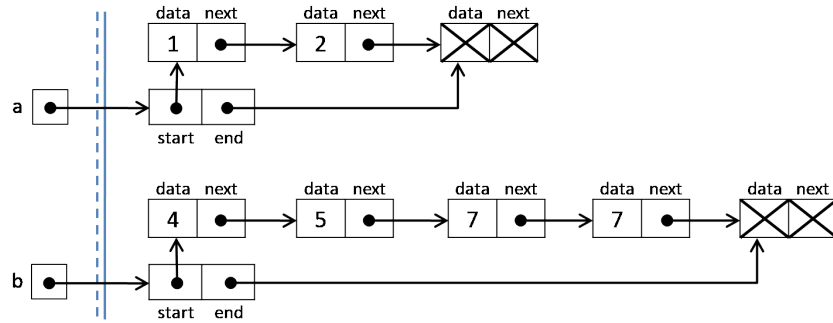
Why do we actually know line 82 is safe?

1.5pts

2.3 Let  $a$  and  $b$  be linked lists with  $m$  and  $n$  data values in them, respectively. For each of the pictures below, draw the final state of the lists after  $\text{mystery}(a, b)$  returns. Put your drawings in the boxes.



*Result:*



*Result:*

What is the final length of linked list  $a$  when

•  $m \geq n$

•  $m < n$

1pt

2.4 Complete the function below that removes the maximum integer from a non-empty linked list of integers. The specification function `gt_listseg(x,s,e)` checks that `x` is strictly larger than every element in the list segment between `s` inclusive and `e` exclusive. You may assume there are no duplicate elements.

```

int remove_max(list_segment* a)

    //@requires _____;

    //@requires _____; // a not empty
    //@requires is_segment(a->start, a->end);
    //@ensures is_segment(a->start, a->end);
    //@ensures gt_listseg(\result, a->start, a->end);
{
    list* first = a->start;
    list* curr = first->next;
    list* prev = first;
    list* max = first;
    list* max_prev = first;

    while ( _____ )
    //@loop_invariant prev->next == curr;
    {
        if (curr->data > max->data) {

            max_prev = _____;

            max = _____;
        }
        prev = _____;

        curr = _____;
    }
    if (max == max_prev)

        _____;

    else _____;
    return max->data;
}

```



0.5pts

- 2.5 The second postcondition of this function is **not** strong enough. Write an alternative function body and provide a non-empty input list segment that causes a contract exploit: the postconditions hold true but it does not remove and return the maximum integer from this linked list.

**Alternative body:**

```
int remove_max(list_segment* a)
// SAME PRECONDITIONS AS ON THE PREVIOUS PAGE
//@ensures is_segment(a->start, a->end);
//@ensures gt_listseg(\result, a->start, a->end);
{

}
}
```

**Input list** (*add as many nodes as you need*):



### 3. Doubly-Linked Lists

Consider the following interface for stacks that store elements of the type `string`:

```
// typedef _____* stack_t;

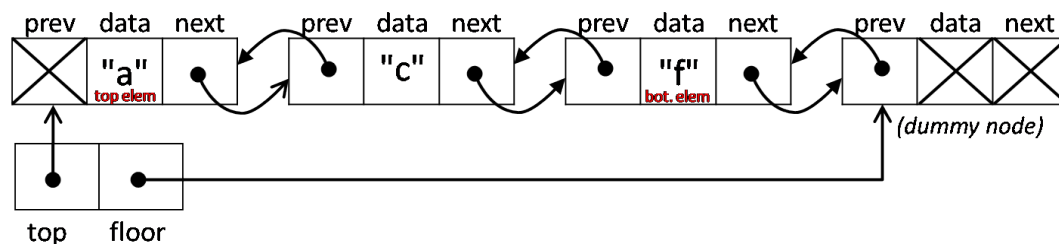
bool stack_empty(stack_t S)           /* 0(1) */
/*@requires S != NULL; @*/ ;

stack_t stack_new()                   /* 0(1) */
/*@ensures \result != NULL; @*/
/*@ensures stack_empty(\result); @*/ ;

void push(stack_t S, string x)        /* 0(1) */
/*@requires S != NULL; @*/
/*@ensures !stack_empty(S); @*/ ;

string pop(stack_t S)                 /* 0(1) */
/*@requires S != NULL; @*/
/*@requires !stack_empty(S); @*/ ;
```

Suppose we decide to implement the stack (of `string`'s) using a doubly-linked list so that each list node contains two pointers, one to the next node in the list and one to the previous (`prev`) node in the list:



```
typedef struct list_node list;
struct list_node {
    string data;
    list* prev;
    list* next;
};

typedef struct stack_header stack;
struct stack_header {
    list* top;
    list* floor; // points to dummy node
};
```

The top element of the stack (if any) will be stored in the first node of the list (pointed to by `top`), and the bottom element of the stack (if any) will be stored in the second-to-last node in the list, with the last node being a “dummy node” (pointed to by `floor`). Intuitively, the bottom element sits on the `floor`.

An empty stack consists of a dummy node only: the `prev`, `data`, and `next` fields of that dummy are all unspecified. A non-empty stack has an unspecified `prev` field for the top, and an unspecified `data` and `next` field for the dummy node.

2pts

3.1 Modify the singly-linked list implementation of stacks below to work with the doubly-linked list representation given above. For each function,

- **either** give the modification(s) that need to be made (e.g. “Insert the statement XXXX after line Y”, “Remove line Z”, “Change line Z to XXXX”, etc),
- **or** write “No change needs to be made” if the original code works for the doubly-linked list implementation.

Do not write to unspecified values unless strictly necessary. You may assume there is an appropriate `is_stack` specification function already defined. Be sure that your modifications still maintain the  $O(1)$  requirement for the stack operations.

```

27 stack* stack_new()
28 // @ensures is_stack(\result);
29 // @ensures stack_empty(\result);
30 {
31     stack* S = alloc(stack);
32     list* L = alloc(list);
33     S->top = L;
34     S->floor = L;
35     return S;
36 }

```

```

38 bool stack_empty(stack* S)
39 // @requires is_stack(S);
40 {
41     return S->top == S->floor;
42 }

```

```

44 void push(stack* S, string x)
45 // @requires is_stack(S);
46 // @ensures is_stack(S);
47 {
48     list* L = alloc(list);
49     L->data = x;
50     L->next = S->top;
51     S->top = L;
52 }

```

```

54 string pop(stack* S)
55 // @requires is_stack(S);
56 // @requires !stack_empty(S);
57 // @ensures is_stack(S);
58 {
59     string e = S->top->data;
60     S->top = S->top->next;
61     return e;
62 }

```

We wish to add a new operation, `stack_bottom`, to our stack **implementation** from the previous part. This operation returns (but does not remove) the bottom element of the stack. It does not modify the input stack at any point. Here's its interface prototype:

```
string stack_bottom(stack_t S)
/*@requires S != NULL && !stack_empty(S); @*/ ;
```

0.5pts

- 3.2 Write this function using the **doubly-linked list** implementation of stacks from the previous part. Be sure that your function is as efficient as possible. (*Remember that the linked list that represents the stack has a dummy node.*)

```
string stack_bottom(stack* S)
/*@requires is_stack(S) && !stack_empty(S);
{

}
}
```

1pt

- 3.3 Next, write the function using the **singly-linked list** implementation of stacks from lecture. (*Recall that the only difference is that these lists lack the `prev` field.*)

```
string stack_bottom(stack* S)
/*@requires is_stack(S) && !stack_empty(S);
{

}
}
```

0.5pts

- 3.4 What is the worst-case asymptotic complexity of each implementation assuming the input stack contains  $n$  elements?

Doubly-linked implementation:  $O(\underline{\hspace{2cm}})$   
 Singly-linked implementation:  $O(\underline{\hspace{2cm}})$

The rest of this question considers the updated implementation of stacks based on doubly-linked lists.

2pts

3.5 Now, consider a broken `is_stack` function for this new stack implementation based on doubly-linked lists.

```
bool is_segment(list* node1, list* node2) {
    if (node1 == NULL) return false;
    if (node1 == node2) return true;
    return is_segment(node1->next, node2);
}

bool is_stack(stack* S) {
    return S != NULL && is_segment(S->top, S->floor);
}
```

Draw a picture of a full stack data structure that

- uses string elements,
- contains at least 4 allocated `list_node` structs
- returns true from `is_stack` (it should pass the first assertion)

BUT fails the unit test below with a segfault or an assertion failure. **Don't use X anywhere:** give specific values for every field. Your diagram should depict pointers (possibly NULL) and string values.

Stack picture:

```
// Unit test that your example above should fail
int main() {
    stack* S = // Code that constructs the example above.
    assert(is_stack(S) && !stack_empty(S)); // This must pass
    string x = stack_bottom(S); // doubly-linked list implementation
    string y = pop(S);
    while (!stack_empty(S)) {
        y = pop(S);
        assert(is_stack(S));
    }
    assert(string_equal(x, y));
    return 0;
}
```