

NEW CLASSICAL INCOME MEASUREMENT: A CHOICE- THEORETIC AXIOMATIC APPROACH*

JEREMY BERTOMEU
Tepper School of Business
Tech and Frew Streets
Carnegie Mellon University
Pittsburgh, PA 15213-3890
bertomeu@cmu.edu

PIERRE JINGHONG LIANG
Tepper School of Business
Tech and Frew Streets
Carnegie Mellon University
Pittsburgh, PA 15213-3890
liangj@andrew.cmu.edu

July 17, 2006

Abstract

At the fundamental level, the key challenge to a theory of income measurement is to resolve the problem caused by soft information, which leads to incomplete preferences within the entity (i.e. some alternatives are not always unambiguously ranked). This paper presents a formal axiomatic foundation for income measurement, building on several recent developments in the economic theory of choice. We first design a meaningful income measure for an entity with incomplete preferences. When there is enough hard information, this measure consists of finitely-many performance criteria (Line-Item Income Measure) which fully represent the incomplete decision problem of the entity. Second, a single-valued income measure (Summary Income Measure) is introduced, which extends classical income to incomplete preferences. Third, these measures can be operational and exhibit recognizable features such as linear aggregation over time and individual operating units.

Keywords: Accounting foundation, Choice, Incompleteness, Income measurement, Representation, Information, Axiom

JEL: B41, D11, D21, D70, M40, M41

*Jeremy Bertomeu acknowledges financial support from the William Larimer Mellon fellowship.

Does a unified theory of accounting exist? Can it exist? What should it look like?

Sidney Davidson, David Green, Chuck Horngren and George Sorter
An Income Approach to Accounting Theory (1964)

Disagreements on income concepts and measurements have been a hallmark in accounting scholarship for at least the last one hundred years. This is a single issue that has captured the imagination of generations of scholars in accounting and economics.¹ One dominant theme in the discourse has been a basic duality of accounting versus economic income. Some economists consider accounting income as only operationally defined and lacking theoretical underpinnings² while some accountants viewed the economic concept of income as too personal and abstract to have a practical, operational application to a business entity.³ In 1964, Davidson, Green, Horngren and Sorter summarized the founding questions of the time, whose answers have been elusive.

Signalling a fundamental shift in the discourse, Bill Beaver and Joel Demski, in their provocative, landmark article titled “The Nature of Income Measurement” published in *Accounting Review* in 1979, focus on the choice-theoretic problem faced by organizations and discuss the difficulty in interpreting accounting income as an objective neo-classical economic income measurement. They argue that the difficulty with this notion of income, which was the dominant “economic measurement” perspective in accounting,⁴ lies in the perfect and complete market assumption implicit in neo-classical accounting. In particular, they argue that a lack of perfect and complete markets would imply the inexistence of an income measure which ranks choices entities face and make. As an alternative, they suggest an information-communication role for income measurement and proceed to lay out a school of thought that has become the “information content” perspective.

In this paper we propose a unifying, foundational theory of income measurement. Our approach integrates existing, seemingly opposing, schools of thought into a single theory built on several axiomatic layers. To begin, we consider a general choice problem for an entity and formalize a notion of soft information, which renders an entity’s preference incomplete. Then we establish a concept of income measurement for an entity with such an incomplete preference. We name the concept New Classical

¹Starting with early work by Canning (1929) and Fisher (1930), followed by Edwards and Bell (1961), and five monographs by Alexander, Bronfenbrenner, Fabricant and Warburton (1973), economists have been active in advancing our understanding of economic income. An excellent volume from the economist perspective was prepared by Parker and Harcourt (1969) and updated in 1991. Seminal early accounting work on income included by Paton (1922), Paton and Littleton (1940), Sterling (1970), among others. Numerous discussions on alternative income measurements populate accounting journals as well as accounting anthologies such as Baxter and Davidson (1962), Zeff and Keller (1964), and Davidson, Green, Horngren and Sorter (1964).

²In particular, Canning (1929) wrote: “[a] diligent search of the literature of accounting discloses an astonishing lack of discussion of the nature of income.” (p.93) In addition, he observed that “what is set out as a measure of net income can never be supposed to be a fact in any sense at all except that it is the figure that results when the accountant has finished applying the procedure which he adopts” (p. 9899). Alexander maintains that the accountant’s quest for objectivity is an “ill-founded” principle to justify accounting practices such as excluding changes in going value from income and that “[t]o the extent that accountants have achieved objectivity and conservatism they have made the measurement of income safer but they have also made it yield a result that only partially achieves the end sought.” (Alexander et al. 1973)

³In economic theory, the formal definition is closely tied to individual consumption. See for example Fisher (1930): “For each individual only those events which come within the purview of his experience are of direct concern. It is these events—the psychic experience of the individual mind—which constitute ultimate income for that individual” or Hicks (1946): “the value of the individual’s consumption *plus* the increment in the money value of his prospect which has accrued during the week” (p.81) or Meade and Stone (1941): “the amount he could have spent on consumption *while maintaining the money value of his capital stock intact.*” These definitions led Lee (1974) to conclude that “[i]n practice, the use of the economic income model would therefore founder on the extreme subjectiveness and inaccuracies of the required prediction[.]”

⁴Feltham (2006) labels this the “truth” approach.

income measurement, which is developed in three steps. First, we construct a vector-valued measure (i.e., consisting in multiple performance criteria) of the entity choices. This measure completely represents the choice problem of the entity given an incomplete preference. In this respect, the measure fulfills the same fundamental role of classical income even though the same task is now much more complex. Second, we construct a single-valued income measure for such an entity. This measure no longer fully represents the underlying (incomplete) preference. In this respect, it should be interpreted as a weaker version of the classical income. However, it does offer a complete ranking of all choices available to the entity. Thus, the accounting income fulfills the additional role of “completing” an incomplete preference. In turn, the new income measure does allow non-unique accounting rules and conventions to play an important role within the organization. Third, the single-valued income measure can be operational and exhibits recognizable features such as linear aggregation over time and individual operating units. In particular, an (abstract) accrual time series can be derived as a property of the periodic income based on a “weak independence” axiom imposed on the underlying set of choices.

Aside from the foundational nature of our results, the paper also synthesizes the literature by bringing in a vast array of accounting concepts/postulates developed in the past century. Almost all important accounting concepts have a place in our theory and play an axiomatic role. These concepts include entity, control, information, quantification, monetary unit, hard/soft information, stewardship, periodicity and linear additivity, which have been presented by Paton and Littleton (1940), Moonitz (1961), Ijiri (1967), Mock (1976), Mattessich (1995), among others, and statements of purpose proposed by the FASB and other institutions and think tanks.

The paper is organized in five sections. In Section 1, we present an outline of our theoretical ideas in a set of core postulates, each followed by summaries of key results, all in a non-technical manner. In Section 2, we lay down the formal building blocks of the theory and motivate the problem of incomplete information. In Section 3, the theory is specialized to revisit two alternative roles of income measurement: classical (economic) income measurement and modern information content conveyance. In Section 4, we bridge the gap between the two approaches by building a robust theory of income measurement. In Section 5, the problem of income measure is decomposed into simpler, more amenable problems and yields a theory of periodization (decomposition by time period) and consolidation (decomposition by activity).

1. Outline of the Theory

In this section, we introduce our theory and summarize results in a non-technical manner. Another purpose of this section is to provide a link to common accounting postulates (sometimes called assumptions or concepts) that have populated the existing literature. First, we list these postulates as non-technical statements of basic premises and assumptions. This is followed by a gradual development of the theory based on these assumptions. A framework of measuring (representing) incomplete preference is developed. Two existing approaches to income measurement are recovered. A unifying approach is proposed by considering a New Classical income measure. Finally, results on its possibility and features are presented.

1.1. Postulates

Now we describe the basic subject and objects of income measurement and some basic assumptions of the accounting process. Following the tradition in accounting scholarship, we call them postulates.

Postulate 1. (economic entity) The environment can be reduced to a set of interpersonal relationships between agents, which can be separated into well-defined agglomerations of contracts.

Postulate 2. (accounting entity) An economic entity can be reduced to a representative agent with some inherent objectives.

Postulate 3. (control) An entity has access to (non-trivial) sets of actions upon its resources in order to attain its objectives.

Postulate 4a. (measurement role) The income measurement is a process for measuring the (potential) consequences of the entity's course of actions; these consequences are to be represented as a simple signal to be apprehended by the individuals in or outside the entity.

Postulate 4b. (information role) The income measurement is a source of information for individuals (in or outside the entity) whose function is to accomplish their objectives; the information is to be conveyed as a simple signal to be apprehended by individuals in or outside the entity.

Postulate 5. (quantification) The entity's current state of affairs can be summarized as a finite (but possibly large) number of quantitative indicators.

Postulate 6. (stewardship) The accounting process nests the managerial decision process as it must lead to actions.

Postulate 7. (time periods) The accounting process must be articulated into meaningful time periods which are used to decompose the activity of the whole as separate one-period flows.

Postulate 8. (prices) There exists outside prices (i.e. independent from the entity) whose nature is fully understood by economic agents and can be used as a measurement unit in the accounting process.

1.2. Accounting Entity

The first three postulates describe the subject and objects of income measurement. The primary field of investigation is a complex environment where individual agents use markets and multilateral contractual agreements to attain a collection of individual objectives. Following Coase (1937), we posit that a set of contracts can be subsumed as a well-defined *economic entity* (Postulate 1). The entity concept⁵ is a legal

⁵The notion of entity is ubiquitous in studies of accounting theory. It appears in the report of the American Institute's special committee on research (1958): "Postulate A-3: Entities (including identification of the entity) Economic activity is carried through specific units or entities. Any report on the activity must identify clearly the particular unit of entity involved" (see Sewell Bray (1966) p.39-40). In Ijiri (1967), the entity is more precisely defined as an object vested with control: "accounting does not exist for a simple collection of individuals such as a mob unless such a collection is considered to constitute an entity"

fiction that allows us to focus our attention away from the complexity of the original environment toward a set of relations involving an artificial "firm."

When applied to accounting, the economic entity concept remains too broad for our purpose. Without further restrictions, the behavior of an entity may be as complex as the original problem, and the relations⁶ among many multilateral contracts may escape a simple comprehension. With Postulate 2, we assume that the entity can be apprehended as a general choice problem of a *representative* agent (or a "principal" used by Ijiri (1967)), giving rise to an Accounting Entity.⁷ Following this reduction, the original environment is simplified to an accounting environment that allows apprehension using language and permits a precise analysis using mathematics.⁸

While we consider the subject of accounting as the suitably reduced accounting entity, we consider the objects of accounting the choices available to the accounting entity. With Postulate 3, we consider an entity with certain well-defined control over resources and actions. The choices about how to use resources (such as past, current, or future transactions) are items to be accounted for. Ijiri (1967) identified the importance of the control axiom and it is an absolutely necessary element to build a reasonable accounting theory.

1.3. Income Measurement

In postulate 4, we confront the fundamental issue of the role of income measurement. With Postulate 4a, we posit that the complexity of the original problem is such that it is possible to distinguish all relevant differences among entity choices and a single accounting income number can be devised to measure the difference. This is the foundation of the so-called "Measurement School." Typically, a perfect and complete market suffice such a (value) measure construction as competitive prices capture all there is to be learned about entity's available choices. As a result, such a measure provides a complete ranking over all possible alternatives.

Postulate 4b, however, emphasizes the sheer complexity of the original problem which make it inherently impossible to fully inform every aspects of the choices to every individual in the entity. These problems may be caused by information -driven frictions which lead to market failures and moral hazard and adverse selection problems within economic organizations. Therefore, the role of accounting is not to measure some absolute notion of value (since it may not exist), but to convey information about the entity in some general sense. This is the foundation of the so-called "Information School."

Due to the complex interactions of the environment, the accounting entity will not possess all possible information that describe perfectly its environment. We focus on the elementary condition of *incomplete*

(p.69). The reference to the original set of contracts is made explicit is Mattessich (1995): "A-5. *Economic Entity*: There is some economic entity (...) represented by a specific accounting system. Such entity consists of economic subjects and economic objects and can enter into contracts."

⁶Mock (1976) defines an "Empirical Relational System": "A relational system where A is a set of empirical objects and R represents a set of empirical relations of the elements in A" (p.11). Ijiri (1967) considers several different relations: "As in representation by language, we have a set of objects and a set of relations among the objects by which we want to discriminate the principal" (p.21).

⁷Vatter (1966) explains that: "Entity theory stems from the legal fiction of the corporate enterprise or a person of its own right" (p.253). The important part of the statement is that the entity is considered as a "person" in the same way as an individual in the economy.

⁸There are many memorable quotes on the relation between accounting and mathematics. Here is one of our favorite, by B.M. Rastall (1908), "[a]ccounting is closely related to the mathematical sciences, resembling and paralleling in a great many ways the science of geometry. Like geometry it rests upon a few fundamental principles (axioms) from which the whole structure in all its ramifications is deduced by logical processes."

*information*⁹ which gives rise to a major distinction between hard and soft information. In our framework, information (with respect to two potential alternatives) is *hard* when there is clear unambiguous information that an alternative is preferred, whereas information is *soft* when the entity is indecisive (due to information incompleteness). The presence of soft information presents a key challenge to a theory of accounting measurement. It has been long recognized in the literature in various forms (e.g., Ijiri (1975)) and is related to the notion of objectivity.¹⁰ The effective realization of a “transaction”, a key guiding principle in the theory of Paton and Littleton (1940), is also sometimes referred to as hard information. However, many commercial and financial transactions are now tied to various forms of long-term contracts, which can lead to its reversal conditional on future outcomes. Finally, in the absence of arbitrage or transactions, diverging opinions among agreed-upon experts (such as auditors, analysts, management, advisors, etc.) typically indicate soft information.

1.4. Two Schools of Income Measurement

In section 3, we reconsider two main schools of income measurement based on the ideas of Postulates 1-5. In classical income measurement, accounting income retains the idea that income fully represents the entity’s preference and provides a complete ranking of all its alternatives. It is implicitly assumed that either the information incompleteness is a second-order issue or can be reasonably reduced (perhaps by a proper analysis of the network of contracts) to recover hard information. An important aid to this measure is the external markets which price the resources and relations involving the entity. Such omnipotent market prices admit the proper application of the stylized theory of capital and income developed by Irving Fisher and other economists.¹¹ In our framework we recover this income measurement as a special case where no soft income exists (see Proposition 3). However, this approach may be infeasible when

⁹The precise distinct between incomplete and imperfect information is discussed later. Here we emphasize that the origin of the incompleteness problem is the aggregated nature of the entity. First, agents subsumed by the entity concept may have different priors but no mechanism to reconcile these priors. Sunder, in Demski, Fellingham, Ijiri, Glover, Liang and Sunder (2002), wrote: “Information relevant to decision making is inherently subjective, and therefore a matter of personal belief and expectations about the future. There is no way of making the estimated cash flows from a project, and the uncertainty associated with them, objective.” and in explaining a crucial and important role of accounting, “Robinson Crusoe, living alone on an island, can count coconuts, and make his decisions to swim, eat or sleep. In the absence of control uses, it is not clear that he does any accounting.” Second, the actions of the entity may have unequal effects on agents which, in the absence of perfect property rights, may induce incomplete rankings over possible actions. A main agenda of the agency theory in accounting has been to explicitly analyze the role of accounting in the presence of a conflict of interests. Third, it is sometimes impossible to design multilateral contracts contingent on all information (incomplete contracts), and thus the aggregation of these contracts may yield unclear objectives. Fourth, the economic environment may be very complex so that recovering a complete set of objectives may be very costly. This is one of the main critiques by practitioners of legislations which make the accounting and reporting process more demanding.

¹⁰The definition of objectivity varies among authors and we do not intend here to take a theoretical stand on what is the right criterion. For example, in Paton and Littleton (1940) as well as most theories in the first half of the century, objectivity is closely tied to *verifiability*; it is fair to say that verifiability still dominates most current disclosure requirements. Ijiri and Jaedicke (1966) and Mock (1976) put substantial focus on the converging opinion of informed experts, where statistical regularity can be used to generate an extended version of objectivity. Finally, a more recent legislation (see for example more recent IAS and FASB statements) pushed toward a wider use of market prices, even when their applicability itself is hard to verify.

¹¹A dominant theme in accounting is to interpret accounting as an approximation of an underlying objective notion of wealth. While (neo-classical) wealth measurement is desirable when an objective benchmark exists, the existence of an objective or monetary criterion when considering the objective of an entity is not guaranteed. See also Boulding (1950): “There is hardly any more subtle or corrupting fallacy in economics than that of misplaced concreteness as applied to values, the view that every good goes through its life with a birth certificate in the form of a price-tag” (p.194). Sunder (1996) explains: “When a well-defined market price exists for the transferred good, there is no economic rationale for the two divisions to be part of single firm” (p.55). An entity is formed endogenously due to the absence of a clear market referential. This does not necessarily imply that markets are useless as measurement standards but that anchoring value using various sources of information is the primary, and non-trivial, role of accounting and cannot be assumed away on pure economic principles.

information incompleteness is important and cannot be reduced. This problem is considered in Beaver and Demski (1979) (when markets are incomplete) who discuss the impossibility of classical income measurement.

A partial solution to this impossibility is to define income measurement in a weaker form, by focusing on its informational role. So income is to provide (a vector of) information about the potential state of affairs. This function is described in Demski and Sappington (1990) and defined as fully revealing income measurement; it is a simple quantitative measurement that unambiguously reveals the information held by the entity. However, no hard and soft income was explicitly considered by Demski and Sappington (1990). Within our framework, we revisit this result by constructing a numerical measure, denoted partially-revealing income measurement, which conveys both soft and hard information but stops short of requiring the measure to further reveal the (binary) relation between two alternative choices or to reveal *all* hard information held by the entity. A partially revealing income measure can always be constructed but may lose some hard information and aggravate the incomplete information problem.

For fully-revealing measurement and neo-classical income to work, a quantification postulate (Postulate 5) is needed. Under Postulate 5, it is possible to reduce the dimensionality of existing informational indicators and recover a fully-revealing income measurement (see Proposition 4) which does not sacrifice hard information as the price for information communication. This notion of income measurement can aggregate various dimensions into a single performance indicator. Generalizing the analysis of Demski and Sappington, we show that a fully-revealing income measurement will exist for very important degrees of (multi-dimensional) incomplete information. By extension, it is possible to design a single common general accounting standard, which achieves full revelation among different entities and thus facilitates the coordination of individuals on a common set of accounting policies.¹²

1.5. New Classical Income Measurement

In sections 4 and 5, we build a new, unifying approach to income measurement, which attempts to resolve the infeasibility of classical income by building on recent choice-theory results on incomplete preferences. With this approach to income measurement, we wish to return to its classical root of representing preferences but also be mindful of the fundamental incomplete-information problem. Hence, we term our approach New Classical Income Measurement. As such, we bridge the gap between the two approaches. Instead of choosing either postulate 4a or 4b, we combine the two into a new classical measure, which contains two specific measures: Line-item Income Measure (LIM) and Summary Income Measure (SIM).

Line-item Income Measurement. We show, in Proposition 5, that a finite number of performance measures (the LIM), which evaluates each alternative based on multiple performance criteria, can be a possible solution to the problem of representing an incomplete preference. These performance criteria are helpful to represent soft versus hard information (as in fully-revealing income measurement) while completely ranking alternatives when information is hard (as in classical income measurement). In particular, if the information is hard with respect to a pair of choices, LIM will assign the preferred choice a higher income score for each element in the vector. If the information is soft, LIM will assign the

¹²A natural construction for a general accounting standard is to focus on the entity that has the most complex economic environment and then, use this entity as a building block for the construction of the standard, i.e. *the design of a general standard is always as complex as the most complex of the firms that it intends to represent.*

pair vectors without identical ranking for all elements. This extended version of income measurement relaxes the assumption of a single-valued income measurement in order to fully represent the original problem. That is, the income measurement indicates when information is soft (disagreement between criteria) as well as the ranking when information is hard (agreement between criteria). This more general construction of income measurement is possible only when there is sufficient hard information in the environment, so that accounting may still fail in complex environments with too much soft information. Inside organizations, the use of LIM is very common as entities use many valuation models to compare their alternatives, and even in financial reports, the net income is one among many other criteria monitored by stakeholders.

Summary Income Measurement. With Postulate 6,¹³ the accounting process must resolve the decision problem of the organization by providing a complete ranking. In response, we introduce a single-valued income measure which we call Summary Income Measure (SIM). Here we turn to a more ambitious role for accounting: “filling the informational gaps.” In this sense, the role of the income measure is to *substitute* for the preferences when the entity is indecisive about certain alternatives. However, this extended measurement concept is different from pure classical income measurement since regions of indecisiveness (or soft information) are resolved by conventions and rules, but not by the addition of unquestionable economic information. SIM points to the idea that accounting “completes” an incomplete preference for the entity, an important role identified in this paper. Our SIM approach corresponds to many existing practices in accounting such as conservatism, non-recognition of goodwill, standard prices for inventory, etc. We show that the consistency of the conventions with existing hard information is the main concern, not whether these conventions are always tied to an underlying notion of value.

1.6. Operational Income Measurement

In accounting, the reduction of the overall measurement problem to a simpler set is crucial. It is applied when the accounts of separate subsidiaries are consolidated or when profit is allocated through different periods. Postulate 7 calls for a linear aggregation in income measurement over time periods, which is key in periodic financial statements. This imposes a more demanding operational requirements on income measurements. In order to apply this reduction, it is necessary to exclude excessive dependence between subsidiaries and periods (see Proposition 9). When time periodicity is properly recovered from an axiom (“weak independence”), the setting allows us to consider the time-series properties of income, such as updating in the accounting process when additional information arrives. Indeed, we show, in Proposition 8, that such an updating process resembles the recognizable accrual process.

Postulate 8 introduces the possibility of an absolute, as opposed to a relative, notion of income measure. In accounting, the use of market value in nominal form¹⁴ or in substantive form of mark-to-market accounting, has been controversial. In our framework, given the linear representation of income, we show it is now possible to identify a unique income measure (corresponding to an absolute concept of value). The entity may use, at its discretion, market (or external) prices to anchor its accounting process.

¹³This postulate has grown in vigor over the last fifty years and is exemplified in one of the clauses of the 2002 Sarbanes-Oxley Act: “the signing officers (...) are responsible for establishing and maintaining internal controls.” (sec. 302). Although the law does not imply that the management is a part of the internal control process, it does explicitly require an active control role for accounting that incorporates choices.

¹⁴The “monetary unit” assumption is common in accounting theory, but has been subjected to debate especially during high inflation periods such as the 70s.

In turn, it is possible to use limited mark-to-market in order to anchor income measurement using an objective notion of value, thus pricing income *as if the assets used for the anchoring process were fully tradable* (see Proposition 10).¹⁵

To summarize, the axiomatic framework allows us to combine the income measurement ideas in both the “measurement school” and the “information school” to converge toward an extended version of income measurement that preserves the notion of soft and hard information while simultaneously representing preferences and finally, to a unique income measurement that ranks all alternatives even in the presence of soft information. Our framework rationalizes existing concepts of income measurements as the process of representation of the objectives of the entity.

2. The Axiomatic Challenge of Income Measurement

2.1. Economic Entity and its Preference

We may now identify the first symbolic elements of the axiomatization. We provide the most elementary representation of the subject and the objects of accounting. The subject is described as an economic entity with certain inherent purpose and the objects to be accounted for are the choices available to the economic entity. To describe the subject and object of accounting, we begin with a description of the environment and introduce an axiom, which we call the Entity Axiom, imposed on the environment such that the entity and its choices allow the accounting process to be meaningful.

Let us denote Ω , a set such that each element (or state) $\omega \in \Omega$ represents an exhaustive description of environment facing an entity including its available alternative choices and their consequences. Conditional on knowing the state ω , the entity has a complete knowledge of its environment and is able to make a choice according to its objectives. Here, we borrow from standard choice theory¹⁶ the notion of a rational preference.

Definition 1. *The binary relation R on a set X is a rational preference¹⁷ if it is:*

- (i) *reflexive, i.e. for x in X , $x R x$.*
- (ii) *transitive, i.e. for x, y and z in X , $x R y$ and $y R z$, then $x R z$.*
- (iii) *complete, i.e. for x and y in X , $x R y$ or $y R x$.*

The symmetric (resp. asymmetric) part of R is defined as I (resp. P) such that, for x and y in X , $x I$ (resp. P) y if $x R y$ and $y R x$ (resp. not $y R x$). If R is reflexive and transitive but not necessarily complete (i.e. an incomplete preorder), we say that R is an incomplete preference.

Axiom 1 formalizes the concept of an entity (Postulates 1-2).

¹⁵This anchoring, although conventional, may be desirable to ensure comparability between different entities as well as more legibility with respect to the outside environment. However, excessive mark-to-market may be undesirable (see Corollary 1), since it may disrupt the original preference of the entity, thus moving away from the fundamental task developed earlier.

¹⁶See for example <http://en.wikipedia.org/wiki/Preference> for a concise presentation.

¹⁷Formally, a preference over X is a subset of $X \times X$. Here, we denote in short-hand: x and y in X , $x R y$ for $(x, y) \in R$. Mathematically, a rational preference is a complete preorder.

A-1. (entity) There exists an entity, that can be fully described as follows. For all $\omega \in \Omega$, there exists a non-empty set X_ω of alternatives for the entity and a rational preference \preceq_ω over X_ω . The rational preference can be called a statement of entity objectives.

Constructively, the entity is obtained by endowing each state with alternatives, X_ω , and a statement of objectives, \preceq_ω . The set of alternatives X_ω corresponds to a set of exhaustive and mutually-exclusive alternatives which represents the possible plans of actions which may be undertaken by the entity (such as investment projects). The objectives of the organization are represented as a rational preference over alternatives (\preceq_ω), such as a ranking of the investment projects. This preference or ranking can be *subjective* in the sense that it may only arise from the specific purpose of the particular entity. Naturally, different entities may have different preference even when facing the same two alternatives.

As in Debreu (1972), the set of contingent states Ω may encompass time and uncertainty, and we delay for now (until section 5) a precise discussion of time and uncertainty and its conceptual implication to accounting concepts such as periodization and consolidation. These two concepts are more demanding on the axiomatic foundation and, later in the paper, we intend to recover periodization and consolidation from additional axioms, as opposed to assuming them up front. We insist here on *economic rationality* as the minimal axiomatic requirement for accounting measurement to be meaningful.

2.2. Accountable Entity and its Induced Preference

A key aspect in our theory is that the entity may have limited knowledge about the exact environment. We model available information by defining a partition of Ω , denoted by \mathcal{W} with a typical element W (so $W \in \mathcal{W}$). If W contains multiple states, the entity may be unsure about its preference or ranking over its choices. Information is incomplete in the sense introduced by Harsanyi (1967) in that the entity does not know the true state, or even a distributional prior over states.¹⁸ However, we impose a basic requirement that the knowledge of the set of available choices for each W is perfect. That is, the entity is fully aware of its available choices even if it may not be fully sure of its ranking over these choices. This requirement is introduced via Axiom 2, which we label the Control Axiom.

A-2. (control) For any $W \in \mathcal{W}$, ω and ω' in W , $X_\omega = X_{\omega'} (\equiv X_W)$.

In Axiom 2, the entity is assumed to be able to fully distinguish events that yield different alternatives.¹⁹ We define an induced event-contingent binary relation \preceq_W as follows.

Definition 2. For a given economic entity $\{X_\omega, \preceq_\omega\}_{\omega \in \Omega}$ and available information W , the preference \preceq_W is defined as follows:

(i) For x and x' in X_W , $x \preceq_W x'$ if $x \preceq_\omega x'$ for all $\omega \in W$.

(ii) If for x and x' in X_W , neither $x \preceq_W x'$ or $x' \preceq_W x$, we say that the entity is indecisive.

¹⁸Incomplete information is more primitive than imperfect information: an incomplete information problem can always be specialized as imperfect information by letting Nature choose randomly over the elements of W . However, a Bayesian prior would be a very demanding axiom and, even if an objective “entity-wide” prior did exist, it would be difficult to observe and communicate in the problem that we investigate here.

¹⁹This requirement is similar to the idea that nodes in the same information set of an extensive form must have the same actions (see Fudenberg and Tirole (1991), p.80). Note that Axioms 1 and 2 correspond to the “Control” axiom in Ijiri (1967).

This definition is related to the construction of preferences for groups in Shubik (1978), but is here applied more generally across contingent states.²⁰ The potentially accountable, or in short-hand *accountable*, entity is an entity with an information structure \mathcal{W} which induces an event-contingent preference.

Axiomatically, we use this accountable entity to represent a reasonable building block for accounting income measurement. Here we refrain from transforming an incomplete information problem into one of imperfect information (such as introducing objective or subjective probabilities). We believe axioms inducing a probability measure (such as the famous Savage axioms in Savage (1972)) seems too strong to place on an entity as opposed to an individual. By confronting the more elementary problem, we place accounting closer to the core problem facing the entity – *decision making under incomplete information*.

Proposition 1 consists in two complementary statements, and shows why, formally, an accountable entity is a meaningful prime material for accounting theory.

Proposition 1. *In an accountable entity, for any W , \preceq_W is an incomplete²¹ preference on X_W . Conversely, let \preceq be a binary relation on a non-empty set X , if*

- (i) \preceq is an incomplete preference on X (conditional rationality), and
- (ii) for any rational preference R on X , there exists $\omega \in \Omega$ such that $X_\omega = X$ and $\preceq_W = R$ (unrestricted preferences).

Then, there exists an information set W in Ω that rationalizes \preceq , i.e. $\preceq = \preceq_W$.

Proof: We prove the first part of the statement. For any $\omega \in W$, for $x \in X$, $x \preceq_\omega x$ (reflexivity of a rational preference), and thus $x \preceq_W x$. For x, y and z in X , assume that $x \preceq_W y$ and $y \preceq_W z$. It must then hold that for all $\omega \in W$, $x \preceq_\omega y$ and $y \preceq_\omega z$; then, by transitivity, $x \preceq_\omega z$. It follows that $x \preceq_W z$. To prove the second part, let us consider a preorder \preceq and construct a set W such that \preceq and \preceq_W coincide. Define K as a subset of $X \times X$ such that for (x, y) in K , neither $x \preceq y$ or $y \preceq x$ holds and, for a particular $(x, y) \in K$, define $\preceq^{(x,y)}$ such that \preceq and $\preceq^{(x,y)}$ coincide for all pairs except for x and y where $x \preceq^{(x,y)} y$. Next, we define $\preceq^{(x,y,+)}$, the transitive closure of $\preceq^{(x,y)}$ such that $x \preceq^{(x,y,+)} y$ and not $y \preceq^{(x,y,+)} x$. By Szpilrajn Theorem (Szpilrajn 1930), there exists a rational preference that contains $\preceq^{(x,y,+)}$. By assumption, there exists $\omega_{(x,y)}$ such that this extension is $\preceq_{\omega_{(x,y)}}$. Construct $W = \bigcup_{(x,y) \in K} \omega_{(x,y)}$. The binary relation \preceq_W will necessarily correspond with \preceq .

Q.E.D.

Proposition 1 describes the nature of the induced preference \preceq_W . Compared with the underlying state-contingent preference (\preceq_ω), the induced preference shares the usual properties of reflexivity and transitivity.²² However, it does not necessarily preserve completeness inherent in the state-contingent

²⁰It should be noted that here we use a “weak” Pareto principle in the sense that according to the entity, an alternative will be strictly preferred although there may be indifference in certain states. This idea, which is not developed here, can be formalized as *safety principle*, but we could not find a clear analogue in accounting textbooks.

²¹Note an incomplete preference is defined weakly (in definition 1) and can be either strictly incomplete or complete (e.g., when W is a singleton).

²²The uncertainty about states preserves transitivity. More precisely, if there is hard information with respect to x to x' , and x'' and x' , then there must be hard information with respect to x and x'' . The accountant can directly compare two alternatives using directly the preference of the entity, rather than having to indirectly seek irrelevant alternatives to achieve such comparisons. At this stage, there is no purpose for accounting to recover hard information from different economic sources, since it is already built in the original economic preference \preceq_W . This finding contrasts with the analysis of Curtis Eaves (1966) who, focusing on the relationship between accounting aggregates, formalizes the use of chains, i.e. sequences of relations that allow to directly move from one element to another. In our setting, and since we discuss alternatives and not accounting rules, the consideration of chains is not necessary. As argued in Chambers (1966), the preference requires no “reasoning” (p.28-31).

preference (\preceq_ω).

The converse of Proposition 1 shows that there exists a theoretical representation of any observed state of affairs of a real entity, that can be made consistent with Axioms 1 to 2. That is, given observed objectives, one can recover²³ a theoretical representation consistent with the deductive setting presented here.²⁴

2.3. Incomplete Preference and its Challenge to Income Measurement

We now confront the problem of incomplete preference and its challenge to income measurement as a representation of entity preferences. To begin, we distinguish hard and soft information.

Definition 3. Consider an information signal $W \in \mathcal{W}$, and two alternatives x and x' in X_W ,

- (i) W is hard information, with respect to x and x' , if $x \preceq_W x'$ or $x' \preceq_W x$.
- (ii) If W is not hard (i.e. the entity is indecisive), we say that the information is soft.

The distinction between hard and soft information is crucial in accounting theory and is natural in the context of our axiomatization. An information is *hard* when it yields an unambiguous ranking among alternatives available to the entity, whereas it is *soft* when, due to incomplete decision-relevant information, the ranking is ambiguous.²⁵ In our framework, the existence of soft information is a consequence of remaining state-uncertainty and a key challenge to accounting theory because soft information causes incompleteness of the induced preference.

Proposition 2. The induced preference \preceq_W is strictly incomplete if and only if W is soft information with respect to some pair x and x' where $x, x' \in X_W$.

To summarize the framework, the subject of income measurement is a well-defined rational accounting entity; the object of income measurement is an information-induced binary relation between two alternative course of action.²⁶ Based on earlier literature on measure theory and accounting theory, when information is complete, the role for income measurement to represent a preference relation is somewhat well-understood (e.g., Debreu (1972) states that there exists a numerical representation of any continuous rational preference). The potential incompleteness of the induced entity preference poses a challenge to the construction of income measures to represent preferences. Even with a well-defined, well-behaved, state-contingent preference, the entity may still face a non-trivial problem of indecisiveness when information is soft, leading to an incomplete preference. A serious consequence is a lack of existence of

²³It is important to note that this discussion differs from the more complex problem of revealed preference, where a rational theory of choice is recovered from observed choices (or constrained optima). However, one may think here that the incomplete preference \preceq_W (taken here as given) is recovered from all possible pairwise choices of the entity (given available information).

²⁴More precisely, the first assumption is that the preference of the entity must be a preorder (reflexive and transitive), which means that the preference uses all available information. The second assumption is that of unrestricted preferences, in that no contingent objective should be excluded. One can interpret this finding as *integrability* (as used in utility theory). We can then consider the preferences \preceq_W for $W \in \mathcal{W}$ as an axiomatically-founded starting point for the analysis without making further restrictions.

²⁵Our notion of hardness of information is consistent with existing notions of “hardness.” In our setting, the classical notions of “verifiability” and “tangibility” (see for example Paton and Littleton (1940), p.19) correspond to means by which an information can be determined to be hard, but not defining attributes of hard information itself.

²⁶This has been long recognized in accounting scholarship. For example, Ijiri (1964, p.28) wrote: “measurement is not concerned with a single object. It is concerned with relations among objects[.]”.

a measure in a traditional sense. We believe this is the key challenge to a theory of income measurement. That is, in a world when information is incomplete, what is the proper role of accounting income measurement?

3. Two Existing Approaches to Income Measurement

In this section, we consider two existing fundamental approaches to income measurement: classical income measurement approach and the information content approach.²⁷ By classical income measurement, we refer to the theoretical approach where an income number is a measure in a pure measure-theory sense. That is, it represents a binary relation and it corresponds to the highly stylized Hicksian or Fisherian income. The benchmark setting with perfect and complete market in Beaver and Demski (1979) is a good example. In our framework, we reframe the classical income measurement approach as using income measurement to measure a rational preference in settings where hard information abound and even the event-contingent preferences are complete.

By information content, we refer to a modern view where an income number is a source of information which may update users' belief about consequences of actions. It is initially proposed by Beaver and Demski (1979) and expanded by Demski and Sappington (1990) (thereafter BD and DS, respectively) and Christensen and Demski (2003). In our framework, the information content approach can be thought of as focusing on using income measurement to convey the event W to outsiders as opposed to the induced binary relation contingent on W .

3.1. Classical Income Measurement

To begin, we formalize the classical income measurement as a simple (single-valued) numerical measure²⁸ that always ranks all available alternative completely.

Definition 4. *A classical income measurement I^c for information set W associates a simple real-valued performance index to each alternative, i.e. it is a function from X_W to \mathbb{R} such that: $x \preceq_W x'$ if and only if $I^c(x, W) \leq I^c(x', W)$.*

Consistent with standard choice theory, the existence of a numerical relation system (such as a real-valued function I) depends on the properties of the empirical relation system (\preceq_W). First, the relation must be a rational preference (i.e., reflexive, transitive, and complete) and second, the preference must be “well-behaved” (i.e., satisfy certain technical assumptions).

A special case of perfect knowledge satisfy the first assumption. So lemma 1 is almost trivial.

Lemma 1. *Suppose $\mathcal{W} = \{\{\omega\} | \omega \in \Omega\}$. Then, under **A-1.** and **A-2.**, the induced preference \preceq_W is always a rational preference.*

²⁷This important distinction is suggested by Chambers (1972) who asks “whether accounting is ideally a measurement device or system, or is, instead some other mode of quantification or description.”

²⁸Note that the definition of classical income depends on W , the information set. Until we define a form of revealing income measurement, we assume that there exists a process by which the current information can be fully revealed; but it is somewhat intriguing that even classical income presupposes some primitive notion of revealing income measurement (although it was introduced later). For the sake of clarity, we follow here a chronological, rather than purely logical, order.

Proof: By assumption, every element $W \in \mathcal{W}$ will be a singleton; then, by Axiom 1, the preference \preceq_W will be complete, and thus will be rational.

Q.E.D.

One more technical assumption is needed which we introduce via Axiom 3.

A-3. (benchmarking) For all $W \in \mathcal{W}$, there exists Z_W countable subset of X_W , such that for any x and x' in X_W , if $x \prec_W x'$, there exists $z \in Z_W$, such that $x \preceq_W z \preceq_W x'$.

Axiom 3 requires the complexity of the entity to be reducible²⁹ to a countable number of pairwise comparisons. It specifies the existence of a benchmark set of projects (for example derived from past history or from internal management accounting) that can be used to separate different alternatives.

Proposition 3. *Under A-1. to A-3., there exists a classical income measurement if W is hard information for all pairs of choices in X_W . Conversely, if W is a soft information, then a classical income measurement does not exist.*

Proof: By Lemma 1, the preference \preceq_W is a rational preference. Adding **A-3.**, the existence of a classical income measure is assured by a standard proof, such as Kreps (1988) (Theorem 3.5, p.25). If W is a soft information, by 2, the preference \preceq_W is incomplete, the inexistence of a classical income measure is also assured by the standard proof.

Q.E.D.

A simple example of this setting is considered in BD. In their model, a choice of the firm is defined as a production plan, i.e. a set of inputs and outputs $V = (v_1, \dots, v_n) \in E \subset \mathbb{R}^n$ (where E is compact). A state ω is given by a complete set of prices for each element of the production plan $P = (p_i)_{i=1}^n$. In this case, the set of alternatives does not depend on the state and can be written $X_\omega = E$ (Axiom 2). Following the neo-classical theory of Arrow and Debreu, BD introduce a conventional definition of income as the value of the production plan, i.e. PV , and assume value-maximization. Given this objective, the preference is defined as follows, for V and V' in E , $V \preceq_P V'$ if $PV \leq PV'$. This “economic” income concept corresponds to the classical income measurement described here. In this situation, and more generally under the conditions of Proposition 3, it is possible to obtain an objective notion of value since the entity will always generate a complete ranking of its alternatives.

To summarize, classical income measurement is possible under complete markets, which allow the entity to fully replicate each of its alternatives based on public prices. To avoid confusion with weaker forms of markets, we shall denote this situation *perfect* mark-to-market, in the sense that markets are sufficient to fully determine the ranking among alternatives.³⁰

²⁹As pointed out in Kreps (1988), a set of sufficient conditions for Axiom 3 is that X_W is a subset of a separable metric space, such as \mathbb{R}^n , and \preceq_W is continuous.

³⁰For example, a small investment fund holding publicly traded assets will generally be able to mark to market its assets. However, this may not be possible for large funds trading on large positions or illiquid assets. Further, firms may own firm-specific assets, such as for example used machines, brands or even human capital, for which market prices, if they exist, remain imperfect or approximative. More generally, under incomplete markets and free-disposal, the positivity of state prices will rank only a subset of alternatives.

A natural limitation of this approach is that the measure does not exist when information (and therefore the event-contingent preference) is incomplete. BD point out that, in general, the set of prices will be incomplete, so the value of the production plan is indeterminate. In the converse of Proposition 3, this impossibility claim may arise more generally. A lack of complete preference is at the heart the problem. The mathematical intuition for this result is that a classical income measure is defined on a completely ordered set (the real line), and thus ranks all alternatives. Then, it will be impossible under classical income measurement to convey the idea that the entity may be indecisive between alternatives.³¹

3.2. Fully Revealing Income Measurement

A partial solution to the general impossibility of classical income measurement is to propose an alternate, perhaps less ambitious, function of income measurement, as described in DS. They propose that the role of income measurement is to objectively communicate the relevant decision-theoretic information, in a cost-effective manner, to individuals who may have a role in the entity. These users would then make more informed decisions with the aid of the information contained in income measurement. A particular benchmark mechanism is to clearly communicate a set of current objectives, or mathematically, which element of \mathcal{W} is realized. In DS, this is called Fully Revealing Income Measurement.

Definition 5. For a given partition \mathcal{W} , a fully (resp. partially) revealing income measurement system is defined as a real-valued injective function from \mathcal{W} (resp. \mathcal{W}') to \mathbb{R} , where \mathcal{W} is finer than (i.e., is a sub-partition of) \mathcal{W}' .

The fineness relation between two partitions is a well-known concept (see Marschak and Miyasawa (1968)). The construction of an fully revealing income measurement requires additional structure on the partition that are being measured. We introduce Axiom 4, the Quantification Axiom.³²

A-4. (quantification) There exists a function ϕ from \mathcal{W} to \mathbb{R}^N (N finite), and such that $\phi(W) = \phi(W')$ implies that $X_W = X_{W'}$.

Axiom 4 imposes that too much information cannot be transmitted via the definition of alternatives.³³ Note that we do not assume that the possible partitions (\mathcal{W} 's), or the number of alternatives given W (i.e., the elements in X_W), are quantified, which is much more demanding. The function ϕ may be described as a finite but very large number of operating measures about the firm and its economic environment.

We now ask whether the information held by the entity, i.e. the set W , can be represented by a single accounting indicator, such as an income measure.

³¹In accounting, contract theory yields an understanding of contractual agreements that constitute the entity, and thus reduces the set W by providing principles that allow us to tighten information on the objectives of the entity generated by these contracts. Information theories of accounting are the reasoning that is used to generate hard information and, in this respect, is placed at the foundation of any non-trivial classical theory of income measurement. Although there is constant progress toward more general informational models of the entity, there is however no guarantee, at the current state of the science, that the informational theory will fully reduce information incompleteness, given that many different effects and several irreducible sources of incompleteness may be at play in the general problem.

³²Quantification is a recurring axiom in accounting; See Ijiri (1964, p. 75), and many others Paton and Littleton (1940), Moonitz (1961), Mock (1976), Mattessich (1995).

³³Further, one can redefine the problem by introducing a number of irrelevant or dominated alternatives so that Axiom 4 is verified.

Proposition 4. *There is always a partially revealing informational income. Further, if ϕ is injective, there exists a fully revealing informational income measure.*

Proof: We prove the second part of the statement first. Since ϕ is injective, it remains for us to construct an injection from \mathbb{R}^N to \mathbb{R} . The problem is equivalent to showing that $(0, 1]^N$ ($N > 1$) and $(0, 1]$ have the same cardinality. Consider x^i in $(0, 1]$ for all $i \in N$; write their unique infinite decimal expansion: $x^i = 0.x_1^i x_2^i \dots$ (such that any real with a finite number of decimals is represented with an infinite number of nines). Now let us define z as the number represented by the infinite decimal expansion $z = x_1^1 \dots x_1^N x_2^1 \dots x_2^N \dots$. Clearly, there exists a bijection between $(0, 1]$ and \mathbb{R} which concludes the proof.³⁴ To obtain the first part of the statement, we define $\mathcal{W}' = \{\bigcup_{\phi(W')=\phi(W)} W' | W \in \mathcal{W}\}$. Clearly \mathcal{W}' is less informative than \mathcal{W} , and then, we can define ϕ' as follows. For any $W \in \mathcal{W}$, $\phi'(\bigcup_{\phi(W')=\phi(W)} W') = \phi(W)$. Since ϕ' is injective, the existence of a partially revealing informational income measure follows from the first part.

Q.E.D.

The nature of a revealing measure depends on “how much” information is to be quantified. Realistically, the set of all possible information sets may be extremely large, in which case a fully-revealing income may be impossible (i.e., ϕ is not injective). Nevertheless, we show that it is still possible to use a coarser measure, which pools together information sets. This simplification entails an informational cost: a partially-revealing measure aggravates the problem of incomplete preferences. A fully revealing income measurement in the sense of DS can be constructed under a more restrictive assumption on ϕ (i.e. the number of possible information sets must be quantified as a finite number of indicators).

The procedure to obtain (fully or partially) revealing measurement is explicit in the Proof, and can be constructed by merging a finite number of information sources in order to reduce the dimensionality of the information.³⁵ We now return to the problem considered in DS to illustrate the proposition. They showed an example where one needs to alter the usual income calculation (via a conservative accounting method) to fully convey underlying state partition. Restated, this is simply to construct an income measure to merge two sources of information (one source on cash flows for period 1, 3 and the other on cash flows for period 2, 4).

Proposition 4 generalizes several results established by DS. First, the accounting standard will hold even when \mathcal{W} is not finite, thus generalizing the existence of a fully revealing standard when there may be a continuum of possible sets W , and discussing the nature of the informational measurement for arbitrary information sets. Second, we differ from DS in that we do not restrict the analysis to a formation resembling classical income. If income is not a measure of value, there is no reason to restrict information revelation to the form of a classical income (such as having an interpretation of “maximum consumption

³⁴The original proof, which we have slightly modified here for presentation purposes, is due to Cantor, who was originally surprised by the result; in a 1877 letter, he writes: “Ich sehe es, aber ich glaube es nicht!”, i.e. “I see it, but I do not believe it.”

³⁵This procedure, although abstract, is not fundamentally different from the set of rules that are considered to construct an aggregate in the accounting process. For example, the proper consideration of depreciation requires merging different sources of information. While it may have been possible to consider a standard that adds separately each measure (which is often one of the properties of a pure “economic” income measure), it may be flawed given an objective of revelation. Note also that the procedure considered in the Proof yields an accounting standard that is strictly increasing in both indicators; i.e. if the indicators are measures of performance, the notion of performance will remain in the accounting standard or, in a different manner, one may still interpret the accounting standard of an aggregate performance measure.

while leaving the entity as well-off as the beginning of the period”). Anchoring revelation of information to classical income may be desirable as a convention, but not as a necessary attribute of income measurement. Third, there may be a trade-off between the simplicity of the accounting measure, which favors a unique real-valued accounting measurement, and the complexity of the underlying environment. Fourth, DS focus on a single firm, implicitly suggesting the adoption of different accounting practices for different firms. While our theory is also presented as a single firm problem, the informational income measurement can be applied more generally, economy-wide, as a unique standard for a large number of problems (in which case the number of information sets will be even more important).

The disadvantage of the RIM approach is that income measurement ceases the function of representing preferences, its traditional role. Income no longer represents a notion of value, but is a simple vector of information. While it may be possible to incorporate additional information about the preference into the RIM, such a construction makes it difficult to directly rank alternatives. In the next section, we integrate these two concepts, by letting the revealing measurement “select” a proper performance measure, and then let the performance measure indicate the ranking over alternatives given available information.

4. New Classical Income Measurement

We now build a new, unifying approach to income measurement, which attempts to resolve the infeasibility of classical income by building on recent choice-theory results on incomplete preferences. With this approach to income measurement, we wish to return to its classical root of representing preferences but also be mindful of the fundamental incomplete-information problem. Hence, we term our approach New Classical Income Measurement, which is developed in two steps. First, we introduce a vector-valued income measurement (I^{LIM}), called a Line-Item Income Measure (LIM), which assigns an incomplete numerical ranking all alternatives but preserves entirely the incomplete preference (\leq_W) defined over X_W . Second, we introduce a single-valued income measurement (I^{SIM}), called Summary Income Measure (SIM), which assigns a complete numerical ranking of all alternative but only partially preserves the incomplete preference (\leq_W) defined over X_W .

4.1. Line-Item Income Measure (LIM)

The first new classic measure, the Line-Item Income Measurement, must satisfy two different measurement functions: (1) indicate the presence of soft information and (2) preserve the entity’s ranking of alternatives in the presence of hard information. In comparison to informational income measurement, this task is more challenging than an objective disclosure of information since it requires the interpretation and representation of a possibly complex set (an incomplete relation). LIM achieves this by assigning each alternative a vector of numerical values (or Items), instead of a single value. Alternative A is preferred (resp. indifferent) to alternative B if and only if A’s vector value is greater than (resp. equal to) B for each element in the vector. The entity is indecisive between two alternatives if the vector-value system is unable to rank the pair, i.e. elements of vector-value disagree in ranking the two alternatives. In our framework, this also indicates the presence of soft information. Line-Item income measurement has relevance in practice as most income statements present income as a series of line items from the

top-line sales numbers to the bottom-line profit figure.³⁶

The New Classical Income must deal with incomplete information in our framework. In general, the task of communicating the entity environment (W) is achieved perfectly or imperfectly through disclosure practices by the entity itself (through financial reporting or corporate voluntary disclosure) or other entities (such as trade organizations, new media, government, or competitors). To represent this (partial) information revelation, we condition the numerical assignment by LIM on a revelation of the event W , denoted as $\phi(W)$. Following the generalization of a fully-revealing income measurement for arbitrary information sets, we take as given the existence of a RIM, and assume that³⁷ $\phi(W)$ reveals W . The conditioning information variable $\phi(W)$ has relevance in practice such as quantitative or qualitative information contained in the notes to financial statements and in management discussions and analysis (MDNA).

We are now equipped to make a statement about income representation. We intend to represent the preference of the entity as a simple set of accounting measures.

Definition 6. We define a *Line-Item Income Measure (LIM)* as a vector-valued function I from $X_W \times \mathbb{R}$ to \mathbb{R}^n (n finite), such that for $W \in \mathcal{W}$, x and x' in X_W , $x \preceq_W x'$ if and only if $I(x, \phi(W)) \leq I(x', \phi(W))$.

A Line-Item Income Measure is defined intuitively as a finite number of performance measures. It takes as argument the current state of affairs, communicated by $\phi(W)$, and an alternative x and, in return, provides a set of numerical (income) measure.³⁸ The advantage of a Line-Item Income Measure is that it will fully represent the original preference of the entity in a simple manner. The key feature of the vector measure is that an agreement of the criteria indicate a presence of hard information and its resulting complete ranking while a disagreement indicates a presence of soft information (or indecisiveness).³⁹

We borrow from the literature on choice theory the following axiom.

A-5. (economic information) For all $W \in \mathcal{W}$, any subset A of X_W such that no pairwise comparison is possible under \preceq_W (i.e. there is no hard information in A) is finite.⁴⁰

Axiom 5 restricts the degree of incompleteness that is acceptable under the current state of affairs; in other words, the organization must have a sufficiently complete statement of objectives in order to allow

³⁶In practice, internally, corporations use many different performance measures which may be considered as real-life examples of accounting performance systems. The valuation of illiquid subsidiaries is done using different methods, such as Net Asset Value (or book value), trading multiples or discount cash flow analysis. To value portfolios, investment firms simultaneously consider current market value as well as Monte-Carlo stress simulations. The performance of production plants is measured under several criteria such as turnover, quality and reliability.

³⁷If only a partially revealing income measure exists, we assume that \mathcal{W} is redefined as a less informative set such that the income measure is fully revealing with respect to \mathcal{W} .

³⁸See for example Mock (1976) (p.67) for an example of multiple performance measures. Further, even with quantitative data, statisticians use different models which are typically hard to compare. An example is by running an OLS regression with substantial common variation in explanatory variables. The model will typically not indicate which of these variables may be insignificant, and which ones are significant.

³⁹The concept of *relevance* (see for example Stamp (1988)) is natural in our framework. If $x \triangleq_W x'$, x and x' will be two distinct alternatives but that are completely equivalent from the point of view of the entity. In other words, we can pool together all alternatives that are equivalent to x , denoted for example $[x]$ (its equivalent class), so that we focus only on *relevant* characteristics. In choice theory, this transformation of X_W is often made, although we keep here the original preference for notational simplicity.

⁴⁰Following the above footnote, this condition can be extended to a finite number of equivalence classes $[x]$. This Axiom is also known as “near-completeness.”

the accounting process to function.

Proposition 5. *Under A-1. to A-5., there exists a Line-Item Income Measure; the number of items (i.e. n) can be chosen to be smaller than the maximum number of alternatives for which there is only soft information.*

Proof: See Ok (2002) (Theorem 2, p.441).

Proposition 5 establishes the existence of a (real-valued) accounting system that fully represents the preferences of the entity. The complex choice problem of the entity is disaggregated into a sequence of line items, whose joint consideration entails no loss of information. While the result is presented as a “possibility”, it is interesting to discuss when accounting may fail. This may occur when there is excessive soft information, which may violate Axiom 5.⁴¹ Then, the result proves that the availability of sufficient (but not complete) hard information is the necessary requirement for a comprehensive accounting system.⁴²

Returning to the example of market incompleteness⁴³ in BD, let us assume that the information set W corresponds to an incomplete set of prices $(p_i)_{i=k+1}^n$, which are chosen in $[\underline{p}, \bar{p}]$. Then, we may consider a set of performance measures of the following form $I_e(V) = \sum_{i=1}^k p_i v_i + \sum_{i=k+1}^n (e_i \underline{p} + (1 - e_i) \bar{p}) v_i$, where $e = (e_{k+1}, \dots, e_n)$ is a vector of ones and zeros. As we showed earlier, the preference of the organization can be represented by considering 2^{n-k} performance indicators, i.e. the LIM is a vector defined by $\{I_e(V) | e \in \{0, 1\}^{n-k}\}$.

4.2. Summary Income Measure (SIM)

While LIM fully represents (or accounts for) the presence of soft information, it does not offer any unambiguous ranking of the available alternatives when soft information is indeed present. Thus, it does not resolve the issue of soft information completely and makes the income measurement somewhat unsatisfying. In this section, we consider a refined New Classical Income Measurement, called Summary Income Measure (SIM), that does indeed offer a complete ranking despite the presence of soft information. In this sense, the New Classical measure closely resembles classical income measurement.

Definition 7. *We define a Summary Income Measure as a (single-valued) function I from $X_W \times \mathbb{R}$ to \mathbb{R} such that for x and x' in X_W , if $x \triangleleft_W x'$ then $I(x, \phi(W)) < I(x', \phi(W))$.*

⁴¹In fact, it is shown in Mandler (2002) that when the incompleteness is too important, the representation holds but using a possibly infinite number of performance standards on \mathbb{R}^{X_W} . There is, in this case, a trade-off between simplicity and representation since the proper consideration of a very large number of measurements may not be of much help to the decision maker.

⁴²Note also that our analysis is fundamentally different from the traditional statistical role of information (in the sense developed in Christensen and Demski (2003)). The measurement does not necessarily correspond to a tangible quantities and is not necessarily statistically informative.

⁴³Many authors compare the impossibility of an accounting standard to Arrow’s impossibility. The argument at play is, however, very different. In BD, as well as in our framework, the inexistence of a representation is due to the incompleteness of the induced social preference: there is no rational social preference that agrees with all individual rankings. In Arrow’s impossibility argument, the would-be social preference would not necessarily need to agree with all individual preferences and, without further restrictions, there may be many valid aggregation rules. By only imposing an axiom that irrelevant alternatives should not influence the social choice, Arrow can, indeed, recover a degenerate (dictatorial) aggregation as one and only possibility.

Compared to LIM, SIM is different in several ways. First, it is single-valued, as opposed to vector-valued, thus offers a complete ranking even if the underlying preference is incomplete. So this income measure “completes” the preference, so to speak. Second, it does not indicate the presence of soft information while LIM does. When information is hard and the entity has a preference between two alternatives, SIM assigns a higher (resp. strictly higher) income number to the preferred (resp. strictly preferred) alternative. When information is soft and the entity is indecisive between two alternatives, SIM “arbitrarily” assigns a higher income number to one of the two. So a summary income measure can be described as a complete ranking over all the alternatives of X_W , rather than an incomplete set of comparisons between its elements. In doing so, the accounting measure does not necessarily represent what is the best interest of the organization if the organization is indecisive, but induces a simple operational decision rule.

We now discuss whether an accountable entity can be endowed with a Summary Income Measure.

Proposition 6. *Under A-1. to A-4., there exists a Summary Income Measure.*

Proof: The proof of this result can be found in Peleg (1970), Theorem 3.1 (p.94).⁴⁴

In Proposition 6, we obtain a simple accounting measure that completes the preference, thus establishing a complete ranking over all alternatives. This representation can be interpreted as a single performance measure and is known in consumer theory as a Richter-Peleg representation. In comparison to Proposition 5, it entails some loss of information with respect to the original “economic” environment. Specifically, if a performance indicator is such that an alternative x is better than another alternative x' , then it is certain that the alternative x' is not strictly better than x . However, the organization may either prefer x to x' or be indecisive. In other words, the income measure does not distinguish between hard and soft information. The summary income measure “hardens” soft information.

The Summary Income Measure presented in Proposition 6 generalizes classical income measurement, in the sense that whenever all information is hard, the SIM will be a classical income measurement. Interestingly, a SIM requires fewer assumptions on the entity under consideration in that Axiom 5 (sufficient hard information) is no longer necessary. This shows that it is possible that an entity could not be fully accounted (in the sense of LIM) but still could be accounted using a set of conventions. In this respect, a Summary Income Measure is a substitute for a Line-Item Income Measure when the latter is infeasible.

SIM clarifies the place of accounting rules and conventions in accounting theory. Here, they play the role of “hardening” with soft information and are, indeed, necessary in constructing an economic measure such as SIM. As such, accountants enjoy a “freedom” (a favorite word used by Ijiri) in the SIM construction (or in how to “complete” an incomplete preference). However, our theory also places bounds on this “freedom” because they must not interfere with its measurement function when hard information is present (after all, all SIM must be classical in that case). Consistency in accounting rules and conventions is key. Note that completing the preference may also generate new problems. Assume that the entity chooses a completion of \preceq_W that is represented by a simple performance measure I . Now suppose that after establishing its internal reporting system, the entity receives further operational information, so

⁴⁴While Peleg assumes continuity, this is not necessary for the existence presented here.

that it “learns” that the state ω , previously in W , is impossible. It may now occur that, for some alternatives x and x' , $x \preceq_{W-\{\omega\}} x'$ (whereas it was previously indecisive). Note that necessarily, $\preceq_{W-\{\omega\}}$ remains reflexive, transitive and antisymmetric. However, this additional information can contradict the previously chosen SIM metric I .⁴⁵

The analysis shows a trade-off between a complete (and historically consistent) income measures and correct and (contemporaneously consistent) measures in a dynamic accounting setting. A complete income measure will be set ex-ante, and (thus) would facilitate proper understanding by remaining on the same rules. It is consistent because rankings remain unchanged. However, it may also perform poorly, in the long-run, in terms of representing the preferences of the entity. As new information arrives, the same income measure may not establish a correct ranking. Alternatively, an adaptive income measure, which would correct itself by changing the assignment rules upon new information arrival, is historically inconsistent, in that the ranking would vary, which impairs its original simplifying role. However, it will always yield the correct ranking.

The trade-off clarifies the role of historical cost accounting. Historical cost establishes a complete ranking by resolving, using current information, regions of economic indecisiveness. Historical cost accounting, then, can play an active stewardship role. However, the arrival of new information may violate the rankings established under historical cost thus creating ex-post inconsistencies. A different approach is to use imperfect prices obtained from the market, which we shall denote *imperfect* mark-to-market. Unlike with perfect mark-to-market introduced previously (which is based for example on replication and arbitrage), these prices are imperfect indicators of preferences, but can be dynamically updated. Imperfect mark to market does not yield a clear stable ranking over a given time horizon, and thus fails as an active stewardship criterion.⁴⁶ However, it is less prone to inconsistencies in preferences. The debate about historical versus mark-to-market accounting is then more a problem about whether entities should offer a Summary Income Measure *across periods*.⁴⁷

To conclude, let us return to the case of incomplete markets in BD. We construct a conventional set of prices $\tilde{p} = (\tilde{p}_i)_{i=1}^n$ to support a SIM in their setting. For $i = 1$ to k , we set \tilde{p}_i equal to the market price p_i . For $i = k + 1$ to n , an accounting price is selected from the possible region, $\tilde{p}_i \in [p, \bar{p}]$, under W . Now, a SIM can be constructed as: $I(V, \phi(W)) = \sum_{i=1}^n \tilde{p}_i v_i$.⁴⁸ Equivalently, a SIM can be constructed as a weighted average of the components of the LIM, i.e. $I(V, \phi(W)) = \sum_e \delta_e I_e(V)$ where $(\delta_e)_e$ is a set of positive weights summing to one. If an additional price, say, p_{k+1} is revealed, the organization will need to reconsider whether to keep or mark to market its accounting rule.

⁴⁵A partial solution would have been to make alternatives with respect to which the entity is indecisive, indifferent in the summary income measure. That is, however, impossible for any non-trivial problem. Consider three distinct alternatives (x, x', y) . Suppose that only W is soft between x 's and y , but $x \prec x'$. But by choosing $I(y) = I(x)$ and $I(y) = I(x')$, the income measure cannot be such that $I(x) < I(x')$.

⁴⁶This problem is related to “the market for excuses” in Watts and Zimmerman (1979), who explain how changes in accounting measures are driven by political, rather than normative, motives. In our framework, a SIM which is theoretically fixed ex-ante should not be altered by the introduction of new information or theories; a LIM, on the other hand, may suggest many excuses for bad ex-post choices.

⁴⁷There are many other real-life practices where these two approaches are combined: (i) depreciation rules typically combine a conventional rate as well as infrequent write-offs or revaluations, (ii) bonds are evaluated by banks using historical rates but are marked to market by investment firms, (iii) inventories can be evaluated based on the most recent production cost (LIFO) or historical production cost (FIFO, Average Cost). Note that mark-to-market can also be used in a summary income procedure, if the entity makes use of current market information to build the summary income measure. The key distinction in our setting is whether the income measure is complete or can be subsequently restated, not its origin.

⁴⁸Further, *any* SIM in this setting can be written as a monotonic transformation of the linear SIM of the form proposed here.

5. Operational Income Measurement

So far, the primary concern of our exploration has been the existence of a new classical income measure. In this section, we explore some desirable features of the measure, taking the measure's existence as given. In particular, we investigate how additional axioms may allow the measure to become a linear aggregation of simpler sub-components.

This is important because typical income measures do have the property of linear aggregation. In practice, this feature of income measurement concerns two important dimensions. First, the analysis of an entity is often decomposed as the sum of unconsolidated components. Second, a major problem in accounting is the role of periodicity, that is, how income and its components are allocated into each individual periods.⁴⁹ However, unlike previous approaches which assume periodicity upfront, we recover these features from axioms imposed on the underlying preferences.⁵⁰

We proceed in two steps. First, we introduce a weak independence axiom which, along with other assumptions, yields a implicitly linear representation of the new classical measure. This allows an interpretation of the income measure as linearly aggregated over time. Within this interpretation, the time series of periodic income, cumulative income, and total income measures are meaningful and each measures are linked intertemporally through an income innovation term which resembles an accrual system.

Second, we introduce a strict independence axiom which allows the new classical income as a linear aggregation of sub-measures. We use this setting to explore the idea that some (but not all) sub-measures may be based on some external reference point such as market values. As a result, the new classical measure may be uniquely identified by an anchoring on market value.

5.1. Periodic Income Measures

For notational simplicity, we shall restrict our attention to linear aggregation over a summary income measure, although the argument extends to line-item income measure in a similar manner. We first derive a basic implicitly linear representation result, which is then developed into a form that resembles a time-series of income with an accrual system, at least in an abstract sense. To simplify notations, we omit now the dependence of the preference and the income measure on W .

5.1.1 Additive income measure

We now interpret each alternative in X , represented by a vector of $x = (x_1, \dots, x_N)$ in $(\underline{x}, \bar{x})^N$, as the collection of various operational attributes of each sub-component. This (somewhat vague) notion of separability may permit a parsimonious representation of the preference. Here, we think of each dimension of X as representing some operational characteristics of each choice that are somehow "separated" from others. One may interpret different sub-components of choice as different subsidiaries at a point in

⁴⁹This accounting concept stands in contrast with the natural economic intuition that output is accounted for only when it is finished. The reporting period in accounting, is not a function of the production technology but an artifact of the accounting art.

⁵⁰Linear aggregation is a prevalent feature of accounting axiomatization. See, for example, Ijiri (1964, chapter 6). In this respect, our approach differs from several other authors. For example, Moonitz (1961) imposes: "Postulate A-4. Time Period (including specification of the time period). Economic Activity is carried on during specifiable periods of time. Any report on that activity must identify clearly the period of time involved." Mock (1976) (chapter 3) proposes an axiomatization of wealth measurement. We contrast with the latter approach in that we start from a more basic ordinal choice-theoretic setting, rather than directly from a cardinal axiomatization, which needs to be clearly explained. More recently, Gaa (1988) (p.75) discusses the imposition of rational preferences in the context of organizations.

time or different time periods of a single entity. In this section, we focus on a time-period interpretation. In accounting theory, the allocation of performance and flows across periods is a crucial problem; it supposes that a complex production problem can be thought of as occurring separately across different time horizons, which can then be linearly aggregated. For simplicity, we assume that the time horizon is bounded (or at least can be approximated by a bounded time horizon).

The following axioms are key to construct an additive representation of a completed preference.

A-6. (non-satiation) The set of alternatives X is a set of the form $(\underline{x}, \bar{x})^N$. For any $x \in X$, one can find $x' \in X$, arbitrarily close to x , such that $x \prec x'$.

A-7. (weak independence) There exists \leq_c , a completion of \leq that verifies **A-1.** to **A-4.** (i.e. a SIM), such that: if $x \stackrel{\Delta}{=} x'$, then $x \stackrel{\Delta}{=} \alpha x + (1 - \alpha)x'$ for any $\alpha \in \mathbb{R}$ such that $\alpha x + (1 - \alpha)x' \in X$.

Axiom 6 is a standard axiom in choice theory. It implies that one can always strictly improve over an alternative with a small change of its characteristics. Axiom 7 is one of the weakest requirements that allows some form of linearity in the representation of the preference. It assumes that indifference (when information is hard) between two alternatives implies indifference among a mixture over these alternatives. It is related to the betweenness axiom (see Starmer (2000), p.344-345) in non-expected utility theory.⁵¹

Proposition 7. Under **A-1.** to **A-7.**, there exists a SIM that can be decomposed (implicitly) as follows, for $x \in X$,

$$I(x) = \sum_{i=1}^N x_i(1 + \alpha_i(I(x)))$$

where: x_i is i^{th} component, $\alpha_i(I(x))$ is an accounting adjustment term.

Proof: From Proposition 6, there exists a SIM which we denote $I(x)$. For each real number \hat{I} , we denote $E_{\hat{I}} = \{x | I(x) = \hat{I}\}$. Non-satiation implies that $E_{\hat{I}}$ is a subset of an hyperplane; if not, the preference would imply indifference over X . Now, note that by Axiom 7, the set $E_{\hat{I}}$ is exactly a hyperplane. We can then write the elements in the hyperplane as follows:

$$\sum_{i=1}^N \beta_i^{\hat{I}} x_i = A_{\hat{I}}$$

Note $A_I = A_{I'}$ must imply $I = I'$ by transitivity. Then, if there exists \hat{I} such that $A_{\hat{I}} = 0$, we can normalize the SIM such $A_0 = 0$. The case with $\hat{I} = 0$ follows by construction. If $\hat{I} \neq 0$, we can rewrite the above equation as follows:

$$\sum_{i=1}^N \frac{x_i \beta_i^{\hat{I}} \hat{I}}{A_{\hat{I}}} = \hat{I}$$

Define $\alpha_i(\hat{I}) = \frac{\beta_i^{\hat{I}} \hat{I}}{A_{\hat{I}}} - 1$. This is the decomposition that we propose.

Q.E.D.

⁵¹In fact, Axiom 7 is a less restrictive version of the betweenness axiom, since we are not interested here in the continuity of the representation, and do not assume that indifference occurs “between” the alternatives.

The SIM is now an implicitly linear combination of a finite number (N) of sub-components. We can interpret each component as a sub-measure for each time-period: a periodic income measure. A key feature of the sub-measure $x_i(1 + \alpha_i(I(x)))$ is that it depends on the overall measure $I(x)$, via the functions α_i , which represents the relative importance of per-period performance. So technically, the overall income measure is only implicitly defined.⁵²

5.1.2 Time series of income measures

As the implicitly linear measure I is viewed as a sum of a time-series, we now further develop the *evolution* of the time series to illustrate its resemblance to the time-series evolution of an accounting report. Notice here, each time-specific attribute (x_i) is weighted by its relative importance to the aggregate income ($I(x)$) before being linearly aggregated into the total income measure over the complete horizon $I(x)$. However, the total income, thus the weights for each periodic income measure, is not realized when each time period ends. If each weighted income is viewed as accounting income for the corresponding period, this introduces a “tentativeness” in each period’s income number. Operationally, one can posit an accounting process where an estimate of $I(x)$ is made, say \hat{I} , and the accountant can *tentatively* compute $x_i(1 + \alpha_i(I(x)))$ and adjust the realized choice/outcome x_i to obtain a “true” income measure for the period. Of course, as the entity moves from one period to the next, the estimate of total income may change and all previous and subsequent weights will need to be adjusted. One can view this process of estimation and re-estimation as an (abstract) form of the more recognizable accounting accrual process.

To elaborate more on the process, let us define, at any given period j , h^j the information available in this period. We assume that this information includes any previous realization (x_1, \dots, x_j) . Then, we define as θ_j , a function that associates each h^j to a (possibly conventional) forecast $(x_1, \dots, x_j, x_{j+1}^j, \dots, x_n^j)$, where (x_1, \dots, x_j) is a past realization and $(x_{j+1}^j, \dots, x_n^j)$ is the actual forecast.⁵³ Each element of this vector is denoted θ_i^j , $i \in \{1, 2, \dots, n\}$, where i is the income period and j denote the period when the forecasts are made. For simplicity, we drop the dependence on x of the income measurement.

Now we can define several income measures based on accounting process: (1) an estimated total income at time period i : I_i ; (2) an estimate of i^{th} period income measure at time period j : I_i^j ; (3) a sum of past periodic income up to period i : I^i , and (4) an revised sum of past income up to period i based on information in the i^{th} -period: I_{updated}^i . They are formally derived as follows (including an important accounting adjustment construct Δ_i^j).

Definition 8. (i) In each period i , the estimated total income measurement is I_i :

$$I_i = \sum_{k=1}^n \theta_k^i (1 + \alpha_k(I_i))$$

(ii) In each period j , the estimate of i^{th} -period income measure is I_i^j :

$$I_i^j = \theta_i^j (1 + \alpha_i(I_j))$$

⁵²This representation is an element of the Chew-Dekel class of utility representations. Note that when the preference is incomplete, a ranking by the SIM will not necessarily imply a weak preference.

⁵³The nature of this forecast is similar to a summary income measure and may be constructed in particular using statistical information (then θ_j is interpreted as an estimator).

So the current income I_i^i is

$$I_i^i = \theta_i^i(1 + \alpha_i(I_i)) = x_i(1 + \alpha_i(I_i))$$

(iii) The historical accumulated income up to period i , I^i :

$$I^i = \sum_{k=1}^i I_k^k$$

(iv) The “continuously updated” accumulated income up to period i , $I_{updated}^i$:

$$I_{updated}^i = \sum_{k=1}^i I_k^i$$

(v) The adjustment to period i 's income based on the information of period j (i.e., income adjustment) Δ_i^j :

$$\Delta_i^j = x_i(\alpha_i(I_j) - \alpha_i(I_i))$$

First, at period i , the vector of actual choices made, (x_1, \dots, x_i) , are known, as well as information to make forecasts $(\theta_i^{i+1}, \dots, \theta_i^n)$. Given these forecasts, it is possible to recover the (estimated) total income over the entire time horizon based on the information available at period i . This estimated total income is denoted I_i and is analogous to a forward-looking value estimate of the entity, such as an equity market value. Note that the recovery of I_i is implicit since it depends on the weights associated to each time period, which depend on the total performance I_i .⁵⁴ Second, we can extract from I_i a particular measure of periodic performance, I_i^i , calculated using the weight associated to the forecasts at period i . This performance measure corresponds to the standard periodic income measurement published by corporations and is possible despite a weak separation between periods. Third, summing all past periodic income, we can obtain a measure of accumulated earnings up to period i . As an alternate accumulated measure, $I_{updated}^i$ uses current information to update all past income numbers before summing them up. Either version of the accumulated measure is analogous to a backward-looking estimate of the entity value, such as retained earnings account in the book-value equity.⁵⁵ Fourth, in order to keep track of changes due to the arrival of new information, we define corrections to the income of period i at period j as Δ_i^j . The following results tie together each periodic measurement concept.

Proposition 8. *The following relations hold:*

(i) *Interim aggregate income and “continuously updated” accumulated income converge to fundamental value: $I_{updated}^n = I_n = I$.*

⁵⁴An example of this setting is well-known in corporate finance. Deriving the value of equity requires knowledge of the proper discount rate, which can be computed from the weighted average cost of capital (WACC). However, the WACC takes as an input the value of equity and thus the discount rate is implicit.

⁵⁵In GAAP, under certain circumstances, restatements of past earnings are required. Other types of adjustments, such as “cumulative effects of accounting changes,” achieve the same effect by adjusting current income instead of restating previous income.

(ii) The “continuously updated” accumulated income is the historical accumulated income plus a cumulative adjustment term

$$I_{updated}^i = \sum_{k=1}^i I_k^k + \sum_{k=1}^i \Delta_k^i = I^i + \sum_{k=1}^i \Delta_k^i$$

(iii) Total income is the sum of all historical periodic income with all adjustments to periodic income:

$$I = \sum_{k=1}^n I_k^k + \sum_{k=1}^n \Delta_k^n$$

Proposition 8 shows a connection between each measurement concept introduced previously. Both the forecasted aggregate income I_n and the cumulative income $I_{updated}^n$ will converge to the (fundamental) total income over the complete horizon I ; so that it is possible to think about both measurements as intermediate steps, given incomplete knowledge of future performance, toward total income. Next, we show that total income can be recovered from our construction of periodic income I_i^i , given that corrections to past income (such as the “true-up” entries) are appropriately considered.

5.2. Income Measurement Anchored on Market Value

In this section, we wish to uniquely identify a summary income measure via an anchoring on market value reference (akin to the Monetary Unit assumption in traditional accounting theory). We begin by introducing the typical (strong) independence axiom to replace the weak independence axiom.

A-8. (independence) Suppose x, x', x'' in X and $\alpha \in \mathbb{R}$ such that $\alpha x + (1 - \alpha)x''$ and $\alpha x' + (1 - \alpha)x''$ in X . Then: $x \stackrel{\Delta}{\sim}_c x'$ implies that $\alpha x + (1 - \alpha)x'' \stackrel{\Delta}{\sim}_c \alpha x' + (1 - \alpha)x''$.

Axiom 8 is the strongest of the axiom that we need to add to the theory, and we do not intend to present it as a truthful representation of reality but rather as an approximation that may be valid in certain cases. The axiom requires the activity of each sub-component to be independent with respect to the decision problem of the entity. When Axiom 8 is satisfied, the choice of the entity can be characterized as a simple sum-of-the-parts analysis.

Proposition 9. Under A-1. to A-8., there exists a summary income measure that can be written as follows, for $x = (x_1, \dots, x_n) \in X$,

$$I(x) = \sum_{i=1}^N \eta_i x_i$$

where $\eta_i > 0$ for all i . And any alternative summary income measure \tilde{I} is an equivalent measure if and only if there is $v > 0$ such that $\tilde{\eta}_i = v\eta_i$ for all i .

Proof: The proof follows from a simple geometric argument (since we are on a subset of \mathbb{R}^N). We consider the case $N = 2$, and then generalize. Let $I_1 > I_2$ two possible utility levels in the SIM representation and, E_{I_1} and E_{I_2} be two different hyperplanes as constructed in the proof of Proposition

7. Pick x_1, x'_1 in E_{I_1} and x_2, x'_2 in E_{I_2} , four different points. Then:

$$y = \alpha x_1 + (1 - \alpha)x_2 \stackrel{\Delta_c}{=} y' = \alpha x_1 + (1 - \alpha)x'_2 \stackrel{\Delta_c}{=} y'' = \alpha x_2 + (1 - \alpha)x'_2$$

From Thales theorem, (yy') is a hyperplane (for now a line) that is parallel to (x_2, x'_2) and $(y'y'')$ is parallel to (x'_1, x_1) . Therefore for y, y' and y'' to be aligned, it is necessary that (x_1, x_2) and (x'_1, x'_2) be parallel. To generalize the result, note that the same construction can be done by choosing the points (x_i) to be a basis for each hyperplane.⁵⁶

Proposition 9 shows two main results. First, income measure is a linear combination of its sub-components. This kind of decomposition into various sub-components is ubiquitous in accounting as well as in financial analysis. Second, one can modify any accounting system by multiplying all weights by a constant, thus what is important is the relative performance of each component, not its absolute performance.⁵⁷ See a related scaling idea in Antle, Demski and Ryan (1994).

The key to the result is, of course, the independence Axiom: the performance of each subsidiary must not interact so that a change in performance impacts the preference of the entity. As a result, each performance will have a fixed operating weight (η_i) which measures the importance of the subsidiary for the complete entity. While one may interpret each dimension of X as a subsidiary, it may also represent a source of financial information.⁵⁸

The second result also implies that the income measure, as such, is unique up to an affine transformation. Now we attempt to uniquely identify the income measure. We introduce, by Axiom 9, the notion of a numeraire bundle as an anchor for the income measure.

⁵⁶A proof of the result can be found in Dubra, Maccheroni and Ok (2004) (Proposition 3, p.125), with an additional continuity axiom and a weaker version of Axiom 8, in that it assumes only convex combinations and can be assumed directly on the incomplete preference. Without completeness, this representation is also known as an Aumann (1962) utility function. The authors also show (p. 123) that there exists a LIM of the form considered previously, although the number of items does not have to be finite.

⁵⁷Although it simplifies the task of representing the choice of the organization, this representation still yields new puzzles which are not considered here. What performance measures should be chosen in order to separate the organization into sub-components? We show here that, to be consistent, this must be done with the objective of preserving our set of axioms, and in particular Axioms 7 and 8. A proper decentralized accounting system then should try to group together sub-components that are related in order to use a simple decomposition.

⁵⁸The regularity that occurs in a linear aggregation allows to consider valuation multiples, which are relative notions of value. Let us consider two entities, which are comparable in the sense that they face the same decision-theoretic problem; however, their internal accounting system (which is only a conventional representation) may differ. We are interested in knowing whether one of the entities achieves a more favorable outcome than the other entity (this is a well-defined question since they face the exact same problem). Of course, the analysis supposes the two entities to be exactly similar, which is the main theoretical assumptions underlying comparables. Suppose as well that the first (resp. second) entity achieves a plan x' (resp. x'') and uses a linear Unitary Income Measure $I(x)$ (resp. $\tilde{I}(x)$). We may write their performance ratio as follows (applying Proposition 9):

$$\frac{I(x)}{\tilde{I}(x)} = \frac{\sum_{i=1}^N \eta'_i x'_i}{\sum_{i=1}^N \eta_i x_i} = v \frac{\sum_{i=1}^N \eta_i x'_i}{\sum_{i=1}^N \eta_i x_i}$$

The performance measurement of a comparable entity will always be *proportional* to the performance measurement of another entity. The factor of proportionality captures differences in the accounting convention. The theory predicts that when comparing two similar entities, although their performance schemes may differ, it is possible to use a benchmark factor of proportionality. An example of this reasoning can be found in the various methods of valuation by multiples. In these methods, one accounting indicator is used to proxy performance (often market price and enterprise value), and the other is used to proxy for operations (often operating earnings, cash flows or revenue). By repeated comparisons, and sufficient knowledge of the external and internal environment, it is possible for the analyst to measure v , and thus to obtain a *relative* pricing notion across firms; the intuition is that each comparable entity is measured in terms of the accounting process of a representative firm.

A-9. (standard price) There exists a price vector p for a bundle vector $x^f \in X$.

In our framework, the numeraire is a reference (or standard) price, not necessarily an effective exchange price; a well-known example of the use of a numeraire includes the various criteria for accounting financial assets, or the use of EVA for the valuation of the subsidiaries inside the firm. In the first case, the numeraire is some standard exchange price. In the second case, the numeraire is the current market risk. However, an internal subsidiary is never traded and thus there is no arbitrage behind the derivation of the effective interest rate. One may also view the (current) market capitalization of the entity as a benchmark standard price for the entity, although all the equity could not possibly be traded simultaneously without affecting prices.

Proposition 10. *Given A-1. to A-9., there is always a unique SIM such that $I(x_f) = p$.*

Proof: The class of all summary income measures can be written as (where η_i is one particular representation):

$$I(x) = \sum_{i=1}^N v\eta_i x_i$$

Evaluating at x^f :

$$I(x^f) = \sum_{i=1}^N v\eta_i x_i^f$$

This yields a particular SIM, denoted I_f and defined as:

$$I_f(x) = \sum_{i=1}^N \frac{I(x^f)}{\sum_{j=1}^N \eta_j x_j^f} \eta_i x_i$$

The Summary Income Measure I_f satisfies $I_f(x_f) = p$. To prove uniqueness, suppose there is another \tilde{I}_f that also satisfies this relation, then it must hold that $\tilde{I}_f(x) = vI_f(x)$ for all $x \in X$. This must hold in particular at x^f which implies that $v = 1$.

Q.E.D.

An entity may use *one* market price in order to anchor its accounting process, even though there is no effective exchange. This representation has two main advantages. First, the use of a market price allows the uniquely select a particular mode of representation; it allows an accountant to resolve the problem of multiplicity of representations. One can speak about “absolute” value only in the sense that it facilitates the choice of a unique representation among many equivalently qualified ones. Second, a money-value based representation is simple to interpret. The entity will measure its state of affairs *as if* it had access to a liquid market where it could trade the reference bundle. Given this reduction, it is possible to think about the organization as a portfolio consisting the reference good and the rest of the activities.

We denote this anchoring use of market prices as conventional mark-to-market. Unlike two previous forms of mark-to-market (perfect and imperfect), the conventional mark-to-market no longer reveals any information on preferences. It is a weakest use of market prices because it merely serves as an extra step of picking a particular summary income measure among all equivalent summary income measures, It may still be very useful as a communication and coordination device (which is of course outside the

scope of our axiomatization). Interestingly, the consideration of the reference good yields a pricing equation for all dimensions in the accounting process. The accounting measurement is made cardinal and unique by the use of a reference bundle. This occurs despite the fact that informative markets do not necessarily exist.⁵⁹ It is important to note that only *one* reference price is used as the anchor. The rationale for using more than a single price is analyzed in the next Corollary.

Corollary 1. *Consider a bundle x^g , there exists only one price p_g such that one can find a SIM with reference (x_f, p_f) such that $I(x^g) = p_g$.*

While there is a complete freedom in choosing the first reference bundle, there is no degree of freedom in choosing a second reference bundle; the addition of a second bundle does not yield additional information. In other words, a summary income measure may never gain from marking to market any number of bundles, and may even, in doing so, disrupt its internal reporting process (if internal and external relative prices do not coincide). Intuitively, prices taken from the outside environment are used only for calibration purposes but do not represent actual trades.

To summarize, we show that mark-to-market is possible, and may be desirable in a decentralized organization, but only for benchmarking purposes and on a very limited scope. Excessive (or even complete) mark-to-market is however undesirable in that it may conflict with the purpose of the entity, in particular when there are substantial differences between market prices and internal shadow prices.

6. Concluding Remarks

In this paper, we provide an integrated theory of income measurement, unifying several schools of thought into a common axiomatic framework. To do so, we put the focus on the income statement, assuming at the outset that the perimeter of the entity (and thus the balance sheet) is clearly apprehended. This simplification remains somewhat unsatisfying, in particular given that accounting entities have grown in size and complexity. An understanding of the proper delineation of the numerous ways of constructing an accounting entity is key in trying to step beyond the income statement and try to explain what determines the preferences and choices of the entity. This fundamental question would lead to a deeper understanding of the balance sheet. Which is the proper perimeter of the Entity? How do we define it? Which set of contracts and in what stage would be included in the balance sheet?⁶⁰ How does the income statement affect it and how does it affect the income statement? How should the components of an entity, and their relative balance of power affect the determinants of income?

References

Alexander, S. S., Bronfenbrenner, M., Fabricant, S. and Warburton, C.: 1973, *Five Monographs on Business Income*, Accounting Classic Series, Lawrence, KS: Scholar Books.

⁵⁹As such, the market enters the internal prices as a convention, not as an effective or potential trading price. Then, variations in the price of the reference good may create variations in the accounting system without a clear economic analogue, which may be disruptive for the process of accounting itself. This is an important issue when using (conventional) mark-to-market accounting, and that was often pointed out in the context of inflation (since the mid-seventies) but also for the valuation of illiquid portfolios in the financial industry.

⁶⁰An early work on the balance-sheet using a contract perspective is Ijiri (1980).

- Antle, R., Demski, J. S. and Ryan, S.: 1994, Multiple sources of information, valuation, and accounting earnings, *Journal of Accounting, Auditing and Finance* **9**(4), 675–696.
- Aumann, R. J.: 1962, Utility theory without the completeness axiom, *Econometrica* **30**(3), 445–462.
- Baxter, W. T. and Davidson, S.: 1962, *Studies in Accounting*, Institute of Chartered Accountants.
- Beaver, W. H. and Demski, J. S.: 1979, The nature of income measurement, *Accounting Review* **54**(1), 38–46.
- Boulding, K. E.: 1950, *A Reconstruction of Economics*, John Wiley and Sons.
- Canning, J. B.: 1929, *Economic Theory of Accountancy: a critical analysis of accounting theory*, Ronald Press, New York, NY.
- Chambers, R. J.: 1966, *Accounting, Evaluation and Economic Behavior*, Prentice Hall.
- Chambers, R. J.: 1972, Measurement in current accounting practice: A critique, *Accounting Review* **47**(3), 488–509.
- Christensen, J. and Demski, J.: 2003, *Accounting Theory: An Information Content Approach*, McGraw-Hill.
- Coase, R.: 1937, The nature of the firm, *Economica* **4**(16), 386–405.
- Curtis Eaves, B.: 1966, Operational axiomatic accounting mechanics, *Accounting Review* **41**(3), 426–442.
- Davidson, S., Green, D. J., Horngren, C. T. and Sorter, G. H.: 1964, *An Income Approach to Accounting Theory: Readings and Questions*, Prentice Hall.
- Debreu, G.: 1972, *Theory of Value : An Axiomatic Analysis of Economic Equilibrium*, Yale University Press.
- Demski, J., Fellingham, J., Ijiri, Y., Glover, J., Liang, P. J. and Sunder, S.: 2002, Some thoughts on the intellectual foundations of accounting, *Accounting Horizons* **16**(3), 157–168.
- Demski, J. S. and Sappington, D. E. M.: 1990, Fully revealing income measurement, *Accounting Review* **65**(2), 363–383.
- Dubra, J., Maccheroni, F. and Ok, E. A.: 2004, Expected utility theory without the completeness axiom, *Journal of Economic Theory* **115**(1), 118–133.
- Edwards, E. O. and Bell, P. W.: 1961, *Theory and Measurement of Business Income*, University of California Press.
- Feltham, G.: 2006, Joel S. Demski: A Leader In Accounting Scholarship, Springer, Norwell, MA, chapter 1, pp. 1–31.
- Fisher, I.: 1930, *The Theory of Interest*, New York: MacMillan.
- Fudenberg, D. and Tirole, J.: 1991, *Game Theory*, MIT Press.
- Gaa, J. C.: 1988, *Methodological Foundations of Standardsetting for Corporate Financial Reporting*, Prentice Hall.
- Harsanyi, J. C.: 1967, Games with incomplete information played by 'bayesian' players, i-iii. part i. the basic model, *Management Science* **14**(3), 159–182.
- Hicks, J.: 1946, *Value and Capital*, Clarendon Press.
- Ijiri, Y.: 1967, *The Foundation of Accounting Measurement*, Prentice Hall.
- Ijiri, Y.: 1975, *Theory of Accounting Measurement*, Vol. 10 of *Studies in Accounting Research*, American Accounting Association, Sarasota, FL.
- Ijiri, Y.: 1980, Recognition of contractual rights and obligations : an exploratory study of conceptual

- issues, FASB Research Report, Financial Accounting Standards Board, Stamford, Conn.
- Ijiri, Y. and Jaedicke, R. K.: 1966, Reliability and objectivity of accounting measurements, *Accounting Review* **41**(3), 474–483.
- Kreps, D. M.: 1988, *Notes on the Theory of Choice*, The Underground Classics in Economics, Westview Press, Boulder, Colorado.
- Lee, T.: 1974, *Income and Value Measurement: Theory and Practice*, The Nelson Series in Accounting and Finance, Nelson, Middlesex, Great Britain.
- Mandler, M.: 2002, Incomplete preferences and rational intransitivity of choice, *Games and Economic Behavior* **50**(2), 255–277.
- Marschak, J. and Miyasawa, K.: 1968, Economic comparability of information systems, *International Economic Review* **9**(2), 137–174.
- Mattessich, R. V.: 1995, *Critique of Accounting*, Quorum Books.
- Meade, J. E. and Stone, R.: 1941, The construction of tables of national income, expenditure, savings and investment, *Economic Journal* **51**(202/203), 216–233.
- Mock, T. J.: 1976, *Measurement and Accounting Information Criteria*, American Accounting Association.
- Moonitz, M.: 1961, *The Basic Postulates of Accounting*, American Institute of CPAs.
- Ok, E. A.: 2002, Utility representation of an incomplete preference relation, *Journal of Economic Theory* **104**(2), 429–449.
- Parker, R. and Harcourt, G.: 1969, *Readings in the Concept & Measurement of Income*, Cambridge University Press.
- Paton, W. A.: 1922, *Accounting Theory*, New York, NY: Ronald Press.
- Paton, W. and Littleton, A.: 1940, *An Introduction to Corporate Accounting Principles*, American Accounting Association, Sarasota, FL.
- Peleg, B.: 1970, Utility functions for partially ordered topological spaces, *Econometrica* **38**(1), 93–96.
- Savage, L. J.: 1972, *The Foundation of Statistics*, Dover Publications, New York, NY.
- Sewell Bray, F.: 1966, *Accounting Postulates and Principles*, Modern Accounting Theory, Prentice-Hall. ed. Backer, Morton.
- Shubik, M.: 1978, On concepts of efficiency, *Policy Sciences* **9**(2), 121 – 126.
- Stamp, E.: 1988, *Edward Stamp: Later Papers (Foundations of Accounting)*, Taylor & Francis.
- Starmer, C.: 2000, Developments in non-expected utility theory developments in non-expected utility theory: The hunt for a descriptive theory of choice under risk, *Journal of Economic Literature* **38**(2), 332382.
- Sterling, R.: 1970, *Theory of the Measurement of Enterprise Income*, Scholars Book Co.
- Sunder, S.: 1996, *Theory of Accounting and Control*, South-Western College Publishing.
- Szpilrajn, E.: 1930, Sur l'extension de l'ordre partiel, *Fund. Math.* **16**, 386–389.
- Vatter, W. J.: 1966, *Corporate Stock Equities*, Modern Accounting Theory, Prentice-Hall. ed. Backer, Morton.
- Watts, R. L. and Zimmerman, J. L.: 1979, The demand for and supply of accounting theories: The market for excuses, *Accounting Review* **54**(2), 273–305.
- Zeff, S. A. and Keller, T. F.: 1964, *Financial Accounting Theory : Issues and Controversies*, McGraw-Hill.