VOLUNTARY DISCLOSURE TO SUSTAIN TACIT COLLUSION*

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Abstract

When facing repeated interactions, firms in an oligopoly can engage in tacit collusion, using the threat of a price war in future periods to sustain higher prices and industry profits in the current period. This paper explores how strategic voluntary disclosures can play an important role as part of a tacit collusion. In each period, one firm receives a signal on market size and must decide whether or not to publicly disclose the information before engaging in price competition in the product market. Two main forces in play are (1) no-disclosure makes it easier for the oligopoly to sustain higher prices because the uninformed firms are uncertain about the market size (and therefore the benefit of deviating from collusion is lower than otherwise); and (2) disclosure makes it easier to coordinate prices if and when the oligopoly wishes to condition equilibrium prices on the market size. We find that, when firms are sufficiently patient such that monopoly prices can be sustained as an equilibrium, no-disclosure is (weakly) preferable to any other disclosure policy. Otherwise and in contrast to the static model, a simple form of partial disclosure can be optimal: the informed firm does not disclose when market size is either too high or too low but discloses for intermediate market sizes, undercutting its competitors when its information is good.

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Repeated interactions among firms are pervasive in most industries. Decisions made in one period are strategic in that they take into account the upcoming encounters with the (same) players in future periods. For example, firms need to decide whether to price-compete more intensely in one period, fully aware of the possibility of a price-war in future periods. In the information arena, when possessing a piece of information about the current environment (market size, industry boom or bust, etc.), firms need to decide whether to make the information public, anticipating the likely competitive response from its competitors in the current and future periods.

The possibility of repeated interactions can give rise to tacit collusion, a phenomenon widely studied in the industrial organization literature (Rotemberg and Saloner (1986), Athey et al. (2004)). Under tacit collusion, long-lived firms can maintain higher profits in current periods (than the competitive outcome) by threatening any firm undercutting its competitors with destructive price-wars in following periods. To sustain tacit collusion, firms must strategically choose their actions to maximize industry profits while keeping the benefit of a deviation below the value lost due to future price-wars. Studying tacit collusion is important because it leads to predictions substantially different from that of a one-shot competitive setting. Specifically, the term tacit collusion refers to agreements that can be sustained by implicit threats of punishments if the agreement is broken (see recent work by Arya, Fellingham and Glover (1997), Stocken (2000) and Huddart, Hughes and Levine (2006)).[1] In a broader context, casual and formal evidence seems to suggest that tacit collusion is fairly widespread, even outside of pure oligopolistic settings. Tacit collusions have been documented among medieval overseas merchants (Greif (1993)), WWI soldiers of different camps (Ashworth (2004)) and security market dealers (Christie and Schultz (1994)); these findings are consistent with behavior in laboratory experiments (see for example, among others, Axelrod and Dion (1988) and other references cited therein).

Most of the existing studies, however, take information as being exogenously given and do not consider how firms may strategically disclose in order to sustain the collusive agreement. Disclosures can play an important role as part of the tacit collusion: they affect the value of a price cut expected by a firm in the oligopoly, or (by increasing the amount of information held by players) the space of actions that the oligopoly can implement. In this paper, we explore this

[1] Tacit collusion is distinct from other forms of price coordination such as information-sharing that do not involve such implicit threats. The terminology may seem misleading, because in a colloquial sense, tacit collusion may be said of any behavior that increases industry profit such as, among other things, the information sharing literature in Vives (2005) and Gal-Or (1985) - yet, in a more narrow academic sense, it is useful to make a distinction between different uses of the term. Models of the latter form, such as information-sharing, do not refer to a system of implicit threats and typically do not use the term of "tacit" or "collusion". What these approaches share with ours, however, is that they are different forms of price/production coordination, either by the best use of efficient threats or by changes to the market structure: in both types of approaches, voluntary disclosure may play an important role.

question formally, and discuss how information may be strategically retained or released as part of the collusive agreement.[2] Specifically, in each period, one firm has an early signal on market size and decides whether or not to publicly disclose it. This is followed by all firms choosing prices simultaneously in the product market. Every firm observes the market size realization and the game moves to the next periods. We assume that all firms are risk-neutral and equally patient about the future (i.e., discount future profits to the same degree). Within this setting, we consider how firms can use the repeated nature of the game to sustain prices higher than in the one-shot game and investigate what forms of disclosure are involved in sustaining these collusive equilibria.

For a given discount rate, we first consider whether the first-best price (i.e., monopoly price) can be sustained in equilibrium with full disclosure (where the informed firm always discloses) or with no disclosure (where the informed never discloses). We show that the first-best equilibria accompanied by no disclosure can be sustained by a wider range of discount rate than those accompanied by full disclosure. The key intuition here is that no disclosure makes the uninformed firms uncertain about the market size and also makes the benefit of deviating from collusion lower than otherwise (i.e., the uninformed firms' incentive compatibility constraint is easier to satisfy with no-disclosure). As a novel feature of the non-disclosure equilibria, the informed firm undercuts its competitors when market size is large, and overprices above its competitors when market size is small.

We then focus on those equilibria where the monopoly price cannot be sustained in equilibrium (i.e., when firms are too impatient). In these equilibria, the firms must lower the deviation benefit by setting a lower equilibrium price (than the monopoly price) in order to maintain the tacit collusion. Knowing the market size may help coordinate which price to charge. Disclosure of market size realizations by the informed firm may make such coordination possible. However, (based on the earlier intuition) disclosure may weaken the incentive for the uninformed firms to cooperate in pricing. Combined, partial disclosure may emerge in equilibrium. We characterize a set of partial-disclosure equilibria in which firms optimally disclose only intermediate realizations of the market size but retain both very good and very bad news (i.e., smooth news announcements). The main trade-off of partial disclosure is providing a balance between the desire to coordinate prices and the desire to provide incentive to the uninformed not to deviate from the collusive arrangement. Specific to equilibria with partial disclosure, undercutting by the informed firm may occur for some high market sizes even after a disclosure.

The model provides several empirical implications, linking firm's market shares and disclosure policies to business shocks. First, the model implies that industries with high concentration ratios and/or low discount rates should exhibit lower levels of disclosure (no-disclosure). In ad-

[2] While we use the term of collusion, other studies use the term of "tacit agreement", "collusive agreement", "tacit coordination" or "implicit collusion".

dition, with tacit collusion, the informed firms market share varies more than the market share of its uninformed competitors; the uninformed firms, on the other hand, do not benefit from large positive shocks to market demand. Second, in industries with lower concentration ratios and/or higher discount rates, the informed firm disclose intermediate news, possibly retaining very good and very bad news. Prices are rigid after a non-disclosure, but are decreasing in total market size shock (e.g., the business cycle) after a disclosure. In other words, we find that the (widely documented) business-cycle price rigidity observed in oligopolies should crucially depend on the disclosures of the informed firm. Further we show that for an oligopoly, there is value in reducing the informativeness of their reported information (as suggested by no- or partial-disclosure). One interpretation of a partial disclosure regime, in which both good and bad news are withheld, is in terms of voluntary conservative accounting practices (non-recognition of goods news in current periods) or earnings management (non-recognition of bad news in current periods). In our model, these accounting policies arise endogenously as a solution maximizing industry profits.

Related Literature

Our model and results are related to several strands of theoretical literature, from which we borrow several building blocks of the model. One such literature is the voluntary disclosure work in accounting. This literature attempts to rationalize discretionary financial disclosure by relaxing assumptions underlying the well-known *unraveling principle* result, starting with the early work by Verrecchia (1983) (assuming an exogenous disclosure cost) and Dye (1985) (assuming uncertain information endowment). In this light, our work can be viewed as introducing a cost of disclosure based on dynamic incentives for tacit collusion: disclosing good news strengthens incentives to price compete in the current period and lessens the incentives to sustain higher prices in future periods. Further, our result addresses the efficiency role of disclosure because in equilibrium, disclosure policies are "self-enforcing" and (industry-) welfare maximizing. Both aspects, we believe, add new dimensions to the existing and expanding accounting literature on voluntary disclosure.

A recent paper, Einhorn and Ziv (2008), focuses on dynamic disclosure with both costly disclosure (similar to Verrecchia (1983)) and a random probability of being informed (similar to Dye (1985)). Similar to our model, they also show future considerations lower the propensity to disclose in the current period. However, the key force in their model is an intertemporal correlation in the information endowment. Disclosing this period updates the market belief about the firm's unknown type (i.e., informed or not). Thus, unlike in our model, the firm's reputation is derived from the market learning, which creates the dis-incentive to disclose. We focus on firm's reputation as any public information that is informative of the player's future payoffs, which is generated by the dynamics of the game. Further, their prediction of disclosure region is similar to that of a static model (i.e., upper tails) while with an endogenous proprietary cost, our model generates a variety of equilibrium disclosure regions.

A second strand relates to repeated competition in oligopolies. Several authors show that, in a dynamic setting in which firms use future rents to sustain higher prices, it can be necessary to set lower prices during booms than busts. The seminal paper in this area is Rotemberg and Saloner (1986) who derive conditions under which competition will be more intense when current market size (and thus expectations about profitability) is large. Starting with Green and Porter (1984), several authors note that periods of price wars may occur on the equilibrium path if some firms have private information. In particular, Athey, Bagwell and Sanchirico (2004) show that collusive equilibria may feature rigid prices that do not depend on the private information, instead of price wars. In these models, the information about the market size in each period is either private or publicly known; in our work, whether such information is made public is set endogenously. The existing work typically takes information as given and does not model how strategically disclosing may be part of a collusive equilibrium.

A closely related to ours in this literature is Huddart et al. (2006). The authors analyze an infinite-horizon Kyle trading environment with multiple informed insiders. They find that insiders can sustain a tacit collusion, in which the first-best one-trader Kyle surplus is achieved. In their model, if an insider deviates from the order flow prescribed by the tacit collusion, other insiders can revert to the lower multiple-trader Kyle surplus; for a discount rate sufficiently low, such future gains are greater than the short-term gains of the deviation. The ability to detect a deviation depends, of course, on the informational environment. When the regulatory environment forces disclosure of all insider trades, the monitoring of a deviation is perfect; while, without such legislation, the aggregate order flow only imperfectly indicates a deviation. Indeed, they find that disclosure of insider trades facilitates tacit collusion. Another related paper is Stocken (2000). In his model, a manager needs to finance a project which, in the absence of any information, would not be financed; however, because the information to be disclosed is soft, he cannot commit to a truthful disclosure. He shows that, even if a lie cannot be perfectly detected, the threat of a loss of credibility in future periods can be sufficient to elicit truthful disclosure.

The driving force in this literature, as well as ours, is that if players deviate to a move to increase their current profit but reduces overall surplus (e.g., a current price cut), other players will shift in future periods to play an equilibrium with lower surplus (e.g., a price war). The effect of current play on future surplus gives rise to implicit incentives, and is key to sustaining equilibria with greater surplus in the repeated game than in the one-shot game. In a contractual setting, Arya et al. (1997) show that implicit incentives can allow for contractibility on information privately known to some agents - in their model, the compensation structure is designed so that, after a deviation in effort in the current period, players will play an equilibrium with lower surplus in the next period. Testing the idea that players condition their strategies on past actions, Schwarts, Young and Zvinakis (2000) find in their experiment that disclosure of past decisions increases cooperation among players.

Finally, another literature focuses on the optimality of information-sharing. Vives (2005) and Gal-Or (1985) show that in Cournot settings with linear demands, no information is voluntarily revealed. In the case of price competition, however, Vives argues that firms voluntarily share information. Some subsequent work in accounting has relied on one-shot duopoly models (of both Cournot and Bertrand varieties) to endogenize disclosure cost, when information disclosed by an incumbent can be used by a potential entrant or competitor. A paper related to ours in this area is Wagenhofer (1990) who considers a one-shot disclosure model, in which a competitor will always take an undesirable action ("deviation") when his expectations about the signal are sufficiently high. We explore this intuition here in the context of product market competition, showing how the benefits of a deviation depend on public information as well as the ability of other firms to price war in future periods. Further, the reason for disclosure in our model is price coordination, while Wagenhofer considers the benefits of disclosure due to capital market considerations. Finally, in a recent paper, Arya and Mittendorf (2005) consider a setting in which more information is unfavorable to industry profits, as in the standard notion of proprietary cost; however, they show that it can lead to informational cascades in which other information providers repeat the information disclosed by the firm but not their private signal, leading overall to less public information.

Most of the existing literature focuses on one-shot market entry settings in which disclosure (and reputations to disclose) do not affect profits beyond the current period. In contrast, this paper studies how future competition responses from rivals affect the current disclosure as well as pricing decisions of an informed firm. To isolate this dynamic effect, we consider a setting where the disclosure is irrelevant in a one-shot problem (i.e., the stage game). As a result, disclosure can help sustain equilibria with collusive prices only when the game is repeated. We are not aware of any previous literature that establishes the role of disclosure as part of a tacit collusive agreement in the product market. It should also be noted that, while we solve for the optimal discount rate consistent with the first-best outcome for the oligopoly, the fact that one can attain efficiency (which is a special case of the standard folk theorem with perfect monitoring) is not the point that we intend to make here. Our focus is the forms of strategies that lead to the highest industry profits, both in terms of disclosure and prices; further, we analyze in details settings in which first-best cannot be attained.

Section I sets up the basic product-market model and Section II lays out the benchmark equilibria to the repeated game including first-best and second best pricing with and without disclosure. In section III, we characterize the partial disclosure equilibria and use the intuitions developed in the benchmark settings to explain the economic forces underlying the partial disclosure equilibria. Section IV concludes.

1. The Model

1.1. Basic Setup

There are N firms ($N \geq 2$) competing in a product market over an infinite time horizon indexed by $t = 0, \ldots, +\infty$. Firms are risk-neutral, face a constant marginal cost normalized to zero and discount payoffs in each period with a discount factor $\delta \in (0, 1)$. The infinite horizon assumption is important for our results: as in the standard literature on repeated games (see Mailath and Samuelson (2006) and cited references), one can also interpret δ as a probability of bankruptcy in each period, so the actual horizon is finite with probability one but the ending date is uncertain.[3] Further, as noted in the introduction, forms of cooperation of the nature studied here seem to be fairly widespread both historically and in laboratory experiments.

In each period, firms face a demand $s_t D(p)$, where s_t represents the size of the market and p is the unit price of the product.

Assumption 1: $pD(p)$ is continuous, strictly increasing on $[0, p^*]$ and strictly decreasing on $[p^*, +\infty)$.

Assumption 2: s_t is i.i.d. and drawn from a continuous distribution with full support over $[0, \overline{s}]$ and finite density $h(s)$ bounded away from zero.

We denote p^* the optimal monopoly price and $\Pi^* = p^* D(p^*)$ the total industry profit when $s = 1$. Further, without loss of generality, we normalize the distribution of s such that $\mathbb{E}_s[s\Pi^*] = 1$. The key assumption in the model is that s_t is not publicly known at the beginning of each period t.[4] We model the arrival of information as an extensive-form game (hereafter, stage

[3]It should also be noted that some of the experimental literature in this area seems to suggest that, even in the extreme case in which the horizon is finite and common knowledge (e.g., most experiments involving repeated prisoner's dilemmas are run with a common knowledge horizon and, even for those with stochastic ending date, there is common knowledge that the session will end in bounded time - see also references in Dal Bo and Frechette (2008)), players seem to adopt tacit collusion arrangement that could only be played with an infinite horizon. In other words, player seem to frame the problem as if they were playing an infinite horizon game, and adopt corresponding strategies. In comparison, a finite horizon would require common knowledge of a (possibly upper bound on) terminal date.

[4]We assume here for simplicity that market sizes are independent across time periods (as in Rotemberg and Saloner (1986)). In practice, however, a high market size in the current period may indicate a high market size in future periods. In this case, it may be easier to elicit cooperation in current periods. Therefore, the informed firm would undercut only periods in which market size in the present is relatively large compared to market size expected in the future. For example, if correlation across market sizes is very large and the discount rate is large

game).

To facilitate the exposition, let $t.i$ ($i = 1, ..., 5$) denote the i^{th} event in period t.

At $t.1$, one firm learns the current market size, s; each firm is equally likely to become informed in each period.[5]

At $t.2$, the informed firm decides to publicly announce s, or to stay silent. We denote this choice as a public message $m \in \{\emptyset, s\}$. We assume that firms cannot openly lie about their prospects and thus s, if disclosed, must be truthful.

At $t.3$, all firms simultaneously choose their prices. The prices may be conditional on s only for the informed firm and for all other firm if s is disclosed.

At $t.4$, the information s becomes publicly known to all firms. It is convenient to interpret this information as *late* information, i.e. after operating decisions (here, prices) have been made.

At $t.5$, total industry profits $s \min p_k D(\min p_k)$ are distributed among all firms charging $\min_k\{p_k\}$. When the period ends, firms observe their profit. The results do not depend on whether or not firms observe the profits of their competitors.

We focus here on standard price competition. The Bertrand game with perfectly elastic demands is useful to capture the essence of competition and focus the discussion on the dynamics of the game, and not on the complementarities between player's actions and total surplus. It is standard in the IO repeated game literature (see for example, among others, Elberfeld and Wolfstetter (1999) and Athey et al. (2004)) and requires no parametric specification of the demand function $D(.)$.[6] Another advantage of the Bertrand assumption in our setting is that it distinguishes our results from the known relationships between forms of imperfect competition and disclosure, previously studied in the information-sharing literature. For convenience, it will be useful to assume that firms can also make a marginal price decrease - a move that we denote undercutting. Formally, we assume that at $t.3$, firms choose both price and a decision $z \in \{share, undercut\}$. The total profit is shared equally among firms charging lowest price if no such firms choose $z = undercut$. If not, the total profit is shared among the undercutting firms (i.e., those charging lowest price and choosing $z = undercut$). The variable z is a mathematical label for a small price deviation which is useful to state and interpret the results.[7]

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as well, the informed firm would undercut (overprice) over low (large) current market sizes. In comparison, if correlation across periods is sufficiently small, the results of the benchmark model carry over.

[5]The analysis is insensitive to whether the identity of the informed firm is common knowledge or private information.

[6]To that extent, the predictions of Bertrand competition may seem extreme (with large variations to firm profits). However, these should not be taken literally but rather as one force among other forces that may affect firm's surplus. In this respect, the current model is not meant as a empirical representation to be matched to data, but rather as an illustration of concepts at play in the real world.

[7]It should be noted that this instrument would have been unnecessary should we have adopted a fine grid of feasible prices (instead of a price chosen of the continuum) - undercutting would correspond to choosing the next lowest price on this grid - however, after considering both formulations, we felt that the combination of continuum and a discrete undercutting choice seemed more elegant in terms of exposition than the grid. Further, in the continuous case, without this extra variable, only the closure of the set of equilibrium payoffs may include the

In stating the model, we make several modeling choices that are designed to isolate the analysis of disclosure from other trade-offs that have been already studied in the existing literature. These choices are meant to extract the new forces at play in our approach and obtain a stripped-down characterization of the optimal disclosure policies. In fact, in a separate supplementary appendix to the paper, we have shown that many of the results of the model can be made robust to several other aspects such as: (i) multiple informed firms, (ii) serially-correlated probability of being informed, (iii) the possibility that no firm is informed, (iv) quantity (Cournot) competition, (v) imperfect monitoring of the firm's private information, (vi) possibly non-truthful disclosure. We discuss three key assumptions below and leave out those extensions from the current version to save space.

First, we follow the financial disclosure literature (e.g., Verrecchia (1983) and Dye (1985)) in assuming that disclosure, when it occurs, is truthful. For example, the firm may be receiving some advance purchases from some of its early clients or purchases from other (unmodelled) products lines which are indicative of high demand in its main market in the current period; these it may not openly falsify without possibly facing an internal accounting investigation. Note as well that, at least in the context of our model, any open lie would be discovered at the end of the period, and thus would trigger a possible legal punishments. If disclosure may be untruthful (e.g., $m \in \{\emptyset\} \cup [0, \overline{s}]$), equilibria involving some disclosure will be more difficult to implement. We have considered a version of the model in which the manager may lie and found that it biases the analysis toward even more non-disclosure.[8] Further, the problem of credibility is a different one from the problem studied here, and we refer the reader to Stocken (2000) for a more formal argument of how capital markets may discipline the manager to make credible reports in a repeated settings.[9]

Second, we assume that s is perfectly observable by the informed firm so that, since realized market size is observable at the end of the stage game, the repeated game is one with perfect monitoring of the disclosure decisions. Our results are unchanged if s is a noisy signal on market size, but can be truthfully disclosed at the end of the stage game; in this case one would replace in our characterizations s by the expectation on market size conditional on signal s.

first-best payoff but, with some additional technical steps, the results would be essentially unchanged. We would say then that the equilibria presented here could be approximated to an arbitrary precision. In other words, the variable z is a mathematical shortcut to avoid a more lengthy presentation of this limiting argument.

[8]In this extension, we show that first-best is still sustained by no-disclosure, so our conclusions are unaffected. In the case of second-best, our conjecture is that there would be a cheap talk equilibrium with partitional information being disclosed in the lines of Fischer and Stocken (2001).

[9]This is with one caveat, also noted in Stocken (2000), that this disciplining mechanism may fail if impatience is too important (which would also prevent the type of phenomena studied here). One possible question is whether capital markets and goods markets may jointly discipline both truthful disclosure and more efficient for collusion purposes (e.g., investors may refuse to finance projects conditional on a disclosure that is truthful but should not have been made); however the degree of coordination between industry competitors and investors that this would require seems considerably more demanding than the argument that we develop here.

However, our results would change if s could not be truthfully disclosed at the end of the stage game, since this would lead to competitors being able to perfectly monitor whether a deviation has taken place (i.e., they would only observe realized market size but not the private signal on which the informed firm's decision was based). An existing literature focus on strategies that best overcome the imperfect monitoring, either via price wars on the equilibrium path (Green and Porter (1984)) or "monitoring-friendly" rigid price strategies (Athey et al. (2004)). In this context, our model describes to what extent voluntary disclosure, when possible, helps solve the imperfect monitoring problem discussed in this literature. Further, we have solved (in the supplementary appendix) a version of the model with imperfect monitoring; one important difference with the current case is that disclosure, because it facilitates monitoring may now be, under certain circumstances, more attractive than non-disclosure.

Third, we restrict the attention to settings in which the monopoly price p^* does not depend on the private information s. While with loss of generality, this assumption is made to focus our attention on a setting in which disclosure is, a-priori, not useful from the perspective of a one-shot monopoly. In contrast, the information sharing literature (e.g., Vives (2005), Gal-Or (1985)) models how disclosure may help firms choose their prices in a one-shot interaction. Extending the model to a dependence on s would (likely) provide some additional benefits to disclosure or non-disclosure, in the direction of the results previously shown in the static information sharing literature. However, this additional trade-off would only add to our analysis and not provide any additional insights beyond what is known in this literature.

1.2. Stage Game Equilibrium

The next Proposition characterizes the set of all Nash equilibria in the stage game. Since the proof of the statement is similar to standard Bertrand competition (Tirole (1988), p.245), it is omitted.

Proposition 1.1. *In all pure-strategy equilibria of the stage game, firms make zero profit. Disclosure m is irrelevant.*

In the stage game, disclosure does not affect price competition: regardless of how the informed firm discloses, firms make zero profit. Note that all firms playing $p = 0$ in all periods is an equilibrium in the repeated version of the game but it is also the worst equilibrium payoff in the game. In our model, any benefit or cost of disclosure must be caused by the repeated nature of the relationship and would not occur in a one-shot interaction.

1.3. Repeated Equilibrium Concept

Now consider the repeated setting. In the repeated setting, firms can condition their actions on outcomes in previous periods to avoid zero profits. Following Athey et al. (2004), we focus on equilibria with public monitoring, i.e. firms only use past public information to enforce an equilibrium. Let p_i denote the price, z_i denote the undercutting decision. When each period ends, we assume that firms observe the market size s, the disclosure m, as well as current prices $\theta = (p_i, z_i)_{i=1}^N$. A strategy σ is a contingent plan of action in the current stage game, for any possible history of (θ, m, s) in past stage games. We will consider a standard equilibrium notion which we shall call pure-strategy strongly symmetric public-monitoring subgame perfect Nash equilibrium (PPNE).

We focus on strongly symmetric equilibria, in the sense of the strategy of firms in each stage game depends only on public histories in past stage games, not on their identity.[10] This restriction makes it impossible to "reward" a firm by switching to continuation paths with asymmetric payoffs, which would require an implausible amount of coordination among players of the game. The equilibrium is with public monitoring since firms do not use their past private information (such as the identity of the informed firm when no disclosure is known) to enforce the equilibrium.[11] Finally, and most importantly, we consider only equilibria that are subgame-perfect, in the sense that anticipated strategies must remain optimal at the beginning of each period t, even for histories that are attained (on the equilibrium path) with probability zero. The formal definition and notations are given in the Appendix.

1.4. Efficient Repeated Equilibrium

One can verify that a strategy profile where each firm chooses zero price every period is indeed a PPNE. However, this PPNE leads to the worst feasible payoff to all firms. We are interested in PPNE equilibria which deliver more profitable payoffs. The ideal payoff would be one in which monopoly profit (Π^*) is achieved every period (and due to symmetry, equally shared among all firms). In such an ideal setting, the total discounted future profit, at any period in time t, is expressed as $\frac{1}{N}(1 + \delta^1 + \delta^2 + ...) = \frac{1}{N(1-\delta)}$, or the expected first-best payoff shared among all firms.

We call this payoff the first-best payoff, which after rescaling with a factor $1 - \delta$, gives rise to a (normalized per-period) payoff of $1/N$. Given that δ is an exogenous parameter, this

[10]This does not preclude different moves in one stage game. Note also that this does not preclude deviations with asymmetric choices off the equilibrium path.

[11]Equilibria with public monitoring are also equilibria with private monitoring, i.e. using strategies that are conditional on private information in past stage games (see Mailath and Samuelson (2006) for a proof). Note also that public monitoring does not mean that firms may not use their private information in the *current* stage game - they should and do in our model. Most games admit even greater sustainable feasible if private monitoring strategies are used. The same holds if symmetric (but not necessarily strongly symmetric) strategies are used.

(standard) normalization is without consequence for the analysis and simplifies notations. The first-best payoff may or may not be achieved by a particular PPNE among potentially many PPNE. We define the efficient equilibria as the set of Pareto dominant PPNE. For a strategy σ, let $u(\sigma)$ denote the payoff (again normalized by $1-\delta$) to a firm when all firms play that strategy.

Definition 1.1. *A PPNE strategy profile σ is efficient if, for any other PPNE σ', $u(\sigma) \geq u(\sigma')$. An efficient strategy profile σ is first-best if it achieves $u(\sigma) = 1/N$.*

An efficient strategy profile is a profile that Pareto dominates any other profiles that can be sustained as a PPNE. This idea is consistent with the idea of tacit collusion and, while we recognize that firms may not necessarily always manage to coordinate on their most attractive strategies, one would expect that a small oligopoly would certainly prefer to coordinate on this equilibrium versus the zero-profit equilibrium described earlier.[12] We define a first-best strategy as a PPNE strategy profile that attains the maximum feasible payoff in the game, i.e. an average profit per period equal to $1/N$. Typically, a first-best PPNE strategy profile will not be unique for a given discount rate. In such cases, we follow standard practice in repeated games and find the strategy profile that will remain an efficient equilibrium for the widest range of discount rates.

2. Repeated Equilibrium Benchmarks

In this section, we analyze and compare PPNE strategy profiles under full and no disclosure regimes. The purpose is to develop the two main intuitions which will be helpful in deriving the main result on partial disclosure. Following the repeated games literature, we use three descriptors of a strategy profile: $\langle cooperation, punishment, transition \rangle$.

1. *cooperation:* This is a strategy mode that describes the action of each firm on the equilibrium path. That is, if no firm deviates from equilibrium play last period, all firms will follow the action prescribed by *cooperation* this period.

2. *punishment:* This a strategy mode that describes the action of each firm off equilibrium path. That is, if any firm deviates from equilibrium play last period, all firms will follow the action prescribed by *punishment* this period and future periods.[13]

[12] Unlike in other games (such as signalling games), repeated games typically do not have other selection criteria that would point out to another equilibrium that yields a profit in-between the zero-profit equilibrium and the efficient equilibrium. Therefore, without more structure, one would expect one of these two equilibria to be played. Further, if firms were able to communicate in a non-binding manner (e.g., trade shows/publications, executive associations, joint ventures, public disclosures), they would most certainly select their most preferred equilibrium outcome.

[13] To prevent a deviation, it is desirable to minimize payoffs after observing an off-equilibrium move, i.e. switch

3. *transition:* describes how each firm move from on-equilibrium (*cooperation*) play into off-equilibrium play (*punishment*).

2.1. First-Best with Full disclosure

We consider next strategy profiles such that the informed firm always chooses $m = s$. A candidate efficient strategy profile for an efficient PPNE is given as follows:

1. *cooperation:* The informed firm discloses ($m = s$). Conditional on a disclosure $m = s$, all firms choose a price $P(s) = p^*$ and $z = share$. Then, they achieve a profit $s\Pi^*/N$.[14] If the informed firm does not disclose, all firms choose a price equal to zero.

2. *punishment:* On the punishment mode, a firm chooses $p = 0$ regardless of its information.

3. *transition:* The game starts at date $t = 0$ with all firms playing Cooperation. Any off-equilibrium move triggers a shift to the Punishment mode.

The strategy described above is a standard "trigger" strategy, in which any deviation from the tacit collusion triggers a move to punishment in future periods, and so as to make the tacit collusion incentive-compatible. Note as well that, given that choosing $p = 0$ is a best response as long as at least one other firm chooses $p = 0$, playing the punishment once the triggers activates is subgame-perfect. That is, once the punishment phase activates, no firm can unilaterally gain from moving away from $p = 0$.[15] Now we focus on conditions under which the above strategy profile can be sustained in equilibrium. In other words, we wish to make sure the prescribed actions are incentive-compatible. Clearly, the informed firm would never deviate to $m = \emptyset$ since this would induce current and future profits equal to zero. To sustain the monopoly price, each firm must prefer choosing $z = share$ to deviating ($z = undercut$).[16] This is written as follows, for all s,

$$(1-\delta)s\Pi^*/N + \delta/N \geq (1-\delta)s\Pi^* + \delta 0 \tag{2.1}$$

to a Punishment. This is formally defined as follows. First, if a firm was playing Punishment at date $t-1$, always stay on Punishment at date t. According to this plan, all firms will achieve zero profit (current and future) once the Punishment stage is reached. Second, for each date t such that Cooperation was played in the previous period, switch to Punishment when an off-equilibrium move is observed.

[14] It can be easily checked that, since after a disclosure all firms are the same, there would be no purpose for different firms to use different prices.

[15] Note that, in other settings, finding the optimal punishment is not as straightforward, and there may exist more effective punishments that are not equilibria of the stage-game; if this were the case, one should make the punishment itself incentive-compatible. In our model, fortunately, the Nash equilibrium of the stage game ($p = 0$) leads to the minimum feasible profit.

[16] Note that in (b), we are considering equilibria in which all firms choose the same price. From Equation (2.1), it can be easily verified that incentive-compatibility binds for the firm receiving the lowest current profit - and thus equilibria in which one firm does not sell (and other firms make a greater profit) in one period are not desirable.

This inequality is most demanding when $s = \overline{s}$. Solving for the threshold in δ yields the following proposition.

Proposition 2.1. *First-best can be attained with full disclosure if and only if $\delta \geq \delta_{fd}$, where:*

$$\delta_{fd} = \frac{(N-1)\overline{s}\Pi^*}{1 + (N-1)\overline{s}\Pi^*} \tag{2.2}$$

The threshold δ_{fd} is increasing in $\overline{s}$ because it is more difficult to sustain high prices when market size is large. As is standard in the literature, we will write our results in terms of the minimum discount factor consistent with first-best. However, it should be noted from a closer inspection of our results, that we could identically look for the minimum size of the oligopoly N consistent with first-best; which would deliver similar characterizations of a threshold in N (this statement will also apply to no-disclosure and partial disclosure). While δ may be interpreted as the firm's inherent cost of capital (e.g., higher in cyclical industries), we would interpret N as the industry's concentration ratio (for example, one popular empirical measure is the Herfindahl index).

2.2. First-Best with No disclosure

Now we focus on strategy profiles such that the informed firm never discloses, regardless of s. Consider the following strategy profile as a potential candidate for an efficient PPNE.

1. *cooperation:* The informed firm does not disclose ($m = \emptyset$). Conditional on no disclosure, all uninformed firms choose $z = share$ and p^*. Conditional on s, the informed firm: (i) overprices by setting $p > p^*$ for $s \in \Omega_1$, (ii) follows the same pricing scheme as the uninformed by setting $z = share$ and $p = p^*$ for $s \in \Omega_2$, (iii) undercuts by setting $z = undercut$ and $p = p^*$ for $s \notin \bigcup_{j=1,2} \Omega_j$. The sets Ω_1 and Ω_2 will be optimally determined in equilibrium. Finally, conditional on any $m \neq \emptyset$, all firms choose a price equal to zero.

2. *punishment:* On the punishment mode, a firm chooses $p = 0$ regardless of its information.

3. *transition:* The game starts at date $t = 0$ with all firms playing Cooperation. Any off-equilibrium move triggers a shift to the Punishment mode. First, the informed firm would switch to punishment if detecting any undercutting by other firms. Second, the uninformed firm would switch to punishment if after observing s at the end of the period if any deviation by the informed and other uninformed firms is detected.

As before, we consider the incentive compatibility of the arrangement. Consider first the prescription for the informed to choose $p > p^*$ when $s \in \Omega_1$. By choosing this action, the informed firm will receive zero profit in the current period and continue on the Cooperation path. By deviating to $z = undercut$ and $p = p^*$ (the best possible deviation), the informed firm can obtain $s\Pi^*$ in the current period, but this will trigger a shift by all firms to the Punishment path (and thus zero profit in future periods). For the recommended action to be optimal, it must hold that: for all $s \in \Omega_1$ (the informed firm overprices),

$$(1-\delta)0 + \delta/N \geq (1-\delta)s\Pi^* + \delta 0 \tag{2.3}$$

This constraint is satisfied when $s \leq \tilde{s} \equiv \frac{\delta}{1-\delta}\frac{1}{N\Pi^*}$. A similar condition is derived for all $s \in \Omega_2$ (the informed firm shares),

$$(1-\delta)s\Pi^*/N + \delta/N \geq (1-\delta)s\Pi^* + \delta 0 \tag{2.4}$$

This constraint is satisfied when $s \leq \hat{s} \equiv \frac{\delta}{1-\delta}\frac{1}{(N-1)\Pi^*}$. Note that undercutting when $x \in [0, \bar{s}]\setminus\bigcup_{j=1,2}\Omega_j$ is always incentive-compatible. In addition to inequalities (2.3), (2.4), it must be incentive-compatible for each uninformed firm not to deviate (to $p = p^* - \epsilon$ and $z = undercut$ with ϵ small).

$$(1-\delta)\left(\int_{\Omega_1} sh(s)ds\frac{\Pi^*}{N-1} + \int_{\Omega_2} sh(s)ds\frac{\Pi^*}{N}\right) + \delta\frac{1}{N} \geq \sup_{\epsilon>0}(1-\delta)E_s[s(p^*-\epsilon)D(p^*-\epsilon)] = (1-\delta) \tag{2.5}$$

In Equation (2.5), the right-hand side corresponds to the expected profit obtained by undercutting all other firms. Since in this case, the uninformed firm deviating does not know s, it will anticipate an expected profit $\mathbb{E}(s)p^*D(p^*) = 1$. The left-hand side corresponds to the profit expected by staying on the Cooperation path, where the profit of the uninformed will depend on s and the strategy of the informed firm.

In the next Proposition, we solve for the optimal Ω_1 and Ω_2 in order to attain first-best.

Proposition 2.2. *First-best can be attained with no disclosure if and only if $\delta \geq \delta_{nd}$, where:*

$$\delta_{nd} = \frac{N(N-1)/\Pi^* - N\int_0^{\tilde{s}} sh(s)ds - (N-1)\int_{\tilde{s}}^{\hat{s}} sh(s)ds}{(N+1)(N-1)/\Pi^* - N\int_0^{\tilde{s}} sh(s)ds - (N-1)\int_{\tilde{s}}^{\hat{s}} sh(s)ds} \tag{2.6}$$

Corollary 2.1. *There exists a unique strategy profile that is a first-best PPNE for any $\delta \geq \delta_{nd}$, it is given as follows:*

(i) For $s \leq \tilde{s}$ (low market size), the informed firm does not sell and only the uninformed sell at a price p^*.

(ii) For $s \in (\hat{s}, \hat{s}]$ (medium market size), total industry profits Π^* are shared equally among all firms.

(iii) For $s > \hat{s}$ (large market size), only the informed firm sells.

In the general case, we show that monopoly prices in every period can potentially be sustained with no disclosure. The equilibrium strategy must prescribe how industry profits are allocated between the informed firm and the uninformed firms as a function of market size. On the one hand, transferring more surplus to the uninformed firms can help avoid deviations to lower prices. On the other hand, it is more difficult to induce the informed firm not to undercut when market size is large. The solution to this trade-off implies an asymmetric allocation of industry profits between the uninformed and the informed firm: when the market size is small (resp. large), the informed firm does not sell (resp. serves the complete market). Unlike with full disclosure, undercutting (when market size is large) occurs on the equilibrium path and does not trigger a price-war.

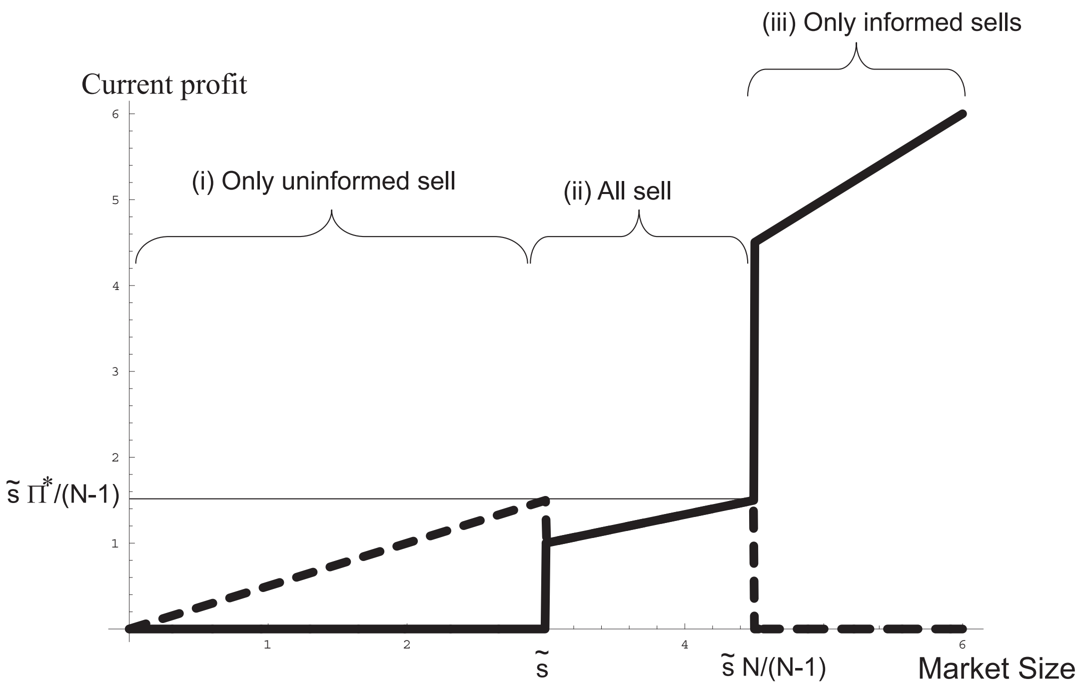

Figure 1. Current Profits: Informed (bold), Uninformed (Dashed)

In Figure 1, we plot the profit of an uninformed firm and the informed firm as a function of s. We make two simple testable empirical predictions. First, because the equilibrium with no disclosure appears to magnify the volatility of firm's earnings in response to market size, one should observe greater cross-sectional earnings variability in industries that feature little or no early disclosure. In this respect, our study suggests a product-market rationale for the use of non-predictable earnings variability as a proxy for accounting quality (as argued empirically in Francis, Lafond, Olsson and Schipper (2005), among others).

Second, a more desirable signal (i.e., higher s) increases the profit of the informed firm but not the profit of competitors. In our model, stock prices should adjust upward given good market conditions, as is intuitive. However, on the other hand, stock prices of uninformed competitors should adjust downward when market conditions are *very* good. In other words, the stock price response of industry competitors to late information (i.e., after operating/price decisions) should exhibit an inverted U-shape, positive for intermediate news, but negative for bad news (since market size will be small) or good news (since the disclosing firm will undercut).

2.3. Value of Secrecy

Now we compare the two strategy profiles considered so far and derive the first main intuition on the value of secrecy.

Corollary 2.2. $\delta_{nd} < \delta_{fd}$.

Comparing δ_{fd} and δ_{nd} reveals that no disclosure can help achieve cooperation for a wider set of parameter values than full disclosure. This is because under no disclosure regime, the oligopoly is better able to dampen the incentives to deviate when the market size is high by leaving most competitors in the dark. Intuitively, when market size is large, disclosing makes deviation more attractive to every firm so firms need to be sufficiently patient to refrain from undercutting. Under no disclosure, $N - 1$ firms do not know whether market size is high and must assume the average market size when contemplating deviation, lowering the benefit of deviating (i.e., the right-hand-side of constraint (2.5) is reduced). In addition, to better elicit cooperative behavior among uninformed firms, the informed firm agrees to give away additional rents when the market is low (thus increasing the left-hand-side of constrain 2.5). In short, no disclosure uses the slack in the incentive-compatibility constraint of the informed firm when market size is low to better motivate the uninformed firms to cooperate. As a result, secrecy is valuable to the oligopoly, not because it necessarily benefits the uninformed firm in the current period, but because it better motivates cooperation among oligopoly members in the long-term. This is the first main intuition derived from the model.

Finally, we discuss whether being an informed firm is good news or bad news in a no-disclosure equilibrium. To do this, we derive conditions under which that the expected profit of an informed firm[17] right before it learns the actual s exceeds the expected profit of an uninformed firm:

$$\frac{\Pi^*}{N}\int_{\tilde{s}}^{\hat{s}} sh(s)ds + \Pi^* \int_{\hat{s}}^{\overline{s}} sh(s)ds \geq \frac{1}{N} \tag{2.7}$$

[17]In Equation 2.7, we write instead $1/N$; however, this is equivalent given that $1/N$ is a weighted average of the profit of the informed firm and the profit of the uninformed firm.

This inequality is satisfied when N is large, δ is small or low realizations of s (lower than $\tilde{s}$) are unlikely. On the other hand, when δ is sufficiently close to one, $\tilde{s} > \overline{s}$ and therefore the uninformed firm is always better-off than the informed firm. That is, being informed is indicative of a low expected own profit for industries with few firms and low discount rates. We take these conditions as representing empirically large mature industries (e.g., automobile, steel). On the other hand, in growth industries, information should be indicative of high profits (e.g., technology).

2.4. Second-Best with Full and No disclosures

Assume now that $\delta < \delta_{fd}$ so that the first-best surplus cannot be sustained as a PPNE in the game with full disclosure. To rule out intuitively unappealing equilibria, we exclude situations in which firms punish another firm for taking an action that ex-post increases every firm's profit. This is modeled as the following Pareto-Consistency (PC) restriction:

Assumption 3: A symmetric PPNE with strategy σ is Pareto-Consistent (PC) if, for any history h^t and h_2^t such that:

(a) h^t and h_2^t differ only over last stage game actions.

(b) The last period profit vector under h_2^t weakly Pareto-dominates the profit vector under h^t.

Then, when the expected equilibrium profit vector in the continuation game with $\sigma_{h_2^t}$ must Pareto-dominate the expected equilibrium profit vector in the continuation game with σ_{h^t}.

Assumption 3 simply states that, if a firm takes an unilateral deviation which increases the ex-post profit of all firms, the deviating firm would not be punished. In other words, an off-equilibrium stage game outcome that is weakly preferred by all players should lead to a strategy in future periods that is also weakly-preferred by all players (so that punishments do not occur). Note that this assumption is irrelevant in first-best since all equilibrium outcomes are efficient. In the rest of the analysis, we restrict the attention to the set of PPNE that can be sustained with a PC-strategy.

Full Disclosure

We analyze first equilibria with full disclosure. There must exist states attained with positive probability such that $P(s) < p^*$. We write V_{fd} ($< 1/N$) the expected surplus received by firms in such an equilibrium:

$$V_{fd} = \frac{\int sP(s)D(P(s))h(s)ds}{N} \tag{2.8}$$

It must be incentive-compatible for all firms to choose $p = P(s)$ and $z = share$ (versus deviating to $z = undercut$):

$$(1-\delta)\frac{sP(s)D(P(s))}{N} + \delta\frac{V_{fd}}{N} \geq (1-\delta)sP(s)D(P(s)) \tag{2.9}$$

Solving for the optimal price for each s yields the next Proposition.

Proposition 2.3. *In an optimal second-best full-disclosure regime,*

1. *For* $s \leq S$, $P(s) = p^*$.
2. *For* $s > S$, $sP(s)D(P(s)) = S\Pi^*$

where S *is the maximal positive* s' *solution to:*

$$s' = \frac{\delta \int_0^{s'} sh(s)ds}{(1-\delta)(N-1) - \delta \int_{s'}^{\bar{s}} h(s)ds} \tag{2.10}$$

Notice that even when firms are not patient enough to achieve the first-best, monopoly profits are earned in some region of s (i.e., $s < \hat{s}'$). Here, disclosure plays an important role of price-coordination. This is the second main intuition in our analysis. In the model, it must be incentive-compatible for firms to stay on the equilibrium path and not to undercut their competitors. When market size is too large, however, the gains from undercutting are too important and thus, at p^*, firms would prefer to undercut. One way firms can avoid such deviations is to agree to a lower price when market size is large, artificially reducing total industry profits and therefore removing incentives to undercut. Disclosing the market size information (s) helps making the price coordination possible.

No Disclosure

Now we go through the case of second-best with no disclosure. Let p_{nd} be the price chosen by the uninformed firms.

Lemma 2.1. *In a PC-strategy with no-disclosure, conditional on not disclosing, the informed firm must choose* p_{nd} *(the same price as the uninformed firms).*

Lemma 2.1 exploits Assumption 3 in the context of our game, essentially excluding PPNE in which the informed firm undercuts by more than p_{nd}. This is because the informed firm would then deviate to undercut by a lesser amount, claiming to its competitors that this price deviation did not hurt them ex-post.[18]

Let us define $\Pi_{nd} = p_{nd}D(p_{nd})$ and the total industry profit expected in such an equilibrium, $V_{nd} = \Pi_{nd} \int sh(s)ds = \Pi_{nd}/\Pi^*$. A lower price can mitigate incentives to deviate by reducing how much profit one firm (informed or not) can take from its competitors. This is one side of the trade-off. On the other hand, a lower price reduces total industry profits V_{nd} and thus reduces the incentive effects of future rents.

Proposition 2.4. *In any no-disclosure PPNE such that $\delta < \delta_{nd}$, $V_{nd} = 0$.*

Choosing prices lower than p^* will never provide incentive benefits with no disclosure. One problem with no disclosure is its lack of flexibility: the *same* price must be used for any realization of s. This feature does not allow firms to adapt their pricing strategies to the environment and thus makes collusion problematic when market size varies too much.

Proposition 2.4 further suggests that collusive industries with high profits but high discount rate (low δ) should feature more disclosure than those with lower discount rate. In the former case, never disclosing is (weakly) suboptimal while in the latter case, always disclosing is (weakly) suboptimal. In other words, the model provides a link between a positive correlation between firms disclosure and discount rate due to product market interactions (i.e., a given discount rate would lead to a given disclosure policy).[19]

3. Partial Disclosure

In this section, we consider cases where partial disclosure may emerge as a repeated equilibrium behavior. The key to partial disclosure is that it combines advantages of both no disclosure (incentive-compatibility of the uninformed) and full disclosure (price-coordination). We explore cases where partial disclosure may dominate both full and no disclosure.

3.1. Efficient Strategy Profile

Consider the following strategy profile for an efficient PPNE:

1. *cooperation:* The cooperation mode is now a combination of the cooperation mode with disclosure and with no-disclosure. For ease of exposition, let the state-contingent disclo-

[18]Doing so, however, may be useful from an ex-ante perspective, given that it makes the price chosen by the informed firm random from the perspective of the uninformed and thus typically reduces the uninformed's incentives to deviate.

[19]In general, firm's discount rate is endogenous in a well-diversified capital market, which in turn, would be affected by the disclosure policy of the firm

sure policy be denoted by a function $m(s) \in \{s, \emptyset\}$ where $m(s) = \emptyset$ means the informed firm does not disclose state s and $m(s) = s$ means the informed firm discloses state s.

Conditional on $m(s) = s$ (disclosure), firms choose a price $P(s)$ and:

a. All firms share the surplus $sP(s)D(P(s))$.

b. Only the informed firm undercuts, and receives the total surplus $sP(s)D(P(s))$

Similarly, let the state-contingent undercutting policy be denoted by a function $z : [\underline{s}, \overline{s}] \rightarrow \{undercut, share\}$ Let $z(s) = share$ means the informed firm shares for state s and $z(s) = undercut$ means the informed firm undercuts. If the informed firm deviates to disclosing s for which $m(s) = 1$, all firms choose a price equal to zero.

Conditional on $m(s) = \emptyset$ (no disclosure), all uninformed firms choose $z(s) = share$ and $p(s) = p_{pd}$. The corresponding profit is denoted $\Pi_{pd} = p_{pd}D(p_{pd})$. Then, the informed firm: (i) overprices by setting $p = p_{pd}$ for $s \in \Omega_1$, (ii) follows the same pricing scheme as the uninformed by setting $z = share$ and $p = p_{pd}$ for $s \in \Omega_2$, (iii) undercuts by setting $z = undercut$ for $s \notin \bigcup_{j=1,2} \Omega_j$.

2. *punishment:* On the punishment mode, a firm chooses $p = 0$ regardless of its information.

3. *transition:* The game starts at date $t = 0$ with all firms playing the Cooperation mode. A shift to the Punishment mode occurs after any deviation (as in Full and No Disclosure).

As in the previous section, we define the following thresholds based on various incentive-compatibility constraints:

1. $\hat{s}_{pd}$ is the maximal market size such that an informed firm not disclosing prefers not to undercut its competitors:

$$(1 - \delta)s\Pi_{pd}/N + \delta V_{pd} \geq (1 - \delta)s\Pi_{pd}$$

   This constraint is satisfied when $s \leq \hat{s}_{pd} \equiv \frac{\delta}{1-\delta} \frac{V_{pd}}{\Pi_{pd}} N/(N-1)$.

2. $\tilde{s}_{pd}$ is the maximal market size such that an informed firm not disclosing accepts to make zero profit in the current period:

$$(1 - \delta)s0 + \delta V_{pd} \geq (1 - \delta)s\Pi_{pd}$$

   This constraint is satisfied when $s \leq \tilde{s}_{pd} \equiv \frac{\delta}{1-\delta} \frac{V_{pd}}{\Pi_{pd}}$.

An immediate analogue to Corollary 2.1 in the context of partial disclosure is that one should always choose $\Omega_1 = [0, \tilde{s}_{pd}]$ and $\Omega_2 = (\tilde{s}_{pd}, \hat{s}_{pd}]$.

3.2. First-Best with Partial Disclosure

Because partial disclosure nests both full disclosure and no disclosure, it holds that the minimum discount rate to attain first-best with partial disclosure, denoted δ_{pd}, should be weakly smaller than δ_{nd}. As Proposition 3.1 shows, however, partial disclosure does not help sustain collusion.

Proposition 3.1. $\delta_{pd} = \delta_{nd}$.

A simple explanation of the result is that at first-best, there is no need to disclose information to coordinate prices because the monopoly price is independent of the private information. To better understand why partial disclosure does not facilitate cooperation; here, it is useful to recall our earlier results. As shown in Proposition 2.1, market size $s > \hat{s}$ cannot be disclosed while maintaining p^*; as a result, these high market sizes should not be disclosed. On the other hand, the market sizes $s \leq \hat{s}$ correspond to the informed firm sharing or giving away its surplus, therefore there is less to gain by the uninformed in undercutting. Indeed, such market sizes help incentive-compatibility in the non-disclosure region and should not be disclosed either. An implication of Proposition 3.1 is that no-disclosure sustains the first-best surplus for the widest possible range of discount rates.

3.3. Second-Best with Partial Disclosure

We consider next second-best equilibria, when $V_{pd} < 1/N$. In this Section, to eliminate situations that would occur only when $\overline{s}$ is sufficiently small, we assume that s has support over $\mathbb{R}^+$.[20] First, we analyze the maximum surplus achievable conditional on disclosure.

Lemma 3.1. *Suppose there exists an efficient partial disclosure PPNE. Then, there exists an efficient PPNE that satisfies:*

(i) If $s \leq \hat{s}_{pd}$ and $m(s) = s$, then $z(s) = share$ and:

$$sP(s)D(P(s)) = \min\left(s\Pi^*, \frac{\delta}{1-\delta}\frac{N}{N-1}V_{pd} \geq sp_{pd}D(p_{pd})\right) \tag{3.1}$$

(ii) If $s \in (\hat{s}_{pd}, \hat{s}_{pd}2(N-1)/N]$ and $m(s) = s$, then $z(s) = undercut$ and:

$$sP(s)D(P(s)) = \min\left(s\Pi^*, \frac{\delta}{1-\delta}V_{pd}\right) \tag{3.2}$$

[20]Without this restriction, there may be equilibria with only two regions: (i) disclosure for high market size, (ii) no disclosure for low market size. We mainly assume $\overline{s}$ is finite in the first section to give a chance to first-best full disclosure.

(iii) If $s > \hat{s}_{pd} 2(N-1)/N$, $m(s) = \emptyset$.

Lemma 3.1 focuses on market sizes for which disclosure by the informed firm should be elicited. One added incentive problem introduced by a partial disclosure PPNE is that for some s, the informed firm can deviate from disclosing ($m(s) = s$) to not disclosing ($m(s) = \emptyset$), attaining a price p_{pd} possibly greater than $P(s)$. When market size is small (case (i)), this deviation is unprofitable, as the informed firm is more concerned about future rents than current small deviation profits. In this case, firms share total surplus equally after a disclosure. This also maximizes the price $P(s)$ at which the product can be sold without one firm deviating to undercut.

When market size is large (case (ii)), however, sharing the total industry surplus after the disclosure is not sufficient to elicit disclosure by the informed firm. To elicit cooperation, the oligopoly can implement a strategy in which the informed firm undercuts after disclosing. Satisfying this incentive-compatibility condition is now costly in terms of total expected surplus. Knowing that the informed firm will undercut, the uninformed firms are more willing to undercut themselves. To elicit cooperation, the total surplus must then be reduced by a factor of $(N-1)/N$.

Finally, when market size is very large (case (iii)), the loss of surplus required to elicit cooperation by the uninformed is too large as compared to the benefits of a deviation to not disclosing. For these market sizes, the informed firm must choose No Disclosure.

Proposition 3.2. *If a partial disclosure PPNE is efficient, it can be constructed as follows: let* $0 < s_0 \leq s_1 \leq s_2 \leq s_3$,

(i) For $s \in [0, s_0) \cup [s_1, s_2) \cup [s_3, +\infty)$, *the informed firm does not disclose.*

(ii) For $s \in [s_0, s_1] \cup [s_2, s_3)$, *the informed firm discloses.*

Unlike in first-best, partial disclosure adds value by providing a balance between the desire to coordinate prices (i.e., setting prices according to $P(s)$) and the desire to provide incentive to the uninformed to not to deviate from the collusive arrangement. We find that equilibria with partial disclosure have a simple structure and feature at most five regions.

First, for extreme market sizes (in the region $[s_3, +\infty)$), the informed chooses not to disclose. When market size is too high, the informed firm cannot be given enough incentives to disclose and thus no-disclosure must be chosen. At the other extreme, when market size is very low (in the region $[0, s_0)$), the benefits from reaching a price p^* (when disclosing) and not p_{nd} (when not disclosing) are very low. Intuitively, by not disclosing low market sizes, the informed firm loses very little industry surplus but makes it more likely that market size conditional on not disclosing (and thus deviation profits) are low.

There is a region of intermediate market sizes such that it may be optimal not to disclose. This region includes only realizations of s in which the informed firm overprices. Here, the incentive benefits of overpricing dominate the loss of surplus generated by not disclosing and charging a lower price. For other moderate market sizes, it is optimal to disclose. For moderately low market sizes (in the region $[s_0, s_1]$), the benefits of disclosing is to able to charge $p^* > p_{nd}$. For moderately high market sizes (in the region $[s_2, s_3]$), disclosure relieves some of the uninformed's incentives to deviate after a non-disclosure.

We give next simpler conditions under which the partial disclosure PPNE simplifies to only two regions.

Corollary 3.1. *The efficient partial disclosure PPNE features only three regions under any one of the following conditions: (a) $\Pi_{nd} = \Pi^*$, (b) $N = 2$, (c) restricting the attention to the most efficient PPNE in the set of PPNE in which the informed firm does not overprice.*

In Corollary 3.1, we illustrate Proposition 3.2 with a thought experiment, shutting down in turn the forces that make each region useful in the Proposition. First, when when $\Pi^* = \Pi_{nd}$, not disclosing does not entail any loss of surplus. In this case, the region $[s_0, s_1]$, whose role was to attain p^* instead of p_{pd} loses its purpose, and thus the PPNE collapses to only three regions. Second, when $N = 2$, any market size $s \geq \hat{s}_{pd}$ can no longer be disclosed. However, all other market sizes below $\hat{s}_{pd}$ are beneficial to induce cooperation by the uninformed after a non-disclosure. As a result, the region $[s_2, s_3]$, whose role was to filter out market sizes to induce deviations after a non-disclosure, is no longer feasible and, again, the equilibrium collapses to only three regions. Finally, when considering only equilibria with no overpricing, the region $[s_1, s_2]$, whose role was to allow the informed firm to overprice, is no longer useful. As before, then, the PPNE collapses to only three regions.

One interpretation of the result is on the issue of voluntary conservatism in accounting. In our model, under full disclosure, a more conservative information system can be interpreted as a distribution over s that pools together signals on $[s_1, \overline{s}]$, so that the manager only observes a pooled message (i.e., market size is between $[s_1, \overline{s}]$), not an exact number (s). Under full disclosure, this always increases the range of discount rates over which first-best can be implemented. In the partial disclosure case, while the manager learns s, he does not disclose the good news which can be interpreted as being conservative. Here the conservatism is ex post, executed by a discretionary disclosure choice by the manager. In practice, the implementation of the partial disclosure can be done through a discretionarily chosen conservatism (such as Microsoft overtly conservative revenue recognition choice). In any case, our paper points to the idea that conservatism can be thought of helping the oligopoly to improve cooperation.

4. Concluding Remarks

In this paper, we explore the relationship between disclosure, collusion and product-market competition. We determine what forms of disclosures maximize industry profits, and relate firm profits to whether a firm is informed, and discloses early or late. In our model, the optimal disclosure policy is endogenous and driven by concerns about future competition. Further, our paper nests endogenous disclosure in a model of collusion. In particular, we find that:

1. Policies with no disclosure are desirable in industries with a low discount rate.
2. Policies with partial or full disclosure are desirable in industries with a high discount rate.
3. In regimes with partial disclosure, informed firms retain very good and very bad information and disclose intermediate news.
4. Disclosure of good market conditions imply high profits for informed firms, but not necessarily for uninformed competitors.

We leave for further work other important aspects of the model which are beyond the scope of our analysis. First, further research is necessary to nest capital market concerns for disclosure when disclosure has real effects on product market competition. Second, if managers are given relative performance contracts, such contracts may interact with incentives to cooperate or undercut in the repeated game. Finally, more work is needed to deepen our understanding of financial disclosure (and its implications for accounting rules) and antitrust laws - one possible direction for the analysis is to model consumer surplus and derive additional implications of accounting rules on consumer welfare.

Appendix: Omitted Proofs

Definition of PPNE: We formally define a pure-strategy strongly symmetric public-monitoring subgame perfect Nash equilibrium (PPNE) using some additional notations. Let $h_t = (s, m, \theta)$ be the outcome of date-t stage game recorded. Let $h^t = (h_0, \dots, h_{t-1})$ be a complete history at date t known to firm i.

Let σ denote firm i's (generic) strategy for the repeated game. This strategy describes firm i's history-dependent stage-game strategy for each period. In particular, after a possible history h^t, σ describe the entire strategy for the rest of the game, we denote $\sigma\ |_{h^t}$ as the continuation strategy for firm i in the subgame starting after history h^t.

Consider a strategy profile where each firm follows the same pure strategy σ. This strategy profile is symmetric because different firms act exactly the same when faced with the same history. Denote expected profit *per period* achieved by each firm by a N-by-1 vector $\pi(\sigma, \dots, \sigma)$ induced by the symmetric strategy profile. Individual components are denoted by $\pi_i, i \in \{1, 2, \ldots, N\}$. Abusing on the notation, we also denote $\pi(\sigma \mid_{h^t}, \ldots, \sigma \mid_{h^t})$ the per-period profit in the subgame after history h^t.[21]

Definition .1. *A strategy profile σ is a PPNE if, for any i, h^t, for any σ',*

$$\pi_i(\sigma \mid_{h^t}, \ldots, \sigma \mid_{h^t}, \ldots, \sigma \mid_{h^t}) \geq \pi_i(\sigma \mid_{h^t}, \ldots, \sigma' \mid_{h^t}, \ldots, \sigma \mid_{h^t})$$

This concludes the definition.□

Proof of Proposition 2.1: The minimum discount rate δ_{fd} must bind Equation (2.1), i.e.

$$(1-\delta_{fd})\overline{s}\Pi^*/N + \delta_{fd}/N = (1-\delta_{fd})\overline{s}\Pi^*$$
$$\delta_{fd}(\overline{s}\Pi^*\frac{N-1}{N} + \frac{1}{N}) = \overline{s}\Pi^* - \overline{s}\Pi^*/N$$
$$\delta_{fd} = \frac{\overline{s}\Pi^*\frac{N-1}{N}}{\overline{s}\Pi^*\frac{N-1}{N} + \frac{1}{N}}$$

Equation (2.2) follows.□

Proof of Proposition 2.2: We prove Corollary 2.1 first, deriving the optimal strategy (i.e., the sets Ω_1 and Ω_2), and then solve for the minimum discount rate δ_{nd} stated in Proposition 2.2.

Note first that any $s \in \Omega_1$ must be such that $s \leq \tilde{s}$ and any $s \in \Omega_2$ must be such that $s \leq \hat{s}$. Therefore, in the left-hand side of Equation (2.5),

$$(1-\delta)(\int_{\Omega_1} sh(s)ds\frac{\Pi^*}{N-1} + \int_{\Omega_2} sh(s)ds\frac{\Pi^*}{N}) + \delta\frac{1}{N} \geq (1-\delta)E_s[sp^*D(p^*)] = (1-\delta)$$

To maximize the left-hand side one should set $\Omega_1 = [0, \tilde{s}]$ and $[\Omega_2 = \tilde{s}, \hat{s}]$ (except possibly over a negligible set). The minimum discount rate consistent with first-best is obtained by binding Equation (2.5).

$$(1-\delta_{nd})(\int_0^{\tilde{s}} sh(s)ds\frac{\Pi^*}{N-1} + \int_{\tilde{s}}^{\hat{s}} sh(s)ds\frac{\Pi^*}{N}) + \delta_{nd}\frac{1}{N} = (1-\delta_{nd})$$

That is:

$$\delta_{nd}(\frac{N+1}{N} - \int_0^{\tilde{s}} sh(s)ds\frac{\Pi^*}{N-1} - \int_{\tilde{s}}^{\hat{s}} sh(s)ds\frac{\Pi^*}{N}) = 1 - \int_0^{\tilde{s}} sh(s)ds\frac{\Pi^*}{N-1} - \int_{\tilde{s}}^{\hat{s}} sh(s)ds\frac{\Pi^*}{N}$$

And solving for δ_{nd}

$$\delta_{nd} = \frac{N(N-1) - N\int_0^{\tilde{s}} sh(s)ds\Pi^* - (N-1)\int_{\tilde{s}}^{\hat{s}} sh(s)ds\Pi^*}{(N+1)(N-1) - N\int_0^{\tilde{s}} sh(s)ds\Pi^* - (N-1)\int_{\tilde{s}}^{\hat{s}} sh(s)ds\Pi^*}$$

This is the desired Equation for δ_{nd}.□

[21] Stating the model in terms of profit per period is common in the repeated games literature (see Mailath and Samuelson (2006) for more details) because it accommodates a discount rate δ equal to 1.

Proof of Corollary 2.2: Suppose that $\delta \geq \delta_{fd}$. Then, Equation (2.1) (incentive-compatibility for the uninformed playing p^*) must be verified. This Equation is the same as Equation (2.4). Therefore, not disclosing and choosing $\Omega_2 = [0, \bar{s}]$ is also incentive-compatible. As shown in Corollary 2.1, there exists another no-disclosure strategy that achieves a strictly lower discount rate.□

Proof of Proposition 2.3: Since it is optimal to set $P(s)$ as close as possible to p^* while still respecting constraint ((2.9)), there must be a threshold, denoted S such that for $s \leq S$, $P(s) = p^*$ and for $s > S$, $P(s) < p^*$.

We solve first for S. Set $P(s) = p^*$ and bind Equation (2.9), i.e.

$$(1-\delta)\Pi^* S = (1-\delta)\frac{\Pi^* S}{N} + \delta V_{fd}$$

Solving for S yields:

$$S = \frac{\delta}{1-\delta}\frac{N}{N-1}\frac{V_{fd}}{\Pi^*}$$

For $s \leq S$, $sP(s)D(P(s)) = s\Pi^*$.

For $s > S$, since Equation (2.9) binds,

$$sP(s)D(P(s)) = \frac{\delta}{1-\delta}\frac{V_{fd}}{N-1}$$

Then:

$$\begin{aligned}
V_{fd} &= \frac{1}{N}(\Pi^* \int_0^S sh(s)ds + \int_S^{\bar{s}} sP(s)D(P(s))h(s)ds) \\
&= \frac{1}{N}(\Pi^* \int_0^S sh(s)ds + \frac{\delta}{1-\delta}\frac{N}{N-1}V_{fd}\int_S^{\bar{s}} h(s)ds) \\
&= \frac{\Pi^*/N \int_0^S sh(s)ds}{1 - \frac{\delta}{1-\delta}\frac{1}{N-1}\int_S^{\bar{s}} h(s)ds} \\
S\Pi^* \frac{1-\delta}{\delta}\frac{N-1}{N} &= \frac{\Pi^*/N \int_0^S sh(s)ds}{1 - \frac{\delta}{1-\delta}\frac{1}{N-1}\int_S^{\bar{s}} h(s)ds}
\end{aligned}$$

Solving for S yields Equation (2.10).□

Proof of Lemma 2.1: Suppose not, and let $p(s)$ be the price chosen by the informed firm. Consider next a deviation to p_{nd} with undercutting. In both cases, the uninformed firms will achieve zero profit. However, the informed firm will achieve a greater profit $sp_{nd}D(p_{nd}) > sp(s)D(p(s))$. This implies that the profit vector in the stage game after this deviation weakly Pareto dominates the profit vector in the stage game on the equilibrium path. By PC, the informed firm must then be expecting a (weakly) greater profit after this deviation has taken place. This deviation is therefore profitable.□

Proof of Proposition 2.4: Consider a no-disclosure PPNE in which uninformed firms choose p_{nd}. Clearly, eliciting undercutting by the informed firm to $P(s) < p_{nd}$ is not incentive-compatible since the uninformed firm would instead choose p_{nd} and $'undercut'$ without affecting the ranking of prices. In a no-disclosure PPNE, then: (i) the uninformed firms choose $p_{nd} < p^*$, (ii) for $s \in \Omega_1$, the informed firm chooses $p > p^{nd}$ (overprices), (iii) for $s \in \Omega_2$, the informed firm chooses $p = p_{nd}$ and share, (iv) for $s \notin \Omega_1 \cup \Omega_2$, the informed firm chooses $p = p_{nd}$ and undercuts.

First, we compute the per-firm surplus V_{nd} in this PPNE,

$$\begin{aligned} V_{nd} &= \int h(s)s\Pi_{nd}ds/N \\ &= \int h(s)s\Pi^* ds\Pi_{nd}/(N\Pi^*) \\ &= \Pi_{nd}/(N\Pi^*) \end{aligned} \tag{A-1}$$

As in first-best, it is optimal to set Ω_1 as the largest possible set such that the informed firm overprices, that is: $s \in \Omega_1$ if and only if $s \leq \tilde{s}_{nd}$ where:

$$\delta V_{nd} \geq (1-\delta_{nd})\tilde{s}_{nd}\Pi_{nd} \tag{A-2}$$

Therefore: $\tilde{s}_{nd} = \frac{\delta}{1-\delta}\frac{V_{nd}}{\Pi_{nd}} = \tilde{s}$ (first-best).

Similarly, $s \in \Omega_2$ if and only if $s \in (\tilde{s}, \hat{s}]$.

Let us now write the incentive-compatibility for the uninformed:

$$(1-\delta)(\int_0^{\tilde{s}} sh(s)ds\frac{\Pi_{nd}}{N-1} + \int_{\tilde{s}}^{\hat{s}} sh(s)ds\frac{\Pi_{nd}}{N}) + \delta\frac{\Pi_{nd}}{\Pi^* N} \geq (1-\delta)\frac{\Pi_{nd}}{\Pi^*} \tag{A-3}$$

Multiplying both sides by Π^*/Π_{nd}, this incentive-compatibility condition is the same as in first-best. Therefore, $\delta \geq \delta_{nd}$.□

Proof of Proposition 3.1: It proves convenient to define a binary function $a(s)$ such that if $m(s) = s$, $a(s) = 0$ and if $m(s) = \emptyset$, $a(s) = 1$. Similarly, define a binary function $b(s)$ such that if $z(s) = undercut$, $b(s) = 0$ and if $z(s) = share$, $b(s) = 1$.

In first-best, $\hat{s}_{pd} = \hat{s}$ and $\tilde{s}_{pd} = s_{pd}$. To verify the PPNE, we need to verify two incentive-compatibility conditions: (i) When $b(s) = 1$, disclosure must be incentive-compatible for the informed, (ii) When $b(s) = 0$, letting the informed firm undercut must be incentive-compatible for the uninformed, (iii) p^* must be incentive-compatible for the uninformed.

(i) For any decision to disclose such that $b(s) = 1$, it must be incentive-compatible for the informed firm not to deviate to not disclosing and undercut, that is:

$$(1-\delta)s \leq (1-\delta)s/N + \delta/N \tag{A-4}$$

This Equation implies that $s \leq \hat{s}$. In other words, $a(s) = 1$ or $b(s) = 0$ for any $s \geq \hat{s}$ and $b(s) = 1$.

(ii) Suppose the informed firm discloses and $b(s) = 0$, it must be incentive-compatible for the uninformed not to undercut, that is:

$$(1-\delta)s \leq \delta/N \tag{A-5}$$

This Equation implies that $s \leq \tilde{s}$. In other words, $a(s) = 1$ or $b(s) = 1$ for any $s \geq \tilde{s}$.

(iii) The incentive-compatibility condition for the uninformed can now be written:

$$(1-\delta)\Pi^*\frac{\int sa(s)h(s)(1_{s\leq\tilde{s}}/(N-1)+1_{s\in(\tilde{s},\hat{s}]}/N)ds}{\int a(s)h(s)ds}+\frac{\delta}{N}\geq(1-\delta)\Pi^*\frac{\int sa(s)h(s)ds}{\int a(s)h(s)ds} \tag{A-6}$$

One can rewrite this inequality as $\int h(s)a(s)\psi(s)ds \geq 0$ where:

$$\psi(s)=s\Pi^*(1-\delta)(1_{s\leq\tilde{s}}/(N-1)+1_{s\in(\tilde{s},\hat{s}]}/N-1)+\delta/N \tag{A-7}$$

The most favorable case to satisfy (A-6) is to maximize $\int h(s)a(s)\psi(s)ds$ in $a(s)$; since this term is linear in $a(s)$, the bang-bang solution prescribes $a(s)=1$ when $\psi(s)>0$ and $a(s)=0$ when $\psi(s)<0$. Next, note that:

$$\begin{aligned}\psi(\hat{s}) &= (1-\delta)\Pi^*\hat{s}\frac{1-N}{N}+\frac{\delta}{N} && \text{(A-8)}\\ &= (1-\delta)\Pi^*\frac{\delta}{1-\delta}\frac{1}{N-1}/\Pi^*\frac{1-N}{N}+\frac{\delta}{N} && \text{(A-9)}\\ &= 0 && \text{(A-10)}\end{aligned}$$

Since $\psi(s)$ is strictly decreasing in s, $\psi(s)>0$ for $s<\hat{s}$. Therefore $a(s)=1$ for any $s<\hat{s}$.

To conclude, (i)-(ii)-(iii) imply that $a(s)=1$ for all s.□

Proof of Lemma 3.1:(i) Conditional on $b(s)=1$, it must be incentive-compatible for all firms to share, that is:

$$(1-\delta)sP(s)D(P(s))/N+\delta V_{pd}\geq(1-\delta)sP(s)D(P(s)) \tag{A-11}$$

If $P(s)=p^*$ satisfies this inequality, it is optimal to set $P(s)=p^*$. Else, maximizing $sP(s)D(P(s))$ requires to bind this inequality and therefore set:

$$sP(s)D(P(s))=\frac{\delta}{1-\delta}\frac{N}{N-1}V_{pd} \tag{A-12}$$

Next, for the informed firm, it must be incentive-compatible to Disclose versus Not Disclose and Undercut. In particular,

$$\begin{aligned}\Pi_{pd}(1-\delta)s &\leq (1-\delta)sP(s)D(P(s))/N+\delta V_{pd}\\ &\leq (1-\delta)\frac{\delta}{1-\delta}\frac{N}{N-1}\frac{V_{pd}}{N}+\delta V_{pd}\\ &\leq \delta V_{pd}\frac{N}{N-1}\end{aligned}$$

As a result, $s\leq\hat{s}_{pd}$.

Finally, we need to verify that $P(s)\geq p_{pd}$. Since $P(s)$ is decreasing in s, it is sufficient to verify that (A-11) is satisfied at equality by $P(s)=p_{pd}$ at $s=\hat{s}_{pd}$.

$$(1-\delta)\hat{s}_{pd}\Pi_{pd}/N+\delta V_{pd}\geq(1-\delta)\hat{s}_{pd}\Pi_{pd} \tag{A-13}$$

Equation (A-13) is true by definition of $\hat{s}_{pd}$.

(ii) Conditional on $b(s) = 0$, it must be incentive-compatible for the uninformed not to undercut, that is:

$$\delta V_{pd} \geq (1-\delta)sP(s)D(P(s)) \tag{A-14}$$

If $P(s) = p^*$ satisfies this inequality, it is optimal to set $P(s) = p^*$. Else, maximizing $sP(s)D(P(s))$ requires to bind this inequality and therefore set:

$$sP(s)D(P(s)) = \frac{\delta}{1-\delta}V_{pd} \tag{A-15}$$

As before, we consider next the incentive-compatibility condition for the informed and compare the profit from disclosing and the profit from not disclosing.

$$\begin{aligned}
\Pi_{pd}(1-\delta)s &\leq (1-\delta)sP(s)D(P(s)) + \delta V_{pd} \\
&\leq (1-\delta)\frac{\delta}{1-\delta}V_{pd} + \delta V_{pd} \\
&\leq 2\delta V_{pd}
\end{aligned}$$

As a result, s must be greater than $\hat{s}_{pd}2(N-1)/N$.□

Proof of Proposition 3.2: Let us define $\psi_{pd}(s)$ as follows:

$$\psi_{pd}(s) = s\Pi_{pd}(1-\delta)(1_{s\leq\tilde{s}_{pd}}/(N-1) + 1_{s\in(\tilde{s}_{pd},\hat{s}_{pd}]}/N-1) + \delta V_{pd} \tag{A-16}$$

Then, as in Proposition 3.1, one can write the incentive-compatibility condition for the uninformed firms after not disclosing as follows: $\int h(s)a(s)\psi_{pd}(s)ds \geq 0$.

ψ is strictly decreasing in s. In addition,

$$\begin{aligned}
\psi(\hat{s}_{pd}) &= \hat{s}_{pd}\Pi_{pd}(1-\delta)(1/N-1) + \delta V_{pd} \\
&= 0
\end{aligned}$$

So that $\psi(.)$ is positive for $s \in [0, \hat{s}_{pd}]$.

We state next the problem of finding the best possible partial disclosure PPNE:

$$\sup V_{pd}$$

s.t.

$$V_{pd} = \frac{1}{N}\int sh(s)\{a(s)\Pi_{pd} + (1-a(s))P(s)D(P(s))\}ds \quad (\lambda) \tag{A-17}$$

$$0 \leq \int a(s)h(s)\psi(s,\Pi_{pd})ds \quad (\mu) \tag{A-18}$$

Let L denote the Lagrangian of this problem. The problem is also subject to the relationships given in Lemma 3.1 which do not depend on $a(s)$ (these constraints are unimportant for our purpose since they do not appear in the Lagrangian when differentiating with respect to $a(s)$). The multiplier λ is readily verified to be strictly positive (if not, V_{pd} large would be a solution to the Lagrangian). Differentiating in $a(s)$ for any s such that disclosure is feasible,

$$\frac{\partial L}{\partial a(s)} = h(s)\{s[\lambda(\Pi_{pd} - P(s)D(P(s)))/N + (1-\delta)\mu\Pi_{pd}(-1 + \frac{1_{s\leq\tilde{s}_{pd}}}{N-1} + \frac{1_{s\in(\tilde{s}_{pd},\hat{s}_{pd}]}}{N})] + \mu\delta V_{pd}\} \equiv h(s)G(s) \quad \text{(A-19)}$$

Note that $a(s) = 1$ when $G(s) > 0$ and $a(s) = 0$ when $G(s) < 0$. We show first that the shadow cost of giving incentives to cooperate to the uninformed after non-disclosure is non-zero.

Lemma A .1. $\mu > 0$.

Proof: Suppose $\mu = 0$. Then: $G(s) = s\lambda(\Pi_{pd} - P(s)D(P(s)))/N$. By Lemma 3.1, $G(s) < 0$ for any $s < \hat{s}_{pd}$. It follows that $a(s) = 0$ for any $s \geq \hat{s}_{pd}$.

For any $s > \hat{s}_{pd}$, the informed firm prefers undercutting to sharing:

$$(1-\delta)s\Pi_{pd} > (1-\delta)s\Pi_{pd}/N + \delta V_{pd}$$

Integrating with respect to $a(s)h(s)$.

$$(1-\delta)\Pi_{pd}\int a(s)h(s)sds > (1-\delta)\Pi_{pd}\int a(s)h(s)sds/N + \delta\int a(s)h(s)ds$$

This implies that choosing p_{nd} is not incentive-compatible for the uninformed; QED.□

In the next Lemma, we analyze the function $G(.)$.

Lemma A .2. *Suppose* $S \leq \tilde{s}_{pd}$. *Then:*

1. $G(0) > 0$.
2. $Sign(G(S)) = -Sign(\frac{\lambda}{N-1} - (1-\delta)\mu)$.
3. $Sign(G(\tilde{s}_{pd})) = -Sign(\frac{\lambda}{N} - (1-\delta)\mu)$.
4. $Sign(\lim_{s\to\tilde{s}_{pd}^+} G(s)) = Sign(G(S))$
5. $G(\hat{s}_{pd}) = 0$.
6. $Sign(\lim_{s\to\hat{s}_{pd}^+} G(s)) = -Sign(G(\tilde{s}_{pd}))$
7. $Sign(G(\frac{N-1}{N}2\hat{s}_{pd})) = -Sign(G(\tilde{s}_{pd}))$.

Proof:(i) $G(0) = \mu\delta V_{pd} > 0$.

(ii) We calculate $G(S)$.

$$\begin{aligned}
G(S) &= \frac{\lambda}{N}(\Pi_{pd}\frac{\delta}{1-\delta}\frac{N}{N-1}\frac{V_{pd}}{\Pi^*} - \frac{\delta}{1-\delta}\frac{N}{N-1}V_{pd}) + \mu\delta V_{pd} + (1-\delta)\mu\frac{1-N}{N}\frac{\delta}{1-\delta}\frac{N}{N-1}\frac{V_{pd}}{\Pi^*}\Pi_{pd} \\
&= \frac{\lambda}{N}\frac{\delta}{1-\delta}V_{pd}(\frac{\Pi_{pd}-\Pi^*}{\Pi^*}) + \mu\delta V_{pd} - \mu\delta V_{pd}\frac{\Pi_{pd}}{\Pi^*} \\
&= \frac{\delta}{1-\delta}V_{pd}\frac{\Pi^*-\Pi_{pd}}{\Pi^*}(\mu(1-\delta) - \frac{\lambda}{N-1})
\end{aligned}$$

(iii) We calculate $G(\tilde{s}_{pd})$.

$$\begin{aligned} G(\tilde{s}_{pd}) &= \frac{\lambda}{N}(\Pi_{pd}\frac{\delta}{1-\delta}\frac{V_{pd}}{\Pi_{pd}} - \frac{\delta}{1-\delta}\frac{N}{N-1}V_{pd}) + \mu(1-\delta)(-1+\frac{1}{N-1})\frac{\delta}{1-\delta}\frac{V_{pd}}{\Pi_{pd}} + \delta\mu V_{pd} \\ &= \frac{lambda}{N}\frac{\delta}{1-\delta}V_{pd}(1-\frac{N}{N-1}) + \delta V\frac{-N+2}{N-1} + \delta\mu V_{pd} \\ &= \frac{1}{N-1}\frac{\delta}{1-\delta}V_{pd}((1-\delta)\mu - \frac{\lambda}{N}) \end{aligned}$$

(iv) We calculate $\lim_{s\to\tilde{s}_{pd}^+} G(s)$.

$$\begin{aligned} \lim_{s\to\tilde{s}_{pd}^+} G(s) &= \frac{\lambda}{N}(\Pi_{pd}\frac{\delta}{1-\delta}\frac{V_{pd}}{\Pi_{pd}} - \frac{\delta}{1-\delta}\frac{N}{N-1}V_{pd}) + \delta\mu V_{pd} + (1-\delta)\mu(-1+1/N)\Pi_{pd}\frac{\delta}{1-\delta}\frac{V_{pd}}{\Pi_{pd}} \\ &= -\frac{\lambda}{N}\frac{\delta}{1-\delta}V_{pd}\frac{1}{N-1} + \delta\mu V_{pd}\frac{1}{N} \\ &= \frac{\delta V_{pd}}{(1-\delta)N}((1-\delta)\mu - \frac{\lambda}{N-1}) \end{aligned}$$

(v) We calculate $G(\hat{s}_{pd})$.

$$\begin{aligned} G(\hat{s}_{pd}) &= \hat{s}_{pd}\lambda(\Pi_{pd} - \Pi_{pd})/N + \mu\Pi_{pd}(1-\delta)(-1+1/N)\hat{s}_{pd} + \mu\delta V_{pd} \\ &= \mu\delta V_{pd} - \mu\delta V_{pd} \\ &= 0 \end{aligned}$$

(vi) We calculate $\lim_{s\to\hat{s}_{pd}^+} G(s)$.

$$\begin{aligned} \lim_{s\to\hat{s}_{pd}^+} G(s) &= \frac{\lambda}{N}(\Pi_{pd}\hat{s}_{pd} - \frac{\delta}{1-\delta}V_{pd}) + \delta\mu V_{pd} - \mu(1-\delta)\Pi_{pd}\hat{s}_{pd} \\ &= \frac{\lambda}{N}(\Pi_{pd}\frac{V_{pd}}{\Pi_{pd}}\frac{\delta}{1-\delta}\frac{N}{N-1} - \frac{\delta}{1-\delta}V_{pd}) + \delta\mu V_{pd} - \mu(1-\delta)\Pi_{pd}\frac{\delta}{1-\delta}\frac{N}{N-1}\frac{V_{pd}}{\Pi_{pd}} \\ &= \frac{\lambda}{N}\frac{\delta}{1-\delta}V_{pd}\frac{1}{N-1} - \delta\mu V_{pd}\frac{1}{N-1} \end{aligned}$$

(vii) We calculate $G(2(N-1)/N\hat{s}_{pd})$.

$$\begin{aligned} G(2(N-1)/N\hat{s}_{pd}) &= \frac{\lambda}{N}(\Pi_{pd}\hat{s}_{pd}2(N-1)/N - \frac{\delta}{1-\delta}V_{pd}) + \delta\mu V_{pd} - \mu(1-\delta)\Pi_{pd}\hat{s}_{pd}2(N-1)/N \\ &= \frac{\lambda}{N}(\Pi_{pd}2\frac{\delta}{1-\delta}\frac{V_{pd}}{\Pi_{pd}} - \frac{\delta}{1-\delta}V_{pd}) + \delta\mu V_{pd} - \mu(1-\delta)\Pi_{pd}2\frac{\delta}{1-\delta}\frac{V_{pd}}{\Pi_{pd}} \\ &= \frac{\lambda}{N}\frac{\delta}{1-\delta}V_{pd} - \delta\mu V_{pd} \\ &= \frac{\delta}{1-\delta}V_{pd}(\frac{\lambda}{N} - \mu(1-\delta)) \end{aligned}$$

QED.

Using Lemma A.2, we can prove the Proposition when $S \leq \tilde{s}_{pd}$. Letting λ vary, there are three cases to consider:

1. Suppose $\lambda \leq (1-\delta)\mu(N-1)$. Then, by Lemma A.2, $Sign(G(S)) \geq 0$, $G(\tilde{s}_{pd}) \geq 0$, $\lim_{s\to\tilde{s}_{pd}^+} G(s) \geq 0$, and $\lim_{s\to\hat{s}_{pd}^+} G(s) \leq 0$ and $G(\hat{s}_{pd}2(N-1)/N) \leq 0$. Thus, the partial disclosure PPNE features disclosure for

$s \in [\hat{s}_{pd}, 2(N-1)/N\hat{s}_{pd}]$ (followed by undercutting) and disclosure otherwise.

2. Suppose that $\lambda \in ((1-\delta)\mu(N-1), (1-\delta)\mu N]$. Then, by Lemma A.2, $Sign(G(s)) \leq 0$, $G(\tilde{s}_{pd}) \geq 0$, $\lim_{s \to \tilde{s}_{pd}^+} G(s) \leq 0$, and $\lim_{s \to \hat{s}_{pd}^+} G(s) \leq 0$ and $G(\hat{s}_{pd} 2(N-1)/N) \leq 0$. Thus, the partial disclosure PPNE features disclosure for $s \in [s_0, s_1] \cup [\tilde{s}_{pd}, 2(N-1)/N\hat{s}_{pd}]$ (where $s_0 \in [0, S]$ and $s_1 \in [S, \tilde{s}_{pd}]$) and non-disclosure otherwise.

3. Suppose that $\lambda > (1-\delta)\mu N$. Then, by Lemma A.2, $Sign(G(S)) < 0$, $G(\tilde{s}_{pd}) < 0$, $\lim_{s \to \tilde{s}_{pd}^+} G(s) < 0$, and $\lim_{s \to \hat{s}_{pd}^+} G(s) > 0$ and $G(\hat{s}_{pd} 2(N-1)/N) > 0$. Thus, the partial disclosure PPNE features disclosure for $s \in [s_0, \hat{s}_{pd}]$ where $s_0 \in (0, S)$ (followed by sharing) and non-disclosure otherwise.

We turn to the other situation in which $S > \tilde{s}_{pd}$. Then, G is decreasing on $[0, S]$. In addition, the proof of (vi) and (vii) in Lemma A.2 remains valid and therefore: $Sign(\lim_{s \to \hat{s}_{pd}^+} G(s)) = Sign(G(\frac{N-1}{N} 2\hat{s}_{pd}))$. There are three cases to consider.

1. Suppose $G(S) \geq 0$. Then, $G(s) \geq 0$ for all $s \leq \tilde{s}_{pd}$. Therefore, for a partial disclosure PPNE to occur, it must hold that $Sign(G(\frac{N-1}{N} 2\hat{s}_{pd})) < 0$. As a result, the partial disclosure PPNE features disclosure for $s \in [\hat{s}_{pd}, 2(N-1)/N\hat{s}_{pd}]$ (followed by undercutting) and disclosure otherwise.

2. Suppose $G(S) < 0$ and $Sign(G(\frac{N-1}{N} 2\hat{s}_{pd})) \leq 0$. Then, the partial disclosure PPNE features disclosure for $s \in [s_0, 2(N-1)/N\hat{s}_{pd}]$, where $s_0 \in (0, S)$ and non-disclosure otherwise.

3. Suppose $G(S) > 0$ and $Sign(G(\frac{N-1}{N} 2\hat{s}_{pd})) > 0$. Then, the partial disclosure PPNE features disclosure for $s \in [s_0, \hat{s}_{pd}]$, where $s_0 \in (0, S)$ and non-disclosure otherwise.□

Proof of Corollary 3.1: Suppose that $\Pi_{pd} = \Pi^*$. Then, by Lemma 3.1, for any $s \leq \hat{s}_{pd}$, $P(s) = p^*$. Therefore $G(s)$ (Equation (A-19)) must be positive for all $s \leq \hat{s}_{pd}$ and $a(s) = 1$ for any $s \leq \hat{s}_{pd}$. Applying Proposition 3.2, the partial disclosure equilibrium features a single disclosure interval $[s_1, 2(N-1)/N\hat{s}_{pd}]$ where $s_1 > \hat{s}_{pd}$.

Suppose that $N = 2$. Then, $\hat{s}_{pd} = 2(N-1)/N\hat{s}_{pd}$. Therefore, no disclosure can be elicited for $s > \hat{s}_{pd}$. It follows that the partial disclosure equilibrium features a single disclosure interval $[s_0, \hat{s}_{pd}]$, where $s_0 < \hat{s}_{pd}$.

Suppose that we consider the best PPNE with partial disclosure, in the class of equilibria that do not feature overpricing by the informed firm. This can be incorporated in Proposition 3.2 by setting $\tilde{s}_{pd} = 0$. This removes all cases such that $S \leq \tilde{s}_{pd}$. However, equilibria with five regions only occur when $S \leq \tilde{s}_{pd}$ (see case 2. in the proof of Proposition 3.2). Thus the partial disclosure PPNE must feature only three regions: disclosure on $[s_0, s_1]$ (where $0 < s_0 < s_1$) and non-disclosure otherwise.□

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