VOLUNTARY DISCLOSURE ALONG BUSINESS CYCLES*

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Abstract

This paper presents a theory that relates business cycles to firms' voluntary disclosure. In the model, firms may be informed about upcoming demand in advance of their competitors and decide whether or not to publicly disclose that information. We examine the cyclical behavior of disclosures, and their association with price-setting behavior and industry profits. We show that, in industries that are highly concentrated and/or feature lower cost of capital, no-disclosure is prevalent and associated with acyclical product prices and higher profits. Otherwise, disclosure occurs in normal times, while no-disclosure occurs prior to either sharp industry expansions or industry declines. Consequently, strategic disclosure can work to reduce early dissemination of information about the cycle.

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Business cycles, defined as the variations in economic activity levels (e.g., demand, production, market prices) experienced by an industry over time, have been the object of an extensive literature in social sciences. However, fairly little is known about the channels through which information about the cycle flows between participants of the product market. In a world where some firms possess private information about the pending cycle, strategic decisions may include information transmission decisions such as voluntary disclosure. As such, corporate voluntary disclosure can affect and be affected by the cycles. On one hand, voluntary disclosure (or lack thereof) can convey industry-wide information to other market participants about the cycle, generating competitive responses which shape the cycle itself. On the other hand, the cycle affects the level and distribution of industry profits which, in turn, will provide incentives or disincentive to disclose firms' private information.

This paper develops a model that accounts for the two-way interactions between industry cycle dynamics and firm voluntary disclosures. We propose a variant of a standard dynamic oligopoly model which incorporates voluntary disclosure into the existing literature on dynamic competition and relates it to the primitives of the industry such as concentration levels, cost of capital and the magnitude of economic shocks. In doing so, our analysis offers several novel testable predictions in terms of how disclosure accompanies aggregate fluctuations.

The idea that the nature of the business cycle may be driven by the informational environment is fairly intuitive. The fluctuations of production, prices, and profits depend on the dissemination of information about common shocks, which is a function of the disclosure behavior. If, say, firms choose to disclose more information about a common shock to the marketplace, such information would allow otherwise uninformed firms to adjust their production and prices in advance of the business cycle shock. Thus, how much the industry as a whole responds to a shock, then, depends on what information is being disclosed. As a result, firms' disclosures will play an important role on how common shocks are translated into fluctuations in production, prices and profit distribution within the industry. Following this logic, the real consequences of the industry-wide shock, and whether changes in prices may precede or lag the common shock, will depend on the firms' external disclosure practices.

We present next an overview of our approach and main results. The basic framework follows the model of Rotemberg and Saloner (1986), hereafter abridged as RS, which is widely-used in the literature on cycles and product markets. In this framework, firms in an oligopoly compete over an infinite horizon with time-varying business cycle shocks. Rotemberg and Saloner show that, when the demand shock is public information, firms can form a tacit agreement featuring counter-cyclical prices, higher during recessions than expansions, as a result of increased competition during booms. As a point of departure from RS, we assume that information about the upcoming cycle may be privately known to an individual firm, who can retain or publicly disclose that information prior to choosing prices on the product market.[1]

We focus first on industries in which either discount rates are low or the number of firms in the market is small. In such industries, the tacit agreement takes the form of a monopoly price and a regime in which the informed firm does not disclose regardless of economic fundamentals. The key intuition here is that incentives to undercut are the greatest when demand is high (and potential profits are large). No-disclosure makes uninformed competitors uncertain about current demand, which lowers the expected benefits of undercutting. On the other hand, the informed firm knows about the upcoming demand and thus in equilibrium *does* undercut its competitors when the demand shock is sufficiently favorable. As a result, the market share of the informed firm is strongly pro-cyclical while the market share of the uninformed firm is counter-cyclical, with actual profit falling during an expansion.

We turn next to industries in which either discount rates are high or the number of firms in the market is large enough. In this case, voluntary disclosure helps coordinate uninformed competitors on a counter-cyclical price schedule. However, disclosure may also weaken the incentive for the uninformed firms to cooperate in pricing. Combined, partial disclosure may emerge in equilibrium. We show that voluntary disclosure occurs for intermediate demand cycle shocks while no-disclosure is preferred for extreme variations in demand. This, in turn, leads

[1] The stylized assumption of a single firm being informed is made for parsimony and to better illustrate information transfers within an industry; for this reason, it is widely-used in the literature (e.g., Darrough (1993)). Naturally, the main insights can be extended to environments in which more than one firm is informed as long as not all firms are informed.

to our prediction that more disclosure should occur in industries featuring more cycle-sensitive pricing policies.

Closely related to our study is a relatively recent literature focusing on the reputational incentives that emerge in a repeated strategic interaction. In general, these reputational incentives can lead to dynamics that are very different from a single-period interaction. Huddart, Hughes and Levine (2006) show that public disclosure of insiders' trading decisions allows multiple insiders to sustain tacit agreements in which they trade less, lower price efficiency and increase their trading gains. Insiders sustain this equilibrium by increasing their trading quantities in future periods if they observe excessive trading by one or more insiders over one period. Baldenius and Glover (2011) examine a three-party tacit agreement between a principal and two agents whenever some performance measures are non-contractible; they show that single-period bonus pools may create incentives for collusion between agents in the repeated interaction.

In the context of voluntary disclosure, Marinovic (2010) examines a model in which, over time, the firm forms its reputation as a function of a sequence of past reports. In his model, the probability that earnings are being misreported perceived by investors, as well as the total level of earnings that may have been misreported, are a function of the entire past history of reports, causing the reporting strategies to also vary. Beyer and Dye (2011) show that managers with reputational concerns tend to disclose more unfavorable information, to increase investors' perceptions that they will be more forthcoming in future periods. Fischer, Heinle and Verrecchia (2012) consider a model in which current investors expect future investors to overweight earnings in their valuation model, implying that they should themselves optimally do so. The paper shows that the rational earnings fixation that emerges results in higher volatility but also higher risk-adjusted surplus for each generation of selling investors. An important difference between these studies and this one is that they focus primarily on financial reporting concerns (i.e., a seller maximizing the perceived value of what he is selling); on the other hand, the focus here is on the interactions between reporting and the product market.

Our paper is also part of an existing literature on disclosure within a competitive environment (e.g., Wagenhofer (1990), Darrough (1993), Evans and Sridhar (2002), Suijs and Wielhouwer (2011)). An important difference with this literature is that disclosure and price-setting behavior are, in the model, self-enforced as a result of a reputational concerns that emerge in a repeated game (a tacit agreement). While there is a large literature in social sciences on tacit agreements in competitive environments (Rotemberg and Saloner (1986), Athey and Bagwell (2001), Mailath and Samuelson (2006)), there are few studies in the accounting literature that focus on more than two periods. A notable exception is Baiman, Netessine and Saouma (2010) who examine the design of a production chain subject to economic shocks that operates over an infinite horizon. However, their focus is different from ours in that their primary focus is on incentive problems rather than information dissemination.

Finally, our paper is related to an emerging empirical literature that examines accounting disclosure quality based on characteristics of market competition, and in particular, as it relates to industry concentration and cycles. In terms of industry concentration, Harris (1998), Cohen (2006) and Balakrishnan and Cohen (2009) provide evidence that less industry competition leads to lower disclosure quality, which is consistent with our theory. Although these studies argue that competition on the product market disciplines disclosure, it is worth noting that the formal theories that are referred to (or those that we know of) typically do not speak about industry concentration and do not provide formal support for this idea. In terms of economic cycles, there is yet very little work that fully documents the relationship between accounting and aggregate shocks and evidence on the subject is mostly available piecemeal within a few recent papers. In particular, Johnson (1999) and Cohen and Zarowin (2008) examine several metrics of accounting quality (e.g., persistence, earning responses and earning management) as a function of macroeconomic conditions.

1. The Model

1.1. Basic Setup

We borrow from the widely-used Rotemberg and Saloner model, or RS, the template for business cycle fluctuations. As in RS, the specific effect of the business cycle is considered from perspective of one representative industry but the model can be easily extended to multiple industries.

There are N firms ($N \geq 2$) competing in a product market over an infinite time horizon indexed by $t = 0, \ldots, +\infty$. Firms are risk-neutral, face a constant marginal cost normalized to zero and discount payoffs in each period with a discount factor $\delta \in (0, 1)$, which one may also interpret as the firm's cost of capital. Implicitly, we abstract away from innovations to marginal costs to focus on common shocks to demand for the firms' products often observed along business cycles.

In each period, firms face a demand $s_t D(p)$, where s_t represents a time-varying mass of potential consumers and $D(p)$ is the per-consumer demand at price p. The function $pD(p)$ is assumed to be continuous and strictly increasing (resp. strictly decreasing) on $[0, p^*]$ (resp. $[p^*, +\infty)$). The price p^* represents the optimal monopoly price and we denote $\Pi^* = p^* D(p^*)$ the maximal industry profit per unit of market size.

This specification is used by Bagwell and Staiger (1997) and captures the key idea that there are more potential consumers who may purchase the product during good times. Importantly, it speaks about the broad variations in market size along the cycle (plausibly, a first-order effect) and separates them from shocks to the average consumer's price elasticity, i.e., $D(.)$ does not depend on s_t.[2] In our study, this assumption is also imposed for a different reason: if p^* were a function of s_t, information would have a value even from the perspective of a monopoly "first-best" problem; by contrast, here, all of the effects are entirely driven by the need for information due to the tacit agreement between competing firms. Naturally, adding these additional features

[2] Price elasticities are likely to be ambiguously related to industry cycles. If an industry is doing well, meaning that more consumers demand the product, it could be the case that price elasticity increases or decreases, depending on the individual price elasticity of the incremental consumers.

to the model would bias the analysis toward providing more information in states where it is more useful for a monopoly, but would provide no incremental insight beyond this observation.

In the next paragraphs, we formally introduce the timeline of the model, the main variables of interest, the nature of competition and the stochastic processes that drive economic shocks. The game is decomposed in time periods, with each period t representing a stage game and $t.i$ denoting the i^{th} event in period t.

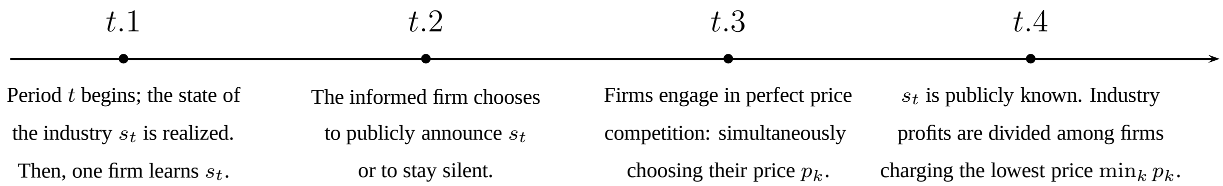

Figure 1: Model Timeline

At $t.1$, an informed firm privately learns market size s_t. We assume that market size shocks s_t are i.i.d., drawn from a continuous distribution with full support over $\mathbb{R}^+$ and a density $h(s)$ bounded away from zero. Without loss of generality, we normalize the distribution of s such that $\mathbb{E}_s[s\Pi^*] = 1$.[3]

The key assumption in the model is that s_t is not publicly known at the beginning of each period t. We assume that one and only one firm learns a signal that is informative on current market size (or state of the industry), s_t at the beginning of each period, call this firm informed, and the others uninformed. This is of course for analytical tractability and similar results can be obtained as long as not all firms know the information. It is unimportant if s_t is a noisy signal on market size; in this case one would have to reinterpret s_t as the expected demand conditional on the signal. Each period, every firm is equally likely to become informed. As a modelling

[3]As in RS (and much of the literature that follows), we make the assumption that demand shocks are independent across time periods. Some studies such as Bagwell and Staiger (1997) have extended the analysis to persistent shocks; extensions of our results are possible with some correlation between shocks, but, for reasons of parsimony, it is not usual in these models to introduce correlation unless time correlation is the sole purpose of the analysis (see for example Green and Porter (1984), Athey and Bagwell (2001), Athey, Bagwell and Sanchirico (2004)). If, consistent with most staggered price models (Calvo (1983)), prices are changed infrequently, time periods may be sufficiently long so that correlations between periods remain low.

choice, the random information endowment is practical to set no ex-ante asymmetry between competitors but (as can seen from the analysis) the forces would be similar if the same firm were repeatedly informed in advance.[4]

At t.2, the informed makes a voluntary disclosure m_t, which can be a choice to publicly announce s_t or to stay silent. In practice, firms release quarterly reports and make voluntary disclosures that contain a fair amount of information about future demand. As an example, Beyer, Cohen, Lys and Walther (2010) find that voluntary earnings forecasts is the variable that explains the largest portion of abnormal stock returns, about three times the variation explained by earnings announcements and pre-announcements, or security analysts' forecasts. MD&A sections, which are often part of mandatory filings, are another venue that is traditionally used by managers to voluntarily provide qualitative information about future sales (Darrough (1993), Bryan (1997)). This noted, the interpretation as a demand shock to the extent that the key assumption is that the environment is with common shocks (Raith (1996)), a central feature to model business cycles. We assume that the firm cannot pre-commit to a disclosure policy but disclosure, when it occurs, is truthful, i.e., $m_t \in \{ND, s_t\}$.[5]

At $t.3$, firms engage in price competition, simultaneously choosing a price p_k that may be conditional on s_t for the informed firm and for all other firm if $m_t = s_t$ is disclosed. It is a useful tool for our exposition to use perfect price competition, which is natural given that the nature of competition is not the main object of analysis; this assumption is used in several classic papers in the area (e.g., Bagwell and Staiger (1997), Athey et al. (2004)) and takes out distracting considerations about information-sharing in various imperfect competition settings which have been well studied in past literature. As any model with price competition, the model is not intended to be factually descriptive as a complete representation of an industry but, rather, focused on "excess profits"; in practice, actual profits may not be zero due to a minimum return

[4]The only additional difficulty under the alternative assumption that the same firm is informed is that there is no single equilibrium that is ex-ante preferred by both the informed and uninformed firms (various tacit agreements feature different allocation of the industry surplus). Even in this case, the equilibrium that maximizes total profits would have similar characteristics to the equilibrium obtained here.

[5]This assumption is consistent with those in the voluntary disclosure literature (e.g., Jovanovic (1982), Verrecchia (1983) and Dye (1985)) in accounting. The question of whether reputational concerns in a repeated game could enforce truthful disclosures is discussed in Stocken (2000).

on current capital, or a variety of other unmodelled reasons that are transversal to the main argument, such as a presence in multiple markets or idiosyncratic noise.

It will be useful to give a formal label to "a small price decrease" which is what occurs when a firm undercuts by a small amount to take away market share from its competitors. Under price competition, it may be desirable to undercut by a very small amount, so as to extract the maximum surplus from consumers. Formally, we assume that firms choose both a price p_k and a decision $z_k \in \{share, undercut\}$. The joint choice of a price and, possibly, a small deviation (p_k, z_k) is what constitutes a pricing strategy.[6]

At $t.4$, the total industry profit $s_t \min_k p_k D(\min_k p_k)$ is shared equally among firms charging lowest price if no such firms choose $z = undercut$. If some among these firms choose to undercut, the total profit is shared among the undercutting firms (i.e., those charging the lowest price and choosing $z = undercut$).[7] Of note, (unlike in most prior literature) the disclosure strategy will not require a formal commitment and is entirely self-enforcing as a result of reputations. In particular, the results are robust to reporting motives: myopic managers who sell their firm could still play the tacit agreement obtained here because any deviation to a myopic report would be immediately detected (e.g., disclose something that should not be disclosed) and trigger a loss of reputation and a sharp decrease in the expected value of the firm.

1.2. Equilibrium Definitions

If we consider the stage game and assume that firms play this stage game only once (shutting down any forward-looking behavior), firms will compete up to the price being equal to marginal cost $p_k = 0$. To see this, note that if total industry profits $s_t \min_k p_k D(p_k)$ were greater than

[6]Some more details can be given for the technically-minded reader. First, in the absence of the variable z_k, we can approximate (arbitrarily closely) the solution provided here using only price p_k; this should be, in practice, equivalent to the undercutting variables and substantially burdens the exposition with no added insights. In addition, the variable z_k is unnecessary if we restrict prices to be chosen in a fine but finite grid (if, say, prices must be written in cents). In this respect, the undercutting variable is no more than a means to make the set of feasible payoffs of the stage game closed and the Pareto frontier of the game reached, while still preserving a continuous price space.

[7]Allowing for multiple levels of undercutting (as in "undercutting the undercutter") would have no effect in our model - this is because when we are considering deviation profits, a firm can do arbitrarily close to its undercutting profit by using a price deviation only.

zero, at least one firm could decrease its own price slightly below $\min_k p_k$ and reap all the market demand (Tirole (1988), p.245). Under perfect price competition, therefore, voluntary disclosure is irrelevant in the single-period game.

Now consider the repeated setting. In the repeated setting, the oligopoly can sustain higher prices by tacitly agreeing to condition future actions on outcomes in previous periods. This is what we refer as a tacit agreement (as originally studied by RS); however, such agreements need to involve potential punishments (such as setting lower prices in future periods) if some firms decide not to follow the tacit agreement and to myopically maximize their current-period profit. For every period, firms use past market sizes $\{s_t\}$, disclosures $\{m_t\}$, as well as past prices $\theta_t = (p_i, z_i)_{i=1}^N$ as conditioning variables for current stage-game actions.

Following the repeated games literature, we simplify any strategy in terms of three descriptors of a strategy profile: $\langle cooperation, punishment, transition \rangle$.

1. *cooperation path:* This is a strategy mode that describes the action of each firm on the equilibrium path. That is, if no firm deviates from equilibrium play last period, all firms will follow the action prescribed by *cooperation* this period.

2. *punishment path:* This a strategy mode that describes the action of each firm off the equilibrium path. That is, if any firm deviates from equilibrium play last period, all firms will follow the action prescribed by *punishment* this period and future periods.[8]

3. *transition:* describes how each firm move from on-equilibrium (*cooperation*) play into off-equilibrium play (*punishment*).

As with other works in this strand, the repeated game we study contains multiple equilibria. We thus adopt the following standard equilibrium selection criterion. First, we restrict the attention to equilibria in which firms condition their actions on only public information available

[8]To prevent a deviation, it is desirable to minimize payoffs after observing an off-equilibrium move, i.e. switch to a Punishment. This is formally defined as follows. First, if a firm was playing Punishment at date $t-1$, always stay on Punishment at date t. According to this plan, all firms will achieve zero profit (current and future) once the Punishment stage is reached. Second, for each date t such that Cooperation was played in the previous period, switch to Punishment when an off-equilibrium move is observed.

in past stage games; these are usually referred as public-monitoring games.[9] Second, we restrict the attention to strongly-symmetric equilibria in which, for all possible histories of past play, the strategies chosen by all players are identical and may only depend on their information in the current period. We denote this solution concept a perfect public monitoring Nash equilibrium (PPNE) and it is a synonym for the notion of tacit agreement described earlier.[10] In reference to antitrust laws, tacit agreements do not involve binding contracts; thus, although they are in theory prohibited by law, they are hard to detect and/or prove in a court of law.

In terms of basic notations, we refer to $\sigma = (\sigma_1, \ldots, \sigma_N)$ as a strategy in the game, i.e. such that σ_k maps for any period j and any public history of actions to firm k's disclosure and price choices in the period j. We then refer to $u_k(\sigma)$ or in short $u(\sigma)$ in a strongly-symmetric equilibrium, as the expected surplus of player k. Also, to minimize the need for technical notations, we refer to Mailath and Samuelson (2006) for a formal presentation of the PPNE (since our notion of PPNE is a special case of their definition).

It should be emphasized that punishments, when they occur, are *not* renegotiation-proof, since firms would be better-off renegotiating away from the punishment. This credible-punishment assumption is entirely standard in the literature on repeated games (Green and Porter (1984), Rotemberg and Saloner (1986), Bagwell and Staiger (1997), Athey and Bagwell (2001), Athey et al. (2004), Mailath and Samuelson (2006), Huddart et al. (2006), Baldenius and Glover (2011)) and, of course, if firms immediately renegotiated any punishments, there would be no hope for a tacit agreement. From a practical sense, the problem with such renegotiations in a competitive setting is that they would likely require some open communication as to the terms of how the punishment is lifted or whether the non-deviating parties should receive some extra sales in the renegotiated equilibrium. It is likely that such open bargaining about the new terms of a collusive agreement would be much more easily detectable by regulators than, as we assume, some implicit coordination at the beginning of the game on the unique equilibrium that

[9]This is not a restriction on the set of feasible actions that can be possibly undertaken; as shown in Mailath and Samuelson (2006), public-monitoring equilibria are Nash equilibria in the complete game. However, there are usually more complex equilibria involving private-monitoring that can achieve even greater profits.

[10]The requirement that the equilibrium be (subgame-)perfect plays no role in our model, given that the min-max optimal punishment coincides the minimum feasible payoff of the stage game.

maximizes ex-ante expected payoffs.[11]

To begin, we consider the punishment path. Since the punishment path is used as a means to provide incentives to cooperate (and never occurs on the equilibrium path), we can always choose the punishment path that minimizes firms' payoffs. In our game, this corresponds to playing (forever) the Nash equilibrium of the stage game in which firms make zero profit and can be interpreted as a complete loss of reputation going forward after players observe an off-equilibrium move (Bar-Isaac and Shapiro (2010), Baldenius and Glover (2011)).

Now that we examined the cooperation mode, we are interested in solving for the choice of the cooperation path that delivers the highest expected surplus. The ideal payoff would be one in which monopoly profit ($s_t\Pi^*$) is achieved every period (and, due to symmetry, equally shared among all firms). In such an ideal setting, the total discounted future profit, at any period in time t, is expressed as $\frac{1}{N}(1+\delta^1+\delta^2+...) = \frac{1}{N(1-\delta)}$, or the expected monopoly payoff shared among all firms.

We call this payoff the monopoly payoff, which after rescaling with a factor $1-\delta$, gives rise to a (normalized per-period) payoff of $1/N$. This ideal payoff may or may not be achieved in the repeated game (and when taking firm's incentives to compete). Imposing incentive-compatibility on the part of firms, we define the efficient tacit agreement as the equilibrium that achieves the highest profit.

Definition 1.1 *A PPNE strategy profile σ is an efficient tacit agreement if for any other PPNE σ', $u(\sigma) \geq u(\sigma')$.*

The efficient tacit agreement represents a natural point of coordination for firms in the oligopoly because it achieves the highest sustainable profit. Further, if firms were able to communicate in a non-binding manner (e.g., trade shows/publications, executive associations, joint ventures, public disclosures), forward-induction arguments could be used to rule out inefficient equilibrium outcomes.

[11]There are a few cases (see references Mailath and Samuelson (2006)) in which a renegation-proof equilibrium can be sustained but these often involve extremely complex asymmetric strategies after a deviation occurs and which seem intuitively unappealing if firms cannot engage in bilateral communications.

Finally, we introduce an additional restriction on the set of PPNEs under consideration (specific to our paper) which we denote Pareto-Consistency. Namely, we require that an off-equilibrium action in a stage-game that Pareto-increases payoffs in the same stage game also Pareto-increases continuation payoffs. This restriction is quite natural as we want to avoid, for example, intuitively implausible equilibria that are sustained by firms starting a price war after a pricing decision that did not ex-post reduce their profit. Abusing slightly on the terminology, we will refer to "equilibria" as PPNE that are efficient tacit agreements over the set of equilibria that satisfy this restriction.[12]

2. No- and Full-Disclosure Benchmarks

2.1. Cooperation Path

In this section, we analyze the cost and benefit of no-disclosure versus those of full-disclosure. Our initial objective is to provide a rough cut at optimal disclosure strategies by considering two extreme cases, and develop the intuitions which will be helpful in explaining the main result of endogenous partial disclosure. Given the repeated model under our analysis is stationary, we now drop the time index on variables when those are no longer needed.

We formally define the equilibrium path under the full-disclosure and no-disclosure benchmarks. The tacit agreement disclosure strategy and the punishment paths are immediate, per our previous observations. We focus next on pricing behavior under the two benchmarks.

One extreme is full disclosure (i.e., disclose all realizations of s). As a benchmark, full-disclosure boils down to information being perfectly known to all market participants as exam-

[12] A formal definition is given next. A strongly symmetric PPNE with strategy σ is Pareto-Consistent if, for any histories of past play h^t and h_2^t such that:

(a) h^t and h_2^t differ only over last stage game actions.

(b) The last period profit vector under h_2^t weakly Pareto-dominates the profit vector under h^t.

Then, when the expected equilibrium profit vector in the continuation game with $\sigma_{h_2^t}$ must Pareto-dominate the expected equilibrium profit vector in the continuation game with σ_{h^t}. This condition is redundant when the equilibrium attains the monopoly payoff.

ined in RS. Conditional on market size s, a price $P(s)$ is set and all firms choose to share.[13] We can then describe the full-disclosure strategy as a function $P(s)$ (to be chosen efficiently).

Another extreme is no-disclosure (i.e., disclose nothing for all s realizations), uninformed firms do not have information, and set a price p_{nd} that does not depend on s. On the cooperation path, three options are available to the informed firm - overprice, share or undercut - for each s realization. As a result, we can define the cooperation path of any no-disclosure equilibrium in terms of a price p_{nd} and three sets Ω_{over}, Ω_{share} and Ω_{under}, as described below.

- The informed firm can set a price strictly higher than p_{nd}, in which case it makes zero profit in the current period; let $s \in \Omega_{over}$ be the set of market sizes such that the informed firm overprices.

- The informed firm can share the market, choosing $p = p_{nd}$ and $z = share$, in which case it makes a current profit $sp_{nd}D(p_{nd})/N$; let $s \in \Omega_{share}$ be the set of market sizes such that the informed firm shares.

- The informed firm can undercut its competitors, choosing $p = p_{nd}$ and $z = undercut.$, in which case it makes a current profit $sp_{nd}D(p_{nd})$; let $s \in \Omega_{under}$ be the set of market sizes such that the informed firm undercuts on the cooperation path.[14]

Below, we give a description of the strategies over the cooperation and punishments, and the equilibrium transitions.

[13] It is never strictly preferred to have one firm undercutting in this case, since this makes the tacit agreement more difficult to sustain for those firms having zero profit (see RS for details).

[14] This is the place where Pareto-consistency is used, because it prevents the uninformed firm (who gets zero profit anyways) to induce a suboptimal current choice of price $P(s) < p_{nd}$ by the informed. Price competition simplifies the algebra but the main force would still be present under other forms of competition. For example, under Cournot competition (e.g., Darrough (1993)) , the informed firm would underproduce instead of overpricing and overproduce instead of undercutting.

	Full-Disclosure Equilibrium	No-Disclosure Equilibrium
cooperation	The informed firm discloses $m = s$ and choose a price $P(s)$ and $z = share$.	The informed firm does not disclose $m = ND$ and chooses $\begin{cases} p > p_{nd} & \text{if } s \in \Omega_{over} \\ p = p_{nd} \text{ and } z = share & \text{if } s \in \Omega_{share} \\ p = p_{nd} \text{ and } z = undercut & \text{if } s \in \Omega_{under} \end{cases}$
	All uninformed firms choose price $P(s)$ and $z = share$. If the informed firm disclose $m = ND$, all firms choose a price equal to zero	All uninformed firms choose a price p_{nd} and $z = share$. If the informed firm discloses $m \neq ND$, all firms choose a price equal to zero.
punishment	On the punishment path, firms chooses $p = 0$ regardless of their information, or current disclosures.	
transition	The game starts at date $t = 0$ with all firms playing the Cooperation path. Any move that does not conform to the Cooperation path triggers a permanent shift to the punishment path.	

Table 1: Strategy Profiles with Full-Disclosure and No-Disclosure

2.2. Incentive Benefit of No-Disclosure

We first examine equilibria in which the tacit agreement can effectively replicate the surplus achieved by a monopoly, which means that, for all s, $P(s) = p_{nd} = p^*$. In that case, firms obtain their ideal symmetric surplus $1/N$. In this setting, we derive our first main intuition that no-disclosure dominates full-disclosure because no-disclosure makes it possible for the tacit agreement to be incentive compatible from the standpoint of uninformed firms. We call this first main intuition the Incentive Benefit of No-Disclosure.

2.2.1 No-disclosure dominates full-disclosure under monopoly pricing

We begin with full-disclosure. For $P(s) = p^*$ to be an equilibrium, no firm shall find it desirable to undercut its competitors, leading to the following incentive-compatibility constraint, for all s,

$$(1-\delta)s\Pi^*/N + \delta/N \geq (1-\delta)s\Pi^* \tag{2.1}$$

The left-hand side has two components. The first component $s\Pi^*/N$ is the current surplus obtained by playing the cooperation path and the second component δ/N is the discounted surplus obtained in future periods (recall that these are expressed in a per-period basis and need to be divided by $1-\delta$ to obtain total expected surplus). The right-hand side has only one component, the deviation profit in the current period; given that such a deviation triggers a permanent price of zero in future periods, there will be zero profit in all future periods. This inequality cannot be satisfied for all $s \in \mathbb{R}^+$ for any $\delta < 1$, which implies that full disclosure may not achieve monopoly profits for all market sizes.

We next compare this benchmark to the no-disclosure case. In this case, depending on the actions of the informed firm, the incentive-compatibility condition is a bit more complicated. Consider first the prescription for the informed firms to overprice ($p > p^*$) for some s-region ($s \in \Omega_{over}$), leading to zero profit in the current period as well continued cooperation path for the informed firm. By deviating to $z = undercut$ and $p = p^*$ (the best possible deviation), the informed firm can obtain $s\Pi^*$ in the current period, but this will trigger a shift by all firms to the punishment path (and thus zero profit in future periods). For the recommended action to be optimal, it must hold that: for all $s \in \Omega_{over}$ (the informed firm overprices),

$$(1-\delta)0 + \delta/N \geq (1-\delta)s\Pi^* \tag{2.2}$$

This constraint is satisfied when $s \leq \tilde{s} \equiv \frac{\delta}{1-\delta}\frac{1}{N\Pi^*}$. A similar condition is derived for all $s \in \Omega_{share}$ (the informed firm shares),

$$(1-\delta)s\Pi^*/N + \delta/N \geq (1-\delta)s\Pi^* \tag{2.3}$$

This constraint is satisfied when $s \leq \hat{s} \equiv \frac{\delta}{1-\delta}\frac{1}{(N-1)\Pi^*}$. It is thus easier to induce sharing than to induce overpricing, as implied by the fact that $\hat{s} > \tilde{s}$. Finally, obviously, equilibrium undercutting is always incentive-compatible for the informed firm.

In addition to inequalities (2.2), (2.3), it must be incentive-compatible for each uninformed firm not to deviate to price slightly lower than the informed firm.

$$(1-\delta)\left(\int_{\Omega_{over}} sh(s)ds\frac{\Pi^*}{N-1}+\int_{\Omega_{share}} sh(s)ds\frac{\Pi^*}{N}\right)+\delta\frac{1}{N}\geq(1-\delta) \qquad (2.4)$$

In Equation (2.4), the right-hand side corresponds to the expected profit obtained by undercutting all other firms. Since in this case, the uninformed firm deviating does not know s, it will anticipate an expected profit $\mathbb{E}(s)p^*D(p^*)=1$. The left-hand side corresponds to the profit expected by staying on the Cooperation path, where the profit of the uninformed will depend on s and the strategy of the informed firm.

Pooling together these constraints, it is established that, to be an equilibrium, the cooperation path must satisfy that: (i) the informed firm overprices (shares) only when it is incentive-compatible to do so (i.e. $s\in\Omega_{over}$ implies that $s\leq\tilde{s}$ and $s\in\Omega_{share}$ implies that $s\leq\hat{s}$), (ii) the uninformed firm does not deviate (i.e., Equation (2.4) is met). The next proposition establishes the existence superiority of no-disclosure regime.

Proposition 2.1 *With full-disclosure, monopoly payoffs cannot be attained.*[15] *With no-disclosure, monopoly payoffs can be attained if and only if* $\delta\geq\delta_{nd}$ *where* $\delta_{nd}<1$.[16]

Compared to full-disclosure, the oligopoly in a no-disclosure regime is better able to dampen the incentives to deviate when current demand is high by leaving most competitors in the dark. Intuitively, when market size is large, disclosing makes deviation more attractive to every firm so firms need to be sufficiently patient to refrain from undercutting. Under no disclosure, $N-1$ firms do not know whether market size is high and must assume the average market size when contemplating deviation, lowering the benefit of deviating (i.e., the right-hand-side of constraint (2.4) is reduced). In addition, to better elicit cooperative behavior among uninformed firms, the informed firm agrees to give away additional rents when the market is low (thus

[15]We have also established the results if the support of s is bounded; in that case, full-disclosure achieves monopoly prices for any $\delta\geq\delta_{fd}$, but that threshold is greater than the minimum discount rate achieving monopoly profits under no disclosure (a result that carries over to the case in which more than one firm is informed).

[16]The threshold δ_{nd} is given by:

$$\delta_{nd}=\frac{N(N-1)/\Pi^*-N\int_0^{\tilde{s}} sh(s)ds-(N-1)\int_{\tilde{s}}^{\hat{s}} sh(s)ds}{(N+1)(N-1)/\Pi^*-N\int_0^{\tilde{s}} sh(s)ds-(N-1)\int_{\tilde{s}}^{\hat{s}} sh(s)ds}$$

increasing the left-hand side of constraint (2.4)). In short, no disclosure uses the slack in the incentive-compatibility constraint of the informed firm when demand is low to better motivate the uninformed firms to cooperate. As a result, secrecy is valuable to the oligopoly, not because it necessarily benefits the uninformed firm in the current period, but because it better motivates cooperation among oligopoly members in the long-term. This is the first main intuition derived from the model.

Generally, our results point to the cyclicality of firm market-share (and profit) which depends on the firms' information endowment (i.e., informed versus uninformed). In our model, the efficient tacit agreement prescribes time-varying market shares for the informed and uninformed firms. On the one hand, transferring more surplus to the uninformed firms can help avoid deviations to lower prices. On the other hand, it is more difficult to induce the informed firm not to undercut when market size is large. The solution to this trade-off implies a pro-cyclical behavior of market share for the informed firm and a counter-cyclical behavior of market shares for the uninformed firms: when demand is low (resp. large), the informed firm does not sell (resp. serves the entire market). Also undercutting (when market size is large) occurs on the equilibrium path and does not trigger a price-war.

While we write results in terms of a minimum discount factor, it should be noted from a closer inspection that we could identically look for the minimum size of the oligopoly N consistent with the monopoly surplus; which would deliver similar characterizations of a threshold in N. While δ may be interpreted as the firm's inherent cost of capital, we would interpret N as the industry's concentration ratio (Herfindahl index).

2.2.2 Properties of no-disclosure equilibria under monopoly pricing

As is usual in the literature, we now solve for the strategy that attains the monopoly surplus for the widest range of discount rates (and thus would be robust to a small amount of uncertainty about discount rates). Observe that the more the informed firm gives away surplus to the uninformed, the easier it is to satisfy Equation (2.4) (this increases the cooperation surplus of the uninformed). Therefore, the equilibrium that achieves cooperation for the widest range of discount rates is the one in which Ω_{over} is set as large as possible, or $s \in [0, \tilde{s})$, followed by Ω_{share}, or $s \in [\tilde{s}, \hat{s})$, with undercutting when nothing else is incentive compatible, or $s \geq \hat{s}$.

Corollary 2.1 *There is a unique strategy that achieves monopoly surplus for any $\delta \geq \delta_{nd}$, it is given as follows:*

(i) For $s \leq \tilde{s}$ (low market size), the informed firm does not sell and only the uninformed sell at a price p^*.

(ii) For $s \in (\tilde{s}, \hat{s}]$ (medium market size), total industry profits Π^* are shared equally among all firms.

(iii) For $s > \hat{s}$ (large market size), only the informed firm sells.

The asymmetric information that remains in the no-disclosure regime has important consequences on the product market. We establish that no-disclosure changes the sensitivity of firm profits to the cycle. Specifically, the analysis suggests that no-disclosure increases the sensitivity of the informed firm to the cycle, and reduces the sensitivity of the profit for firms that did not anticipate the shock. A favorable but unanticipated shock, as shown above, can lead to a reduction in firms' profit.

Finally, we discuss whether being an informed firm is good news or bad news in a no-disclosure equilibrium. To do this, we derive conditions under which the expected profit of an informed firm[17] right before it learns the actual s exceeds the expected profit of an uninformed firm:

$$\frac{\Pi^*}{N}\int_{\tilde{s}}^{\hat{s}} sh(s)ds + \Pi^* \int_{\hat{s}}^{+\infty} sh(s)ds \geq \frac{1}{N} \tag{2.5}$$

This inequality is satisfied when N is large, δ is small or low realizations of s (lower than $\tilde{s}$) are unlikely. On the other hand, when δ is sufficiently close to one, or N is small, the uninformed firm is always better-off than the informed firm. That is, being informed is indicative of a

[17] In Equation 2.5, we write instead $1/N$; however, this is equivalent given that $1/N$ is a weighted average of the profit of the informed firm and the profit of the uninformed firm.

low expected own profit for industries with few firms and low discount rates. We take these conditions as representing empirically large mature industries (e.g., automobile, steel). By contrast, in growth industries, information should be indicative of high profits (e.g., technology).

2.3. Price Coordination Benefit of Full-Disclosure

We turn next toward the case in which monopoly profits may not be achieved by the industry. We will then show that disclosure may play a role in terms of coordinating prices (a role that it did not have under monopoly pricing since prices were not a function of s). We develop this observation further by establishing two results, (a) that an efficient full-disclosure pricing policy specifies prices $P(s)$ that vary with s and (b) that no profitable tacit agreement is now possible under no-disclosure.

We begin with full-disclosure. Note that, if monopoly prices are not always sustained, it must be the case that $P(s) < p^*$ for some states s. Let V_{fd} ($< 1/N$) be the expected surplus received by firms in such an equilibrium:

$$V_{fd} = \frac{\int sP(s)D(P(s))h(s)ds}{N} \tag{2.6}$$

Similar to the previous case (but using V_{fd} instead of $1/N$), it must be incentive-compatible for all firms to choose $p = P(s)$ and $z = share$ (versus deviating to $z = undercut$):

$$(1-\delta)\frac{sP(s)D(P(s))}{N} + \delta\frac{V_{fd}}{N} \geq (1-\delta)sP(s)D(P(s)) \tag{2.7}$$

Comparing the above incentive-compatibility constraint to that for monopoly pricing case (equation 2.1), a key difference is that the deviation payoff (i.e., the right-hand-side) is now a function of the prices ($P(s)$) which can be state-dependent and optimally chosen as a part of the tacit agreement. Solving for the optimal price for each s yields the following full-disclosure benchmark.

Proposition 2.2 *In an efficient full-disclosure equilibrium,*

1. *For* $s \leq S$, $P(s) = p^*$.

2. *For* $s > S$, $sP(s)D(P(s)) = S\Pi^*$

where S *is the maximal positive* s' *solution to:*

$$s' = \frac{\delta \int_0^{s'} sh(s)ds}{(1-\delta)(N-1) - \delta \int_{s'}^{\overline{s}} h(s)ds} \tag{2.8}$$

Notice that even when firms are not patient enough to achieve the monopoly surplus, monopoly profits are earned in some region of s (i.e., $s < S$). In the other region, equilibrium prices ($P(s) < p^*$) are a function of state-variable s. Here, disclosure plays an important role of price-coordination. In the model, it must be incentive-compatible for firms to stay on the equilibrium path and not to undercut their competitors. When market size is too large, however, the gains from undercutting are too important and thus, at p^*, firms would prefer to undercut. One way firms can avoid such deviations is to agree to a lower price when market size is large, artificially reducing total industry profits and therefore removing incentives to undercut. This is precisely the insight from RS where market size is information is *assumed* to be public knowledge. In our paper, it leads the informed firm to voluntarily disclosing the business cycle information (s) in order to help making the price coordination possible. This is the second main intuition in our analysis.

Given our intuition that the value of disclosure is that of price coordination and that price coordination is required to set a cyclical pricing policy, we may then observe that no-disclosure fails to facilitate the tacit agreement if firms are too impatient to sustain monopoly prices for all s-realizations (i.e., δ is too low).

Proposition 2.3 *If* $\delta < \delta_{nd}$*, any no-disclosure equilibrium yields zero profit for all firms.*

Combined, the propositions in this section depict a stark picture of no-disclosure. On one hand, it gives rise to monopoly payoff and dominates full-disclosure when firms are patient enough. On the other, when firms are less patient, it immediately reduces profit to zero and cannot sustain any in-between profit levels. The problem with no disclosure is its lack of flexibility: the *same* price must be used for any realization of s. This feature does not allow firms to adapt their pricing strategies to the environment and thus makes the tacit agreement problematic. Proposition 2.3 further suggests that industries with high profits but high discount rate (low δ) should feature more disclosure than those with lower discount rate. In the former case, never disclosing is (weakly) suboptimal while in the latter case, always disclosing is (weakly) suboptimal. In other words, the model provides some support for an association between time-series price variations, high discount rate and greater levels of voluntary disclosure.[18]

In summary, the preceding section discusses two fundamental roles of disclosure highlighted our repeated setting. First, no-disclosure keeps uninformed firm in the dark in order to lower its deviation benefit, thus making it easier for the uninformed firms to cooperate stay on the equilibrium path. This benefit of no-disclosure appears when discount rate is rather low (i.e., firms care more about the future) or, equivalently, when the number of firms in an oligopoly is small for a given discount rate. Second, disclosure allows the firms to set state-contingent prices if and when it is necessary to lower prices to levels below monopoly prices in order to maximize expected industry profit. This benefit of disclosure appears when discount rate is rather high (i.e., firms care less about the future) or, equivalently, when the number of firms in an oligopoly is large for a given discount rate. However, This does not say, however, that full disclosure is the optimal disclosure policy in the tacit agreement in high-discount rate (or less concentrated industries) situations. Partial disclosure may not play a role when $\delta < \delta_{nd}$, as discussed next.

3. Partial Disclosure

3.1. Cooperation Path

In this section, we consider cases where partial disclosure may emerge as a repeated equilibrium behavior. The key to partial disclosure is that it combines advantages of both no disclo-

[18]In general, firm's discount rate is endogenous in a well-diversified capital market, which in turn, would be affected by the disclosure policy of the firm

sure (incentive-compatibility of the uninformed) and full disclosure (price-coordination). We explore cases where partial disclosure may dominate both full and no disclosure.

Partial disclosure combines characteristics of the strategies under full-disclosure and no-disclosure. Let us define $s \in \Omega$ as the set of market sizes that are not disclosed, we can then define the tacit agreement under partial disclosure as follows.

Table 2: Strategy Profile with Partial Disclosure

	Disclosure Region ($s \notin \Omega$)	No Disclosure Region ($s \in \Omega$)
cooperation	The informed firm discloses $m = s$ and choose a price $P(s)$ and $z = share$.	The informed firm does not disclose $m = \emptyset$ and choose $\begin{cases} p > p_{pd} & \text{if } s \in \Omega_{over} \subset \Omega \\ p = p_{pd} \text{ and} & \\ z = share & \text{if } s \in \Omega_{share} \subset \Omega \\ p = p_{pd} \text{ and} & \\ z = undercut & \text{if } s \in \Omega_{under} \subset \Omega \end{cases}$
	All uninformed firms choose price $P(s)$ and $z = share$. If the informed firm does not disclose $m = ND$, all firms choose a price equal to zero	All uninformed firms choose a price p_{pd} and $z = share$. If the informed firm discloses $m \neq ND$, all firms choose a price equal to zero.
punishment	On the punishment path, firms chooses $p = 0$ regardless of their information, or current disclosures.	
transition	The game starts at date $t = 0$ with all firms playing the Cooperation path. Any move that does not conform to the Cooperation path triggers a permanent shift to the punishment path.	

3.2. Partial Disclosure Equilibria

Building on earlier intuitions, it can be shown that partial disclosure does not facilitate the tacit agreement that sustains the monopoly surplus (simply because no price coordination is used when p^* is set every period).[19] Let us next assume that the monopoly surplus is not attainable, and denote $V_{pd} < 1/N$ firm's surplus in the efficient partial disclosure equilibrium. Under partial disclosure, it is useful to define thresholds on s to help define pricing choices for an informed firm not disclosing. Denote $\Pi_{pd} = p_{pd} D(p_{pd})$, $\hat{s}_{pd}$ and $\tilde{s}_{pd}$ as follows

[19] We have omitted the proof to save space. Although the statement is intuitive, the formal proof is not entirely trivial and is available on request from the authors.

- $\Pi_{pd} = p_{pd}D(p_{pd})$ is the industry profit in the per-consumer no-disclosure region:

$$\Pi_{pd} = \frac{sp_{pd}D(p_{pd})}{s} = p_{pd}D(p_{pd})$$

- $\hat{s}_{pd}$ is the maximal state s in the no-disclosure region such that an informed firm prefers not to undercut its competitors:

$$(1-\delta)s\Pi_{pd}/N + \delta V_{pd} \geq (1-\delta)s\Pi_{pd}$$

  This constraint is satisfied when $s \leq \hat{s}_{pd} \equiv \frac{\delta}{1-\delta}\frac{V_{pd}}{\Pi_{pd}}N/(N-1)$.

- $\tilde{s}_{pd}$ is the maximal state s in the no-disclosure region that an informed firm overprice competitors in the current period:

$$(1-\delta)s0 + \delta V_{pd} \geq (1-\delta)s\Pi_{pd}$$

  This constraint is satisfied when $s \leq \tilde{s}_{pd} \equiv \frac{\delta}{1-\delta}\frac{V_{pd}}{\Pi_{pd}}$.

In the disclosure region, the firms arrange a state-dependent price schedule and share industry profits equally. However, these state-dependent prices must satisfy certain properties in order to deter deviation (to either no-disclosure by the informed firm or undercutting by any firm). The following lemma describe these properties.

Lemma 3.1 *The prices over disclosure region, $P(s)$, must satisfy:*

i. If $s \leq \hat{s}_{pd}$ and informed firm discloses $m(s) = s$, then,

$$sP(s)D(P(s)) = \min\left(s\Pi^*, \frac{\delta}{1-\delta}\frac{N}{N-1}V_{pd} \geq sp_{pd}D(p_{pd})\right) \tag{3.1}$$

ii. If $s \in (\hat{s}_{pd}, \hat{s}_{pd}2(N-1)/N]$ and informed firm discloses $m(s) = s$, then,

$$sP(s)D(P(s)) = \min\left(s\Pi^*, \frac{\delta}{1-\delta}V_{pd}\right); \tag{3.2}$$

iii. and the disclosure region can never include states where $s > \hat{s}_{pd} 2(N-1)/N$.

Lemma 3.1 focuses on the tradeoffs in coordinating prices if and when disclosure is made. One added incentive problem introduced by a partial disclosure regime is that for some s, the informed firm can deviate from disclosing ($m(s) = s$) to not disclosing ($m(s) = \emptyset$), attaining a price p_{pd} possibly greater than $P(s)$.[20] So prices and disclosure region must be set to ensure the informed has no incentive to deviate from the cooperation disclosure choice. When demand is small (case (i)), this deviation is unprofitable, as the informed firm is more concerned about future rents than current small deviation profits. In this case, firms share total surplus equally after a disclosure. This also maximizes the price $P(s)$ at which the product can be sold without one firm deviating to undercut.

When market size is large (case (ii)), however, sharing the total industry surplus after the disclosure is not sufficient to elicit disclosure by the informed firm. To elicit cooperation, the oligopoly can implement a strategy in which the informed firm undercuts after disclosing.[21] Satisfying this incentive-compatibility condition is now costly in terms of total expected surplus. Knowing that the informed firm will undercut, the uninformed firms are more willing to undercut themselves: satisfying their incentive-compatibility requires to reduce overall profit by a factor of $(N-1)/N$.

Finally, when market size is very large (case (iii)), the loss of surplus required to elicit cooperation by the uninformed is too large as compared to the benefits of a deviation to not disclosing. For these market sizes, the informed firm must choose no-disclosure.

Combining the constraints and payoffs in both the disclosure and no-disclosure regions, the tacit agreement chooses the optimal disclosure and no-disclosure regions (i.e.,$\mathbb{R}^+ \setminus \Omega$ and Ω) and corresponding prices (both state-contingent $P(s)$ and no-disclosure price p_{pd}) to maximize the

[20]This deviation is not possible in either full- or no-disclosure equilibria where deviations by the informed firm on disclosure choice is always observed immediately and punished. Here, observing a no-disclosure, the uninformed firm is not sure whether s is in the no-disclosure region (thus the informed is not deviating) or s is actually in the disclosure region but the informed has deviated.

[21]Note that this does not contradict strong-symmetry, which is assumed across stage games: within one stage game, an informed firm is different from an uninformed firm at the beginning of the period, and thus may undercut. Also, our results carry over (although with some loss in surplus) if we restrict the attention to strategies in which the informed firm does not undercut after disclosing.

ex ante expected per-period firm profit V_{pd}. The following Proposition describes the resulting equilibrium.

Proposition 3.1 *If a partial disclosure equilibrium is efficient, it can be constructed as follows: let* $0 < s_0 \leq s_1 \leq s_2 \leq s_3$*,*

(i) For $s \in [0, s_0) \cup [s_1, s_2) \cup [s_3, +\infty)$*, the informed firm does not disclose.*

(ii) For $s \in [s_0, s_1] \cup [s_2, s_3)$*, the informed firm discloses.*

Unlike when monopoly surplus can be attained, partial disclosure adds value by providing a balance between the desire to coordinate prices (i.e., setting prices according to $P(s)$) and the desire to provide incentive to the uninformed to not to deviate from the collusive arrangement. We find that equilibria with partial disclosure have a simple structure and feature at most five regions.

For extreme market sizes (in the region $[s_3, +\infty)$), the informed firm chooses not to disclose. When market size is too high, the informed firm cannot be given enough incentives to disclose and thus no-disclosure must be chosen. At the other extreme, when market size is very low (in the region $[0, s_0)$), the benefits from reaching a price p^* (when disclosing) and not p_{nd} (when not disclosing) are very low. Intuitively, by not disclosing low market sizes, the informed firm loses very little industry surplus but makes it more likely that market size conditional on not disclosing (and thus deviation profits) are low.

There is a region of intermediate market sizes such that it may be optimal not to disclose. This region includes only realizations of s in which the informed firm overprices. Here, the incentive benefits of overpricing dominate the loss of surplus generated by not disclosing and charging a lower price. For other moderate market sizes, it is optimal to disclose. For moderately low market sizes (in the region $[s_0, s_1]$), the benefits of disclosing is to able to charge $p^* > p_{nd}$. For moderately high market sizes (in the region $[s_2, s_3]$), disclosure relieves some of the uninformed firms' incentives to deviate after a no-disclosure.[22]

[22]This form of partial disclosure is remindful of Wagenhofer (1990), who pointed out that no-disclosure may occur for extreme shocks. Yet, our main intuition is different in that Wagenhofer considers an exogenously-

We give next simpler conditions under which the partial disclosure equilibrium simplifies to only three regions.

Corollary 3.1 *The efficient partial disclosure PPNE features only three regions under any one of the following: (a)* $\Pi_{nd} = \Pi^*$*, (b)* $N = 2$*, (c) restricting the attention to the most efficient PPNE in the set of PPNE in which the informed firm does not overprice.*

In Corollary 3.1, we illustrate Proposition 3.1 with a thought experiment, shutting down in turn the forces that make each region useful in the Proposition. First, when when $\Pi^* = \Pi_{nd}$, not disclosing does not entail any loss of surplus. In this case, the region $[s_0, s_1]$, whose role was to attain p^* instead of p_{pd} loses its purpose, and thus the equilibrium collapses to only three regions. Second, when $N = 2$, any market size $s \geq \hat{s}_{pd}$ can no longer be disclosed. However, all other market sizes below $\hat{s}_{pd}$ are beneficial to induce cooperation by the uninformed after a no-disclosure. As a result, the region $[s_2, s_3]$, whose role was to filter out market sizes to induce deviations after a no-disclosure, is no longer feasible and, again, the equilibrium collapses to only three regions. Finally, when considering only equilibria with no overpricing, the region $[s_1, s_2]$, whose role was to allow the informed firm to overprice, is no longer useful; again, the equilibrium collapses to only three regions.

4. Concluding Remarks

In this paper, we explore the relationship between disclosure, industry cycles and product-market competition. We determine what forms of disclosure maximize industry profits, and relate firm profits to whether a firm is informed and discloses that information early. In our model, the optimal disclosure policy is endogenous and driven by concerns about future competition. In particular, we find that:

specified entry cost and financial reporting motives. Our setting, on the other hand, recovers both costs and benefits endogenously as a result of product-market competition and, in that respect, links them to testable characteristics of the product market. Note also that our form of partial disclosure may prescribe an interior region of non-disclosure.

1. Policies with no disclosure are desirable in industries with a low discount rate or high concentration.

2. Policies with partial or full disclosure are desirable in industries with a high discount rate or low concentration.

3. In regimes with partial disclosure, informed firms retain very good and very bad information and disclose intermediate news.

4. Disclosure of good market conditions imply high profits for informed firms, but not necessarily for uninformed competitors.

We hope that our study will provide some first steps - with a model that puts the focus on the product market - to understand how and why disclosure interacts with economic cycles. Cycles are a central area of interest for both firms and policy makers; perhaps almost as important as the cycles themselves is information about the cycle. As we have shown using a standard paradigm in industry cycle research, cycles will have important effects on product-market driven incentives to disclose or retain information. However, broadening the scope to other disclosure paradigms, and most notably mandatory disclosure and financial reporting motives, it is clear that more research is necessary to fully understand how information provided by firms accompanies the economic cycle.

Appendix: Omitted Proofs

Proof of Proposition 2.1: We derive the optimal strategy (i.e., the sets Ω_{over}, Ω_{share}, and Ω_{under}), and then solve for the minimum discount rate δ_{nd} stated in Proposition 2.1.

Note first that any $s \in \Omega_{over}$ must be such that $s \leq \tilde{s}$ and any $s \in \Omega_{share}$ must be such that $s \leq \hat{s}$. Therefore, in the left-hand side of Equation (2.4),

$$(1-\delta)(\int_{\Omega_{over}} sh(s)ds\frac{\Pi^*}{N-1} + \int_{\Omega_{share}} sh(s)ds\frac{\Pi^*}{N}) + \delta\frac{1}{N} \geq (1-\delta)E_s[sp^*D(p^*)] = (1-\delta)$$

To maximize the left-hand side one should set $\Omega_{over} = [0, \tilde{s}]$ and $[\Omega_{share} = \tilde{s}, \hat{s}]$. The minimum discount rate consistent with monopoly pricing is obtained by binding Equation (2.4).

$$(1-\delta_{nd})(\int_0^{\tilde{s}} sh(s)ds\frac{\Pi^*}{N-1} + \int_{\tilde{s}}^{\hat{s}} sh(s)ds\frac{\Pi^*}{N}) + \delta_{nd}\frac{1}{N} = (1-\delta_{nd})$$

That is:

$$\delta_{nd}(\frac{N+1}{N} - \int_0^{\tilde{s}} sh(s)ds\frac{\Pi^*}{N-1} - \int_{\tilde{s}}^{\hat{s}} sh(s)ds\frac{\Pi^*}{N}) = 1 - \int_0^{\tilde{s}} sh(s)ds\frac{\Pi^*}{N-1} - \int_{\tilde{s}}^{\hat{s}} sh(s)ds\frac{\Pi^*}{N}$$

And solving for δ_{nd}

$$\delta_{nd} = \frac{N(N-1) - N\int_0^{\tilde{s}} sh(s)ds\Pi^* - (N-1)\int_{\tilde{s}}^{\hat{s}} sh(s)ds\Pi^*}{(N+1)(N-1) - N\int_0^{\tilde{s}} sh(s)ds\Pi^* - (N-1)\int_{\tilde{s}}^{\hat{s}} sh(s)ds\Pi^*}$$

This is the desired Equation for δ_{nd}.□

Proof of Proposition 2.2: Since it is optimal to set $P(s)$ as close as possible to p^* while still respecting constraint ((2.7)), there must be a threshold, denoted S such that for $s \leq S$, $P(s) = p^*$ and for $s > S$, $P(s) < p^*$.

We solve first for S. Set $P(s) = p^*$ and bind Equation (2.7), i.e.

$$(1-\delta)\Pi^* S = (1-\delta)\frac{\Pi^* S}{N} + \delta V_{fd}$$

Solving for S yields:

$$S = \frac{\delta}{1-\delta}\frac{N}{N-1}\frac{V_{fd}}{\Pi^*}$$

For $s \leq S$, $sP(s)D(P(s)) = s\Pi^*$.

For $s > S$, since Equation (2.7) binds,

$$sP(s)D(P(s)) = \frac{\delta}{1-\delta}\frac{V_{fd}}{N-1}$$

Then:

$$\begin{aligned} V_{fd} &= \frac{1}{N}(\Pi^* \int_0^S sh(s)ds + \int_S^{\overline{s}} sP(s)D(P(s))h(s)ds) \\ &= \frac{1}{N}(\Pi^* \int_0^S sh(s)ds + \frac{\delta}{1-\delta}\frac{N}{N-1}V_{fd}\int_S^{\overline{s}} h(s)ds) \\ &= \frac{\Pi^*/N\int_0^S sh(s)ds}{1-\frac{\delta}{1-\delta}\frac{1}{N-1}\int_S^{\overline{s}} h(s)ds} \\ S\Pi^*\frac{1-\delta}{\delta}\frac{N-1}{N} &= \frac{\Pi^*/N\int_0^S sh(s)ds}{1-\frac{\delta}{1-\delta}\frac{1}{N-1}\int_S^{\overline{s}} h(s)ds} \end{aligned}$$

Solving for S yields Equation (2.8).□

Proof of Proposition 2.3: In a no-disclosure PPNE, then: (i) the uninformed firms choose $p_{nd} \leq p^*$, (ii) for $s \in \Omega_{over}$, the informed firm chooses $p > p^{nd}$ (overprices), (iii) for $s \in \Omega_{share}$, the informed firm chooses $p = p_{nd}$ and share, (iv) for $s \notin \Omega_{over} \cup \Omega_{share}$, the informed firm chooses $p = p_{nd}$ and undercuts.

First, we compute the per-firm surplus V_{nd} in this PPNE,

$$\begin{aligned} V_{nd} &= \int h(s)s\Pi_{nd}ds/N \\ &= \int h(s)s\Pi^* ds\Pi_{nd}/(N\Pi^*) \\ &= \Pi_{nd}/(N\Pi^*) \end{aligned} \tag{A-1}$$

As before, it is optimal to set Ω_{over} as the largest possible set such that the informed firm overprices, that is: $s \in \Omega_{over}$ if and only if $s \leq \tilde{s}_{nd}$ where:

$$\delta V_{nd} \geq (1-\delta_{nd})\tilde{s}_{nd}\Pi_{nd} \tag{A-2}$$

Therefore: $\tilde{s}_{nd} = \frac{\delta}{1-\delta}\frac{V_{nd}}{\Pi_{nd}} = \tilde{s}$ (does not depend on p_{nd}).

Similarly, $s \in \Omega_{share}$ if and only $s \in (\tilde{s}, \hat{s}]$.

Let us now write the incentive-compatibility for the uninformed:

$$(1-\delta)(\int_0^{\tilde{s}} sh(s)ds\frac{\Pi_{nd}}{N-1} + \int_{\tilde{s}}^{\hat{s}} sh(s)ds\frac{\Pi_{nd}}{N}) + \delta\frac{\Pi_{nd}}{\Pi^* N} \geq (1-\delta)\frac{\Pi_{nd}}{\Pi^*} \tag{A-3}$$

Suppose that $\Pi_{nd} > 0$. Then, multiplying both sides by Π^*/Π_{nd}, this incentive-compatibility condition is the same as that required in Proposition 2.1. Therefore, $\delta \geq \delta_{nd}$.□

Proof of Lemma 3.1: To prove the result, we define two auxiliary variables that describe disclosure and pricing strategies on the cooperation path. First, let $a(s)$ be a binary function such that $a(s) = 0$ if the firm discloses and $a(s) = 0$ if the firm does not disclose. Second, let $b(s)$ be a binary function such that $b(s) = 0$ if the informed firm undercuts and $b(s) = 1$ if the informed firm shares (we do not need to give a label to overpricing).

(i) Conditional on $b(s) = 1$, it must be incentive-compatible for all firms to share, that is:

$$(1-\delta)sP(s)D(P(s))/N + \delta V_{pd} \geq (1-\delta)sP(s)D(P(s)) \tag{A-4}$$

If $P(s) = p^*$ satisfies this inequality, it is optimal to set $P(s) = p^*$. Else, maximizing $sP(s)D(P(s))$ requires to bind this inequality and therefore set:

$$sP(s)D(P(s)) = \frac{\delta}{1-\delta}\frac{N}{N-1}V_{pd} \tag{A-5}$$

Next, for the informed firm, it must be incentive-compatible to 'Disclose' versus 'Not Disclose and Undercut.' In particular,

$$\begin{aligned}
\Pi_{pd}(1-\delta)s &\leq (1-\delta)sP(s)D(P(s))/N + \delta V_{pd} \\
&\leq (1-\delta)\frac{\delta}{1-\delta}\frac{N}{N-1}\frac{V_{pd}}{N} + \delta V_{pd} \\
&\leq \delta V_{pd}\frac{N}{N-1}
\end{aligned}$$

As a result, $s \leq \hat{s}_{pd}$.

Finally, we need to verify that $P(s) \geq p_{pd}$. Since $P(s)$ is decreasing in s, it is sufficient to verify that (A-4) is satisfied at equality by $P(s) = p_{pd}$ at $s = \hat{s}_{pd}$.

$$(1-\delta)\hat{s}_{pd}\Pi_{pd}/N + \delta V_{pd} \geq (1-\delta)\hat{s}_{pd}\Pi_{pd} \tag{A-6}$$

Equation (A-6) is true by definition of $\hat{s}_{pd}$.

(ii) Conditional on $b(s) = 0$, it must be incentive-compatible for the uninformed not to undercut, that is:

$$\delta V_{pd} \geq (1-\delta)sP(s)D(P(s)) \tag{A-7}$$

If $P(s) = p^*$ satisfies this inequality, it is optimal to set $P(s) = p^*$. Else, maximizing $sP(s)D(P(s))$ requires to bind this inequality and therefore set:

$$sP(s)D(P(s)) = \frac{\delta}{1-\delta}V_{pd} \tag{A-8}$$

As before, we consider next the incentive-compatibility condition for the informed and compare the profit from disclosing and the profit from not disclosing.

$$\begin{aligned}
\Pi_{pd}(1-\delta)s &\leq (1-\delta)sP(s)D(P(s)) + \delta V_{pd} \\
&\leq (1-\delta)\frac{\delta}{1-\delta}V_{pd} + \delta V_{pd} \\
&\leq 2\delta V_{pd}
\end{aligned}$$

As a result, s must be greater than $\hat{s}_{pd}2(N-1)/N$.□

Proof of Proposition 3.1: It is useful (as before) to write the incentive-compatibility for the uninformed. Conditional on a realization of s, the cooperation payoff to the uninformed is given by:

$$(1-\delta)\Pi_{pd}s(1_{s\leq\tilde{s}_{pd}}\frac{1}{N-1} + 1_{s\in(\tilde{s}_{pd},\hat{s}_{pd}]}\frac{1}{N}) + \delta V_{pd}$$

Conditional on non-disclosure, the uninformed firm makes a conditional expectation, which yields the following Incentive-compatibility.

$$(1-\delta)\Pi_{pd}\mathbb{E}(s(1_{s\leq\tilde{s}_{pd}}\frac{1}{N-1} + 1_{s\in(\tilde{s}_{pd},\hat{s}_{pd}]}\frac{1}{N})|a(s)=1) + \delta V_{pd} \geq \Pi_{pd}\mathbb{E}(s|a(s)=1)$$

We can rewrite this expression in terms of an auxiliary function $\psi_{pd}(.)$

$$\mathbb{E}(\psi(s)|a(s)=1) \geq 0$$

where: $\psi(s) = s\Pi_{pd}(1-\delta)(1_{s\leq\tilde{s}_{pd}}/(N-1) + 1_{s\in(\tilde{s}_{pd},\hat{s}_{pd}]}/N-1) + \delta V_{pd}$.

$\psi(.)$ is strictly decreasing in s. In addition,

$$\begin{aligned}
\psi(\hat{s}_{pd}) &= \hat{s}_{pd}\Pi_{pd}(1-\delta)(1/N-1) + \delta V_{pd} \\
&= 0
\end{aligned}$$

So that $\psi(.)$ is positive for $s \in [0, \hat{s}_{pd}]$.

We state next the problem of finding the best possible partial disclosure PPNE:

$$\sup V_{pd}$$

s.t.

$$V_{pd} = \frac{1}{N}\int sh(s)\{a(s)\Pi_{pd} + (1-a(s))P(s)D(P(s))\}ds \quad (\lambda) \tag{A-9}$$

$$0 \leq \int a(s)h(s)\psi(s,\Pi_{pd})ds \quad (\mu) \tag{A-10}$$

Let L denote the Lagrangian of this problem. The problem is also subject to the relationships given in Lemma 3.1 which do not depend on $a(s)$ (these constraints are unimportant for our purpose since they do not appear in the Lagrangian when differentiating with respect to $a(s)$). The multiplier λ is readily verified to be strictly positive (if not, V_{pd} large would be a solution to the Lagrangian). Differentiating in $a(s)$ for any s such that disclosure is feasible,

$$\frac{\partial L}{\partial a(s)} = h(s)\underbrace{\{s[\lambda(\Pi_{pd} - P(s)D(P(s)))/N + (1-\delta)\mu\Pi_{pd}(-1 + \frac{1_{s\leq\tilde{s}_{pd}}}{N-1} + \frac{1_{s\in(\tilde{s}_{pd},\hat{s}_{pd}]}}{N})] + \mu\delta V_{pd}\}}_{G(s)} \tag{A-11}$$

Note that $a(s) = 1$ when $G(s) > 0$ and $a(s) = 0$ when $G(s) < 0$. We show first that the shadow cost of giving incentives to cooperate to the uninformed after non-disclosure is non-zero.

Lemma A .1 $\mu > 0$.

Proof: Suppose $\mu = 0$. Then: $G(s) = s\lambda(\Pi_{pd} - P(s)D(P(s)))/N$. By Lemma 3.1, $G(s) < 0$ for any $s < \hat{s}_{pd}$. It follows that $a(s) = 0$ for any $s \geq \hat{s}_{pd}$.

For any $s > \hat{s}_{pd}$, the informed firm prefers undercutting to sharing:

$$(1-\delta)s\Pi_{pd} > (1-\delta)s\Pi_{pd}/N + \delta V_{pd}$$

Integrating with respect to $a(s)h(s)$.

$$(1-\delta)\Pi_{pd}\int a(s)h(s)sds > (1-\delta)\Pi_{pd}\int a(s)h(s)sds/N + \delta\int a(s)h(s)ds$$

This implies that choosing p_{nd} is not incentive-compatible for the uninformed. QED.

Lemma A .2 *Suppose $S \leq \tilde{s}_{pd}$. Then:*

1. $G(0) > 0$.

2. $Sign(G(S)) = -Sign(\frac{\lambda}{N-1} - (1-\delta)\mu)$.

3. $Sign(G(\tilde{s}_{pd})) = -Sign(\frac{\lambda}{N} - (1-\delta)\mu)$.

4. $Sign(\lim_{s \to \tilde{s}_{pd}^{+}} G(s)) = Sign(G(S))$

5. $G(\hat{s}_{pd}) = 0$.

6. $Sign(\lim_{s \to \hat{s}_{pd}^{+}} G(s)) = -Sign(G(\tilde{s}_{pd}))$

7. $Sign(G(\frac{N-1}{N} 2\hat{s}_{pd})) = -Sign(G(\tilde{s}_{pd}))$.

Proof:(i) $G(0) = \mu\delta V_{pd} > 0$.

(ii) We calculate $G(S)$.

$$
\begin{aligned}
G(S) &= \frac{\lambda}{N}(\Pi_{pd}\frac{\delta}{1-\delta}\frac{N}{N-1}\frac{V_{pd}}{\Pi^*} - \frac{\delta}{1-\delta}\frac{N}{N-1}V_{pd}) + \mu\delta V_{pd} + (1-\delta)\mu\frac{1-N}{N}\frac{\delta}{1-\delta}\frac{N}{N-1}\frac{V_{pd}}{\Pi^*}\Pi_{pd} \\
&= \frac{\lambda}{N}\frac{\delta}{1-\delta}V_{pd}(\frac{\Pi_{pd}-\Pi^*}{\Pi^*}) + \mu\delta V_{pd} - \mu\delta V_{pd}\frac{\Pi_{pd}}{\Pi^*} \\
&= \frac{\delta}{1-\delta}V_{pd}\frac{\Pi^*-\Pi_{pd}}{\Pi^*}(\mu(1-\delta) - \frac{\lambda}{N-1})
\end{aligned}
$$

(iii) We calculate $G(\tilde{s}_{pd})$.

$$
\begin{aligned}
G(\tilde{s}_{pd}) &= \frac{\lambda}{N}(\Pi_{pd}\frac{\delta}{1-\delta}\frac{V_{pd}}{\Pi_{pd}} - \frac{\delta}{1-\delta}\frac{N}{N-1}V_{pd}) + \mu(1-\delta)(-1+\frac{1}{N-1})\frac{\delta}{1-\delta}\frac{V_{pd}}{\Pi_{pd}} + \delta\mu V_{pd} \\
&= \frac{lambda}{N}\frac{\delta}{1-\delta}V_{pd}(1-\frac{N}{N-1}) + \delta V\frac{-N+2}{N-1} + \delta\mu V_{pd} \\
&= \frac{1}{N-1}\frac{\delta}{1-\delta}V_{pd}((1-\delta)\mu - \frac{\lambda}{N})
\end{aligned}
$$

(iv) We calculate $\lim_{s \to \tilde{s}_{pd}^{+}} G(s)$.

$$
\begin{aligned}
\lim_{s \to \tilde{s}_{pd}^{+}} G(s) &= \frac{\lambda}{N}(\Pi_{pd}\frac{\delta}{1-\delta}\frac{V_{pd}}{\Pi_{pd}} - \frac{\delta}{1-\delta}\frac{N}{N-1}V_{pd}) + \delta\mu V_{pd} + (1-\delta)\mu(-1+1/N)\Pi_{pd}\frac{\delta}{1-\delta}\frac{V_{pd}}{\Pi_{pd}} \\
&= -\frac{\lambda}{N}\frac{\delta}{1-\delta}V_{pd}\frac{1}{N-1} + \delta\mu V_{pd}\frac{1}{N} \\
&= \frac{\delta V_{pd}}{(1-\delta)N}((1-\delta)\mu - \frac{\lambda}{N-1})
\end{aligned}
$$

(v) We calculate $G(\hat{s}_{pd})$.

$$\begin{aligned} G(\hat{s}_{pd}) &= \hat{s}_{pd}\lambda(\Pi_{pd}-\Pi_{pd})/N + \mu\Pi_{pd}(1-\delta)(-1+1/N)\hat{s}_{pd} + \mu\delta V_{pd} \\ &= \mu\delta V_{pd} - \mu\delta V_{pd} \\ &= 0 \end{aligned}$$

(vi) We calculate $\lim_{s\to\hat{s}_{pd}^+} G(s)$.

$$\begin{aligned} \lim_{s\to\hat{s}_{pd}^+} G(s) &= \frac{\lambda}{N}(\Pi_{pd}\hat{s}_{pd} - \frac{\delta}{1-\delta}V_{pd}) + \delta\mu V_{pd} - \mu(1-\delta)\Pi_{pd}\hat{s}_{pd} \\ &= \frac{\lambda}{N}(\Pi_{pd}\frac{V_{pd}}{\Pi_{pd}}\frac{\delta}{1-\delta}\frac{N}{N-1} - \frac{\delta}{1-\delta}V_{pd}) + \delta\mu V_{pd} - \mu(1-\delta)\Pi_{pd}\frac{\delta}{1-\delta}\frac{N}{N-1}\frac{V_{pd}}{\Pi_{pd}} \\ &= \frac{\lambda}{N}\frac{\delta}{1-\delta}V_{pd}\frac{1}{N-1} - \delta\mu V_{pd}\frac{1}{N-1} \end{aligned}$$

(vii) We calculate $G(2(N-1)/N\hat{s}_{pd})$.

$$\begin{aligned} G(2(N-1)/N\hat{s}_{pd}) &= \frac{\lambda}{N}(\Pi_{pd}\hat{s}_{pd}2(N-1)/N - \frac{\delta}{1-\delta}V_{pd}) + \delta\mu V_{pd} - \mu(1-\delta)\Pi_{pd}\hat{s}_{pd}2(N-1)/N \\ &= \frac{\lambda}{N}(\Pi_{pd}2\frac{\delta}{1-\delta}\frac{V_{pd}}{\Pi_{pd}} - \frac{\delta}{1-\delta}V_{pd}) + \delta\mu V_{pd} - \mu(1-\delta)\Pi_{pd}2\frac{\delta}{1-\delta}\frac{V_{pd}}{\Pi_{pd}} \\ &= \frac{\lambda}{N}\frac{\delta}{1-\delta}V_{pd} - \delta\mu V_{pd} \\ &= \frac{\delta}{1-\delta}V_{pd}(\frac{\lambda}{N} - \mu(1-\delta)) \end{aligned}$$

QED.

Using Lemma A.2, we can prove the Proposition when $S \le \tilde{s}_{pd}$. Letting λ vary, there are three cases to consider:

1. Suppose $\lambda \le (1-\delta)\mu(N-1)$. Then, by Lemma A.2, $Sign(G(S)) \ge 0$, $G(\tilde{s}_{pd}) \ge 0$, $\lim_{s\to\tilde{s}_{pd}^+} G(s) \ge 0$, and $\lim_{s\to\hat{s}_{pd}^+} G(s) \le 0$ and $G(\hat{s}_{pd}2(N-1)/N) \le 0$. Thus, the partial disclosure PPNE features disclosure for $s \in [\hat{s}_{pd}, 2(N-1)/N\hat{s}_{pd}]$ (followed by undercutting) and disclosure otherwise.

2. Suppose that $\lambda \in ((1-\delta)\mu(N-1), (1-\delta)\mu N]$. Then, by Lemma A.2, $Sign(G(s)) \le 0$, $G(\tilde{s}_{pd}) \ge 0$, $\lim_{s\to\tilde{s}_{pd}^+} G(s) \le 0$, and $\lim_{s\to\hat{s}_{pd}^+} G(s) \le 0$ and $G(\hat{s}_{pd}2(N-1)/N) \le 0$. Thus, the partial disclosure PPNE features disclosure for $s \in [s_0, s_1] \cup [\tilde{s}_{pd}, 2(N-1)/N\hat{s}_{pd}]$ (where $s_0 \in [0, S]$ and $s_1 \in [S, \tilde{s}_{pd}]$) and non-disclosure otherwise.

3. Suppose that $\lambda > (1-\delta)\mu N$. Then, by Lemma A.2, $Sign(G(S)) < 0$, $G(\tilde{s}_{pd}) < 0$, $\lim_{s \to \tilde{s}_{pd}^+} G(s) < 0$, and $\lim_{s \to \hat{s}_{pd}^+} G(s) > 0$ and $G(\hat{s}_{pd} 2(N-1)/N) > 0$. Thus, the partial disclosure PPNE features disclosure for $s \in [s_0, \hat{s}_{pd}]$ where $s_0 \in (0, S)$ (followed by sharing) and non-disclosure otherwise.

We turn to the other situation in which $S > \tilde{s}_{pd}$. Then, G is decreasing on $[0, S]$. In addition, the proof of (vi) and (vii) in Lemma A.2 remains valid and therefore: $Sign(\lim_{s \to \hat{s}_{pd}^+} G(s)) = Sign(G(\frac{N-1}{N} 2\hat{s}_{pd}))$. There are three cases to consider.

1. Suppose $G(S) \geq 0$. Then, $G(s) \geq 0$ for all $s \leq \tilde{s}_{pd}$. Therefore, for a partial disclosure PPNE to occur, it must hold that $Sign(G(\frac{N-1}{N} 2\hat{s}_{pd})) < 0$. As a result, the partial disclosure PPNE features disclosure for $s \in [\hat{s}_{pd}, 2(N-1)/N\hat{s}_{pd}]$ (followed by undercutting) and disclosure otherwise.

2. Suppose $G(S) < 0$ and $Sign(G(\frac{N-1}{N} 2\hat{s}_{pd})) \leq 0$. Then, the partial disclosure PPNE features disclosure for $s \in [s_0, 2(N-1)/N\hat{s}_{pd}]$, where $s_0 \in (0, S)$ and non-disclosure otherwise.

3. Suppose $G(S) > 0$ and $Sign(G(\frac{N-1}{N} 2\hat{s}_{pd})) > 0$. Then, the partial disclosure PPNE features disclosure for $s \in [s_0, \hat{s}_{pd}]$, where $s_0 \in (0, S)$ and non-disclosure otherwise.□

Proof of Corollary 3.1: Suppose that $\Pi_{pd} = \Pi^*$. Then, by Lemma 3.1, for any $s \leq \hat{s}_{pd}$, $P(s) = p^*$. Therefore $G(s)$ (Equation (A-11)) must be positive for all $s \leq \hat{s}_{pd}$ and $a(s) = 1$ for any $s \leq \hat{s}_{pd}$. Applying Proposition 3.1, the partial disclosure equilibrium features a single disclosure interval $[s_1, 2(N-1)/N\hat{s}_{pd}]$ where $s_1 > \hat{s}_{pd}$.

Suppose that $N = 2$. Then, $\hat{s}_{pd} = 2(N-1)/N\hat{s}_{pd}$. Therefore, no disclosure can be elicited for $s > \hat{s}_{pd}$. It follows that the partial disclosure equilibrium features a single disclosure interval $[s_0, \hat{s}_{pd}]$, where $s_0 < \hat{s}_{pd}$.

Suppose that we consider the best PPNE with partial disclosure, in the class of equilibria that do not feature overpricing by the informed firm. This can be incorporated in Proposition 3.1 by setting $\tilde{s}_{pd} = 0$. This removes all cases such that $S \leq \tilde{s}_{pd}$. However, equilibria with five regions only occur when $S \leq \tilde{s}_{pd}$ (see case 2. in the proof of Proposition 3.1). Thus the partial disclosure PPNE must feature only three regions: disclosure on $[s_0, s_1]$ (where $0 < s_0 < s_1$) and non-disclosure otherwise.□

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