Endogenous Precision of Performance Measures and Limited Managerial Attention*

Pierre Jinghong Liang
Tepper School of Business
Carnegie Mellon University
liangj@andrew.cmu.edu

Lin Nan
Krannert School of Management
Purdue University
lnan@purdue.edu

September 2013

Abstract

In this paper, we model two drivers that underlie the economic trade-off that shareholders face in designing incentives for optimal effort allocation by managers. The first driver is the presence of a performance-reporting task, by which we mean managers' ability to exert effort to improve the precision (or quality) of their performance measures. The second is limited managerial attention, where performing one task may have an adverse effect on the cost-efficiency of performing another. We show that the subtle interactions of the two drivers may alter the characteristics of incentive provision. First, we show that the interaction may lead to a positive relation between the strength of the incentive and the variance of the performance measures. Second, the interaction may cause an informative performance signal to not be used in equilibrium incentive contracts. In particular, we show that it is possible that the principal will not use a signal whose precision can be improved by the manager in order to discourage the manager from diverting attention to the performance-reporting task. Finally, we apply the model to a specific project-selection setting and show that, in order to induce the agent to choose higher-risk, higher-return projects, the principal may need to raise the bonus rate when the choice of project is unobservable.

*We wish to thank Jon Glover for initial encouraging discussions. We thank Mark Bagnoli, Jeremy Bertomeu, John Christensen, Ron Dye, Bjorn Jorgensen, John O'Brien, Sri Sridhar, Susan Watts, Martin Wu, and other participants at the 2009 AAA Annual Meetings at New York City, the 2009 Carnegie Mellon University Accounting Conference, and seminars at Purdue University and Tsinghua University for their comments.

Endogenous Precision of Performance Measures and Limited Managerial Attention

1 Introduction

In a modern firm, a well-motivated management team has become a vital source of organizational success. One important component of designing managerial incentives is to assure optimal allocation of managerial effort over multiple tasks (see Roberts 2004, p. 140-153). Among the crucial tasks important to managers, the performance-reporting task (both internal and external) prompts particular interest from the press, policy makers, and the academic accounting profession. While many widely publicized cases have been negative (e.g., Enron and Worldcom), most managerial reporting efforts are legitimate and do generally improve the informativeness of reported firm performance. For example, academic studies have shown that managers may engage in earnings smoothing to improve the informativeness of their earnings about their firms' true performance. Managers also work on accruals quality by improving the precision and informativeness of the accounting accrual estimates.[1] As a result, the precision of measured performance is an endogenous variable and should play an important role in incentive designing.

At the same time, the performance-reporting task may compete for the limited managerial attention with other productive tasks. For example, managers are responsible for maintaining and improving internal control over financial reporting (ICFR). The Sarbanes–Oxley Act in 2002 (SOX), especially Section 404, demands significant attention from the management of public companies.[2] In this light, managers face a trade-off between productive efforts such as identifying real investment opportunities and "non-productive" effort such as performance-reporting tasks. This trade-off has received attention in the business press.[3] More broadly, limited attention is a widespread issue when managing large organizations. In a classic work, Herb Simon points out

> "... the scarce resource is not information; it is processing capacity to attend to information. Attention is the chief bottleneck in organizational activity, and the bottleneck becomes narrower and narrower as we move to the tops of organizations, where parallel processing capacity becomes less easy ..." (Simon 1973, 270).

In this paper, we formally model the two drivers underlying the economic trade-off in managerial effort allocation. The first driver is the presence of a performance-reporting task, by which we

[1] See, for example, Tucker and Zarowin 2006; Subramanyam 1996; Hunt, Moyer, and Shevlin 2000; and Francis, LaFond, Olsson, and Schipper 2005.

[2] In particular, "management should evaluate the design of the controls to determine whether they adequately address the risk that a material misstatement in the financial statements would not be prevented or detected in a timely manner. ... that the evaluation of evidence about the operation of controls should be based on assessments of the controls' associated risk." (KPMG 2007)

[3] In a testimony on Capital Hill in April 2005, SEC Chairman Donaldson commented that complying with SOX Section 404 has been time-consuming and expensive for most companies, as confirmed by surveys (see Stovall 2008). Sayther (2003) claims that compliance demands steal CFOs' focus and leave less time and fewer resources for strategic thinking; Stone (2005) reports comments by industry insiders that SOX is siphoning away CEO creativity and forces CEOs to worry more about compliance and losing their jobs than figuring out how to invest in growth for the future.

mean managers' ability to exert personally costly effort to improve the precision (or quality) of their performance measures. The second is limited managerial attention, where performing one task may have an adverse effect on the cost-efficiency of performing another. Roberts (2004) notes that the presence of multiple tasks and their competition for managerial attention are important factors when designing incentives to optimally motivate management (see 142–143). In our paper, we use these two drivers to analyze three specific and long-standing issues involving management control and motivation. First, how should the strength of the incentive relate to the precision of performance measures? Second, how should firms choose from among multiple performance measures? And lastly, how should firms design incentives to motivate managers to choose the desirable investment choice?

Along these three important questions in designing managerial motivations, the results of our model reflect the subtle interaction of the two drivers we identified. First, we show that, within an LEN setting, the strength of the incentives and the variance of the performance measures may be positively related. This contrasts common intuition, but may explain mixed empirical findings of the relation (see Prendergast 2002, Lafontaine and Bhattacharyya 1995, and others). This finding offers a multi-task-based rationale for explaining the mixed empirical results on the relation between the variance of performance measures and the equilibrium incentive strength. Second, the interaction may cause an informative performance signal to not be used in equilibrium incentive contracts; specifically, we show that it is possible that the principal will not use a signal whose precision can be improved by the manager in order to discourage the manager from diverting attention to the performance-reporting task. Our finding here offers a novel explanation of why informative signals are left unused, complementing other competing reasons such as incomplete contracts and subjective performance measures. Finally, we apply the model to a specific project-selection setting and show that the presence of the performance-reporting task by the agent causes the principal to raise the incentive strength in order to induce the agent to choose higher-risk, higher-return projects. This occurs because a stronger incentive will motivate the agent to exert more performance-reporting effort to reduce the measurement risk of a project with a high cash-flow risk, which may offset the increase in the risk premium caused by the strong incentive weight.

Specifically, we use an agency model similar to the single-period, multi-task model of Feltham and Xie (1994), which is further examined by Christensen, Sabac, and Tian (2010). The main distinguishing features of our model are (1) that the agent may exert a personally costly performance-reporting effort to improve the accuracy of the measured performance, which is a noisy signal of future cash flows (and thus of the productive effort) and (2) that the two efforts compete for limited managerial attention in the sense that exerting more effort in one may lead to a higher marginal cost of exerting effort in another. In other words, the two tasks in our setting (referred to as "productive task" and "performance-reporting task") are linked together in two respects. First, they affect the same performance measure, with one affecting the mean and the other affecting the precision; second, the performance-reporting effort may affect the agent's marginal cost of the productive effort. The family of performance signals in this setting are most likely those generated by a sophisticated information system such as an accounting information system (for internal as well as external use), which requires active managerial attention in order to maintain its precision.

Within the model, a key trade-off emerges. When designing the optimal incentive contract in this environment, the principal must consider subtle interactions induced by the two drivers. Any pay-for-performance scheme using the performance measure will induce the agent to exert performance-reporting as well as productive effort, since a risk-averse agent would enjoy a reduced variance in his compensation. The principal also enjoys the reduced variance, as compensation costs (those due to the risk premium) would be lower. Thus, the induced response from the agent is desirable from the principal's perspective. However, this induced response may also complicate the problem if the performance-reporting effort has a spillover effect on the moral-hazard problem involving the productive effort. This would take place if exerting performance-reporting effort would increase the marginal cost of the agent's productive effort, which indeed makes the moral-hazard problem more severe. This is an undesirable aspect of the response induced from the agent. When facing such a problem, the principal must balance the benefits and costs from the desirable as well as the undesirable aspects of multi-tasking. This key trade-off underlies the three main results described previously.

In our model, we have made two key modeling choices (or assumptions), which require some elaboration. First, we assume that the same manager exerts both the productive effort as well as the performance-reporting task; we believe this to be a realistic assumption. In practice, it is likely that forces such as the choice of exchange and the legal environment (possibly beyond managerial control) impact the quality of the reporting system. However, to achieve the stated reporting goal, all performance-reporting systems require managerial efforts, such as improving the precision of accruals estimates and smoothing earnings based on managerial predictions of future profitability in order to improve the informativeness of financial reports. All of these efforts require managerial expertise and are difficult to be centralized to the principal. Additionally, in practice it is more likely that a dedicated employee or team, such as a CFO and the financial reporting and compliance teams, will be deployed to work on performance-reporting tasks. Our model does not consider this aspect and focuses on the task-allocation tension placed upon a single agent such as a CEO. However, we examine an extension of the setting in which a second agent exists who only puts effort toward performance reporting; using this extension, we show that the fundamental tension remains as long as the CEO can exert unobservable reporting effort. As our second key modeling choice, we assume that the marginal cost of performing one task may be increasing in the effort level of performing the other. This same force has been studied with different perspectives by Geanakoplos and Milgrom (1991), Darrough and Melumad (1995), and Peng and Roell (2008). While we believe the assumption to be realistic, we wish to emphasize the importance of this assumption to our result. In particular, it is critical to the first two results of the analysis. That is, standard intuition from standard analysis does carry over to the new setting with the addition of the performance-reporting task; it is the assumption of limited attention that causes the effort-reallocation effect, which alters the standard intuition. For the last result on project selection, the key assumption is the presence of the performance-reporting task, not limited attention.

Previous agency studies of multi-tasking, such as Holmstrom and Milgrom (1991); Feltham and Xie (1994); Zhang (2003); and Christensen, Sabac, and Tian (2010), usually focus on productive efforts and assume exogenous variance (and covariance) of performance measures. From these studies we learn of the importance of goal congruence, or the delicate balance between tasks, in incentive provision. Balance continues to be important in our paper, but we learn that a special type of balance exists between the tasks we study: productive versus performance reporting. As a result, the right balance between these tasks may lead to a different implication in our setting than in the commonly studied settings of past studies. Taking additional signal as an example, Feltham and Xie (1994) examine a similar model to Holmstrom and Milgrom's setting, and show that any informative additional signal can reduce risk and non-congruity (see extensions by Christensen, Sabac, and Tian 2010).[4] In contrast, our paper shows that the right balance between the two tasks is better preserved by discarding an otherwise-useful signal. Multiple tasks may be uniformly widespread in managerial settings, but context also matters when inducing task-balance.

Standard moral-hazard models usually predict a negative association between risk and incentives. However, empirical studies show mixed evidence and the existence of a positive association in some contexts. Recently, several theoretical studies have explored this positive association, including Prendergast (2002); Rajan and Saouma (2006); Hemmer (2006); Dutta (2008); and Liang, Rajan, and Ray (2008).[5] Existing work provide us with explanations of mostly economic and technological nature. For example, Dutta (2008) considers an additional information risk from the uncertainty about the manager's expertise, and Liang, Rajan, and Ray (2008) find that additional design choices, such as team size, also affect the apparent risk-incentive relation. Our paper, however, looks for explanations based on accounting measurements, which add to our understanding and bring more relevance to the accounting profession.

The literature on incentive provision and project selection, such as Lambert (1986), Sung (1995), and Dutta and Reichelstein (2003), among many others, have taught us that both asymmetric information and moral hazard are important. For example, in Lambert (1986), the principal and the agent may not agree on the choice of the "best" project due to private information acquisition. In Dutta and Reichelstein (2003), knowing which is the right project, the principal will be concerned about moral hazard and, thus, will design optimal private incentives in order to induce the agent to select the right project. Without breaking out of the existing moral-hazard framework, we make the point that even if the principal knows which project (risky or safe) to induce, the equilibrium contract may still depend on context. In our case, when the agent has the ability to improve the precision of the performance measurement, a high-powered incentive scheme may be needed

[4] The value of additional signals has also been a focus of agency work since its early years. Holmstrom (1979) pioneered this inquiry and established the early standard result called the *Informativeness Criterion*. In accounting, this work is followed by Antle and Demski (1988); Demski (1994); Feltham and Xie (1994); Feltham and Wu (2000); Arya, Glover, and Radhakrishnan (2005); and Christensen, Sabac, and Tian (2010), among others.

[5] Hughes (1982); Danielsson, Jorgensen, and Vries (2002); Baker and Jorgensen (2005); and Bertomeu (2008) also consider the agent's ability to change the risk profile of the firm output, and thus the agent's performance measure. In all of these papers, limited managerial attention is not a key research issue.

to induce the agent to pick the risky project (because a stronger incentive also induces higher managerial effort to reduce the risk). In contrast, Sung (1995) shows that the principal would lower the sensitivity of the incentive to motivate the selection of a riskier project precisely because, in that setting, the manager cannot influence the risk through his effort. Again, project selection is a uniformly widespread managerial task, but context matters when considering incentive provision.

Finally, to the extent that the precision of the performance measure is related to the predictive power of accounting measures (which underlies the notion of accounting quality in many empirical accounting investigations), our model points to the endogenous nature of such an empirical notion. In other words, the precision of performance measures (and thus their predictive power) is a result of both exogenous environmental conditions as well as the manager's performance-reporting effort induced by equilibrium contracts. Environmental changes not only will directly affect the predictive power (or accounting quality) of the accounting measures, but will also have an indirect effect via the equilibrium performance-reporting tasks, especially when the managers have limited attention. Empirical studies on the predictive power of accounting measures usually focus on earnings persistence (e.g., Dechow and Dichev 2002), while our model provides testable predictions of the precision of accounting measures for future empirical studies to examine.

The remainder of the paper is organized as follows: Section 2 lays out the basic model and analyzes the key economic tension caused by the introduction of the two drivers. Section 3 analyzes the relation between incentive strength and performance variance and shows the forces that cause a positive relation. Sections 4 analyzes the contracting value of an additional signal with performance-reporting task and limited attention. Section 5 examines the role of performance-reporting effort in a project-selection setting. Section 6 summarizes our findings and concludes this report.

2 Basic Model

We consider a single-period, two-task, LEN agency setting in which a risk-neutral principal is contracting with a risk-averse agent. The agent provides two-dimensional effort, denoted $\{e_1, e_2\}$, where $e_i \in \mathbb{R}^+$, at a personal cost $C(e_1, e_2)$.[6] The agent's productive effort, denoted e_1, raises expected output, denoted x. We assume a constant return to scale $x'(e_1) = q > 0$, and that the noise of x follows a zero-mean normal distribution, $\varepsilon_x \sim N(0, \sigma_x^2)$. We also assume that the output

[6] We focus on the single-agent setting in the model. However, the key assumption is that the agent's unobservable managerial efforts contain these two dimensions. Even if the principal assigns the two tasks to two agents separately, one agent may still choose to execute both tasks because the efforts are unobservable and the agent still has an incentive to improve the precision of the performance measures as well as improving production (see related work on teams in Huddart and Liang [2005] and in Liang, Rajan, and Ray [2008]).

We also explicitly examined a setting in which the performance-reporting effort is assigned to a CFO, while the CEO can exert both productive effort and performance-reporting effort. We are able to show that inducing no performance-reporting effort from the CEO is not optimal. A detailed analysis of this setting is included in the Appendix. This result may suggest that, empirically, when reporting effort is unobservable and the CEO's and CFO's reporting efforts are substitutes, perfect task specialization between the CEO and CFO is unlikely. This job-design question may be of potential interest for future studies.

x is realized too late for contracting, but there is a contractible signal y that is a noisy signal of x:

$$y = x + \varepsilon_y = qe_1 + \varepsilon_x + \varepsilon_y,$$

where ε_y is a zero-mean, normally distributed random variable with variance $V(e_2, \sigma^2)$. That is, $\varepsilon_y \sim N\left(0, V(e_2, \sigma^2)\right)$. We also assume that $\varepsilon_x, \varepsilon_y$ are stochastically independent. We regard e_2 as the agent's performance-reporting effort, exerted to reduce the error in his performance measures.[7] These activities generally include any choices or decisions that managers make to improve the accuracy of their performance measures regarding their managerial abilities or efforts. We assume that a higher e_2 leads to a more accurate performance measure (i.e., $V_{e_2} \equiv \frac{\partial}{\partial e_2} V(e_2, \sigma^2) < 0$).[8] In addition, we assume typical regularity conditions: $\frac{\partial V^2(e_2,\sigma^2)}{\partial e_2^2} \geq 0$, $V_{e_2}|_{e_2 \to 0} = -\infty$ and $V_{e_2}|_{e_2 = +\infty} = 0$. Parameter σ^2 is a known constant; it can be regarded as the exogenous factor of the performance-measure variance and is unaffected by e_2.[9] This parameter captures the idea that output (and thus managerial productive effort) may be harder or easier to measure for a given amount of

[7]Dye and Sridhar (2007) and Stocken and Verrecchia (2004) also look at the case in which the precision of a disclosed estimate or that of a firm's accounting-reporting system is a choice variable. In Dye and Sridhar's study, a risk-averse initial owner discloses an estimate of the mean future cash flow to risk-neutral investors. Their study shows that whether the initial owner's precision choice is private or public and whether her disclosure is voluntary or mandatory lead to different equilibria of risk allocation between the owner and the investors. Their paper focuses on the allocational effects, while our paper focuses on the interaction between the agent's productive effort and precision choice. Stocken and Verrecchia's study examines the interaction between the manager's choice of the precision of a firm's accounting-reporting system and the manager's disclosure management decision. It shows that the manager may not choose the most precise reporting system when he has the option to manipulate the financial report. Again, their study does not consider the effect of precision on the choice of productive effort.

[8]In our paper, we focus on the agent's effort to improve the precision of performance measures. Notice that we assume the performance-reporting effort (e_2) only reduces noise associated with the performance measure (σ_y); it does not affect either the expectation or the variance (i.e., risk) of the underlying cash flow (x). In Section 5, we introduce a third managerial choice, which determines the risk-return profile of the cash flow (i.e., project selection). With this assumption, we rule out cases in which effort designed to reduce measured risk (such as e_2) in our model may also affect the real variables (such as $E[x]$ and $Var[x]$). Arguably, all performance-reporting tasks may affect real variables in practice. Incorporating these effects will undoubtedly complicate the model, but doing so may unveil some additional interactions. For example, adding a real effect to e_2 would add to the model an element of goal congruence. That is, because both e_1 and e_2 affect the expected output, an added tension would emerge concerning the optimal combination of these two efforts in both first- and second-best cases (see Feltham and Xie 1994). As to how goal congruence would affect the tension between productive vs. performance-reporting tasks, adding a productive aspect to e_2 may make inducing more e_2 slightly more attractive (compared with cases in which e_2 does not have a positive/productive real effect). This attraction may make our main result (that the risk-incentive relation may be positive) less or more likely to survive, depending on how the spillover between e_1 and e_2 responds to the level of e_2 -productivity on output. If the spillover is very high but e_2's real effect is also high, the principal might not redirect attention away from e_2 (unlike the case we show in Section 3), making the positive relation less likely to emerge.

Alternatively, if we allow the agent to garble the performance measures in this model through e_2 (i.e., higher e_2 increases, rather than decreases, the performance variance), the results of analyzing incentive-risk relation and additional signals may still remain or may be even strengthened. In the relation between incentive and risk, as the performance-reporting effort brings less benefit (potential garbling in addition to inducing a higher marginal cost of productive effort), the principal would be more strongly motivated to induce less performance-reporting effort and more productive effort, which may lead to the positive relation between the variance σ^2 and incentive. When considering an additional signal, potential garbling through e_2 may make the principal more likely to ignore the signal whose precision can be manipulated.

[9]Notice that when the manager exerts zero performance-reporting effort, the performance-measure variance is $V(e_2 = 0, \sigma^2) > 0$, which is not necessarily σ^2. Here, σ^2 only represents an exogenous determinant of the performance-measure variance, not the variance with zero performance-reporting effort.

performance-reporting effort for different firms in different industries during different periods of time. Let $V_{\sigma^2} \equiv \frac{\partial}{\partial \sigma^2} V(e_2, \sigma^2) > 0$.

As usual, we assume that the agent's personal effort cost $C(e_1, e_2)$ is increasing and weakly convex in both e_1 and e_2. Furthermore, we assume that $C_{12}(e_1, e_2) \equiv \frac{\partial^2}{\partial e_1 \partial e_2} C(e_1, e_2) \geq 0$ to highlight the interaction between the two actions. In particular, a positive cross-partial derivative implies limited managerial attention, where a higher level of one effort increases the marginal cost of performing the other effort.[10] When $C_{12}(e_1, e_2) > 0$, a spillover is present between the cost of two actions; when $C_{12}(e_1, e_2) = 0$, the cost is separable between the two efforts and there is no spillover.[11] For notational convenience, we also define $C_{11} \equiv \frac{\partial^2 C(e_1, e_2)}{\partial e_1^2}$, $C_{22} \equiv \frac{\partial^2 C(e_1, e_2)}{\partial e_2^2}$.

In our model, the same agent who exerts productive effort is the one who also provides e_2, not the principal or a third party such as a dedicated employee. As we mentioned in Section 1, we examined an extension of the setting with a second agent (e.g., a CFO team) whose only task is performance reporting; we show that the fundamental tension remains as long as the original agent (e.g., a CEO) has the capacity to exert unobservable reporting effort. The detailed analysis is available in the Appendix.

The principal offers a linear contract on y, with a fixed wage α and a bonus rate β on the performance measure y.

$$w = \alpha + \beta y.$$

The time line of the events is:

[10] Peng and Roell (2008) record a recent example of limited managerial attention, that "in the real world, the time constraint is one of the most important constraints faced by managers. And they do complain of the significant amount of time and attention they are forced to devote to public relations and reassuring the stock market (in Europe, prominent business leaders have pointed out that the threat of a takeover, now that corporate control is more contestable than it used to be, is having the unfortunate side effect of distracting management from running the underlying business). This time cost comes out clearly in the London Stock Exchange's *A Practical Guide to Listing*: 'Both the flotation process itself and the continuing obligations—particularly the vital investor relations activities ... —use up significant amounts of management time which might otherwise be directed to running the business ... It is vital that you maintain your company's profile, and stimulate interest in its shares on a continuing basis. Many listed companies, even relatively small ones, employ specialist financial public relations and investor relations advisors on a retainer basis to keep the business on the financial pages and in the minds of investors. ... However, you cannot leave press or investor relations to your advisers. Top executives will commonly devote at least a couple of days a month to developing and nurturing such contacts. ...This must be regarded as time well-spent. ... As a publicly-quoted company, it is a core element of running your business properly and responsibly.'(pp. 11, 47–48.)"

[11] Formally, we assume $C(e_1, e_2)$ is continuous and differentiable over $\left(\mathbb{R}^+\right)^2$, where $C_{e_1}(.), C_{11} \geq 0$, and $C_{e_2}(.), C_{22} \geq 0$. In some examples, we may consider a specific cost function to illustrate economic intuition using closed-form solutions. In these examples, we consider $C(e_1, e_2) = f(e_2)e_1^2$, where $f(e_2) > 0$. In this case, condition $C_{12} = f'(e_2)2e_1 > 0$ reflects the limited managerial attention. In the example for separable costs, we consider $C(e_1, e_2) = L(e_1) + K(e_2)$, which has the property $C_{12} = 0$. Finally, we assume $C_{11}C_{22} - (C_{12})^2 \geq 0$ to satisfy the second-order condition.

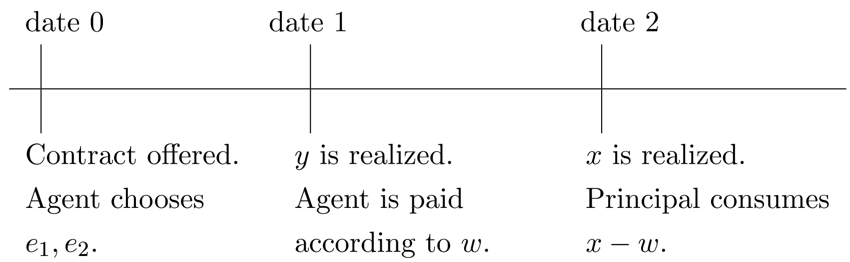

Figure 1: Time line.

The agent's preference is represented by a negative exponential utility function ($-e^{r(w-C(e_1,e_2))}$) with Arrow–Pratt measure of risk aversion r. This allows the standard transformation of the agent's problem into

$$\max_{e_1,\ e_2} \alpha + \beta E[y] - \frac{r}{2}\beta^2[V(e_2,\sigma^2) + \sigma_x^2] - C(e_1,e_2),$$

which yields a standard incentive constraint on the equilibrium choice of e_1 (in equilibrium, this constraint always binds):

$$C_{e_1}(e_1,e_2) = q\beta. \tag{1}$$

In addition, it yields an additional incentive constraint on the equilibrium choice of e_2:[12]

$$-\frac{r}{2}\beta^2 V_{e_2}(e_2,\sigma^2) - C_{e_2}(e_1,e_2) = 0. \tag{2}$$

Notice that from (2), if $\beta > 0$, then the optimal e_2 is positive. Intuitively, when performance measure y is used in the contract, the agent will always be incentivized to exert performance-reporting effort (e_2) to reduce the variance of that measure.[13] Conditions (1) and (2) implicitly define the agent's best response (e_1 and e_2) to a given choice β by the principal.

Before moving on to the principal's incentive design problem, we made some key observations about the agent's effort-allocation trade-off. From the first-order condition (1), we deduce that

$$\frac{de_1}{d\beta} = \frac{q - C_{12}\frac{de_2}{d\beta}}{C_{11}(e_1,e_2)}. \tag{3}$$

Notice that from Eq. (3), when $C_{12} = 0$, we have $\frac{de_1}{d\beta} > 0$. Furthermore, if the reporting effort is fixed ($\frac{de_2}{d\beta} = 0$), it is easy to verify that the agent will also exert more productive effort as β

[12] To ensure that the two first-order conditions characterize the maximum, we compute and verify that the Hessian

$$\begin{bmatrix} -C_{11} & -C_{12} \\ -C_{12} & -\frac{r}{2}\beta^2\frac{\partial V^2(e_2,\sigma^2)}{\partial e_2^2} - C_{22} \end{bmatrix}$$

is indeed negative-definite, given that $C_{11}C_{22} > (C_{12})^2$. In the specific examples we use later in the paper, second-order conditions are satisfied with details given in the Appendix.

[13] Formally, for any given positive bonus weight β, a manager choosing $e_2 = 0$ is not optimal because at $e_2 = 0$, the marginal benefit is proportional to $-V_{e_2}(e_2,\sigma^2) = +\infty$ and the marginal cost is $C_{e_2}(e_1,e_2) < +\infty$. By continuity, the manager can always find an $e_2 > 0$ to equate the marginal benefit and the marginal costs.

increases ($\frac{de_1}{d\beta} = q/C_{11} > 0$). Similarly, from the first-order condition (2), we deduce

$$\frac{de_2}{d\beta} = \frac{r\beta V_{e_2}(e_2, \sigma^2) + C_{12}\frac{de_1}{d\beta}}{-\frac{r}{2}\beta^2\frac{\partial V^2(e_2,\sigma^2)}{\partial e_2^2} - C_{22}},$$

which implies that when productive effort is fixed ($\frac{de_1}{d\beta} = 0$) or when $C_{12} = 0$, we have $\frac{de_2}{d\beta} > 0$ and the performance measure will become more informative (i.e., $V(e_2, \sigma^2)$ will decrease) as β increases.[14] However, when the agent exerts both productive and reporting efforts and there is a spillover between these two efforts, the agent's equilibrium responses to the incentive for both actions will no longer always be positive. Reallocation of efforts may involve substitution between tasks when incentives are strengthened or weakened. For example, the agent may increase his reporting effort at the cost of decreased productive effort when the bonus rate changes.

Without loss of generality, the reservation wage for the agent is set at zero. The principal will set the fixed wage α so that the agent's individual rationality constraint binds.[15] The principal's problem is

$$\max_{\beta} E[x(e_1)] - \frac{r}{2}\beta^2[V(e_2, \sigma^2) + \sigma_x^2] - C(e_1, e_2), \tag{PP}$$

yielding a first-order condition for optimal choice of incentive β:

$$[q - C_{e_1}(e_1, e_2)]\frac{de_1}{d\beta} - r\beta[V(e_2, \sigma^2) + \sigma_x^2] + \left[-\frac{r}{2}\beta^2 V_{e_2}(e_2, \sigma^2) - C_{e_2}(e_1, e_2)\right]\frac{de_2}{d\beta} = 0, \tag{4}$$

with the associated second-order condition, denoted SOC_β, being negative.[16] If (2) is satisfied in equilibrium (thus eliminating the third term in Eq. [4]), when substituting $q\beta$ for $C_{e_1}(e_1, e_2)$ using Eq. (1), (4) can be written as the implicit function

$$\beta = \frac{q^2}{q^2 + r[V(e_2, \sigma^2) + \sigma_x^2]C_{11}/[1 - C_{12}\frac{de_2}{d\beta}\frac{1}{q}]}. \tag{5}$$

This implicit function of β is different from standard agency models because of the new term in its denominator, $[1 - C_{12}\frac{de_2}{d\beta}\frac{1}{q}]$. Our model differs from traditional multi-task models in several ways. First, the performance-reporting task e_2 is endogenous to the moral-hazard problem of the productive task e_1. Notice that the first-best action combination is clearly $\langle e_1^{FB} > 0, e_2^{FB} = 0\rangle$,

[14] We thank an anonymous referee for bringing this point to our attention.

[15] This is because the principal can always adjust the fixed wage α, without affecting any incentive constraints, to make sure the agent takes the contract by setting $\alpha = -\beta E[y] + \frac{r}{2}\beta^2[V(e_2, \sigma^2) + \sigma_x^2] + C(e_1, e_2)$.

[16] Specifically, substituting the agent's first-order conditions (Eqs. [1] and [2]) into (4) and differentiating (4) with respect to β again, we have

$$SOC_\beta \equiv -q\frac{de_1}{d\beta} - r[V(e_2, \sigma^2) + \sigma_x^2] + q(1-\beta)\frac{d^2e_1}{d\beta^2} - r\beta[V_{e_2}(e_2, \sigma^2)\frac{de_2}{d\beta}.$$

To ensure the first-order condition characterizes a global maximum, we assume

$$SOC_\beta < 0.$$

This condition is verified for specific examples we use later in the paper.

while the second-best is $\langle e_1^{FB} > e_1^{SB} > 0, e_2^{SB} > 0 \rangle$. In other words, without the moral-hazard problem with respect to e_1 (e.g., if the principal could contract directly on e_1), the principal would not demand any effort from the agent to reduce the error in his performance metric. Second, managerial attention (e.g., $C_{12}(e_1, e_2)$) is a key factor when determining the optimal choice of performance-reporting effort. If there is no limited managerial attention, the agent will increase both his productive effort and his performance-reporting effort as the principal offers a higher incentive. That is, higher performance-reporting effort can only lessen the agency problem.[17]

However, when $C_{12}(e_1, e_2) > 0$—that is, if the marginal cost of the productive effort, $C_{e_1}(e_1, e_2)$, is an increasing function of e_2—the agency issue becomes more complicated. In particular, inducing the agent to provide performance-reporting effort leads to an interaction (or a spillover) effect on the agent's choice of productive effort. From the agent's perspective, one obvious effect is that inducing a higher e_2 choice makes the agent lower his e_1 choice for a given bonus rate (such that condition [1] holds). From the principal's perspective, inducing a higher e_2 choice makes e_1 marginally more costly (i.e., a higher $C_{e_1}(e_1, e_2)$). On the one hand, the principal would like to increase the bonus rate β to motivate a higher e_2 to obtain a more precise performance measure (i.e., a lower $V(e_2, \sigma^2)$), which amounts to a "less severe" moral-hazard problem. On the other hand, a higher e_2 leads to a higher marginal cost of motivating e_1, which results in a "more severe" moral-hazard problem and would press the principal to lower the optimal bonus rate β. This two-way interaction is a result caused by the combination of (i) induced demand for the performance-reporting task and (ii) limited managerial attention.

We use this two-task model to address three long-standing issues in management control, and we show that there are subtleties in extending standard results to settings in which the agent can influence the variance of his own performance measures. In Section 3, we investigate how the presence of the performance-reporting task and limited attention (spillover effect) affect the characteristics of the optimal incentive provision. We show that, unlike the setting in which performance variance is exogenous, in our model the relation between incentive strength (β) and performance variance ($\sigma^2 + \sigma_x^2$) may be positive. In Section 4, we introduce an additional performance signal whose precision is not affected by e_2; we then derive conditions where it is efficient for the principal to discard the signal with endogenous precision in a setting with the spillover effect, even if the signal is informative. Both the analysis of the possible positive risk-incentive relationship and the analysis of the additional signal show that the spillover effect may cause cases where a legitimate effort to reduce the performance-measure noise becomes undesirable and aggravates agency problems. Finally, in Section 5 we examine the effect of the agent's performance-reporting effort without the spillover and apply the model to a specific project-selection setting. We show that the principal can motivate a riskier project selection with a higher incentive when the agent's project-selection choice is unobservable. This is because a higher incentive will motivate the agent to exert more

[17] Furthermore, it is easily verified that the optimal e_2 supplied by the agent at the solution to (PP) is identical to the solution of a slightly modified problem (PP') where e_2 is supplied by the principal (at the same cost, separate from the cost of e_1). In other words, without spillover costs, there is no conflict of interest with respect to the provision of e_2.

performance-reporting effort in order to reduce the noise in the measurement of project outcomes, and that may offset the increase in the risk premium due to a higher incentive.

3 Incentive-Variance Relation

In this section, we examine the relation between incentive and variance with the presence of the two-way interaction. Recall that, in standard LEN moral-hazard models, the precision of the performance measure is typically unaffected by the agent's effort. In these settings, risk and incentive are typically predicted to be negatively related. That is, the principal offers a lower bonus rate when the agent's performance is measured with high variance (risk). Our model nests such a prediction as a special case. Consider the case in which e_2 is a known constant denoted by E (and thus not a choice of the agent). The principal's trade-off in this case is captured by the following special case of Eq. (5):

$$\beta = \frac{q^2}{q^2 + r[V(E, \sigma^2) + \sigma_x^2]C_{11}(e_1, E)}. \tag{6}$$

The negative relation between incentive and signal variance is intuitive: the principal lowers incentive rates in response to higher variance in the performance measure imposed on a risk-averse agent. Indeed, from Eq. (6), an increase in measurement noise σ^2 leads to a decrease in β.[18] The key is that such an increase in σ^2 does not generate a response in the agent's choice of e_2, which would have affected β indirectly.

Outside this special case, an increase in σ^2 would induce a response from the agent's performance-reporting effort (e_2). Anticipating this change in the agent's performance-reporting effort, the principal would react by adjusting the incentive provision (i.e., bonus rate β). As discussed in Section 2, the presence of e_2 creates a subtle, two-way interaction effect on the incentive rate. The overall impact of an exogenous change in σ^2 on incentive rate β is more complicated than it is in the standard setting.

3.1 Endogenous Precision without Spillover Effect

We first consider the case where the agent can influence performance-measure precision through e_2, but e_2 does not spillover on the marginal cost of e_1. For example, if the effort cost function is additively separable (i.e., in the form of $C = L(e_1) + K(e_2)$), then there is no spillover ($C_{12} = 0$) and the agent would consider each task separately, because the benefit and cost of each task are separable in his choice problem.

Following the basic setup, when we totally differentiate (1) and (2) and divide them by $d\beta$, we get

[18] This intuition can be shown to hold even if C_{11} is a function of e_1 (and thus β), making Eq. (6) implicit in β. In the standard LEN model, the cost of effort is usually quadratic, making C_{11} a constant and making Eq. (6) explicit in β.

$$q - C_{11}\frac{de_1}{d\beta} - C_{12}\frac{de_2}{d\beta} = 0, \tag{7}$$
$$-rV_{e_2}\beta - C_{12}\frac{de_1}{d\beta} - (\frac{r}{2}\frac{\partial V^2(e_2,\sigma^2)}{\partial e_2^2}\beta^2 + C_{22})\frac{de_2}{d\beta} = 0.$$

To reduce the mathematical complexity while maintaining the basic economic intuition, we assume $\frac{\partial V^2(e_2,\sigma^2)}{\partial e_2^2} = 0$ to better illustrate the main point of the analysis.[19] With $\frac{\partial V^2(e_2,\sigma^2)}{\partial e_2^2} = 0$, (7) gives us

$$\frac{de_1}{d\beta} = \frac{qC_{22} + rC_{12}V_{e_2}\beta}{\Delta}, \tag{8}$$
$$\frac{de_2}{d\beta} = -\frac{qC_{12} + rC_{11}V_{e_2}\beta}{\Delta}, \text{ where } \Delta \equiv C_{11}C_{22} - C_{12}^2.$$

With (1), (2), (8), and $C_{12} = 0$, the first-order condition for the optimal choice of incentive β (Eq. [4]) can be transformed into

$$(1-\beta)\frac{q^2}{C_{11}} - r\beta[V(e_2,\sigma^2) + \sigma_x^2] = 0. \tag{9}$$

Further differentiating this first-order condition with respect to the measurement noise parameter allows us to compute the comparative statics, $\frac{\partial\beta}{\partial\sigma^2}$. In other words, we assess how the principal adjusts β when σ^2 increases: that is, the principal needs to adjust β accordingly so that the new marginal benefit equals the new marginal cost. The following proposition shows that the incentive-variance relationship remains negative as in the standard LEN agency model.[20]

Proposition 1 *With endogenous variance but no spillover effect ($V_{e_2} < 0$ and $C_{12} = 0$), the incentive-variance relation is negative; i.e.,* $\frac{\partial\beta^*}{\partial\sigma^2} < 0$.

Proof. All proofs appear in the Appendix. ■

Intuitively, as σ^2 increases, the agent's working environment becomes harder to measure (for example, a technology change makes the firm's competitive environment harder to predict), imposing more risk on the agent; in response, the principal lowers the bonus rate. This intuition is captured by Eq. (9), where the marginal cost gets larger because the performance measure gets noisier and the principal must pay a higher risk premium. The only way to reduce the risk premium is to offer a lower β. With a lower incentive, the marginal benefit, $(1-\beta)\frac{q^2}{C_{11}}$, also increases, and the new

[19] We thank an anonymous referee for suggestions on simplifying the math of our analysis. This assumption is not the driving assumption for our results, but helps simplify the mathematical complexity in our analysis of incentive-variance relation. All results in Section 3 hold without this assumption. In addition, Sections 4 and 5 are not restricted by this assumption. The more general analysis without this simplifying assumption was in an earlier version of our paper and is available upon request.

[20] More specifically, $SOC_\beta \frac{\partial\beta}{\partial\sigma^2} - r\beta V_{\sigma^2} = 0$, where SOC_β is the second-order condition discussed in Footnote 16. Since the second-order condition for β must be negative and $V_{\sigma^2} > 0$, the principal must decrease the bonus rate β as σ^2 increases. Therefore, the incentive-risk relationship remains negative.

optimal β matches the marginal benefit with the marginal cost. The trade-off here is similar to that in a standard agency setting without endogenous precision; even in the presence of the reporting task, the incentive-risk relation remains negative.

In summary, Proposition 1 shows that, with no spillover effect, the reporting task reduces the variance of the performance measure, but not enough to change the trade-off between risk and incentives that we know from models in which informativeness is not influenced by the agent. Therefore, the negative incentive-risk relation remains.

The performance-measurement variance captured by empirical data may more likely be the endogenous variance, $V(e_2, \sigma^2) + \sigma_x^2$, than the exogenous variance, $\sigma^2 + \sigma_x^2$. Therefore, we also examine the relation between the endogenous incentive β and endogenous variance $V(e_2, \sigma^2)$ caused by a change in the exogenous σ^2. We find that, with no spillover effect, a sufficient condition for this relationship to be negative is $\frac{\partial V^2(e_2,\sigma^2)}{\partial e_2 \partial \sigma^2} > 0$. This relationship between β and $V(e_2, \sigma^2)$, however, can be positive when $\frac{\partial V^2(e_2,\sigma^2)}{\partial e_2 \partial \sigma^2}$ is sufficiently negative.

Corollary 1 *With endogenous variance but no spillover effect ($V_{e_2} < 0$ and $C_{12} = 0$), the relationship between the incentive and $V(e_2, \sigma^2)$ is negative when $\frac{\partial V^2(e_2,\sigma^2)}{\partial e_2 \partial \sigma^2} > 0$, and it may be positive when $\frac{\partial V^2(e_2,\sigma^2)}{\partial e_2 \partial \sigma^2}$ is sufficiently negative.*

An increase in σ^2 results in a lower incentive β, and a lower incentive in turn results in a lower level of performance-reporting effort ($\frac{de_2}{d\beta} = -\frac{rC_{11}V_{e_2}\beta}{\Delta} > 0$) and a higher endogenous variance V. Especially, if the performance-reporting effort is less effective in reducing the noise in the performance measures when σ^2 increases (i.e., $\frac{\partial V^2(e_2,\sigma^2)}{\partial e_2 \partial \sigma^2} > 0$), then an increase in σ^2 leads to lower performance-reporting effort and thus a higher V. Therefore, $\frac{\partial V^2(e_2,\sigma^2)}{\partial e_2 \partial \sigma^2} > 0$ is a sufficient condition for a positive relationship between σ^2 and $V(e_2^*, \sigma^2)$, which implies a negative relation between β^* and $V(e_2^*, \sigma^2)$. On the other hand, if the performance-reporting effort becomes much more effective in reducing the variance when σ^2 increases (i.e., $\frac{\partial V^2(e_2,\sigma^2)}{\partial e_2 \partial \sigma^2}$ is sufficiently negative), an increase in σ^2 will lead to higher performance-reporting effort, which may result in an overall lower endogenous variance and a positive relationship between β^* and $V(e_2^*, \sigma^2)$.

3.2 Endogenous Variance and Spillover Effect

Now we consider the case with the spillover effect. That is, in this case the manager can affect the performance-measure variance through his effort e_2, but performance-reporting effort increases the marginal cost of his productive effort e_1. With $C_{12} > 0$, the first-order condition for the optimal choice of incentive β (Eq. [4]) becomes

$$q(1-\beta)\frac{qC_{22} + rC_{12}V_{e_2}\beta}{\Delta} - r\beta[V(e_2, \sigma^2) + \sigma_x^2] = 0.$$

As σ^2 increases, the principal must adjust β accordingly so that the new marginal benefit equals the new marginal cost. Therefore, we have $SOC_\beta \frac{\partial \beta}{\partial \sigma^2} + \frac{q(1-\beta)rC_{12}}{\Delta}\frac{\partial V^2(e_2,\sigma^2)}{\partial e_2 \partial \sigma^2} - r\beta V_{\sigma^2} = 0$. Different from the no-spillover case in Section 3.1, with spillover between the two efforts, $\frac{\partial \beta}{\partial \sigma^2}$ can become positive if $\frac{\partial V^2(e_2,\sigma^2)}{\partial e_2 \partial \sigma^2} > 0$ and is sufficiently large.

What makes this result different from the no-spillover case is the critical role of limited attention: C_{12}. In the presence of spillover effects ($C_{12} > 0$), the incentive-risk relation depends on the sign and magnitude of $\frac{\partial V^2(e_2,\sigma^2)}{\partial e_2 \partial \sigma^2}$: the relative effectiveness of the performance-reporting task at reducing measurement error (V_{e_2}) when the environment is hard to measure (high σ^2) versus when it is easier to measure (low σ^2). Suppose that the performance-reporting effort becomes less effective in reducing the variance as σ^2 becomes larger (i.e., V_{e_2} becomes less negative for larger σ^2).[21] Since the manager's marginal benefit from exerting effort e_2 is lower, the manager puts less effort in the performance-reporting task; this lowers the marginal cost of productive effort. Therefore, the incentive required to motivate the productive effort becomes lower and the relation between σ^2 and the incentive β becomes positive.

The following proposition summarizes our findings on the positive incentive-risk relation:[22]

Proposition 2 *With endogenous variance and spillover effect ($V_{e_2} < 0$ and $C_{12} > 0$),*

$$\frac{\partial \beta^*}{\partial \sigma^2} > 0 \text{ if } \frac{\partial V^2(e_2,\sigma^2)}{\partial e_2 \partial \sigma^2} \text{ is positive and sufficiently large.}$$

When the relationship between the incentive and σ^2 is positive, we find that the relation between the incentive and the endogenous variance $V(e_2, \sigma^2)$ can also be positive if the spillover effect is significant (i.e., C_{12} is sufficiently large).

Corollary 2 *With endogenous variance and spillover effect, when $\frac{\partial \beta^*}{\partial \sigma^2} > 0$, the relation between the incentive and the endogenous variance $V(e_2, \sigma^2)$ is positive when C_{12} is sufficiently large.*

When the spillover effect is strong, as σ^2 increases, the manager will exert less performance-reporting effort since $\frac{\partial V^2(e_2,\sigma^2)}{\partial e_2 \partial \sigma^2} > 0$. Less performance-reporting effort, combined with a higher σ^2 leads to a higher $V(e_2, \sigma^2)$. As a result, the relations between the incentive and both the exogenous σ^2 and the endogenous variance V can be positive.

The preceding analyses in Corollaries 1 and 2 have implications on empirical analysis in managerial accounting research. Our model indicates that the relation between the exogenous precision of the performance metric and the strength of managerial incentive depends on the limited-attention effect and the manager's ability to influence the performance-measure variance. Our analysis sheds new light on the reason underlying the mixed findings on the relation between risk and incentives. Furthermore, our paper shows that, when performing empirical research, controlling for cross-sectional differences in the spillover (C_{12}) may be important.

---

[21] In practice, the manager's e_2 effort may become less effective when facing high risk in business. When addressing the risk management in industries that rely on R&D and innovations, Elsum (2008) comments that "one size does not fit all—distinctly different management frameworks are required for success in research, development and/or innovation with high compared with low uncertainty. Most organizations find this difficult to cope with."

[22] We can show that an explicit sufficient condition for $\frac{\partial \beta^*}{\partial \sigma^2} > 0$ is $\frac{\partial V^2(e_2,\sigma^2)}{\partial e_2 \partial \sigma^2} > \max\{0, \frac{r\Delta(-V_{e_2})V_{\sigma^2}}{q^2 C_{22} - r\Delta(\sigma^2 + \sigma_x^2)}\}$. A detailed analysis is presented in the proof in the Appendix.

Analyzing both Propositions 1 and 2 shows that the driving force of the results is indeed that the marginal cost of productive effort increases as a result of increased reporting effort. Even without the spillover effect, the reporting effort does not have sufficient influence to change the standard incentive-risk relationship; with spillover effect, the benefit of reporting effort is even lower, as it makes the productive effort more costly. Although reporting effort is meant to improve the informativeness of the performance measure and helps to mitigate agency costs, its benefit is sometimes outweighed by its cost when the reporting effort also increases the marginal cost of the manager's productive effort. Although the reporting effort in our model appears constructive, it may aggravate the agency problem when we consider the spillover effect.

4 The Value of an Additional Signal

We now examine the effect of endogenous performance-measure variance and the spillover effect in a setting with an additional signal. Traditional agency models with exogenous variances predict that any informative signal about the agent's productive effort, no matter how imperfect, should be used in contracting to improve the efficiency. However, including endogenous variances and the spillover effect between productive effort and performance-reporting effort, we find that in some cases an informative signal may be discarded by the principal in contracting.

Compared with signals generated by a sophisticated accounting-information system, the precision of certain other signals (such as hours worked, output quantities, cash flows, or stock price) are less affected by managers' performance-reporting task. Here, we abstract away from the richness in the different sensitivities of such signals to managerial reporting efforts and instead explore the extreme case of signals with precision unaffected by the management. This exploration allows us to qualitatively compare the optimal use of two different signals with such a distinctive difference, and it offers new insights into the value of an additional signal, a vital theoretical interest in agency theory since Holmstrom (1979).

To begin, we modify the model to include an additional performance measure z. Both z and y are noisy measures of x :

$$
\begin{aligned}
y &= x + \varepsilon_y, \\
z &= x + \varepsilon_z.
\end{aligned}
$$

However, unlike for y, the variance of z (σ_z^2) cannot be reduced through the agent's effort.[23] That

[23] An alternative specification would be to assume that the additional signal (z) is informative about e_2 (e.g., $z = e_2 + \varepsilon_z$). Reporting effort (e_2) is arguably even harder to measure in reality than productive effort (e_1); however, for completeness of our analysis, we examined this setting. Doing so, we find that if the incentive on signal z cannot be negative, the principal will not use signal z when e_2 is not effective enough in reducing the performance variance. However, if we allow the incentive on signal z to be negative, the principal will use signal z and impose a negative incentive on z to lower e_2.

We thank an anonymous reviewer for bringing this point to our attention. A detailed analysis is available upon request.

is,

$$\begin{bmatrix} \varepsilon_y \\ \varepsilon_z \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} V(e_2, \sigma^2) & 0 \\ 0 & \sigma_z^2 \end{bmatrix} \right).$$

The principal offers a linear contract on y and z. As in the previous setting, α is a fixed wage and β is the bonus rate on y. In addition, the contract also assigns a bonus rate δ on z:

$$w = \alpha + \beta y + \delta z.$$

We first examine the optimal use of the two signals in two benchmarks. In the first benchmark, both variances of performance measures y and z are exogenous, as in most standard agency models. In the second benchmark, the variance of y can be reduced by e_2, but the costs of e_1 and e_2 are separable. In these two benchmarks, we find that both measures are useful (that is, the principal is better off by including both measures in the contract) as long as their variances are non-degenerate. Then, we consider a setting where the variance of y is endogenous and e_2 has a spillover effect on the marginal cost of e_1. In this setting, we show that excluding measure y from the contract may be efficient even if the variance of y is non-degenerate. The reason is, again, that using y would, via e_2, induce a higher marginal cost of productive effort e_1, and the incentive benefit of y cannot offset this cost increase in the presence of another performance signal.

4.1 Benchmark Settings

Consider the following two settings:

- In the first benchmark, we return to a simpler setting where the agent's effort does not affect the variance of performance measures. This setting is consistent with standard agency studies, such as those by Holmstrom (1979) and Feltham and Xie (1994). Without loss of generality, we parameterize this benchmark by setting $V(e_2, \sigma^2) = \sigma^2$ for simplicity.[24] We label this setting *exogenous variance.*

- In the second benchmark, the agent is able to exert e_2 to reduce the variance of the performance measure y. However, the personal cost of the agent's effort is separable in e_1 and e_2 ($C_{12} = 0$). Without loss of generality, we parameterize this benchmark by setting $C(e_1, e_2) = L(e_1) + K(e_2)$. We label this setting *separable costs.*

Lemma 1 summarizes the optimal use of the two performance measures in these two benchmark settings.

Lemma 1 *Under either exogenous variance setting $V(e_2, \sigma^2) = \sigma^2$ or the separable cost setting $C(e_1, e_2) = L(e_1) + K(e_2)$,*

$$\beta^*,\ \delta^* > 0 \Longleftrightarrow V(e_2, \sigma^2),\ \sigma_z^2 < +\infty \text{ for all } e_2. \tag{10}$$

[24] We assume $V(e_2, \sigma^2) = \sigma^2$ for simplicity, but any fixed variance V is valid for our analysis.

In Lemma 1, the result of the first benchmark is a reproduction of the conclusions of the standard-agency studies by Holmstrom (1979), Banker and Datar (1989), and Feltham and Xie (1994). The standard agency models with exogenous variances show that any informative signal about the agent's productive effort, no matter how imperfect, can be used in contracting to improve the principal's welfare. The key argument is that the principal will always use a signal as long as its variance is finite, because the principal can always place a sufficiently small weight on the signal to balance the marginal cost from higher risk premium against the marginal benefit from a higher productive effort.

The result of the second benchmark shows that the standard agency conclusion still holds with an endogenous variance, as long as the cost of the performance-reporting effort is separable from the cost of productive effort (i.e., no spillover). Again, the principal can always choose a proper weight on the signal to balance the marginal cost and benefit. However, the second benchmark includes an additional marginal cost: the cost of performance-reporting effort. Nevertheless, when the bonus weight is close to zero, the marginal benefit always outweighs the marginal cost; thus, the principal can always benefit from slightly increasing the bonus weight from zero.

4.2 Additional Signal Setting with Spillover

Now, we examine a setting with the spillover effect; that is, a setting where exerting performance-reporting effort e_2 may affect the marginal cost of productive effort e_1. For simplicity, we use a specific example, supposing $V(e_2, \sigma^2) = \frac{\sigma^2}{e_2}$ and $C(e_1, e_2) = \frac{1}{2}(c_1 + ke_2)e_1^2$. This example helps us to provide an explicit analysis and clearly illustrate the intuition. A general analysis is available in the Appendix.

For this specific example, we define $\widehat{\beta}$, $\widehat{\delta}$ as the optimal incentives on y and z, respectively, and $\widehat{e}_1$, $\widehat{e}_2$ as the optimal efforts made by the agent. Assuming an interior solution, the optimal bonus coefficients β and δ are

$$\widehat{\beta} = \frac{q(q - \sigma\sqrt{kr}) - q^2\widehat{\delta} - r\widehat{\delta}\sigma_x^2 c_1}{q^2 - \sigma^2 kr + r\sigma_x^2 c_1}, \quad \text{and } \widehat{\delta} = \frac{q^2(1 - \widehat{\beta}) - r\widehat{\beta}\sigma_x^2 c_1}{q^2 + c_1 r(\sigma_z^2 + \sigma_x^2)}.$$

Even with a spillover effect between the two types of effort, the signal with an exogenous precision will always be used in the optimal contract. To see this, suppose $\widehat{\delta} = 0$, then we have $\widehat{\beta} = \frac{q(q-\sigma\sqrt{kr})}{(q+\sigma\sqrt{kr})(q-\sigma\sqrt{kr})+r\sigma_x^2 c_1}$; it can then be easily verified that the marginal benefit of increasing δ is higher than the marginal cost at $\delta = 0$. Therefore, the principal can improve her payoff by increasing δ from zero. In other words, signal z is always used in the contract, consistent with the intuition in Lemma 1.

However, $\widehat{\beta}$ is no longer guaranteed to be positive with the non-separable costs of e_1 and e_2. Supposing $\widehat{\beta} = 0$, then we have $\widehat{\delta} = \frac{q^2}{q^2 + c_1 r(\sigma_z^2 + \sigma_x^2)}$. We can show that, when the precision of signal z is high enough, the marginal benefit of using signal y is lower than its marginal cost, and the corner solution $\widehat{\beta} = 0$ is indeed optimal.[25] That indicates that the signal y, although informative, may be

[25] In this paper we only consider non-negative bonus rates on signals about e_1, since e_1 is productive effort. This can

ignored in the contracting. This conclusion is different from those of standard-agency models and that of the separable-costs case without the spillover effect (shown in Lemma 1).[26] We summarize this result in the following proposition.

Proposition 3 *In the case of endogenous variance with an additional signal, suppose* $V(e_2, \sigma^2) = \frac{\sigma^2}{e_2}$ *and* $C(e_1, e_2) = \frac{1}{2}(c_1 + ke_2)e_1^2$*. It is then optimal to ignore signal* y *and only use signal* z *in the compensation contract if signal* z *is precise enough. That is,* $\{\widehat{\beta} = 0, \widehat{\delta} > 0\}$ *iff* σ_z^2 *is small enough.*[27]

On the surface, ignoring the signal y may seem to be an undesirable move, since an informative signal is unused and the manager may be less motivated. However, in many cases not all signals are used; for example, a firm's information system collects many financial and non-financial metrics, not all of which are used in the top managers' compensation even if the signals are all, presumably, informative about the managers' productive efforts. There may be many sound reasons why a particular signal is not used in equilibrium managerial contracts. In our model, ignoring signal y also has the desirable consequence of drawing the managers' attention away from performance-reporting tasks and towards productive tasks. When signal z is precise enough, the desirable effect of ignoring y dominates the undesirable effect in the trade-off, and the principal finds it efficient not to use signal y.

Allocation between the two effort choices is the important underlying tension in this case; we see that, by introducing an additional performance measure z, the principal can redirect the agent's attention from performance reporting to production (it can be shown that $\widehat{e}_1 > e_1^*$ and $\widehat{e}_2 < e_2^*$). When the performance-reporting effort spills over to the cost of productive effort, including y in contracting draws the agent's attention to performance reporting, thus making productive tasks more costly. An additional performance measure that cannot be modified by the agent may help the principal alleviate the tension in managerial attention. Furthermore, we see that sometimes it is efficient for the principal to exclude the performance measure y from contracting ($\widehat{\beta} = 0$). This

be regarded as an implicit constraint $\beta \geq 0$. If we incorporate this constraint in the program and examine the Kuhn–Tucker conditions, we see that this condition is not binding in most cases. However, when $\sigma_z^2 + \sigma_x^2 < \frac{q^2\sigma\sqrt{rk}}{rc_1(q-\sigma\sqrt{rk})}$, the first-order condition with respect to β is not zero, and its Kuhn–Tucker multiplier is zero, while the condition $\beta \geq 0$ is binding with its Kuhn–Tucker multiplier being positive. Thus, β must be zero and cannot deviate from zero.

Given $\beta = 0$, we only need to check the second-order condition with respect to δ. The second-order derivative of the principal's objective function with respect to δ is $-\frac{q^2}{c_1} - r\left(\sigma_z^2 + \sigma_x^2\right) < 0$. Therefore, the second-order condition is satisfied, and $\{\widehat{\delta} = \frac{q^2}{q^2 + c_1 r(\sigma_z^2 + \sigma_x^2)}, \widehat{\beta} = 0\}$ is indeed the global maximum when $\sigma_z^2 + \sigma_x^2 < \frac{q^2\sigma\sqrt{rk}}{rc_1(q-\sigma\sqrt{rk})}$.

[26] To elaborate, in standard agency settings, the marginal benefit is positive when the bonus weight is close to zero, while the marginal cost approaches zero because both the risk premium and the manager's personal cost of efforts are quadratic. In the separable-costs case with no spillover, the marginal cost of increasing the incentive on signal z approaches zero as the bonus weight β approaches zero due to the quadratic form of personal-effort cost, but the total marginal cost does not go to zero because of the term $r\delta\sigma_x^2$ due to the covariance between the signals (see Eq. [15] in the Appendix). However, the total marginal cost is always outweighed by the marginal benefit, and it is still efficient to include signal y in the contract. When the spillover occurs between effort choices, not only does the total marginal cost of increasing β not approach zero as β approaches zero, but it can also outweigh the marginal benefit (see Eq. [24] in the Appendix).

[27] The explicit condition for $\{\widehat{\beta} = 0, \widehat{\delta} > 0\}$ is $\sigma_z^2 + \sigma_x^2 < \frac{q^2\sigma\sqrt{rk}}{rc_1(q-\sigma\sqrt{rk})}$. See the proof in the Appendix.

happens when the benefit of variance reduction from the reporting effort cannot outweigh a higher marginal cost of the productive effort. In other words, the reporting effort on a net basis becomes a negative externality. In this case, the principal will avoid using a measure whose informativeness is affected by the manger's reporting effort if another sufficiently good performance measure exists: by doing so she can induce the productive effort without motivating the undesirable reporting effort.

5 Project Selection and Endogenous Variance

We now apply the baseline model of endogenous variance to study project selection. We show that, in contrast to predictions of previous studies, the principal may motivate riskier projects by offering a higher incentive when the performance variance can be reduced through the agent's effort and there is no spillover effect; this occurs because the agent's performance-reporting effort can reduce the noise in the measurement of the project outcome and this may offset the increase in the risk premium due to a higher incentive.

To facilitate the project-selection choice, we enrich the model by assuming there are two mutually exclusive types of projects, H- and L-projects. The H-project has a higher profitability than the L-project, but also higher risk. The expected return of a project also depends on the manager's productive effort e_1. Formally,

$$x_i = q_i e_1 + \varepsilon_{xi}, i \in \{H, L\},$$

where $q_H > q_L, \varepsilon_{xi} \sim N(0, \sigma_{xi}^2)$ and $\sigma_{xH}^2 > \sigma_{xL}^2$. Again, we assume that the output of the project is realized too late for contracting, but there is a contractible signal y that is a noisy signal of the output,[28]

$$y_i = x_i + \varepsilon_{y_i}, i \in \{H, L\}.$$

In addition, we assume that the original performance measure for the outcome of the high-risk project (H-project) is noisier, but the original performance-measure noise can be reduced through the manager's e_2 effort. That is,

$$\begin{aligned} \varepsilon_{yH} &\sim N\left(0, \frac{\sigma_{yH}^2}{e_2}\right), \\ \varepsilon_{yL} &\sim N\left(0, \frac{\sigma_{yL}^2}{e_2}\right), \text{ where } \sigma_{yH}^2 > \sigma_{yL}^2. \end{aligned}$$

We further assume that the cost function of effort is $C(e_1, e_2) = \frac{c}{2} e_1^2 + k e_2$. Notice that the cost function is separable in the two efforts and there is no spillover. The key for the differing prediction of project selection in our study versus previous ones is the endogenous variance rather than the spillover effect. In addition, we assume that the manager's project selection is costless.

We use these assumptions to capture the variations in the underlying risk of cash flows (σ_x^2) and performance measures (σ_y^2). Some empirical evidence may suggest that firms undertaking high-risk

[28] Our result does not change if $y_i = q_i e_1 + \varepsilon_{yi}$.

projects are motivated to improve the informativeness of their performance measurements. For example, the current financial-reporting model is alleged to be particularly ill-suited for high-tech industries such as pharmaceuticals, computers, and telecommunications. In other words, earnings for these industries are much noisier performance measures than those in traditional industries. However, Francis and Schipper (1999) find that, compared with traditional industries, high-tech industries do not show lower value relevance of their financial information. Similarly, Collins, Maydew, and Weiss (1997) show that the combined value relevance of earnings and book values has not declined over the past 40 years as we shift from an industrialized economy to a high-tech, service-oriented economy. Instead, value relevance appears to have increased slightly. This evidence may indicate that firms with high-risk projects are engaged in improving the informativeness or precision of their financial information.

5.1 Observable Project-Selection Benchmark

We first look at a benchmark case where the manager's project-selection decision is observable. For purely computational ease, we assume proportionality among six exogenous parameters in the following fashion: $\frac{\sigma_{yH}}{q_H^2} = \frac{\sigma_{yL}}{q_L^2} \equiv \phi$ and $\frac{\sigma_{xH}}{q_H} = \frac{\sigma_{xL}}{q_L} \equiv \psi$. That is, the risk in cash flows ($\sigma_{x.}^2$) and the noise in performance measures ($\sigma_{y.}^2$) are proportional to the expected productivity ($q_.$). With these simplifying assumptions, we show that the principal offers

$$\alpha_H^* = \left(\frac{1 - c\sqrt{2rk}\phi}{1 + rc\psi^2}\right)^2 \frac{q_H^2(rc\psi^2 - 1)}{2c} + \frac{1 - c\sqrt{2rk}\phi}{1 + rc\psi^2}\sqrt{rk}(1 + \frac{\sqrt{2}}{2})\sigma_{yH}, \ \beta_H^* = \frac{1 - c\sqrt{2rk}\phi}{1 + rc\psi^2}$$

to motivate the H-project, and offers

$$\alpha_L^* = \left(\frac{1 - c\sqrt{2rk}\phi}{1 + rc\psi^2}\right)^2 \frac{q_L^2(rc\psi^2 - 1)}{2c} + \frac{1 - c\sqrt{2rk}\phi}{1 + rc\psi^2}\sqrt{rk}(1 + \frac{\sqrt{2}}{2})\sigma_{yL}, \beta_L^* = \frac{1 - c\sqrt{2rk}\phi}{1 + rc\psi^2}$$

to motivate the L-project. Notice that, although $\beta_H^* = \beta_L^*$ due to our simplification, the contract that motivates the H-project offers a higher fixed wage ($\alpha_H^* > \alpha_L^*$).[29]

In this setting where the principal observes the manager's project-selection decision, her payoff when she motivates the H-project is

$$PP_H = \frac{1 - c\sqrt{2rk}\phi}{2c(1 + rc\psi^2)}(q_H^2 - 2c\sigma_{yH}\sqrt{2rk} + c\sqrt{2rk}\phi q_H^2),$$

and her payoff when she motivates the L-project is

$$PP_L = \frac{1 - c\sqrt{2rk}\phi}{2c(1 + rc\psi^2)}(q_L^2 - 2c\sigma_{yL}\sqrt{2rk} + c\sqrt{2rk}\phi q_L^2).$$

[29] The assumptions $\frac{\sigma_{yH}}{q_H^2} = \frac{\sigma_{yL}}{q_L^2} \equiv \phi$ and $\frac{\sigma_{xH}}{q_H} = \frac{\sigma_{xL}}{q_L} \equiv \psi$ are only made to simplify our calculation and do not affect our results. We focus on the comparision between (α_H^*, β_H^*) in the observable setting and $(\alpha_H^{'*}, \beta_H^{'*})$ in the unobservable setting. Whether β_H^* is equivalent to β_L^* in the observable setting does not influence our analysis.

The principal prefers the H-project if and only if $PP_H > PP_L$, which implies $\phi < \frac{1}{c\sqrt{2rk}}$. Intuitively, the principal finds the risk premium too high relative to the expected return when ϕ is too high. Even though the principal is risk-neutral, delegating the decision to a risk-averse agent makes the principal act as if she is risk-averse when it comes to project selection.

5.2 Unobservable Project Selection

Now, we suppose the manager's project-selection decision is unobservable. In this setting, if the principal offers the same contract as in the observable setting, the manager may not choose the principal's desired project. Let us assume that the principal prefers the H-project. If the principal still offers (α_H^*, β_H^*) to the manager in this unobservable setting, the manager may find it optimal to choose the L-project. To see this, notice that the manager's certainty equivalent when choosing the H-project is

$$\begin{aligned} CE_H &= \alpha_H^* + \beta_H^* q_H e_{1H}^* - \frac{r}{2}\beta_H^{*2}(\sigma_{xH}^2 + \frac{\sigma_{yH}^2}{e_{2H}^*}) - \frac{c}{2}e_{1H}^{*2} - ke_{2H}^* \\ &= \alpha_H^* + \frac{q_H^2}{2c}\beta_H^{*2} - \frac{r\sigma_{xH}^2}{2}\beta_H^{*2} - \sqrt{2rk}\sigma_{yH}\beta_H^*, \end{aligned}$$

and his certainty equivalent if he chooses the L-project when offered (α_H^*, β_H^*) is

$$\begin{aligned} CE_L &= \alpha_H^* + \beta_H^* q_L e_{1L}^* - \frac{r}{2}\beta_H^{*2}(\sigma_{xL}^2 + \frac{\sigma_{yL}^2}{e_{2L}^*}) - \frac{c}{2}e_{1L}^{*2} - ke_{2L}^* \\ &= \alpha_H^* + \frac{q_L^2}{2c}\beta_H^{*2} - \frac{r\sigma_{xL}^2}{2}\beta_H^{*2} - \sqrt{2rk}\sigma_{yL}\beta_H^*. \end{aligned}$$

When $CE_H < CE_L$, the manager will choose the L-project, even if the principal desires the riskier project. The fundamental tension introduced by unobservable project selection (by the agent) is the potential conflict of interests between the agent and the principal regarding the choice of projects. Even if the act of choosing a project is not personally costly, there may exist a moral-hazard problem that affects the project choice; this type of moral hazard is referred to as an "induced moral hazard" by Baiman and Demski (1980).

A sufficient condition for the manager to choose the L-project ($CE_H - CE_L < 0$) is $\phi > \frac{1}{3c\sqrt{2rk}}$.[30] Therefore, when $\frac{1}{3c\sqrt{2rk}} < \phi < \frac{1}{c\sqrt{2rk}}$, the manager will choose the safer L-project when offered the contract to motivate the H-project in the observable setting, even though the principal desires the riskier H-project.

If the principal wants to motivate the manager to choose the H-project in this unobservable setting, she must offer a different contract. Solving the principal's program gives us $\beta_H'^* = \frac{2c\sqrt{2rk}\phi}{1-rc\psi^2}$ and $\alpha_H'^* = \beta_H'^{*2}\frac{q_H^2(rc\psi^2-1)}{2c} + \beta_H'^*\sqrt{rk}(1+\frac{\sqrt{2}}{2})\sigma_{yH}$. Obviously, $\beta_H'^* > \beta_H^*$ and $\alpha_H'^* > \alpha_H^*$; that is, to induce the manager to choose the riskier project desired by the principal, the principal must offer

[30] When $\phi > \frac{1}{3c\sqrt{2rk}}$, we have $1 - c\sqrt{2rk}\phi < 2c\sqrt{2rk}\phi$ and, thus, $1 - c\sqrt{2rk}\phi < 2c\sqrt{2rk}\phi + 2rc^2\psi^2\sqrt{2rk}\phi$, which can be rewritten as $\frac{\beta_H^{*2}}{2c}(q_H^2 - q_L^2) < \sqrt{2rk}\beta_H^*(\sigma_{yH} - \sigma_{yL})$. Therefore, we have $CE_H < CE_L$.

a higher incentive as well as a higher fixed compensation. With the higher incentive and the higher fixed compensation, the manager's performance-reporting effort, $e_{2H}^{\prime *} = \beta_H^{\prime *}\sigma_{yH}\sqrt{\frac{r}{2k}}$, is higher too.

Proposition 4 *With endogenous variance, when $\frac{1}{3c\sqrt{2rk}} < \phi < \frac{1}{c\sqrt{2rk}}$, the principal must offer both a higher fixed wage and a higher incentive than those in the benchmark where the project selection can be observed in order to motivate the manager to choose the riskier project (i.e., $\beta_H^{\prime *} > \beta_H^*$ and $\alpha_H^{\prime *} > \alpha_H^*$). Furthermore, the induced performance-reporting effort is higher: $e_{2H}^{\prime *} > e_{2H}^*$.*

This result contrasts with Sung (1995)'s prediction that the principal would lower the incentive to motivate the manager to take a riskier project. The reason for this difference is because, in our model, the manager has the ability to reduce the performance-measurement risk, which he cannot do in Sung (1995). In Sung's setting, as the incentive increases, the manager remains reluctant to take a riskier project with a higher return because the risk premium increases as a result of an increased incentive. In our setting, however, the increase in incentive β also induces a higher managerial effort to reduce the risk, which may offset the increase in the risk premium due to a higher β. Therefore, it is more likely that the principal will be able to motivate a riskier project selection with a higher incentive in our setting. Notice that, different from the previous settings where we concentrated on the spillover effect, the result in this project-selection setting is driven by the informativeness effect of the reporting effort. In this induced moral-hazard problem, the principal benefits from the agent's reporting effort.

6 Conclusion

This paper focuses on the agent's effort to improves the precision in his performance measures. In our model the agent exerts two types of effort. One is the productive effort that increases the expected output of the firm, and the other is the performance-reporting effort that increases the quality of the manager's own performance measures. In addition, these two types of effort may compete for limited managerial attention. This research identifies a complication in the manager's effort allocation. Specifically, our analysis illustrates that the incentive contract shows a mixed risk-incentive relation. Furthermore, when we consider the addition of the spillover effect, we find that sometimes an informative signal is discarded to avoid increasing the marginal cost of productive effort. We also apply our model to a specific project-selection setting and show that, when the project-selection choice is unobservable, the principal may raise the incentive in order to motivate the manager to choose a riskier project.

We carried out the main analysis in this study in a tractable LEN framework. Considering future work, we would be interested to see if our result regarding additional signals holds in generalized non-linear contracts. In addition, a multiple-period version of this model that allows for an inter-temporal performance-reporting effort may elicit additional features. For example, Christensen, Feltham, and Sabac (2005) examine a multi-period setting and focus on the predictive power of the positive inter-temporal correlation of the performance measures, and future studies may examine the role of the manager's performance-reporting effort in such a multi-period model.

References

[1] Antle, R., and J. Demski. 1988. The controllability principle in responsibility accounting. *The Accounting Review* 63: 700–718.

[2] Arya, A., J. Glover, and S. Radhakrishnan. The controllability principle in responsibility accounting: Another look. In *Essays on Accounting Theory in Honour of Joel S. Demski*, edited by R. Antle, F. Gjesdal, and P. J. Liang. New York, NY: Springer.

[3] Baiman, S., and J. Demski. 1980. Economically optimal performance evaluation and control systems. *Journal of Accounting Research* 18 (Supplement): 184–220.

[4] Baker, G. and B. Jorgensen. 2005. *Volatility, noise, and incentives.* Working paper, Harvard University.

[5] Banker, R. D., and S. M. Datar. 1989. Sensitivity, precision, and linear aggregation of signals for performance evaluation. *Journal of Accounting Research* 27 (1): 21–39.

[6] Bertomeu, J. 2008. *Risk Management, executive compensation and the cross-section of corporate earnings.* Working paper, Northwestern University. Available at SSRN: http://ssrn.com/abstract=1276412.

[7] Bruggen, A., and F. Moers. 2007. The role of financial incentives and social incentives in multi-task settings. *Journal of Management Accounting Research* 19: 25–50.

[8] Christensen, P., G. A. Feltham, and F. Sabac. 2005. A contracting perspective on earnings quality. *Journal of Accounting and Economics* 39: 265–294.

[9] Christensen, P., F. Sabac, and J. Tian. 2010. Ranking performance measures in multi-task agencies. *The Accounting Review* 85 (5): 1545–1575.

[10] Collins, D. W., E. L. Maydew, and I. S. Weiss. 1997. Changes in the value-relevance of earnings and book values over the past forty years. *Journal of Accounting and Economics* 24: 39–67.

[11] Danielsson, J., B. Jorgensen, and C. de Vries. 2002. Incentives for effective risk management. *Journal of Banking and Finance* 26: 1407–1425.

[12] Darrough, M. N., and N. D. Melumad. 1995. Divisional versus company-wide focus: The trade-off between allocation of managerial attention and screening of talent. *Journal of Accounting Research* 33 (Supplement): 65–94.

[13] Dechow, P. M., and I. D. Dichev. 2002. The quality of accruals and earnings: The role of accrual estimation errors. *The Accounting Review* 77 (Supplement): 35–59.

[14] Demski, J. S. 1994. *Managerial Uses of Accounting Information*, Chapter 22, Kluwer Academic Publishers, USA. ISBN: 0792398475

[15] Demski, J. S., and R. Dye. 1999. Risk, return, and moral hazard. *Journal of Accounting Research* 37 (1): 27–55.

[16] Dutta, S. 2008. Managerial expertise, private information, and pay-performance sensitivity. *Management Science* 54 (3): 429–442.

[17] Dutta, S., and S. Reichelstein. 2003. Leading indicator variables, performance measurement and long-term versus short-term contracts. *Journal of Accounting Research* 41 (5): 837–866.

[18] Dye, R., and S. Sridhar. 2007. The allocational effects of the precision of accounting estimates. *Journal of Accounting Research* 45 (4): 731–769.

[19] Elsum, I. 2008. Managing' high uncertainty innovation. Presented at the Commonwealth Scientific and Industrial Research Organization (CSIRO) Workshop on Supporting Risk-Aware Research, July 11, Australia.

[20] Feltham, G. A., and M. G. H. Wu. 2000. Public reports, information acquisition by investors, and management incentives. *Review of Accounting Studies* 5: 155–190.

[21] Feltham, G. A., and M. G. H. Wu. 2001. Incentive efficiency of stock versus options. *Review of Accounting Studies* 6: 7–28.

[22] Feltham, G. A., and J. Xie. 1994. Performance measure congruity and diversity in multi-task principal/agent relations. *The Accounting Review* 69 (3): 429–453.

[23] Francis, J., R. LaFond, P. Olsson, and K. Schipper. 2005. The market pricing of accruals quality. *Journal of Accounting and Economics* 39: 295–327.

[24] Francis, J., and K. Schipper. 1999. Have financial statements lost their relevance? *Journal of Accounting Research* 37 (2): 319–352.

[25] Geanakoplos, J., and P. Milgrom. 1991. A theory of hierarchies based on limited managerial attention. *Journal of the Japanese and International Economics* 5: 205–225.

[26] Hemmer, T. 2006. On the subtleties of the principal-agent model. In *Essays in Accounting Theory in Honour of Joel S. Demski*, edited by R. Antle, F. Gjesdal, and P. J. Liang. Nowell, MA: Springer.

[27] Holmstrom, B. 1979. Moral hazard and observability. *The Bell Journal of Economics* 10 (1): 74–91.

[28] Holmstrom, B., and P. Milgrom. 1991. Multitask principal-agent analysis: Incentive contracts, assets ownership, and job design. *Journal of Law, Economics, & Organization* 7: 24–52.

[29] Huddart, S., and P. J. Liang. 2005. Profit sharing and monitoring in partnerships. Journal of Accounting and Economics 40: 153–187.

[30] Hughes, J. 1982. Agency theory and stochastic dominance. *Journal of Financial and Quantitative Analysis* 17 (3): 341–361.

[31] Hughes, J. S. and S. Pae. 2004. Voluntary disclosure of precision information. *Journal of Accounting and Economics* 37: 261–289.

[32] Hunt, A., S. Moyer, and T. Shevlin. 2000. *Earnings volatility, earnings management, and equity value.* Working paper, University of Washington.

[33] KPMG LLP. 2007. Defining the Issues. No. 07-20. Available at: http://www.us.kpmg.com/definingissues.

[34] Lafontaine F., and S. Bhattacharyya. 1995. The role of risk in franchising. *Journal of Corporate Finance* 2 (1–2): 39–74.

[35] Lambert, R. A. 1986. Executive effort and selection of risky projects. *Rand Journal of Economics*17 (1): 77–88.

[36] Liang, P. J., M. Rajan, and K. Ray. 2008. Optimal team size and monitoring in organizations. *The Accounting Review* 83 (3): 789–822.

[37] London Stock Exchange. 2002. A Practical Guide to Listing. Available at: http://www.londonstockexchange.com/en-gb/products/companyservices/.

[38] Meth, B. 1996. Reduction of outcome variance: Optimality and incentives. *Contemporary Accounting Research* 13 (1): 309–328.

[39] Peng, L., and A. Roell. 2008. Manipulation and equity-based compensation. *American Economic Review: Papers and Proceedings* 98 (2): 285–290.

[40] Penno, M. 1996. Unobservable precision choices in financial reporting. *Journal of Accounting Research* 34 (1): 141–149.

[41] Prendergast, C. 2002. The tenuous trade-off between risk and incentives. *Journal of Political Economy* 110 (5): 1071–1102.

[42] Rajan, M. V. and R. E. Saouma. 2006. Optimal information asymmetry. *The Accounting Review* 81 (3): 677–712.

[43] Roberts, J. 2004. *The Modern Firm: Organizational Design for Performance and Growth.* Oxford University Press, USA. ISBN: 019829376.

[44] Sayther, C. A. 2003. Report card on Sarbanes–Oxley: One year later. *Financial Executive* 19 (7): 6.

[45] Simon, H. A. 1973. Applying information technology to organizational design. *Public Administration Review* 33 (3): 268–278.

[46] Stocken, P. C. and R. E. Verrecchia. 2004. Financial reporting system choice and disclosure management. *The Accounting Review* 79 (4): 1181–1203.

[47] Stone, A. 2005. SOX: Not so bad after all? *Business Week* (August 1).

[48] Stovall, D. C. 2008. Sox compliance: Cost and value. *The Business Review, Cambridge* 11 (2): 107–113.

[49] Subramanyam, K. R. 1996. The pricing of discretionary accruals. *Journal of Accounting and Economics* 22: 249–281.

[50] Sung, J. 1995. Linearity with project selection and controllable diffusion rate in continuous-time principal-agent problems. *RAND Journal of Economics* 26 (4): 720–743.

[51] Donaldson, W. H. 2006. Testimony delivered to the House Committee on Financial Services, April 21. Available at: www.sec.gov.

[52] Securities and Exchange Commission (SEC). 2007. *Commission Guidance Regarding Management's Report on Internal Control Over Financial Reporting Under Section 13(a) or 15(d) of the Securities Exchange Act of 1934.* SEC Releases No. 33-8810. Washington D.C.: SEC. Available at: www.sec.gov.

[53] Securities and Exchange Commission (SEC). 2007. *Amendments to Rules Regarding Management's Report on Internal Control Over Financial Reporting.* SEC Release No. 33-8809. Washington D.C.: SEC. Available at: www.sec.gov.

[54] Tucker, J. W. and P. A. Zarowin. 2006. Does income smoothing improve earnings informativeness? *The Accounting Review* 81 (1): 251–270.

[55] Zhang, L. 2003. Complementarity, task assignment, and incentives. *Journal of Management Accounting Research* 15: 225–246.

Appendix

Proof of Proposition 1

With (1), (2), and (8), (4) can be rewritten into a quadratic equation of β :

$$-rqC_{12}V_{e_2}(e_2,\sigma^2)\beta^2 - \{q^2C_{22} + r[(V(e_2,\sigma^2)+\sigma_x^2)\Delta - qC_{12}V_{e_2}(e_2,\sigma^2)]\}\beta + q^2C_{22} = 0. \tag{11}$$

With $C_{12} = 0$, from (11) we see the optimal solution for β is

$$\beta^* = \frac{q^2}{q^2 + r[V(e_2,\sigma^2)+\sigma_x^2]C_{11}(e_1)}.$$

It is easy to verify that

$$\frac{\partial\beta^*}{\partial\sigma^2} = -\frac{rq^2C_{11}V_{\sigma^2}}{[q^2 + r(V+\sigma_x^2)C_{11}]^2} < 0.$$

Proof of Corollary 1

To examine the relationship between β^* and $V(e_2^*,\sigma^2)$, first notice that a direct effect of an increase in σ^2 is a higher V ($V_{\sigma^2} > 0$). Also, since $\frac{\partial\beta^*}{\partial\sigma^2} < 0$, an increase in σ^2 results in a lower β; additionally, it results in a lower level of e_2 because $\frac{de_2}{d\beta} = -\frac{rC_{11}V_{e_2}\beta}{\Delta} > 0$, which in turn leads to a higher V. In addition, if $\frac{\partial V^2(e_2,\sigma^2)}{\partial e_2\partial\sigma^2} > 0$, then an increase in σ^2 makes the performance-reporting effort less effective in reducing the variance; this change leads to a lower e_2 and thus a higher V. In other words, $\frac{\partial V^2(e_2,\sigma^2)}{\partial e_2 \partial \sigma^2} > 0$ is a sufficient condition for a positive relationship between σ^2 and $V(e_2^*, \sigma^2)$. Because we have $\frac{\partial \beta^*}{\partial \sigma^2} < 0$ with $C_{12} = 0$, there is a negative relationship between β^* and $V(e_2^*, \sigma^2)$.

When $\frac{\partial V^2(e_2,\sigma^2)}{\partial e_2 \partial \sigma^2}$ is sufficiently negative, the relationship between σ^2 and $V(e_2^*, \sigma^2)$ can be negative, which may lead to a positive relationship between β^* and $V(e_2^*, \sigma^2)$.

Proof of Proposition 2

With $V_{e_2} < 0$ and $C_{12} > 0$, from (11) the optimal solution for β is

$$\beta^* = \frac{q^2 C_{22} + r[(V+\sigma_x^2)\Delta - qC_{12}V_{e_2}] - \sqrt{\{q^2C_{22} + r[(V+\sigma_x^2)\Delta - qC_{12}V_{e_2}]\}^2 + 4rq^3 C_{12}C_{22}V_{e_2}}}{-2rqC_{12}V_{e_2}}.$$

Taking derivative of β^* with respect to σ^2, we get

$$\frac{\partial \beta^*}{\partial \sigma^2} = \frac{q^2 C_{22} \frac{\partial V^2(e_2,\sigma^2)}{\partial e_2 \partial \sigma^2}(1-\beta^*) - r\Delta[(-V_{e_2})V_{\sigma^2} + \frac{\partial V^2(e_2,\sigma^2)}{\partial e_2 \partial \sigma^2}(V+\sigma_x^2)]\beta^*}{(-V_{e_2})\sqrt{\{q^2C_{22} + r[(V+\sigma_x^2)\Delta - qC_{12}V_{e_2}]\}^2 + 4rq^3 C_{12}C_{22}V_{e_2}}}.$$

The denominator of $\frac{\partial \beta^*}{\partial \sigma^2}$ is positive; therefore the sign of $\frac{\partial \beta^*}{\partial \sigma^2}$ depends on the numerator, $q^2 C_{22} \frac{\partial V^2(e_2,\sigma^2)}{\partial e_2 \partial \sigma^2}(1-\beta^*) - r\Delta[(-V_{e_2})V_{\sigma^2} + \frac{\partial V^2(e_2,\sigma^2)}{\partial e_2 \partial \sigma^2}(V+\sigma_x^2)]\beta^*$. It is easy to verify that the numerator is positive if $\frac{\partial V^2(e_2,\sigma^2)}{\partial e_2 \partial \sigma^2} > \frac{r\Delta(-V_{e_2})V_{\sigma^2}\beta^*}{(1-\beta^*)q^2C_{22} - r\Delta(V+\sigma_x^2)\beta^*}$.

In addition, from the analysis in Section 3.2, $\frac{\partial V^2(e_2,\sigma^2)}{\partial e_2 \partial \sigma^2}$ must be positive to have $\frac{\partial \beta^*}{\partial \sigma^2} > 0$.

To get a sufficient condition for $\frac{\partial \beta^*}{\partial \sigma^2} > 0$, notice that $\Gamma \equiv \frac{r\Delta(-V_{e_2})V_{\sigma^2}\beta^*}{(1-\beta^*)q^2C_{22} - r\Delta(V+\sigma_x^2)\beta^*} = 0$ when $\beta = 0$, and that $\Gamma = \frac{(-V_{e_2})V_{\sigma^2}}{-(V+\sigma_x^2)} < 0$ when $\beta = 1$. In addition, $\frac{\partial \Gamma}{\partial \beta} > 0$ if $\beta < \frac{1}{2}$, and $\frac{\partial \Gamma}{\partial \beta} \leq 0$ if $\beta \geq \frac{1}{2}$. Thus, Γ maximizes at $\beta = \frac{1}{2}$ and its maximum value is $\frac{r\Delta(-V_{e_2})V_{\sigma^2}}{q^2C_{22} - r\Delta(V+\sigma_x^2)}$. Further, $V \leq \sigma^2$. Therefore, an explicit sufficient condition for $\frac{\partial \beta^*}{\partial \sigma^2} > 0$ is $\frac{\partial V^2(e_2,\sigma^2)}{\partial e_2 \partial \sigma^2} > \max\{0, \frac{r\Delta(-V_{e_2})V_{\sigma^2}}{q^2C_{22} - r\Delta(\sigma^2+\sigma_x^2)}\}$.

Proof of Corollary 2

To examine the relationship between β^* and $V(e_2^*, \sigma^2)$, first notice that a direct effect of an increase in σ^2 is a higher V ($V_{\sigma^2} > 0$). Also, since $\frac{\partial V^2(e_2,\sigma^2)}{\partial e_2 \partial \sigma^2} > 0$, an increase in σ^2 makes the performance-reporting effort less effective in reducing the variance; this change leads to a lower e_2 and thus a higher V. In addition, if C_{12} is sufficiently large, we have $\frac{de_2}{d\beta} = -\frac{qC_{12} + rC_{11}V_{e_2}\beta}{\Delta} < 0$, which implies that a higher β leads to a lower level of e_2 and thus a higher V. In other words, when the relationship between the incentive and σ^2 is positive, a sufficient condition for a positive relationship between β^* and $V(e_2^*, \sigma^2)$ is that C_{12} is sufficiently large.

Proof of Lemma 1

(1) Benchmark 1: Exogenous Variance ($V(e_2, \sigma^2) = \sigma^2$).

The agent chooses his productive effort e_1 to maximize his payoff $\alpha + \beta E[y] + \delta E[z] - \frac{r}{2}\beta^2(V + \sigma_x^2) - \frac{r}{2}\delta^2(\sigma_z^2 + \sigma_x^2) - r\beta\delta\sigma_x^2 - C(e_1)$. From the first-order condition with respect to e_1, we have $C_{e_1}(e_1) = q(\beta + \delta)$, $\frac{de_1}{d\beta} = \frac{q}{C_{11}(e_1)}$ and $\frac{de_1}{d\delta} = \frac{q}{C_{11}(e_1)}$.

The principal's problem is:

$$\max_{\beta,\delta} qe_1^* - \frac{r}{2}\beta^2(\sigma^2+\sigma_x^2) - \frac{r}{2}\delta^2(\sigma_z^2+\sigma_x^2) - r\beta\delta\sigma_x^2 - C(e_1^*).$$

The principal's first-order conditions show that:

$$\begin{aligned}\frac{q^2}{C_{11}(e_1)} - r\beta(\sigma^2+\sigma_x^2) - r\delta\sigma_x^2 - \frac{q^2(\beta+\delta)}{C_{11}(e_1)} &= 0,\\ \frac{q^2}{C_{11}(e_1)} - r\delta(\sigma_z^2+\sigma_x^2) - r\beta\sigma_x^2 - \frac{q^2(\beta+\delta)}{C_{11}(e_1)} &= 0.\end{aligned}$$

From the principal's first-order conditions, we have:

$$\begin{aligned}\beta^* &= \frac{q^2/C_{11}}{r(\sigma^2+\sigma_x^2)\frac{\sigma_z^2}{\sigma^2} + r\sigma_x^2 + q^2(\frac{\sigma_z^2}{\sigma^2}+1)/C_{11}} > 0,\\ \delta^* &= \frac{(q^2\frac{\sigma_z^2}{\sigma^2})/C_{11}}{r(\sigma^2+\sigma_x^2)\frac{\sigma_z^2}{\sigma^2} + r\sigma_x^2 + q^2(\frac{\sigma_z^2}{\sigma^2}+1)/C_{11}} > 0.\end{aligned}$$

Therefore, in Benchmark 1, β^* and δ^* are both positive as long as $\sigma_z^2, \sigma^2, \sigma_x^2 < +\infty$. In addition, β^* and δ^* have a relation that $\beta^*\sigma^2 = \delta^*\sigma_z^2$.

(2) Benchmark 2: Separable Cost ($C(e_1,e_2) = L(e_1)+K(e_2)$; following convention, the $'$ symbol indicates derivatives such as $L'(e_1)$ and $K'(e_2)$).

The agent's problem with the additional signal is

$$\max_{e_1,\ e_2} \alpha + \beta E[y] + \delta E[z] - \frac{r}{2}\beta^2[V(e_2,\sigma^2)+\sigma_x^2] - \frac{r}{2}\delta^2(\sigma_z^2+\sigma_x^2) - r\beta\delta\sigma_x^2 - C(e_1,e_2). \tag{12}$$

The first-order condition with respect to e_1 shows that

$$L'(e_1) = q(\beta+\delta).$$

If we totally differentiate this first-order condition, we get $\frac{de_1}{d\beta} = \frac{q}{L''(e_1)}$ and $\frac{de_1}{d\delta} = \frac{q}{L''(e_1)}$.

The first-order condition with respect to e_2 is

$$-\frac{r}{2}\beta^2 V_{e_2}(e_2,\sigma^2) - K'(e_2) = 0. \tag{13}$$

Eq. (13) implies that if $\beta > 0$, then $e_2^* > 0$ must be true, since $V_{e_2} = -\infty$ at $\beta = 0$.

The principal's problem is

$$\max_{\beta,\delta} E[x(e_1)] - \frac{r}{2}\beta^2[V(e_2,\sigma^2)+\sigma_x^2] - \frac{r}{2}\delta^2(\sigma_z^2+\sigma_x^2) - r\beta\delta\sigma_x^2 - L(e_1) - K(e_2). \tag{14}$$

The problem yields a first-order condition for optimal choice of incentive β:

$$\frac{q^2}{L''} - \frac{q^2(\beta+\delta)}{L''} - r\beta[V(e_2,\sigma^2)+\sigma_x^2] - r\delta\sigma_x^2 + \left[-\frac{r}{2}\beta^2 V_{e_2}(e_2,\sigma^2) - K'(e_2)\right]\frac{de_2}{d\beta} = 0. \qquad (15)$$

$\frac{q^2}{L''}$ is the marginal benefit of increasing β, and

$$\frac{q^2(\beta+\delta)}{L''} + r\beta[V(e_2,\sigma^2)+\sigma_x^2] + r\delta\sigma_x^2 + \left[-\frac{r}{2}\beta^2 V_{e_2}(e_2,\sigma^2) - K'(e_2)\right]\frac{de_2}{d\beta}$$

is the marginal cost. If marginal benefit is lower than marginal cost, then the optimal β will be zero, which is a corner solution.

If (13) is satisfied (that is, if $e_2^* > 0$), according to (13), $-\frac{r}{2}\beta^2 V_{e_2}(e_2,\sigma^2) - K'(e_2) = 0$, yielding

$$\beta = \beta\left(e_2,\delta,\sigma^2\right) \equiv \frac{q^2(1-\delta) - r\delta\sigma_x^2 L''}{r[V(e_2,\sigma^2)+\sigma_x^2]L'' + q^2}. \qquad (16)$$

The first-order condition with respect to δ yields

$$\frac{q^2}{L''} - \frac{q^2(\beta+\delta)}{L''} - r\delta(\sigma_z^2+\sigma_x^2) - r\beta\sigma_x^2 = 0, \qquad (17)$$

$$\delta = \Delta(e_2,\beta,\sigma_z^2) \equiv \frac{q^2(1-\beta) - r\beta\sigma_x^2 L''}{r(\sigma_z^2+\sigma_x^2)L'' + q^2}.$$

There are four possible cases to consider:

1. $\beta^* = 0, \delta^* = 0$: This case leads to $e_1^* = 0, e_2^* = 0$. This cannot be the optimal solution, since (17) shows a marginal benefit of $\frac{q^2}{L''} > 0$ and zero marginal cost. It can be improved by increasing δ slightly.

2. $\beta^* > 0, \delta^* = 0$: (15) becomes $(1-\beta)\frac{q^2}{L''} - r\beta[V(e_2,\sigma^2)+\sigma_x^2] = 0$, which gives $\beta^* = \frac{q^2}{r(V+\sigma_x^2)L''+q^2}$. If we substitute β^* to (17), we see that the net between the marginal benefit and the marginal cost, $[1 - \frac{q^2}{r(V+\sigma_x^2)L''+q^2}]\frac{q^2}{L''} - \frac{r\sigma_x^2 q^2}{r(V+\sigma_x^2)L''+q^2} = \frac{rVq^2}{r(V+\sigma_x^2)L''+q^2}$, is always positive. That is, the marginal benefit is always higher than the marginal cost, and the principal can improve by increasing δ. Therefore $\beta^* > 0, \delta^* = 0$ cannot be the optimal solution.

3. $\beta^* = 0, \delta^* > 0$: This case leads to $e_2^* = 0$. From (17), when $\beta^* = 0$ we have $\delta^* = \frac{q^2}{r(\sigma_z^2+\sigma_x^2)L''+q^2} > 0$. Substituting δ^* into (15), evaluated at $e_2^* = 0$, $\beta^* = 0$ holds when

$$\frac{rq^2(\sigma_z^2+\sigma_x^2)}{r(\sigma_z^2+\sigma_x^2)L''+q^2} \leq \frac{rq^2\sigma_x^2}{r(\sigma_z^2+\sigma_x^2)L''+q^2} + K'(e_2)\frac{de_2}{d\beta}\,|_{\beta=0}\,. \qquad (18)$$

Because $K'(e_2)|_{\beta=0} = 0$ when costs are separable, inequality (18) doesn't hold, and it is impossible to have $\beta^* = 0, \delta^* > 0$.

4. $\beta^* > 0, \delta^* > 0$: This case leads to $e_1^* > 0, e_2^* \geq 0$. When (13) holds ($e_2^* > 0$), the optimal β^* and δ^* are:

$$\beta^* = \frac{q^2/L''}{r(V+\sigma_x^2)\frac{\sigma_z^2}{V} + r\sigma_x^2 + q^2(\frac{\sigma_z^2}{V}+1)/L''}, \tag{19}$$

$$\delta^* = \frac{(q^2\frac{\sigma_z^2}{V})/L''}{r(V+\sigma_x^2)\frac{\sigma_z^2}{V} + r\sigma_x^2 + q^2(\frac{\sigma_z^2}{V}+1)/L''}. \tag{20}$$

When (13) shows a greater marginal cost of e_2 than its marginal benefit ($e_2^* = 0$), we have the optimal β^* and δ^* decided by (19) and (20).

General Analysis of Additional Signal Setting with Spillover

The manager's problem with the additional signal is

$$\max_{e_1,\ e_2} \alpha + \beta E[y] + \delta E[z] - \frac{r}{2}\beta^2[V(e_2,\sigma^2)+\sigma_x^2] - \frac{r}{2}\delta^2(\sigma_z^2+\sigma_x^2) - r\beta\delta\sigma_x^2 - C(e_1,e_2).$$

The first-order condition with respect to e_1 shows that the optimal e_1 satisfies

$$q\,(\beta+\delta) = C_{e_1}(e_1,e_2).$$

In addition, it yields an additional incentive constraint on the equilibrium choice of e_2:

$$-\frac{r}{2}\beta^2 V_{e_2}(e_2,\sigma^2) - C_{e_2}(e_1,e_2) = 0. \tag{21}$$

The principal's problem is now

$$\max_{\beta,\delta} E[x\,(e_1)] - \frac{r}{2}\beta^2[V(e_2,\sigma^2)+\sigma_x^2] - \frac{r}{2}\delta^2(\sigma_z^2+\sigma_x^2) - r\beta\delta\sigma_x^2 - C(e_1,e_2). \tag{PP2}$$

It yields a first-order condition for optimal choice of incentive δ; this optimal choice, after substituting the incentive constraint for e_1 (i.e., $q\,(\beta+\delta) = C_{e_1}(e_1,e_2)$), can be written as

$$q\frac{de_1}{d\delta} - r\delta(\sigma_z^2+\sigma_x^2) - q\,(\beta+\delta)\,\frac{de_1}{d\delta} - r\beta\sigma_x^2 = 0. \tag{22}$$

From $q\,(\beta+\delta) = C_{e_1}(e_1,e_2)$, we have $qd\delta = C_{11}de_1 + C_{12}de_2$, which implies that $q = C_{11}\frac{de_1}{d\delta} + C_{12}\frac{de_2}{d\delta}$. $-\frac{r}{2}\beta^2 V_{e_2}(e_2,\sigma^2) - C_{e_2}(e_1,e_2) = 0$ implies that $\frac{de_2}{d\delta} = 0$. Therefore, $\frac{de_1}{d\delta} = \frac{q}{C_{11}(e_1,e_2)}$. Substituting $\frac{de_1}{d\delta}$, the first-order condition for optimal choice of incentive δ becomes

$$\delta = \Delta(e_2,\beta,\sigma_z^2) \equiv \frac{q^2(1-\beta) - r\beta\sigma_x^2 C_{11}(e_1,e_2)}{r(\sigma_z^2+\sigma_x^2)C_{11}(e_1,e_2) + q^2}.$$

Additionally, the first-order condition with respect to β, after substituting $q\,(\beta+\delta) = C_{e_1}(e_1,e_2)$

and $\frac{de_1}{d\beta}$, can be written as

$$\frac{q(q - C_{12}\frac{de_2}{d\beta})}{C_{11}} - \frac{q(\beta+\delta)(q - C_{12}\frac{de_2}{d\beta})}{C_{11}} - r\beta(V + \sigma_x^2) - r\delta\sigma_x^2 + \left[-\frac{r}{2}\beta^2 V_{e_2} - C_{e_2}\right]\frac{de_2}{d\beta} = 0. \quad (23)$$

If we assume $C(e_1, e_2) = e_1^2 f(e_2)$, $\frac{de_2}{d\beta} = 0$, and if in equilibrium $-\frac{r}{2}\beta^2 V_{e_2}(e_2, \sigma^2) - C_{e_2}(e_1, e_2) = 0$ is satisfied, (23) becomes

$$\frac{q^2}{C_{11}(e_1, e_2)} - \frac{q^2(\beta+\delta)}{C_{11}(e_1, e_2)} - r\beta[V(e_2, \sigma^2) + \sigma_x^2] - r\delta\sigma_x^2 + 0 = 0,$$

where the $\frac{q^2}{C_{11}(e_1,e_2)}$ term is the marginal benefit of increasing β. The rest of the terms are the marginal costs. The above equation is reduced to

$$\beta = \frac{q^2(1-\delta) - r\delta\sigma_x^2 C_{11}(e_1, e_2)}{r[V(e_2, \sigma^2) + \sigma_x^2]C_{11}(e_1, e_2) + q^2}.$$

In this case, it is easy to see that both signals are used in contracts, similar to the benchmark cases. However, if (21) is not satisfied in equilibrium (that is, if $e_2^* = 0$, a corner solution), we show it to be possible that $\beta^* = 0$ and $\delta^* > 0$. Suppose the solution of $\beta = 0$ and $\delta > 0$. From (22) we learn that $\delta = \Delta(e_2, \beta = 0, \sigma_z^2) \equiv \frac{q^2}{r(\sigma_z^2+\sigma_x^2)C_{11}(e_1,e_2)+q^2} < 1$. From (23), we learn that, at $\beta = 0$, the marginal net benefit (i.e., marginal benefit minus all marginal costs) is

$$\frac{q^2(1-\delta)}{C_{11}(e_1, e_2)} - \left[\frac{q(1-\delta)C_{12}(e_1, e_2)}{C_{11}(e_1, e_2)} + C_{e_2}(e_1, e_2)\right]\frac{de_2}{d\beta} - r\delta\sigma_x^2. \quad (24)$$

Let us consider each term in the marginal net benefit evaluated at the contemplated solution $\beta = 0$ and $\delta > 0$:

- The $\frac{q^2(1-\delta)}{C_{11}(e_1,e_2)}$ term is always positive, since $\delta < 1$.
- At the contemplated $\beta = 0$, e_2 is a corner solution ($e_2 = 0$), but $C_{12}(.)$ and C_2 are both positive because e_1 is positive (since a positive δ induces some productive effort). Furthermore, notice that $\frac{de_2}{d\beta}|_{\beta=0,\delta>0} > 0$.[31] Therefore, $\left[\frac{q(1-\delta)C_{12}(e_1,e_2)}{C_{11}(e_1,e_2)} + C_{e_2}(e_1, e_2)\right]\frac{de_2}{d\beta}$ is positive.
- $r\delta\sigma_x^2$ is always positive.

If the second and the third terms dominate, $\beta = 0$ is indeed optimal because the marginal net benefit (Eq. [24]) is negative. As a result, $\beta = 0$ and $\delta > 0$ can be sustained as an equilibrium.[32] As long as $\frac{q^2(1-\delta)}{C_{11}(e_1,e_2)} < \left[\frac{q(1-\delta)C_{12}(e_1,e_2)}{C_{11}(e_1,e_2)} + C_{e_2}(e_1, e_2)\right]\frac{de_2}{d\beta} + r\delta\sigma_x^2$, signal y will be ignored. The important factor, again, is the spillover effect between the two efforts.

[31] Notice that when the first-order condition with respect to e_2 ($-\frac{r}{2}\beta^2 V_{e_2}(e_2, \sigma^2) - C_{e_2}(e_1, e_2) = 0$) is not satisfied at the corner solution $e_2^* = 0$, $\frac{de_2}{d\beta} \neq 0$ even with the assumption $C(e_1, e_2) = e_1^2 f(e_2)$.

[32] See the proof of Proposition 3 for a discussion of second-order conditions.

Intuitively, the principal would always use z. No matter whether signal y is used or not ($\beta > 0$ or $\beta = 0$), the marginal benefit of increasing δ is always greater than the marginal cost. However, at $\beta = 0$ the marginal benefit of increasing β may be less than the marginal cost because of the spillover effect. In other words, if the principal uses y in the contract at all, the marginal cost from risk-sharing is zero, but the marginal cost from limited attention is positive, which may outweigh the positive marginal benefit. Therefore, sometimes it is efficient for the principal to ignore signal y ($\beta^* = 0$), even though signal y is informative.

Similar to the second part of the proof of Lemma 1, there are four possible cases for this analysis. Cases 1, 2, and 4 follow nearly identical arguments. Therefore, we only provide details for Case 3.

If $\beta^* = 0$ and $\delta^* > 0$, then $e_2^* = 0$. From (22), when $\beta^* = 0$ we have $\delta^* = \frac{q^2}{r(\sigma_z^2+\sigma_x^2)C_{11}+q^2} > 0$. If we substitute δ^* into (24), we have

$$\frac{qr(\sigma_z^2+\sigma_x^2)}{r(\sigma_z^2+\sigma_x^2)C_{11}+q^2} - [\frac{qr(\sigma_z^2+\sigma_x^2)C_{12}}{r(\sigma_z^2+\sigma_x^2)C_{11}+q^2} + C_{e_2}]\frac{de_2}{d\beta} \mid_{\beta=0,\delta=\frac{q^2}{r(\sigma_z^2+\sigma_x^2)C_{11}+q^2}}.$$

As long as

$$\frac{qr(\sigma_z^2+\sigma_x^2)}{r(\sigma_z^2+\sigma_x^2)C_{11}+q^2} \leq [\frac{qr(\sigma_z^2+\sigma_x^2)C_{12}}{r(\sigma_z^2+\sigma_x^2)C_{11}+q^2} + C_{e_2}]\frac{de_2}{d\beta} \mid_{\beta=0,\delta=\frac{q^2}{r(\sigma_z^2+\sigma_x^2)C_{11}+q^2}}, \quad (25)$$

$\beta^* = 0$, $\delta^* > 0$ is sustained as an equilibrium. Notice that this condition requires a non-negative $\frac{de_2}{d\beta}$ at $\beta = 0$. In (21), we see that a slight increase of β from $\beta = 0$ will increase the marginal benefit of e_2 tremendously (from zero to positive infinity). Therefore, we have $\frac{\partial e_2}{\partial \beta} \mid_{\beta=0\to 0^+} > 0$.

In addition, we see that a slight decrease of β from $\beta = 0$ will increase the marginal benefit of e_2 from zero to positive infinity. That is, we have $\frac{\partial e_2}{\partial \beta} \mid_{\beta=0\to 0^-} < 0$. Therefore, a decrease of β from 0 to 0^- will only increase the marginal cost (which includes a higher risk premium and a higher marginal cost of e_1); in this case, the principal would not choose a negative β. In other words, β must be non-negative.

Proof of Proposition 3

Now, using a specific example we show the existence of a case where signal y can be ignored. In the example with $V(e_2, \sigma^2) = \frac{\sigma^2}{e_2}$ and $C(e_1, e_2) = \frac{1}{2}(c_1 + ke_2)e_1^2$, the manager's problem is:

$$\max_{e_1,\ e_2} \alpha + \beta E[y] + \delta E[z] - \tfrac{r}{2}\beta^2[V(e_2) + \sigma_x^2] - \tfrac{r}{2}\delta^2(\sigma_z^2 + \sigma_x^2) - r\beta\delta\sigma_x^2 - \tfrac{1}{2}(c_1 + ke_2)e_1^2.$$

The manager's first-order conditions, in closed form, are

$$\begin{aligned} \hat{e}_1 &= \frac{\beta(q - \sigma\sqrt{rk}) + q\delta}{c_1}, \\ \hat{e}_2 &= \frac{c_1\sigma\beta}{\beta\left(q - \sigma\sqrt{rk}\right) + q\delta}\sqrt{\frac{r}{k}}. \end{aligned}$$

The principal's problem is

$$\max_{\beta,\delta}\ q\widehat{e}_1 - \frac{1}{2}\left(c_1 + k\widehat{e}_2\right)\widehat{e}_1^2 - \frac{r}{2}\beta^2\frac{\sigma^2}{\widehat{e}_2} - \frac{r}{2}\delta^2\sigma_z^2 - \frac{r}{2}\beta^2\sigma_x^2 - \frac{r}{2}\delta^2\sigma_x^2 - r\beta\delta\sigma_x^2.$$

After substituting the manager's first-order conditions (FOCs), the problem becomes

$$\max_{\beta,\delta}\ \frac{q\beta(q-\sigma\sqrt{rk})+q^2\delta}{c_1} - \frac{\left[\beta(q-\sigma\sqrt{rk})+q\delta\right]^2}{2c_1} - \frac{\beta\sigma\sqrt{rk}\left[\beta(q-\sigma\sqrt{rk})+q\delta\right]}{c_1} - \frac{r}{2}\delta^2(\sigma_z^2+\sigma_x^2) - \frac{r}{2}\beta^2\sigma_x^2 - r\beta\delta\sigma_x^2.$$

It leads to a first-order condition and an interior solution (if binding) of β, as follows:

$$\begin{aligned} 0 &= \frac{q(q-\sigma\sqrt{rk})}{c_1} - \frac{\left[\beta(q-\sigma\sqrt{rk})+q\delta\right]q}{c_1} - \frac{(q-\sigma\sqrt{rk})\beta\sigma\sqrt{rk}}{c_1} - r\beta\sigma_x^2 - r\delta\sigma_x^2, \\ \widehat{\beta} &= \frac{q(q-\sigma\sqrt{kr}) - q^2\widehat{\delta} - rc_1\widehat{\delta}\sigma_x^2}{q^2 - \sigma^2 kr + r\sigma_x^2 c_1}, \end{aligned}$$

and a first-order condition with respect to δ *(FOC-δ):

$$\begin{aligned} 0 &= \frac{q^2}{c_1} - \frac{\beta q(q-\sigma\sqrt{rk})+q^2\delta}{c_1} - \frac{\beta\sigma\sqrt{kr}q}{c_1} - r\delta\sigma_z^2 - r\delta\sigma_x^2 - r\beta\sigma_x^2, \\ \widehat{\delta} &= \frac{q^2(1-\widehat{\beta}) - rc_1\widehat{\beta}\sigma_x^2}{q^2 + c_1 r(\sigma_z^2+\sigma_x^2)}. \end{aligned}$$

The second-order condition is described in the following Hessian:

$$H = \begin{bmatrix} \frac{\partial}{\beta^2}Obj & \frac{\partial}{\beta\delta}Obj \\ \frac{\partial}{\delta\beta}Obj & \frac{\partial}{\delta^2}Obj \end{bmatrix} = \begin{bmatrix} -\frac{1}{c_1}\left(q^2 - rk\sigma^2\right) - r\sigma_x^2 & -\frac{q^2}{c_1} - r\sigma_x^2 \\ -\frac{q^2}{c_1} - r\sigma_x^2 & -\frac{q^2}{c_1} - r(\sigma_z^2+\sigma_x^2) \end{bmatrix}.$$

Notice that $\frac{\partial}{\beta^2}Obj < 0$ under the maintained assumption of $q - \sigma\sqrt{rk}$. The sufficient condition for the principal minors to have alternate signs is $c_1/k > 1/\sigma_x^2 + 1/\sigma_z^2$. As a result, the global maximum can be assured as long as the spillover cost k is not too high.

If we suppose that $\widehat{\delta} = 0$, then we have

$$\widehat{\beta} = \frac{q(q-\sigma\sqrt{kr}) - q^2\widehat{\delta}}{q^2 - \sigma^2 kr + r\sigma_x^2 c_1} = \frac{q(q-\sigma\sqrt{kr})}{\left(q+\sigma\sqrt{kr}\right)(q-\sigma\sqrt{kr}) + r\sigma_x^2 c_1}.$$

FOC-δ then becomes

$$\frac{q^2}{c_1} - \frac{\widehat{\beta}q(q-\sigma\sqrt{rk})}{c_1} - r\widehat{\beta}\sigma_x^2 = \frac{q^2[(q+\sigma\sqrt{rk})(q-\sigma\sqrt{rk}) - (q-\sigma\sqrt{rk})^2] + rc_1\sigma_x^2\sigma\sqrt{rk}}{c_1[(q+\sigma\sqrt{rk})(q-\sigma\sqrt{rk}) + rc_1\sigma_x^2]} > 0,$$

a contradiction of the hypothesis $\widehat{\delta} = 0$; thus, $\widehat{\delta}$ must be positive in equilibrium.

If we suppose that $\widehat{\beta}=0$, then $\widehat{\delta}=\frac{q^2}{q^2+c_1 r(\sigma_z^2+\sigma_x^2)}$. Substituting this expression into the first-order condition with respect to β, we have the marginal benefit net of the marginal cost, evaluated at the proposed solution, equal to

$$\begin{aligned}
& \frac{q(q-\sigma\sqrt{rk})}{c_1}-\frac{0+q\widehat{\delta}}{c_1}[q-\sigma\sqrt{rk}]-\frac{0+q\widehat{\delta}}{c_1}\sigma\sqrt{rk}-\frac{r\sigma_x^2 q^2}{q^2+c_1 r(\sigma_z^2+\sigma_x^2)} \\
= & \frac{q}{c_1[q+c_1 r(\sigma_z^2+\sigma_x^2)]}\{[q^2+c_1 r(\sigma_z^2+\sigma_x^2)](q-\sigma\sqrt{rk})-q^3\}-\frac{r\sigma_x^2 q^2}{q^2+c_1 r(\sigma_z^2+\sigma_x^2)}.
\end{aligned}$$

Thus, a sufficient condition for the marginal benefit to be less than the marginal cost and thus $\widehat{\beta}=0$ is optimal is $[q^2+c_1 r(\sigma_z^2+\sigma_x^2)](q-\sigma\sqrt{rk})-q^3<0$, or equivalently, $c_1 r(\sigma_z^2+\sigma_x^2)<\frac{q^2\sigma\sqrt{rk}}{q-\sigma\sqrt{rk}}$. Recall that β must be non-negative, which can be regarded as the implicit constraint $\beta\geq 0$. If we incorporate this constraint in the program and examine the Kuhn–Tucker conditions, we see in most cases that this condition is not binding. However, when $c_1 r(\sigma_z^2+\sigma_x^2)<\frac{q^2\sigma\sqrt{rk}}{q-\sigma\sqrt{rk}}$, the first-order condition with respect to β is not zero and its Kuhn–Tucker multiplier is zero, while the condition $\beta\geq 0$ is binding with its Kuhn–Tucker multiplier being positive. In this case β must be zero and will not deviate from zero.

Given $\beta=0$, we only need to check the second-order condition with respect to δ. The second-order derivative of the principal's objective function with respect to δ is $-\frac{q^2}{c_1}-r\left(\sigma_z^2+\sigma_x^2\right)<0$. Therefore, the second-order condition is satisfied, and $\{\widehat{\delta}=\frac{q^2}{q^2+c_1 r(\sigma_z^2+\sigma_x^2)},\widehat{\beta}=0\}$ is indeed the global maximum when $c_1 r(\sigma_z^2+\sigma_x^2)<\frac{q^2\sigma\sqrt{rk}}{q-\sigma\sqrt{rk}}$.

Proof of Proposition 4

In the benchmark case where the manager's project-selection decision is observable, if the manager chooses the H-project, his certainty equivalent is $\alpha+\beta q_H e_{1H}-\frac{r}{2}\beta^2(\sigma_{xH}^2+\frac{\sigma_{yH}^2}{e_{2H}})-\frac{c}{2}e_{1H}^2-ke_{2H}$ and his optimal effort choices are $e_{1H}^*=\frac{\beta q_H}{c}$ and $e_{2H}^*=\beta\sigma_{yH}\sqrt{\frac{r}{2k}}$. Similarly, if the manager chooses the L-project, then his optimal choices are $e_{1L}^*=\frac{\beta q_L}{c}$ and $e_{2L}^*=\beta\sigma_{yL}\sqrt{\frac{r}{2k}}$.

If the principal wants the manager to choose the H-project, her design program is

$$\max_{\beta_H} q_H e_{1H}^*-\frac{r}{2}\beta^2(\sigma_{xH}^2+\frac{\sigma_{yH}^2}{e_{2H}^*})-\frac{c}{2}e_{1H}^{*2}-ke_{2H}^*.$$

Substituting e_{1H}^* and e_{1L}^* into the program, we have

$$\max_{\beta_H}\frac{q_H^2\beta_H}{c}-\frac{r}{2}\beta_H^2\sigma_{xH}^2-\beta_H\sigma_{yH}\sqrt{2rk}-\frac{q_H^2\beta_H^2}{2c}.$$

The principal's optimal β_H is $\beta_H^*=\frac{1-c\sigma_{yH}\sqrt{2rk}/q_H^2}{1+rc\sigma_{xH}^2/q_H^2}$ and optimal fixed payment is $\alpha_H^*=\left(\frac{1-c\sqrt{2rk}\phi}{1+rc\psi^2}\right)^2\frac{q_H^2(rc\psi^2-1)}{2c}+\frac{1-c\sqrt{2rk}\phi}{1+rc\psi^2}\sqrt{rk}(1+\frac{\sqrt{2}}{2})\sigma_{yH}$. Similarly, if the principal wants the manager to choose the L-project, she will offer $\beta_L^* = \frac{1-c\sigma_{yL}\sqrt{2rk}/q_L^2}{1+rc\sigma_{xL}^2/q_L^2}$ and $\alpha_L^* = \left(\frac{1-c\sqrt{2rk}\phi}{1+rc\psi^2}\right)^2 \frac{q_L^2(rc\psi^2-1)}{2c} + \frac{1-c\sqrt{2rk}\phi}{1+rc\psi^2}\sqrt{rk}(1+\frac{\sqrt{2}}{2})\sigma_{yL}$. As we assume $\frac{\sigma_{yH}}{q_H^2} = \frac{\sigma_{yL}}{q_L^2} \equiv \phi$ and $\frac{\sigma_{xH}}{q_H} = \frac{\sigma_{xL}}{q_L} \equiv \psi$, we have $\beta_H^* = \beta_L^* = \frac{1-c\sqrt{2rk}\phi}{1+rc\psi^2}$.

In this setting where the principal observes the manager's project-selection decision, her payoffs when the manager chooses the H-project and the L-project are, separately:

$$\begin{aligned} PP_H &= \frac{1-c\sqrt{2rk}\phi}{2c(1+rc\psi^2)}(q_H^2 - 2c\sigma_{yH}\sqrt{2rk} + c\sqrt{2rk}\phi q_H^2), \\ PP_L &= \frac{1-c\sqrt{2rk}\phi}{2c(1+rc\psi^2)}(q_L^2 - 2c\sigma_{yL}\sqrt{2rk} + c\sqrt{2rk}\phi q_L^2). \end{aligned}$$

The principal prefers the H-project if and only if $PP_H > PP_L$, which implies that $\phi < \frac{1}{c\sqrt{2rk}}$.

Now, we suppose that the manager's project-selection decision is unobservable. In this setting, if the principal still offers (α_H^*, β_H^*), the manager's certainty equivalent when choosing the H-project is

$$\begin{aligned} CE_H &= \alpha_H^* + \beta_H^* q_H e_{1H}^* - \frac{r}{2}\beta_H^{*2}(\sigma_{xH}^2 + \frac{\sigma_{yH}^2}{e_{2H}^*}) - \frac{c}{2}e_{1H}^{*2} - ke_{2H}^* \\ &= \alpha_H^* + \frac{q_H^2}{2c}\beta_H^{*2} - \frac{r\sigma_{xH}^2}{2}\beta_H^{*2} - \sqrt{2rk}\sigma_{yH}\beta_H^*, \end{aligned}$$

and his certainty equivalent when choosing the L-project is

$$\begin{aligned} CE_L &= \alpha_H^* + \beta_H^* q_L e_{1L}^* - \frac{r}{2}\beta_H^{*2}(\sigma_{xL}^2 + \frac{\sigma_{yL}^2}{e_{2L}^*}) - \frac{c}{2}e_{1L}^{*2} - ke_{2L}^* \\ &= \alpha_H^* + \frac{q_L^2}{2c}\beta_H^{*2} - \frac{r\sigma_{xL}^2}{2}\beta_H^{*2} - \sqrt{2rk}\sigma_{yL}\beta_H^*. \end{aligned}$$

When $CE_H < CE_L$, the manager will choose the L-project, even if the principal desires the riskier project.

A sufficient condition for $CE_H - CE_L < 0$ is $\phi > \frac{1}{3c\sqrt{2rk}}$. Therefore, when $\frac{1}{3c\sqrt{2rk}} < \phi < \frac{1}{c\sqrt{2rk}}$, the manager will choose the safer L-project when offered the same contract as in the observable setting, even though the principal desires the riskier H-project.

If the principal wants to motivate the manager to choose the H-project in this unobservable setting, she must offer a different β. Her design program becomes

$$\max_{\beta_H} \frac{q_H^2\beta_H}{c} - \frac{r}{2}\beta_H^2\sigma_{xH}^2 - \beta_H\sigma_{yH}\sqrt{2rk} - \frac{q_H^2\beta_H^2}{2c},$$

$$s.t. \frac{\beta_H^2}{2c}(q_H^2 - q_L^2) \geq \frac{r\beta_H^2}{2}(\sigma_{xH}^2 - \sigma_{xL}^2) + \sqrt{2rk}\beta_H(\sigma_{yH} - \sigma_{yL}).$$

Solving this program gives us $\beta_H'^* = \frac{2c\sqrt{2rk}\phi}{1-rc\psi^2}$ and $\alpha_H'^* = \beta_H'^{*2}\frac{q_H^2(rc\psi^2-1)}{2c} + \beta_H'^*\sqrt{rk}(1+\frac{\sqrt{2}}{2})\sigma_{yH}$. It is easy to verify that $\beta_H'^* = \frac{2c\sqrt{2rk}\phi}{1-rc\psi^2} > \beta_H^* = \frac{1-c\sqrt{2rk}\phi}{1+rc\psi^2}$, and $\alpha_H'^* > \alpha_H^* = \beta_H^{*2}\frac{q_H^2(rc\psi^2-1)}{2c} + \beta_H^*\sqrt{rk}(1+$

$\frac{\sqrt{2}}{2})\sigma_{yH}$.

A Setting with a CFO and a CEO

As in the main setting, we assume the managers' effort levels are not observable and not contractible. Instead, the principal compensates the managers based on a noisy signal y,

$$y = x + \varepsilon_y = qe_1 + \varepsilon_x + \varepsilon_y.$$

We denote the CFO's effort on performance reporting to be e_F, and denote the CEO's efforts on productive activities and performance reporting to be e_1 and e_2, respectively, as before. Furthermore, we assume

$$\begin{aligned} V(e_2, \sigma^2) &= \frac{\sigma^2}{e_2 + e_F}, \\ C(e_1, e_2) &= \frac{1}{2}(c_1 + ke_2)e_1^2 \ , \\ C(e_F) &= \frac{1}{2}k_F e_F, \end{aligned}$$

where $C(e_1, e_2)$ is the cost function for the CEO's efforts and $C(e_F)$ is the cost function for the CFO's effort. Notice that in this specific example we have the "spillover" in the CEO's cost function. We also assume that the compensation contracts for the CEO and the CFO are, respectively

$$\begin{aligned} w_{ceo} &= \alpha + \beta y, \\ \text{and } w_{cfo} &= \alpha_F + \beta_F y. \end{aligned}$$

The CEO chooses his efforts to maximize his expected utility

$$\max_{e_1,\ e_2} \alpha + \beta E[y] - \frac{r}{2}\beta^2[\frac{\sigma^2}{e_2 + e_F} + \sigma_x^2] - \frac{1}{2}(c_1 + ke_2)e_1^2,$$

with FOCs

$$\begin{aligned} c_1 e_1 + ke_1 e_2 &= q\beta, \\ -\frac{r}{2}\beta^2\left[-\frac{\sigma^2}{(e_2 + e_F)^2}\right] - \frac{k}{2}e_1^2 &= 0. \end{aligned}$$

The CFO chooses his effort to maximize his utility

$$\max_{e_F} \alpha_F + \beta_F E[y] - \frac{r}{2}\beta_F^2[\frac{\sigma^2}{e_2 + e_F} + \sigma_x^2] - \frac{1}{2}k_F e_F,$$

with FOC

$$-\frac{r}{2}\beta_F^2\left[-\frac{\sigma^2}{(e_2+e_F)^2}\right]-\frac{k_F}{2}=0.$$

Reducing the FOCs gives us

$$\begin{aligned}
e_1^* &= \frac{\beta}{\beta_F}\sqrt{\frac{k_F}{k}}>0,\\
e_2^* &= q\beta_F\sqrt{\frac{k}{k_F}}-c_1\geq 0,\\
e_F^* &= \frac{\beta_F}{\sqrt{k_F}}\left[\sigma\sqrt{r}-q\sqrt{k}\right]+c_1\geq 0.
\end{aligned}$$

Now, we prove that in equilibrium we do not have $e_2^*=0$.

Proof. If $e_2^*=0$, we must have

$$e_2^*=q\beta_F\sqrt{\frac{k}{k_F}}-c_1=0,$$

and thus the principal should choose

$$\beta_F^*=\frac{\sqrt{k_F}c_1}{q\sqrt{k}}.$$

The principal's objective function is

$$\begin{aligned}
&\max_{\beta,\beta_F} qe_1-\frac{1}{2}(c_1+ke_2)e_1^2-\frac{r}{2}\beta^2[\frac{\sigma^2}{e_2+e_F}+\sigma_x^2]\\
&-\frac{1}{2}k_Fe_F-\frac{r}{2}\beta_F^2[\frac{\sigma^2}{e_2+e_F}+\sigma_x^2].
\end{aligned}$$

Substituting e_1^*, e_2^*, and e_F^* into the principal's objective function, we have

$$\begin{aligned}
&\max_{\beta,\beta_F} q\frac{\beta}{\beta_F}\sqrt{\frac{k_F}{k}}-\frac{1}{2}(c_1+kq\beta_F\sqrt{\frac{k}{k_F}}-kc_1)\frac{\beta^2}{\beta_F^2}\frac{k_F}{k}\\
&-\frac{1}{2}k_F\frac{\beta_F}{\sqrt{k_F}}\left[\sigma\sqrt{r}-q\sqrt{k}\right]-\frac{1}{2}k_Fc_1-\frac{r}{2}(\beta^2+\beta_F^2)[\frac{\sigma\sqrt{k_F}}{\beta_F\sqrt{r}}+\sigma_x^2].
\end{aligned}$$

The FOCs with respect to β and β_F are, respectively

$$\begin{aligned}
FOC_{\beta_F} &: \sqrt{k_F}[-q\beta\frac{1}{k}-\frac{1}{2}q\beta^2\sqrt{k}+q\beta^2k\sqrt{k}+\frac{1}{2}\sigma\sqrt{r}\beta^2]\frac{1}{\beta_F^2}+(1-k)k_Fc_1\frac{\beta^2}{\beta_F^3}-r\sigma_x^2\beta_F+\sqrt{k_F}(\frac{1}{2}q\sqrt{k}-\sigma\sqrt{r})=\\
FOC_{\beta} &: q\sqrt{\frac{k_F}{k}}\frac{1}{\beta_F}-\{[(1-k)c_1+kq\sqrt{\frac{k}{k_F}}\beta_F]\frac{k_F}{k\beta_F^2}+\frac{\sigma\sqrt{rk_F}}{\beta_F}+r\sigma_x^2\}\beta=0.
\end{aligned}$$

Now, let us assume that the optimal β_F is $\frac{\sqrt{k_F}c_1}{q\sqrt{k}}$; that is, it is optimal to have $e_2^*=0$. Substituting $\beta_F = \frac{\sqrt{k_F} c_1}{q\sqrt{k}}$ into FOC_β, we solve the corresponding optimal β to be:

$$\beta = \frac{q^2}{q^2 k + q\sigma\sqrt{rk} + rc\sigma_x^2}.$$

It is easy to verify that this equation is not the solution to FOC_{β_F} if we also substitute $\beta_F = \frac{\sqrt{k_F} c_1}{q\sqrt{k}}$ into FOC_{β_F}. Therefore, $\beta_F = \frac{\sqrt{k_F} c_1}{q\sqrt{k}}$ cannot be the optimal β_F^*, and in equilibrium we do not have $e_2^* = 0$. ■