

Informational Feedback Effect, Adverse Selection, and the Optimal Disclosure Policy *

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September 2011

*This paper was previously titled as "Learn from and disclose to the stock market". Comments from participants at the 2009 Chicago-Minnesota Theory Conference, the 2010 Information, Markets and Organizations Conference at Harvard University, the 2010 Danish Center for Accounting and Finance Conference, and workshops at City University of Hong Kong, National Chengchi University (Taiwan), Pennsylvania State University, University of Alberta, and the University of Chicago are appreciated. We would like to thank in particular Jeremy Bertomeu, John Core, Douglas Diamond, Paul Fischer, Jonathan Glover, Milton Harris, Zhiguo He, Steve Huddart, Florin Sabac and Haresh Sapra for helpful discussions. Gao also acknowledges the generous financial support from Chicago Booth.

Abstract

Trading in a secondary stock market not only redistributes wealth among investors but also generates information that guides subsequent real decisions. We provide a disclosure model that reflects both functions of the secondary market. By partially preempting traders' information advantage established from information acquisition, disclosure reduces incentives for private information acquisition. The resulting reduction in information acquisition has two opposite effects on firm value. On one hand, it narrows the information gap between informed and uninformed traders and improves the liquidity of firm shares. On the other hand, it reduces the informational feedback from the stock market to real decisions. The optimal disclosure policy is determined by the trade-off between liquidity enhancement and the investment efficiency. The model explains why the firm value could be higher in an environment that promotes disclosure and private information production at the same time and why growth firms are endogenously more opaque than value firms.

JEL classification: G14, K22, M41, M48

Key Words: Disclosure, Adverse Selection, Informational Feedback Effect, Securities Regulation

1 Introduction

Disclosure has been an important part of corporate policy and the foundation of securities regulation in the United States since its inception in 1930's. One major theoretical support for disclosure to a secondary market is that it levels the playing field. By making firms' otherwise private information public, disclosure discourages traders from private information acquisition. The reduction in private information acquisition attracts liquidity to the secondary market and eventually results in higher firm value in the primary market. At the heart of this theory is that the private information that guides traders' trading decisions is the root cause of adverse selection and illiquidity in the secondary market.

However, the same private information also guides real decisions when transmitted to relevant decision makers. Through the secondary market trading, traders' private information is impounded into stock price and passed on to the firm's stakeholders. Capital providers, employees, major customers and suppliers, and the firm manager may look into stock price when making relevant decisions that affect the firm value. In other words, the private information produced by traders for their own trading also improves the informational efficiency of stock price, which in turn feeds back to real decisions and resource allocation. We term the effect of private information on real decisions as the informational feedback effect.

In this paper, we study a model of disclosure with the informational feedback effect. In the model, a firm sets a disclosure policy at the time of issuing shares in the primary market. The shares are then traded between investors who have liquidity needs and a speculator who could collect information. The firm's disclosure partially preempts the speculator's information advantage and makes the private information acquisition less profitable. As a result, the speculator acquires less information, which has two opposite effects on the firm value. On one hand, the reduction in private information acquisition results in a smaller informational gap between liquidity investors and the speculator. With

a more leveled playing field, the liquidity investors lose less to the informed speculator and thus are willing to pay more for the shares in the primary market. Thus, disclosure raises firm value by enhancing liquidity. On the other hand, when the speculator acquires less information, the stock price may become less informative. When the firm looks to stock price for guidance, the more it has disclosed, the less it gleans news from stock price. As a result, the investment decisions that rely on the information in stock price become less efficient. Thus, disclosure lowers firm value by weakening the informational feedback from stock price. Hence, the optimal disclosure policy trades off these two effects on firm value.

This trade-off can also be viewed from an incentive provision perspective. When the speculator has a competitive advantage in generating certain information valuable to the firm, incentives must be provided to the speculator to generate such information. Tolerating more adverse selection in the secondary market enables the speculator to profit more from its private information acquisition and thus provides him with stronger incentives to produce the private information.

The explicit consideration of the informational feedback effect enriches the disclosure literature. Our comparative statics results suggest that the firm value is higher in an environment with a *lower* cost of private information acquisition if the informational feedback effect is sufficiently strong. This is consistent with the institutional feature of the securities market in the United States that encourages private information production and promotes disclosure at the same time. In contrast, if disclosure policy focused mainly on leveling the playing field, facilitating private information production and promoting disclosure would be self-contradictory.

Moreover, disclosure is often advocated to both improve liquidity and enhance price discovery. We show that the forces underlying liquidity enhancement and price discovery are actually opposite when private information production is endogenous. Liquidity enhancement results from less private information acquisition while price discovery could be improved by private information production.

Our model also generates new testable prediction on the relation between firm growth

and equilibrium disclosure. In particular, the model predicts that growth firms are *endogenously* more opaque than value firms. Learning from stock price is more important for growth firms and as a result growth firms disclose less to attract more private information production.

The critical assumption of our model is that the stock market could produce information that is new to the manager of the firm and that the incremental information production by the market could be significant enough to influence the firm's disclosure policy. To some readers, this assumption is an immediate implication of the efficient market hypothesis that stock price is the most informative source of information. Nonetheless, we provide further motivation for this assumption.

Theoretically, the stock market has competitive advantage in producing some types of information, an idea dated back to Hayek (1945). First, while the manager has a great deal of information about his firm, the efficiency of the firm's investment decisions depends also on information about actions of other firms and factors in the economy- or industry-level, which is dispersed among firm outsiders and could be aggregated through the trading process. Second, the corporate bureaucracy could be inefficient in collecting some information that exists within the firm's scope, such as information that is difficult to be standardized, hard to interpret, or incentive incompatible with the information possessors (e.g., Rajan and Zingales (2003)). The profit-driven trading in an anonymous stock market could have competitive advantage in eliciting such information. Finally, given the disperse nature of information, the stock market provides a venue for whoever good at information production to supply her talents to the firm. The traders' profit-seeking trading motive saves the firm extra search cost or incentive cost typically associated with other information generation mechanisms.

Empirically, Rajan and Zingales (2003) survey the evidence that the stock market provides information that affects resource allocation. More direct evidence about the manager using the information in stock price to guide his investment decisions has also started to emerge. Chen, Goldstein, and Jiang (2007) shows that the sensitivity of a firm's

investment decision to its own stock price *increases* in the level of information asymmetry in the secondary market, suggesting that the private information that creates the adverse selection in the stock market also guides the manager's investment decisions. For the large scale investments, firms tend to reverse merger and acquisition decisions when confronted by negative market reactions (e.g., Luo (2005)) and those who do not are more likely to become the next targets (e.g., Mitchell and Lehn (1990)). In addition, the development of the prediction markets also lends indirect support to the importance of the informational feedback effect (see Wolfers and Zitzewitz (2004) and section 4.2 of this paper for more discussions).

In addition to the theoretical and empirical support, the informational feedback effect has been contended to be significant enough to affect many other important corporate policies, including insider trading (Fishman and Hagerty (1992), Khanna, Slezak, and Bradley (1994)), public v.s. private financing (Subrahmanyam and Titman (1999)), project selection (Dye and Sridhar (2002), Goldstein, Ozdenoren, and Yuan (2010)), securities design and capital structure (Fulghieri and Lukin (2001)), and market-based policy making (Sunder (1989), Bond, Goldstein, and Prescott (2010)). Therefore, the informational feedback effect could be strong enough to affect disclosure policy.

The interactions between public disclosure and private incentive to acquire information have been studied in the literature.¹ However, the informational feedback to the investment decisions subsequent to the trading in our model is new to this literature. Further, our model does not rely on the substitution of disclosure and private information production. We have chosen this as a starting point because the leveled playing field is often advocated as one major rationale for disclosure. If disclosure is complimentary to private information production, then more disclosure exacerbates the adverse selection problem and at the same time improves the investment decision. The trade-off are reversed but still exist.

¹See, for example, Diamond (1985), Kim and Verrecchia (1994), Demski and Feltham (1994), and McNichols and Trueman (1994).

Our paper complements the literature on the real effects of accounting disclosure that emphasizes on the two-way impacts between firm decisions and capital market pricing (see Kanodia (2007) for a review of the literature). Our paper contributes to this literature by introducing an additional link from the secondary stock market to the firm's subsequent real decisions: the firm's real decisions respond to the stock price because it transmits the traders' private information to the decision makers through the faceless, profit-driven trading process.

Our paper is also related to a large literature on the monitoring benefit of the secondary stock market (Diamond and Verrecchia (1982), Holmstrom and Tirole (1993), and Baiman and Verrecchia (1995)). In this literature, the stock price influences the manager's decisions because the firm links his compensation to the stock price to exploit the informativeness of the stock price. The monitoring role is absent from our model because we assume away any intra-firm agency conflict. The major difference between the monitoring role and the informational feedback role of the stock price is that each exploits a different type of information. The monitoring role relies on the backward-looking information about the past action of the manager, while the informational feedback role takes advantage of the forward-looking information. In fact, information about the future often impedes the monitoring role of the stock price (Paul (1992)).

Section 2 describes the model. Section 3 highlights the basic trade-off of disclosure on liquidity cost and investment efficiency. We then use the trade-off to analyze its implications for securities regulation and the endogenous opaqueness of growth firms. In Section 4 we discuss two extensions to the baseline model. First, we consider decision makers outside the firm who glean information from stock prices. Second, we compare the informational feedback effect with other mechanisms of information production such as prediction markets. Section 5 concludes. Detailed proofs are presented in the Appendix.

2 The Model

We start with a model in which disclosure mitigates adverse selection among traders. We then incorporate the informational feedback role of the secondary market into the model to study its effects on the optimal disclosure policy. Towards this goal, we explicitly model two features of the secondary market. First, some information that is otherwise unknown to the firm could be produced by the market and transmitted to the firm through stock price. Second, the firm uses the information in stock price to guide its real decisions

In the baseline model, the firm learns from stock price and makes investment decisions. In Section 4.1 we extend the model to show that as long as stock price transmits information to some stakeholders of the firm whose decisions affect the firm value, the same trade-off of liquidity provision and investment efficiency for disclosure remains. However, letting the firm be the decision maker has one advantage of making the model cleaner. Because the firm as a decision maker could still utilize the undisclosed information, disclosure affects the information available to the decision maker (the firm) only through the learning from stock price.

All parties are risk neutral and the risk-free rate of gross return is normalized to be 1. The timeline of the model consists of four dates, as depicted in Figure 1.

Date	1	2	3	4
Firm commits to a disclosure level and issues shares to original investors.		Speculator acquires a signal; Firm makes disclosure; liquidity shocks realized; Firm shares traded in secondary market.	Firm observes stock price and chooses investment.	Cash flow is realized.

Figure 1: The Timeline

At date 1, the firm owns a stochastic technology that produces cash flow at date 4. The firm issues equity shares to a continuum of ex ante identical investors (original investors) in the primary market. The total mass of investors is normalized to be 1 and the total number of shares is normalized to be 1 share per capita. In pricing the shares, the original investors at date 1 expect that they will have stochastic liquidity shocks at date 2 that can only be satisfied by trading in the secondary market. In addition, the firm sets the disclosure policy at date 1. The disclosure policy commits the firm to fully disclose its information at date 2 with probability $\beta \in (0, 1)$ before the secondary market opens. With probability $1 - \beta$, nothing is disclosed. β thus measures the quality of disclosure.

At date 2, the secondary market for the equity shares opens after the disclosure by the firm. In addition to the original investors, a speculator who could acquire information at a cost enters the secondary market and the order flow is balanced by a market maker through a Kyle-type setting. The market-maker and the speculator are assumed not to participate in the primary market at date 1.²

At date 3, the firm chooses an investment level K based on all information available to the firm, including the price from the secondary market. At date 4, the cash flow is realized and consumption takes place.

Having completed the timeline, we elaborate on the technology and the information structure. The firm consists of one asset-in-place (AIP) and one growth opportunity, whose future cash flows are influenced by a stochastic technology captured by random variable $\tilde{\mu}$. Assume $\tilde{\mu}$ is normally distributed with mean zero and variance σ_μ^2 , i.e., $\tilde{\mu} \sim N(0, \sigma_\mu^2)$. In particular, the terminal cash flow from the AIP is $\tilde{A} = A_0 + \tilde{\mu}$, where A_0 is the certain component of the cash flow. In contrast, the terminal cash flow from the growth

²This is a common simplification used in the literature to induce illiquidity pricing in the primary market (e.g., Baiman and Verrecchia (1995), Bertomeu, Beyer, and Dye (2011)). Since the market maker and the speculator are risk neutral and do not suffer from liquidity shocks on date 2, their participation in the date-1 market would drive out the original investors and eliminate the liquidity discount in share prices, thus muting the incentives to use disclosure to address date-2 adverse selection concern. Diamond and Verrecchia (1991) shows that the same illiquidity pricing is preserved with the participation of the speculators and the market makers in the primary market, provided that the speculators experience stochastic liquidity shocks at date 2 and the market makers are risk averse.

opportunity is

$$\tilde{G} = \tilde{\mu}K - \frac{1}{2g}K^2,$$

where K is the firm's investment decision made at date 3. Both \tilde{A} and \tilde{G} share the same source of uncertainty $\tilde{\mu}$.³ The difference is that the distribution of \tilde{G} is endogenous to the investment decision K while the distribution of \tilde{A} is fixed exogenously.

Between date 1 and date 2, the speculator expends resources to acquire a signal \tilde{y} . $\tilde{y} = \tilde{\mu} + \tilde{\varepsilon}_y$ where $\tilde{\varepsilon}_y$ is normally distributed with mean zero and variance σ_y^2 , i.e., $\tilde{\varepsilon}_y \sim N(0, \sigma_y^2)$. Defining the quality of signal \tilde{y} as $\gamma \equiv \sqrt{\frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_y^2}}$, the cost of information acquisition is $C(\gamma) = \frac{c}{2}\gamma^2$. The more resources the speculator spends, the more precise her signal is. We assume that $c > \frac{\sigma_n \sigma_\mu}{2}$ so that the equilibrium information acquisition is interior, i.e., $\gamma^* \in (0, 1)$.

The firm privately learns a signal \tilde{z} at no cost. \tilde{z} reveals $\tilde{\mu}$ perfectly with probability $f \in (0, 1)$ and is uninformative at all with probability $1 - f$. Exogenous parameter f measures the quality of the firm's internally available information. Since the firm's date-1 choice of disclosure level β commits the firm to disclose its information (\tilde{z}) perfectly with probability β , the actual disclosure at date 2, denoted as \tilde{x} , has the following property:

$$\tilde{x} \equiv \begin{cases} \tilde{\mu} & \text{with probability } \beta f \\ \emptyset & \text{with probability } 1 - \beta f \end{cases}$$

where \emptyset denotes the empty set. To avoid discussing various corner solutions, we assume that the firm incurs a direct cost of disclosure $\frac{w}{2}\beta^2$ with $w > \frac{[4c - g\sigma_\mu^2(1-f)](1-f)f\sigma_n^2\sigma_\mu^2}{8c^2} > 0$ so that the equilibrium disclosure choice is interior, i.e., $\beta^* \in (0, 1)$.

Also at date 2, the original investors experience an aggregate liquidity shock, denoted as \tilde{n} which is assumed to be normally distributed with mean zero and variance σ_n^2 , i.e., $\tilde{n} \sim N(0, \sigma_n^2)$.⁴ Due to the information advantage of the speculator, the original investors

³This assumption is only for simplicity and could be relaxed. What is necessary is that the sources of uncertainty for \tilde{A} and \tilde{G} are correlated.

⁴One interpretation could be that the liquidity shock requires each investor i , $i \in [0, 1]$, to place a

who have to trade in the secondary market lose to the speculator on average. Anticipating this trading disadvantage, they demand a higher liquidity discount when pricing the firm's shares at date 1. This is the channel through which the adverse selection in the secondary market at date 2 is related to the disclosure policy at date 1. The firm who maximizes the proceeds from the issuance of shares in the primary market has incentives to commit to a disclosure policy at date 1 to mitigate the adverse selection at date 2.

The original investors and the speculator trade through a market-maker in a Kyle-type setting. The total order flow thus consists of the information-based order $d(\tilde{x}, \tilde{y})$ from the speculator and the liquidity-based order \tilde{n} from the original investors. The market maker observes the total order flow $\tilde{n} + \tilde{d}$ but cannot distinguish the two components. The market maker then sets a price P to clear the market and break even.

A distinguishing feature of this model with the informational feedback effect is that the cash flow from the growth opportunity at date 4 depends not only directly on $\tilde{\mu}$ but also on the investment decision at date 3 which in turn depends on the information about $\tilde{\mu}$ in the price at date 2. As a result, the price of the growth opportunity at date 2 is non-linear in $\tilde{\mu}$, making it not tractable to infer information about $\tilde{\mu}$ from the price of the growth opportunity. Following the modeling device in Subrahmanyam and Titman (1999) to simplify the analysis, we assume that the AIP is publicly traded but the growth opportunity is not. As discussed on page 1050 of Subrahmanyam and Titman (1999), this assumption has no substantive effect on the main results but allows for closed-form solutions and the explicit characterization of the information content of stock prices.⁵ Since the terminal cash flow of the AIP and the growth opportunity are subjected to the same sources of uncertainty ($\tilde{\mu}$), the inference about $\tilde{\mu}$ made from the price of AIP could

market order of $\tilde{n} + \tilde{\varepsilon}_i$ where \tilde{n} represents the market-wide shock and is common to all investors and $\tilde{\varepsilon}_i$ represents non-systematic, mean-zero *iid* shocks. The market-wide shock \tilde{n} is normally distributed with mean zero and variance σ_n^2 . The idiosyncratic shocks across investors sum to zero ($\int_{i \in [0,1]} \tilde{\varepsilon}_i di = 0$ with probability one). Thus, the total order from investors sums to \tilde{n} .

⁵A more direct but complex solution to this technical issue is use a setting with binary signals and actions (e.g., Goldstein and Guembel (2008), Bond, Goldstein, and Prescott (2010)) in which the price is discrete and the inference could be made even in the presence of the informational feedback effect.

be used in the investment decision for the growth opportunity at date 3.⁶

As a result, the market maker prices the firm's AIP at date 2 with his knowledge about the disclosure \tilde{x} and the total order flow $\tilde{n} + \tilde{d}$; in particular, we have

$$P = E_{\tilde{\mu}}[\tilde{A}|\tilde{x}, \tilde{n} + \tilde{d}(\tilde{x}, \tilde{y})].$$

As discussed in Introduction, the informational feedback effect requires that the stock price contain information that is new to the firm. That is, P is not redundant when the firm chooses investment K at date 3. This has been operationalized through the information structure summarized by Table 1. In the last row of Table 1, with probability $1 - f$, the firm does not learn anything internally about $\tilde{\mu}$, but the price P contains information about $\tilde{\mu}$ that originates ultimately from the speculator's private information acquisition \tilde{y} . As a result, stock price is not a redundant source of information to the firm. In addition, in the first row of Table 1, with probability $f\beta$, \tilde{x} completely preempts the speculator's information advantage \tilde{y} . Thus, from the perspective at date 1 when the disclosure policy is made, the information produced by the speculator is correlated but not a subset of the firm's information.

Probability	Firm information \tilde{z}	Firm disclosure \tilde{x}	Speculator information \tilde{y}	Price \tilde{P}
$f\beta$	$\tilde{\mu}$	$\tilde{\mu}$	$\tilde{\mu} + \tilde{\varepsilon}_y$	$P(\tilde{\mu})$
$f(1 - \beta)$	$\tilde{\mu}$	\emptyset	$\tilde{\mu} + \tilde{\varepsilon}_y$	$P(\tilde{y})$
$1 - f$	\emptyset	\emptyset	$\tilde{\mu} + \tilde{\varepsilon}_y$	$P(\tilde{y})$

Table 1 Information Structure

⁶One interpretation is that the firm spins off its AIP division to go public and retains control over the growth opportunity privately at date 1. The information in the stock price of the spin-off AIP is useful for the decisions about the growth opportunity given the correlation between the common factors that drive the two businesses.

3 Results

3.1 Preliminary Analysis

In this section, we derive the firm's objective function when choosing the date-1 disclosure level. At date 2, after the speculator's acquisition of y and the firm's disclosure x , the speculator submits an order $d(x, y)$ to maximize its expected gross profit. If the disclosure reveals μ perfectly (the first row in Table 1), which occurs with probability $f\beta$, the speculator does not trade and expects zero gross profit. Otherwise, we have a standard Kyle model with one informed speculator, one market-maker, and liquidity traders. The following Lemma is obtained by using standard solution techniques for Kyle-model (proof omitted but available upon request).

Lemma 1 *If the firm's disclosure is ineffective at date 2 (the second and third rows in Table 1), i.e., $x = \emptyset$,*

1. *the speculator's order is $d(\emptyset, y) = \frac{\sigma_n}{\sigma_u} \gamma y$;*
2. *the price is $P(x = \emptyset, y) = A_0 + \frac{\gamma}{2} \frac{\sigma_\mu}{\sigma_n} (d + n) = A_0 + \frac{\gamma^2}{2} y + \frac{\gamma}{2} \frac{\sigma_\mu}{\sigma_n} n$.*

From Lemma 1, we can derive the speculator's information acquisition decision and the liquidity cost to the firm. When $x = \emptyset$, which occurs with probability $1 - f\beta$, the acquisition of signal y affords the speculator an informational advantage and results in adverse selection in the secondary market. The speculator's informational advantage over the market maker is measured by the difference of their respective residual uncertainty about μ at date 2 and can be shown as $Var[\tilde{\mu}|x, d(x, y)] - Var[\tilde{\mu}|x, y] = \frac{\sigma_\mu^2}{2} \gamma^2$.

With this informational advantage, the speculator expects a *gross* profit of $E_y[dE[\tilde{\mu} - \tilde{P}|y]] = \frac{\sigma_n \sigma_\mu}{2} \gamma$ from the trading with information acquisition γ . Thus, before the revelation of x and y when the speculator decides on the information acquisition policy $\gamma(\beta)$, the expected gross profit as a function of β and γ is $\pi(\beta; \gamma) = (1 - f\beta) \frac{\sigma_n \sigma_\mu}{2} \gamma$. The speculator's information acquisition decision is to choose γ to maximize the *net* profit $\pi(\beta; \gamma) - \frac{c}{2} \gamma^2$.

Therefore, the speculator's optimal information acquisition is

$$\gamma^*(\beta) \equiv \arg \max_{\gamma \in (0,1)} \pi(\beta; \gamma) - \frac{c}{2}\gamma^2 = (1 - \beta f) \frac{\sigma_n \sigma_\mu}{2c}. \quad (1)$$

Because of the zero-sum nature of the trading at date 2, the speculator's gross profit is equal to the original investors' trading loss. Thus, the date-1 original investors' expected trading loss is

$$\Pi(\beta) \equiv \pi(\beta; \gamma^*(\beta)) = (1 - f\beta) \frac{\sigma_n \sigma_\mu}{2} \gamma^*(\beta). \quad (2)$$

Anticipating $\Pi(\beta)$, the original investors price the AIP at $E_{\tilde{\mu}}[A] - \Pi(\beta)$ at date 1, making $\Pi(\beta)$ the liquidity cost to the firm.

We now turn to the investment decision. At date 3, based on its information set $\{z, P\}$, the firm's investment decision is $K^*(z, P) \equiv \arg \max_K KE[\tilde{\mu}|z, P] - \frac{1}{2g}K^2 = gE[\tilde{\mu}|z, P]$ and the date-3 expected value of the growth opportunity is $K^*E[\tilde{\mu}|z, P] - \frac{1}{2g}K^{2*} = \frac{g}{2}(E[\tilde{\mu}|z, P])^2$. Our information structure allows us to solve $E[\tilde{\mu}|z, P]$ in closed-form. When the firm receives the information internally (the first and second rows in Table 1), *i.e.*, $z = \mu$, which occurs with probability f , the firm knows μ perfectly and ignores P in making the investment decision. When the firm does not receive any information internally (the last row in Table 1), *i.e.*, $z = \emptyset$, the firm gleans information from P and the posterior belief about $\tilde{\mu}$ could be obtained by Bayes rule: $\tilde{\mu}|(z, P) \sim N(P(\emptyset, y), (1 - \frac{\gamma^2}{2})\sigma_\mu^2)$. At date 1, the expected value of the growth opportunity is calculated as the expectation taken over all realizations of z and P :⁷

$$\Psi(\beta) = \frac{g\sigma_\mu^2}{2} \left(f + (1 - f) \frac{\gamma^2}{2} \right). \quad (3)$$

⁷Explicitly, notice that

$$\Psi(\beta) = E_{\{z, P\}} \left[\frac{g}{2} (E[\tilde{\mu}|z, P])^2 \right] = \frac{g}{2} (f E_{\tilde{\mu}}[\mu^2] + (1 - f) E_P[(E[\tilde{\mu}|P(\emptyset, y)])^2]) = \frac{g\sigma_\mu^2}{2} \left(f + (1 - f) \frac{\gamma^2}{2} \right)$$

In summary, at date 1 when the firm chooses disclosure policy β to maximize firm value, the firm value could be written as follows:

$$V(\beta) \equiv E_{\tilde{\mu}}[A] - \Pi(\beta) + \Psi(\beta) - \frac{w}{2}\beta^2. \quad (4)$$

$E_{\tilde{\mu}}[A] - \Pi(\beta)$ is the value of AIP, $\Psi(\beta)$ is the value of the growth opportunity and $\frac{w}{2}\beta^2$ is the direct cost of disclosure. Accordingly, the optimal disclosure policy is determined by the following first order condition:⁸

$$\frac{d}{d\beta}V(\beta) = -\frac{d\Pi(\beta)}{d\beta} + \frac{d\Psi(\beta)}{d\beta} - w\beta = 0. \quad (5)$$

3.2 The Basic Trade-off

With the preparation above, we examine in detail the firm's disclosure policy at date 1.

Lemma 2 *Higher disclosure level leads to lower information acquisition by the speculator in equilibrium, that is, $\frac{d\gamma^*(\beta)}{d\beta} < 0$.*

Lemma 2 is straightforward from equation 1. The speculator's acquisition of signal y affords him an informational advantage in trading only if the firm's disclosure x is not informative. When the firm's disclosure improves, the costly private information acquisition becomes less profitable and is pulled back.

This reduction in private information acquisition, resulting from disclosure, has two opposite effects on the firm value. It levels the playing field among traders on one hand but reduces the firm's investment efficiency on the other. This is the basic trade-off of the disclosure policy at date 1.

Proposition 1 *In addition to the effect on the direct cost, disclosure, by reducing the speculator's private information acquisition, has two countervailing effects on firm value.*

⁸Since both $\Pi(\beta)$ and $\Psi(\beta)$ are both quadratic in β , the second order condition for maximization is satisfied.

1. it reduces the liquidity cost the firm incurs, that is, $\frac{d\Pi(\beta)}{d\beta} < 0$;
2. it reduces the value of the growth opportunity, that is, $\frac{d\Psi(\beta)}{d\beta} < 0$.

Proposition 1 is proved by differentiating equation 2 and 3 with respect to β . Disclosure's first effect on the firm value is positive as more disclosure reduces the liquidity cost. This benefit of disclosure has been well established in the literature and is often labeled as "leveling the playing field".⁹ Disclosure reduces not only the chance that the speculator has an informational advantage (i.e., $1 - f\beta$ is lower) but also the magnitude of the informational advantage when it exists (i.e., $\frac{\gamma^{*2}(\beta)}{2}\sigma_\mu^2$ is lower). As a result, higher disclosure level reduces adverse selection and enhances liquidity in the secondary market, which improves the firm value at date 1. The speculator's costly information acquisition redistributes wealth from some investors (and eventually from the firm) to the speculator and generates a negative externality on the firm, which motivates the preemptive disclosure.

However, the reduction in private information acquisition, which saves the firm liquidity cost, compromises the firm's investment decisions, as suggested by Part 2 of Proposition 1. The firm's investment decisions are more efficient the more the firm knows about μ . In the case the firm does learn from stock price P (the last row in Table 1), the equilibrium informativeness of P , from the firm's perspective, is measured by the reduction of the firm's uncertainty about μ :

$$Var[\tilde{\mu}] - Var[\tilde{\mu}|P(x = \emptyset, y)] = \frac{\gamma^{*2}(\beta)}{2}\sigma_\mu^2. \quad (6)$$

Thus, price discovery is determined directly by the speculator's information acquisition decision. As disclosure lowers $\gamma^*(\beta)$, the stock price becomes less informative *to the firm*. When the firm looks into stock price for guidance on investment decisions, the more it has disclosed, the more it sees its own information and the less it learns from the stock price.

⁹See, e.g., Diamond and Verrecchia (1991) and Easley and O'Hara (2004). Leuz and Wysocki (2007) provides a survey of this literature.

As a result, higher disclosure level reduces the firm's price discovery in the secondary market and makes investment decision less efficient.

This effect of disclosure on firm value, resulting from the informational feedback effect, is new to the disclosure literature. It creates a positive externality of the speculator's private information acquisition to the firm. Motivated entirely by trading profits, the speculator generates a private signal that is used once for trading at date 2, and then again (with some noise) by the firm in making the investment decision at date 3. Thus, the informational feedback effect imparts a positive social value to the profit-driven speculative information acquisition. Since preemptive disclosure reduces private information acquisition, disclosure has an endogenous cost arising from the foregone investment efficiency.

The basic trade-off of the disclosure policy highlights the dual functions of the secondary market. Not only does the secondary market provide liquidity to traders, it also generates information that could improve investment efficiency. Preemptive disclosure could not serve both functions at the same time. A disclosure policy that maximizes firm value does not narrowly promote a more leveled playing field. Put differently, the informational feedback is not provided to the firm for free. Eventually the firm pays for the information production service by the speculator in the form of the increased liquidity cost of its shares resulting from reduced disclosure. The more valuable the information provided by the speculator, the more the firm's disclosure policy is pulled back from fully addressing the liquidity concern.

To see the significance of incorporating the informational feedback effect into the consideration of disclosure policy, we explore two implications of the basic trade-off identified in Proposition 1. First, we use the model to reconcile the institutional feature of the US securities market that encourages firm disclosure and facilitates private information acquisition at the same time. Second, we use the model to explain why growth firms could be endogenously opaque.

3.3 Private Information Acquisition and Firm Value

Does the firm benefit from an increase in parameter c , the speculator's cost of private information acquisition? From the perspective of "leveling-the-playing-field" alone, the answer is "Yes" because the adverse selection problem in the secondary market is mitigated by an increase in the private information acquisition. However, when the informational feedback effect is taken into account, the answer changes.

Proposition 2 *Defining V^* as the date 1 firm value in equilibrium and $\hat{g} \equiv \frac{2c}{\sigma_\mu^2(1-f)}$.*

$$\frac{d}{dc}V^* > 0 \quad \text{iff } g < \hat{g}.$$

Recall that the firm's investment decision is $K^*(z, P) = gE[\tilde{\mu}|z, P]$. A larger g means that the firm's growth opportunity is more responsive to information, making the feedback effect more important to the firm. We label g as the firm's growth prospect. The firm value is improved by an increase in c if and only if g is small and thus the informational feedback effect is relatively weak. A higher c induces the speculator to decrease information acquisition, which, by Proposition 1, leads to both a lower liquidity cost and lower investment efficiency. Whether the firm value increases as a result of a higher c thus depends on the strength of each effect. When the investment opportunity is important and the benefit of the feedback from stock price is large, the investment efficiency dominates the liquidity cost and the firm is better off with a lower, rather than a higher, c .

This result is significant for understanding a firm's disclosure policy in the broad context of securities regulation. Even though the disclosure improves the firm value by discouraging private information acquisition, the firm value could increase in an environment that facilitates private information acquisition. This seems paradoxical from the perspective that focuses disclosure narrowly on leveling-the-playing-field, but is consistent with the disclosure environment in the United States that promotes disclosure and en-

courages private information acquisition at the same time.¹⁰ Alternatively, to the extent that private information production is viewed as a proxy for the health of a stock market and actively pursued as a desirable goal by firms and regulators alike, the informational feedback effect could be inferred as significant in practice.

3.4 Growth and Disclosure Level

The basic trade-off in Proposition 1 points to growth factors that strengthen the informational feedback effect, which in turn creates incentives for firms to reduce disclosure level in order to preserve the speculator’s incentive to acquire information. In our model, growth is represented by

$$\Psi(\beta) = \frac{g\sigma_{\mu}^2}{2} \left(f + (1-f) \frac{(\gamma^*(\beta))^2}{2} \right).$$

Each of the relevant exogenous parameters, g , f , and σ_{μ}^2 , captures one facet of a growth firm. Their effects on disclosure policy are summarized by the following proposition.

Proposition 3 *Ceteris paribus,*

1. *firms with higher growth prospect (higher g) disclose less;*
2. *firms that are more likely to learn information from the stock price (lower f) disclose less; and*
3. *firms with higher uncertainty (higher σ_{μ}^2) disclose less if and only if g is sufficiently large.*

Proposition 3 adds new predictions about the relation between growth and disclosure policy. As growth prospect g increases, information about the profitability of the growth

¹⁰The legal literature has established that the tenet of securities regulation in the United States has shifted to the “efficiency enhancement model” since 1970’s as part of the triumph of the Efficient Market Hypothesis (e.g., Stout (1988), Mahoney (1995)). Under the guidance of this new doctrine institutions and policies have been designed to facilitate the information production in the secondary market. This doctrine has been employed in the public discourse of a wide array of prominent issues, such as insider trading, regulation FD, short sales, program trading, and the regulation of financial institutions.

opportunity becomes more valuable to a firm. Thus, the firm reduces disclosure level to make the information acquisition by the speculator more profitable, which in turn incentivizes the speculator to acquire more information.

Not only is prospective information more important for growth firms, but also growth firms are more likely to have less information generated internally. Thus, growth firms have a low f . A low f makes the speculator's information more valuable to the firm, giving the firm an incentive to lower disclosure level to encourage the speculator's information acquisition. At the same time, a lower f increases liquidity costs by aggravating the adverse selection problem, which provides firms with incentives to increase disclosure level. As a consequence, disclosure level measured by β can be increasing or decreasing in parameter f . However, measured by total disclosure level $f\beta$, disclosure level is everywhere increasing in f , consistent with the idea that growth firms are more opaque overall.

Growth firms face more uncertainty relevant to its future decisions, parameterized by the variance of the uncertainty $\tilde{\mu}$ in our model. This parameter affects the value of the information to both the firm and the speculator. On one hand, as σ_μ^2 increases, the marginal benefit of learning by the firm becomes larger. The firm reduces disclosure to encourage more information acquisition by the speculator. On the other hand, as σ_μ^2 increases, the speculator's information acquisition becomes more profitable because her information gives her a bigger informational advantage. This leads to a higher liquidity cost for the firm and induces the firm to improve disclosure level. Since the first effect increases in g while the second is independent of g , the first effect dominates the second as g is large. Hence, the firm's disclosure level increases in σ_μ^2 if and only if g is small. In sum, growth firms may choose to be more opaque in the hope of learning more information from their own stock prices.

4 Extensions

4.1 Who Learns?

We have assumed that the firm is the decision maker who benefits from the information in its own stock price. However, the basic idea that preemptive disclosure could reduce firm value through its suppression of information production incentive is more general. As long as the information in the stock price influences decisions, made by the firm or outsiders, that affect firm value, the firm's disclosure policy will balance its effects on liquidity enhancement and decision efficiency.

To illustrate, suppose an outsider takes an action K at date 3 to maximize his own payoff $\tilde{G} = \tilde{\mu}K - \frac{1}{2g}K^2$ and the firm benefits from the outsider's decision by an amount $H(\tilde{G}) = h\tilde{G}$ where $h > 0$. The effect of disclosure on liquidity is not affected by this change. So we only look at the effect of disclosure on decision efficiency.

Proposition 4 *When an outsider looks to stock price to guide his decisions and the improvement in these decisions indirectly benefits the firm ($h > 0$), disclosure reduces the efficiency of decisions (thus the firm value) if either the firm's internal information is sufficiently limited (f is sufficiently small) or the speculative information acquisition is sufficiently efficient (c is sufficiently small).*

To understand Proposition 4, note that action K is improved with the decision maker's better knowledge about $\tilde{\mu}$ at date 3, just as in the baseline model. The outsider's knowledge about μ at date 3 is affected by both the firm's disclosure and the informativeness of stock price. When the disclosure is informative (the first row in Table 1), the outsider learns μ perfectly. When the disclosure is not informative (the second and third rows in Table 1), the outsider's knowledge about μ comes solely from stock price. The informativeness of the stock price for the outsider is measured by the resolution of uncertainty occasioned by stock price P :

$$Var[\tilde{\mu}] - Var[\tilde{\mu}|P(x = \emptyset, y)] = \frac{\gamma^{*2}(\beta)}{2}\sigma_{\mu}^2. \quad (7)$$

The only difference between the knowledge of the firm and of the outsider about μ occurs when the firm has undisclosed information (the second row in Table 1). In this case, the firm could still use the undisclosed information in making decisions but the outsider can only rely on the stock price. As a result, in addition to its negative effect on decision making as studied in the baseline model, disclosure has an additional, direct effect: it increases the information available to the outside decision maker by directly supplying him with disclosed information. This direct channel changes the efficiency of the decision making from $\frac{g\sigma_\mu^2}{2} \left(f + (1-f) \frac{\gamma^{*2}(\beta)}{2} \right)$ to $\frac{g\sigma_\mu^2}{2} \left(f\beta + (1-f\beta) \frac{\gamma^{*2}(\beta)}{2} \right)$. The net effect of disclosure on the outsider's decision making then depends on the relative importance of the direct and indirect channels.

Proposition 4 shows that when the firm's internal information is scarce (low f) or the market information production is more efficient (low c), the information directly provided by increased disclosure is dominated by the reduced learning from stock price and disclosure reduces the efficiency of the outsider's decisions. To the extent that the firm benefits from these decisions, the disclosure policy still trades off its benefit of saving liquidity cost against the cost of reduced learning from stock price.

Because the direct channel from disclosure to decision efficiency does not alter the basic trade-off for the disclosure policy studied in Proposition 1, we have chosen to let the firm be the decision maker in the baseline model to make the model cleaner.

Decisions made by outsiders and guided by information in a firm's stock price could also reduce firm value, which amounts to $h < 0$. One example is that competitors and labor unions use information gleaned from the firm's disclosure and the stock price to the firm's disadvantage (e.g., the proprietary cost in Verrecchia (1983)). To illustrate we label $H(\tilde{G})$ as proprietary cost for the firm by assuming that $h < 0$.

Corollary 1 *When an outsider makes investment decisions which hurt the firm ($h < 0$), disclosure reduces, rather than increases, proprietary cost if the firm's internally generated information is sufficiently limited (f is sufficiently small) or the speculative information*

acquisition is sufficiently efficient (c is sufficiently small).

The intuition is similar to that in Proposition 4. Nonetheless, this extension adds a novel perspective to the literature on the proprietary cost of disclosure. That is, more disclosure could lower proprietary cost, a similar result to Arya and Mittendorf (2005) but with a different mechanism. Even though disclosure provides information to the competitors, it also reduces the information the competitors could learn from the stock price. The net effect of disclosure on the competitors learning should take into account of both channels.

4.2 Who is the Most Efficient Information Provider?

We have demonstrated that information production by the secondary market is not free for the firm in that the firm eventually pays for the information it learns from the stock price in the form of a higher liquidity cost. We assess the comparative efficiency of this market mechanism of information production. To start we establish a benchmark in which the firm has the same information production technology as the speculator.

Proposition 5 *If the firm could use the same technology the speculator has to acquire information, the firm chooses information production $\hat{\gamma}$. Compared with this benchmark case the information production by the speculator in our baseline model is too low when the growth prospect is high and too high when the growth prospect is low.*

Proposition 5 reveals the suboptimal nature of the information production through the secondary stock market. The efficiency loss originates from the misalignment of the speculator's private incentive with the firm's. The speculator's profit-driven information acquisition has the negative externality on the firm value through the liquidity cost and the positive externality through the investment decision, but the speculator internalizes neither of them. Since in our model the investment value of information increases with growth prospect but the trading profit (or liquidity cost) does not, the speculator's in-

centive produces too little information when the net externality is positive and too much when the net externality is negative.

Despite its inefficiency, the advantage of information production through financial markets is highlighted in the comparison with its alternatives. One alternative is that the firm could hire outside consultants or set up internal organizations to produce information. These mechanisms suffer from the well-known and well-studied agency problems in a contractual relationship. Thus, the market mechanism has competitive advantage for information that is subject to severe agency issues, such as information that is difficult to be quantified, not incentive-compatible for direct revelation by the information owner/producer, and information whose most efficient provider could be not easily identified.

Another alternative is to use prediction markets to produce forward-looking information, a tool that has become increasingly popular (see Wolfers and Zitzewitz (2004) for a survey of prediction markets). Part of the demonstrated success of a prediction market is attributed to its ability to overcome the "comprehensiveness" problem of the stock price (e.g., Bresnahan, Milgrom, and Paul (1992)), a problem abstracted away in our model.¹¹ However, the number one practical problem for prediction market is that they suffer lack of market depth and thus incentive for information acquisition (e.g., Wolfers and Zitzewitz (2006)). As illustrated in our model, for markets to produce information, it is indispensable to provide participants with incentive to acquire information. In financial market such incentive is provided mainly by trading profits that are affected by market depth and disclosure policy.

¹¹Take as an example that the firm announces a merger proposal. The market participants' information about the size of synergy of the deal will be reflected in the stock price reaction. The comprehensiveness problem arises from two sources. First, the stock price reaction is also affected by other contemporary factors that are orthogonal to the merger proposal. This issue is absent because in our model the only source of uncertainty is relevant for both pricing and for the investment decision. Second, the stock price also anticipates the probability that the deal could go through, which is partly determined by the market reaction. Thus, a mild reaction could indicate either that the synergy is believed to be moderate or that the market believes that the synergy is so negative that the deal will be abandoned or stopped. A security in prediction markets could be defined narrowly over the merger event to mitigate this comprehensiveness issue.

Two lessons from the predictions markets corroborate the importance of our result. First, the popularity and success of prediction markets attest to the importance of the informational feedback effect. Second, the incentive issue with prediction markets shows that the information production by market relies crucially on private incentives. Thus, leveled playing field could hurt firm value if the informational feedback effect is important for the firm.

5 Conclusion

Disclosure has been the foundation of securities regulation in the United State since its inception in 1930's. One major theoretical support for disclosure to a secondary market is that it levels the playing field. At the heart of this theory is the notion that private information acquisition is the root cause of adverse selection in the secondary market and disclosure improves firm value by reducing incentives for private information acquisition.

While widely accepted, this theory of disclosure seems incomplete when disclosure is viewed as an integral part of the broad market infrastructure. More private information production by traders is often viewed as a proxy for the health of a stock market and thus a desirable goal pursued by firms and regulators alike. The underlying idea is that the private information produced by traders for their trading also guides resource allocation when it is transmitted to relevant decision makers through stock price. As a result, the same private information production that exacerbates adverse selection and illiquidity in the secondary stock market is also the ultimate source of the information market participants look to guide their real decisions. Thus, the disclosure policy that maximizes the firm value balances the dual effects of disclosure on liquidity enhancement and decision efficiencies.

In other words, the secondary market plays two functions at the same time. On one hand, the secondary market provides liquidity to investors. By providing a venue where investors could take different positions based on their information and liquidity needs, the secondary stock market provides liquidity to investors and redistributes wealth among

investors. On the other hand, through the trading in the secondary market, stock prices aggregate information from every corner of the economy and market participants look to the stock prices for information to improve their decisions. That is, the stock prices both *reflect* and *affect* firm value.

Private information acquisition has opposite effects on these two functions of the secondary market. It impedes the liquidity provision function but improves the informational feedback function. The leveling-the-playing-field theory focuses exclusively on the liquidity provision function of the secondary market. The presence of the informational feedback function creates an endogenous cost for preemptive disclosure. Alternatively, the illiquidity in the secondary market induced by the private information could be viewed as the cost for the secondary market to fulfill its informational feedback function.

One major benefit of explicitly considering the informational feedback effect in a theory of disclosure is that it reconciles the joint promotion of disclosure and private information acquisition in securities regulation, which is paradoxical when we focus only on the liquidity provision function of the secondary market. It also explains why growth firms are more likely to be endogenously opaque.

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6 Appendix

6.1 Proof of Proposition 2

For notation, we use subscripts to denote partial derivatives, *i.e.*, $X_Y \equiv \frac{\partial X}{\partial Y}$ and $X_{YY} \equiv \frac{\partial^2 X}{\partial Y^2}$, and write the total derivative as $\frac{dX}{dY}$. We analyze the firm disclosure choice (β) at date 1. From eqn. 4, the firm’s decision problem at date 1 is

$$\max_{\beta \in (0,1)} V(\beta) = A_0 - \Pi(\beta) + \Psi(\beta) - \frac{w}{2}\beta^2$$

The first-order condition determines the optimal disclosure policy β^* :

$$\begin{aligned} 0 &= V_\beta^* \equiv \left. \frac{\partial V(\beta)}{\partial \beta} \right|_{\beta=\beta^*} \\ &= \Psi_\beta^* - \Pi_\beta^* - \beta^* w \\ &= -\frac{(g\sigma_\mu^2(1-f) - 4c)}{2} f(1 - \beta^* f) \frac{\sigma_n^2 \sigma_\mu^2}{4c^2} - w\beta^* \end{aligned} \quad (8)$$

Similar to the definition of V_β^* , Ψ_β^* and Π_β^* are defined as Ψ_β and Π_β being evaluated at $\beta = \beta^*$.

The second-order condition is

$$\begin{aligned} V_{\beta\beta}^* &\equiv \Psi_{\beta\beta}^* - \Pi_{\beta\beta}^* - wf \\ &= \frac{(g\sigma_\mu^2(1-f) - 4c)}{2} f^2 \frac{\sigma_n^2 \sigma_\mu^2}{4c^2} - w. \end{aligned}$$

The assumption on w on page 8 ensures that $V_{\beta\beta}^* < 0$.
Further, the same assumption also ensures that $\beta^* \in (0, 1)$ because

$$V_{\beta|\beta=0} = \frac{(g\sigma_{\mu}^2(1-f) - 4c)}{2} \left(\frac{\sigma_n\sigma_{\mu}}{2c} \right) \left(-\frac{f\sigma_n\sigma_{\mu}}{2c} \right) > 0 \quad (9)$$

and

$$V_{\beta|\beta=1} = \frac{(g\sigma_{\mu}^2(1-f) - 4c)}{2} \left((1-f)\frac{\sigma_n\sigma_{\mu}}{2c} \right) \left(-\frac{f\sigma_n\sigma_{\mu}}{2c} \right) - w < 0.$$

Define $V^* \equiv V(\beta^*)$. Now we compute comparative statics of V^* with respect to c . By the envelope theorem,

$$\frac{dV^*}{dc} = V_c^* = \frac{\gamma^{*2}(\beta^*)}{2c} (2c - g\sigma_{\mu}^2(1-f))$$

Define \hat{g} as

$$\hat{g} \equiv \frac{2c}{\sigma_{\mu}^2(1-f)}. \quad (10)$$

We conclude that $\frac{d}{dc}V^* \geq 0$ if and only if $g \leq \hat{g}$.

6.2 Proof of Propositions 3

We now study the determinants of the optimal disclosure policy β^* by the implicit function theorem: differentiating the first-order condition (eqn. 8) with respect to relevant parameters.

The impact of growth prospect g on the optimal disclosure policy β^*, β_g^* , is determined by

$$\Psi_{\beta g}^* + V_{\beta\beta}^*\beta_g^* = 0.$$

Thus, $\beta_g^* < 0$ because $\Psi_{\beta g}^* = \frac{\sigma_{\mu}^2}{2}(1-f)\gamma^*\gamma_{\beta}^* < 0$ and because $V_{\beta\beta}^* < 0$ by the second-order condition.

$$\begin{aligned} \beta_{\sigma_{\mu}^2}^* &= -\frac{\Psi_{\beta\sigma_{\mu}^2}^* - \Pi_{\beta\sigma_{\mu}^2}^*}{V_{\beta\beta}^*} \\ &= -\frac{1}{V_{\beta\beta}^*} \frac{\sigma_n^2(1-\beta^*f)f}{4c^2} (2c - g(1-f)\sigma_{\mu}^2). \end{aligned}$$

$\beta_{\sigma_{\mu}^2}^* > 0$ if and only if $g < \hat{g}$. \hat{g} is defined in eqn. 10.

For the impact of the firm's own information endowment f on its disclosure quality,

we consider the total amount of disclosure by the firm $f\beta^*$, instead of β^* alone.

$$\begin{aligned}
(f\beta^*)_f &= \beta^* + f\beta_f^* = \frac{\beta^*(\Psi_{\beta\beta}^* - \Pi_{\beta\beta}^* - w) - f(\Psi_{\beta f}^* - \Pi_{\beta f}^* - \frac{w\beta^*}{f})}{V_{\beta\beta}^*} \\
&= \frac{\beta^*(\Psi_{\beta\beta}^* - \Pi_{\beta\beta}^*) - f(\Psi_{\beta f}^* - \Pi_{\beta f}^*)}{V_{\beta\beta}^*} \\
&= -\frac{1}{V_{\beta\beta}^*} \frac{\sigma_n^2 \sigma_\mu^2 (1 - \beta^* f) f}{8c^2} (4c - (1 - f)g\sigma_\mu^2 + fg\sigma_\mu^2) \\
&> 0
\end{aligned}$$

6.3 Proof of Propositions 4

When the decision maker of investment K is not the firm, the only difference in the computation of Ψ is that with probability $f\beta$, not f , the decision maker has perfect information and with probability $1 - f\beta$, not $1 - f$, the decision benefits from the information in price. So the ex ante value to the outside decision maker, denoted Ψ' , is

$$\Psi' = E_{\tilde{z}, \tilde{P}} \left[\frac{g}{2} (E[\mu|z, P])^2 \right] = \frac{g\sigma_\mu^2}{2} \left(f\beta + (1 - f\beta) \frac{(\gamma^*(\beta))^2}{2} \right).$$

The ex ante benefit to the firm is

$$E[\tilde{H}] = E[h\tilde{G}] = h\Psi'.$$

Thus,

$$\begin{aligned}
\frac{dE[\tilde{H}]}{d\beta} &= \frac{hg\sigma_\mu^2}{2} \left[f - f \frac{(\gamma^*(\beta))^2}{2} + (1 - f\beta)\gamma^*\gamma_\beta^* \right] \\
&= \frac{hg\sigma_\mu^2}{2} \left(f - \frac{3}{2} (\gamma^*)^2 f \right) \\
&= \frac{fhg\sigma_\mu^2}{2} \left(1 - \frac{3}{2} (1 - \beta f)^2 \left(\frac{\sigma_n \sigma_\mu}{2c} \right)^2 \right)
\end{aligned}$$

So we have

$$\frac{dE[\tilde{H}]}{d\beta} < 0 \quad \text{if and only if} \quad (\gamma^*)^2 = \left((1 - \beta f) \frac{\sigma_n \sigma_\mu}{2c} \right)^2 > \frac{2}{3}.$$

It is straightforward that $\left((1 - \beta f) \frac{\sigma_n \sigma_\mu}{2c} \right)^2$ decreases in f and c .

6.4 Proof of Propositions 5

If the firm could use the same technology the speculator has to acquire information the firm solves

$$\max_{\gamma} \frac{g\sigma_{\mu}^2}{2} (f + (1-f)\gamma^2) - \frac{c}{2}\gamma^2.$$

Within the parameter regime defined by assumptions on parameters w and c on page 8, the solution to $\hat{\gamma}$ is binary:

$$\hat{\gamma} = \begin{cases} 1 & \text{if } \frac{c}{\sigma_{\mu}^2(1-f)} < g < \frac{4c}{\sigma_{\mu}^2(1-f)} \\ 0 & \text{if } 0 < g < \frac{c}{\sigma_{\mu}^2(1-f)} \end{cases} .$$

Recall in the baseline model, $\gamma^* \in (0, 1)$. Thus, compared with the benchmark $\hat{\gamma}$, the information production in our baseline model γ^* is too low when the growth prospect is high and too high when the growth prospect g is low.