The Economic Consequences of Expanding Accounting Recognition*

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Abstract

By its recognition rules, traditional accounting system excludes measuring economic events that are hard-to-measure or stemming from future activities. The increasing use of fair value accounting expands the accounting recognition. We investigate the economic consequences to the reporting firm of an expanding recognition scope in accounting measurements. Using a simple model of endogenous investment whose payoffs are measured by either a restrictive (Partial accounting with high precision) or an expanded recognition rule (Full accounting with low precision), we show that, in the process of expanding accounting recognition, firms' internal investment efficiency and external share-price risk premium may *not* necessarily be a trade-off. In particular, we show that the consequences of moving from Partial to Full accounting depend on the investment environment (e.g., growth prospects) as well as the inherent measurement characteristics (e.g., measurement precision). For example, even with low measurement precision, Full accounting may generate a lower risk-premium in the firm's share price than Partial accounting. More surprisingly, an expanded recognition may lead to a higher investment efficiency and a lower risk-premium at the same time. The underlying driving force is that endogenous investment choices make endogenous the total uncertainty of the firm's cash flows as well as the resolution of the uncertainty due to the accounting report.

Key words: accounting measurement systems; measurement scope; accounting choice

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1 Introduction

From the dual perspectives of internal investment efficiency and external stock price risk premium, this paper investigates the economic consequences to the reporting firm of expanding the recognition scope in its accounting measurements. Within the accounting measurement structure, a crucial recognition issue is whether or not accounting excludes, from its measurement consideration, certain economic activities potentially value-relevant to the firm.[1] Some recognition criteria exclude expected future economic benefits/sacrifices that are inherently hard to measure or lack verifiable evidence even if they have already been triggered by past events (e.g., expected benefits of current labor forces/human capital, internally generated goodwill, or certain contingent economic liabilities). Some others exclude expected benefits/sacrifices originating from anticipated future events (e.g., anticipated but not-yet-incurred loan losses).[2] Over the past decades, the scope of accounting recognition has been expanding to include more and more hard-to-measure future economic events, mostly through the use of fair value accounting.[3] The expansion is stated/justified to be a response to the increasing demand from investors for more value-relevant information about the future cash flows of an entity, particularly when much of the firm value emanates from future events and is thus hard to measure. These future events are precisely those excluded from the traditional financial statements.

Within the accounting measurement structure, the expansion in recognition takes two primary forms. First, by changing the measurement method to fair value, accounting measurements reflect more changes in the anticipated future economic events involving the assets or liabilities already recognized on the balance sheet. The change in goodwill accounting from the amortization model to the impairment model is a suitable example here, so is the fair-value-option treatment on equity investment in affiliates (e.g., 20-50% equity stake).[4] Further, the expanded use of fair value may also

[1] As a workhorse in accounting standards, recognition governs the inclusion and exclusion of economic events, and the included events are then subject to prescribed measurement methods. It is reasonable to conclude that the recognition-measurement pair is the primary tool to implement any accounting idea. That is, any accounting idea is implemented through a prescription of particular recognition rules followed by measurement methods.

[2] These recognition criteria can trace their roots in the traditional accounting recognition of assets and liabilities, which emphasizes past transactions and measurability/reliability as the main drivers for recognition (see, for example, the definition of assets and liabilities in Statement of Financial Accounting Concepts No. 6, "Elements of Financial Statements," and the fundamental recognition criteria in Statement of Financial Accounting Concepts No. 5, "Recognition and Measurement in Financial Statements of Business Enterprises").

[3] In the joint FASB/ISAB conceptual framework project, the revised definition of assets and liabilities removes the explicit reference to past transactions, paving the way for more accounting recognition.

[4] Consider the typical triggers in a goodwill-impairment review, such as adverse changes in legal factors, regulation,

force the recognition of anticipated future events otherwise excluded from accounting measurements. For example, under the current US hedge accounting rules, a specific contractual commitment to purchase/sell products at a future date would be recognized if it is paired with an effective fair value hedge (such as a derivative contract). Second, the increasing use of expectations (i.e., estimates) may also induce a similar effect of expanding the recognition of future events. For example, consider the FASB's recent exposure draft on the contemplated change in the accounting for loan losses. Under the current "incurred loss" model, only expected losses over a specific time horizon that pass a "probable" threshold are recognized, resulting in a prohibition of recognizing loan losses on the same day of the loan origination. The newly proposed "current expected credit loss" (CECL) model removes the "probable" threshold and expands the time horizon of the expected losses. As a result, it is technically feasible that a loan loss may be recorded on day-one.[5]

Much of the expansion agenda has had its share of controversies. Critics of the expansion focus on the high inherent noise in the measurement of newly recognized accounting items and/or on the potential for manipulation and abuse, making these measures unreliable. For example, Kothari et al. (2010) justify the limited accounting recognition based on the idea that including cash flows with high uncertainty and contingent on future events are not suitable for accounting to serve the demand for audited financial statements by contracting parties. As a result, they are suspicious of the "survival value" of the income recognition rules based on "one-period change in fair-value-based net asset values" and caution "against expanding fair-value measurements to balance sheet items for which liquid secondary markets do not exist." In this light, the policy

and competition, a loss of key personnel, and an expectation that the reporting unit will be sold or disposed of. These triggers imply a significant change in the future events affecting the entity. In measuring the impairment, the current standard considers a quoted market price of the reporting unit as the best available measure, while other present values or earnings multiples can be used. Direct market-price drops (or those implied by earnings multiples), which typically precede or accompany an impairment of goodwill, are likely to reflect the expected losses from the profitable future events or actions which were previously expected (such as expansion options or synergies due to complementary operations).

[5] According to the exposure draft, the expected credit loss is defined as: "An estimate of all contractual cash flows not expected to be collected from a recognized financial asset (or group of financial assets) or commitment to extend credit." The proposed rules consider more forward-looking information than is permitted under the current U.S. GAAP. In particular, the proposed CECL model requires the estimated loan losses be based on relevant information about not only past events and current conditions (as required in the current incurred loss model) but also "reasonable and supportable forecasts that affect the expected collectability of the financial assets' remaining contractual cash flows." For example, in addition to evaluating the borrowers' current creditworthiness, the CECL model also requires an evaluation of the forecasted direction of the economic cycle. Because the expansion of the time horizon, the estimate makes use of the time value concept such that expected losses are discounted at the asset's effective interest rate. Further, estimates should reflect both the possibility that a credit loss results and the possibility that no credit loss results.

debate on fair value brings the theoretical attention to the fundamental distinction between the two opposite ends of the recognition consideration in accounting measurements: the traditional accounting model which focuses on the reliably-measured consequences of past actions, on one end, and on the other, the newer accounting model which gives full consideration of all consequences of both past and anticipated future actions. The distinction between these two measurement models can be economically significant for firms whose future cash flows are hard to measure and/or who are facing high growth prospects.

In this paper, we provide a simple model where such a distinction between measurements has both accounting and economic meanings. In particular, we build two alternative accounting measurements designed to highlight the difference described above. These alternative measurements generate informational differences in the resulting accounting measures. We then embed the accounting model into a standard economic model in which the accounting measurement choices may change both distributional and allocational efficiency in the economy. Specifically, we consider a risk-neutral firm making an investment decision, which exposes the firm to stochastic future cash flows. Along the measurement dimension, some cash flows are easy to measure (or pass an evidential threshold), while other cash flows are hard to measure. Before the realization of the cash flows, the firm must report a measured accounting value to risk-averse equity investors who determine competitively the share price of the firm. Economically, the accounting report resolves some uncertainty about the future cash flows and thus increases the share price by reducing the risk premium collectively required by investors (defined as the difference between the expected future cash flows and the price).

There are two alternative accounting regimes: *Partial accounting* measures the easy-to-measure future cash flows from the investment with high precision, whereas *Full accounting* measures all the anticipated future cash flows (i.e., both the easy-to-measure and hard-to-measure cash flows from the investment) with low precision. The key feature of the model is the fundamental difference in the informational properties induced by the two accounting regimes. This difference is especially large for (i) firms *with higher growth prospects*, which, in our model, are characterized by more hard-to-measure future cash flows, and (ii) firms *with higher profitability risks*, which are characterized by higher volatility of the same hard-to-measure cash flows. Investors understand such a structural difference and make rational inferences based on the reported accounting value to price the firm.

From the firm's point of view, the two accounting measures affect the risk premium and investment efficiency differently, leading to different economic consequences to the firm. Our model delivers the following results. When the recognition scope expands from Partial accounting to Full accounting,

1. the investment becomes more efficient for firms with high growth prospects;

2. the risk premium in the share price, measured either in dollars or in percentages, may be lowered regardless of the measurement noise of Full accounting. Further, the risk premium is more likely to become smaller under Full accounting for firms with higher future profitability risks; and

3. for firms with high growth prospects and high future profitability risks, it is possible that both the investment becomes more efficient and the firm's share-price risk premium becomes smaller at the same time even if Full accounting has a high measurement noise.

The first consequence is familiar and straightforward. Full accounting better aligns the accounting signal with the benefit from the investment. Therefore, it induces more efficient investment than Partial accounting for firms with high growth prospects where most of the cash flows from the investment are hard-to-measure and excluded from the recognition of Partial accounting.

The rest of the consequences are less intuitive and come from an *indirect* effect of the accounting measurements. While the accounting measurements directly change the risk premium by resolving part of a *given* level of the ex ante cash-flow uncertainty (i.e., direct effect), they also change the level of the ex ante cash-flow uncertainty to begin with (i.e., indirect effect), because the accounting measurements also change the firm's investment, which has an impact on the ex ante cash-flow uncertainty.[6] This indirect effect opens another channel through which the accounting measurements can affect the risk premium. Under either accounting regime, this *indirect* effect may dominate the direct effect, causing the risk-premium to be, counter-intuitively, always decreasing in the accounting noise. When the managerial myopia is not too low, the indirect effect makes the

[6] To give an analogy here, think of the ex ante cash-flow uncertainty as a whole pie, which consists of two pieces, one representing the resolved uncertainty by the accounting measurement, and the other representing the unresolved/remaining uncertainty (i.e., the risk premium). Given a certain size of the whole pie, a higher measurement noise would increase the size of the piece representing the risk premium. However, at the same time, the higher measurement noise would decrease the size of the whole pie by reducing the investment. As a result, the net impact of the higher measurement noise on the size of the piece representing the risk premium is unclear and depends on the dominant effect.

risk premium generally elevated under Partial accounting because the hard-to-measure cash-flow uncertainty magnified by the investment is left unchecked by the Partial accounting signal. As a result, *regardless of the noise level*, Full accounting, by virtue of measuring all the cash flows, may lead to a lower risk premium (than Partial accounting) in the presence of the indirect effect. More importantly, higher uncertainty in the hard-to-measure cash flows (i.e., higher future profitability risk) could strengthen this risk-premium distinction between Partial accounting and Full accounting by increasing the chance this case happens. Combining both the investment-efficiency and risk-premium results, in our model, a firm with a high growth prospect and a high future profitability risk can find Full accounting preferable even with a high measurement noise, a feature commonly criticized upon fair value measures. Thus, to these firms, the process of expanding accounting recognition does not necessarily generate a trade-off between the two concerns.

In our paper, the cash flow characteristics (easy-to-measure vs. hard-to-measure) present a challenge to measuring an entity's activities, because accounting must deal with the scope choice issue (i.e., inclusion or exclusion of the cash flow as a measurement object) in addition to other measurement issues (e.g., measurement precision). As such, our paper makes an attempt to explicitly model the expanding recognition scope in accounting measurements. Our paper's central accounting concern follows a broad theme in the modeling work on accounting measurement structure.[7] In the recent strands of this theme closely related to our paper, Dye (2002) views classification as a foundational accounting measurement function, and its possible manipulation has implications in the equilibrium accounting standards, which he terms, tellingly, "Nash" standards. Dye and Sridhar (2004) focus on accounting aggregation and the resulting trade-off between relevance and reliability. Along a similar line, Liang and Wen (2007) focus on input- versus output-based accounting measures and their differential effects on equilibrium investment. Among other studies highlighting the importance of accounting structure, Arya et al. (2000) revive the earlier linear algebra work on the double-entry bookkeeping structure into a modern light. Ohlson (1995) and Feltham and Ohlson (1995) bring valuation theory to clean-surplus accounting. Liang and Zhang

[7] There was an older literature on accounting properties such as its axiomatic structure (Mattessich 1964), algebraic representation (Butterworth 1972), objectivity and reliability (Ijiri 1975), and relevance and timeliness (Feltham 1972). This area remains less-explored. Professor Ron Dye attributed the lack of progress partially to Demski's (1973) "General Impossibility" observation "that Blackwell's (1951) theorem, as applied to accounting, indicates that in evaluating two non-comparable accounting information systems, one can always find a pair of decisions problems in which one information system is preferred for one decision problem and the other is preferred for the second decision problem" (page 52 in Dye 2002).

(2006) study the effects of flexible and rigid accounting regimes when firms face inherent or incentive uncertainties. Bertomeu and Magee (2009) model accounting-standard setters in a strategic setting and study how political pressures may affect the accounting regulation. Caskey and Hughes (2011) examine how different fair value measures affect the efficiency of debt contracts. Marinovic (2011) investigates the efficiency implications of three alternative accounting methods in the context of corporate acquisitions. Some other studies focus on the financial reporting quality choice (e.g., Dye and Sridhar 2007; Bertomeu and Magee 2011).

While our paper is mostly concerned with measurement structure, the economic trade-off is studied within an investment decision. Along this dimension, our paper is related to the literature on the real effect of accounting. The real effect literature, pioneered by Kanodia (1980), develops the notion that disclosure of accounting information has an impact not only upon market prices but also upon corporate production/investment decisions (e.g., Kanodia and Lee 1998; Beyer and Guttman 2011). Our paper follows this line of research and presents both direct and indirect (i.e., real investment) effects of accounting structure on the risk premium in share prices and allows us to evaluate accounting recognition using both internal investment efficiency as well as external share-price risk-premium. Given the foundational role of recognition in accounting, modeling the expanding recognition scope of accounting measurements enhances the real effect literature.

The results in our paper also have implications for the empirical work on the relation between accounting quality and cost of capital. Prior literature has documented mixed evidence on the association between earnings quality and cost of capital (e.g., Botosan 1997; Botosan and Plumlee 2002; Francis et al. 2004; Beyer et al. 2010). The comparative statics results of our model indicate that such factors as the managerial myopia (i.e., the extent the manager cares about the current share price vs. future share price), the future profitability risk, the intensity of the use of market-based accounting measures (i.e., Partial accounting or Full accounting), and the firms' growth prospect may help explain the mixed empirical findings. For example, our results imply that, if the managerial myopia is low (high), the cost of capital decreases (increases) in the accounting quality. Our results also imply that when the firms' growth prospect is high and/or their future profitability risk is not too large, it is less likely to have a positive relation between cost of capital and accounting quality for firms in the industries with less use of market-based accounting measures (i.e., Partial accounting) than in the industries with more use of market-based accounting measures (i.e., Full accounting).

The rest of the paper proceeds as follows. Section 2 describes the model. Section 3 presents the main analyses and results of the model, and Section 4 provides key discussions on the model assumptions, relation to cost-of-capital studies, and policy implications. Section 5 concludes the paper.

2 Model

A risk-neutral entrepreneur owns a technology, which requires an initial investment. Before making the investment, on date-0, the entrepreneur chooses between two accounting measurement systems, *Partial accounting* and *Full accounting*, for the firm she is going to establish. On date-1, the entrepreneur chooses her private investment $I \in \mathbb{R}^+$ to establish the firm. The chosen accounting system generates a public signal y on date-2^-. On date-2, the entrepreneur sells a fixed portion, $\beta \in (0,1)$, of her ownership in the firm to outside investors, and the market price P is determined based on all publicly available information. All cash flows are realized on date-3. We denote the total cash flows on date-3 by x. Figure 1 summarizes the sequence of events.

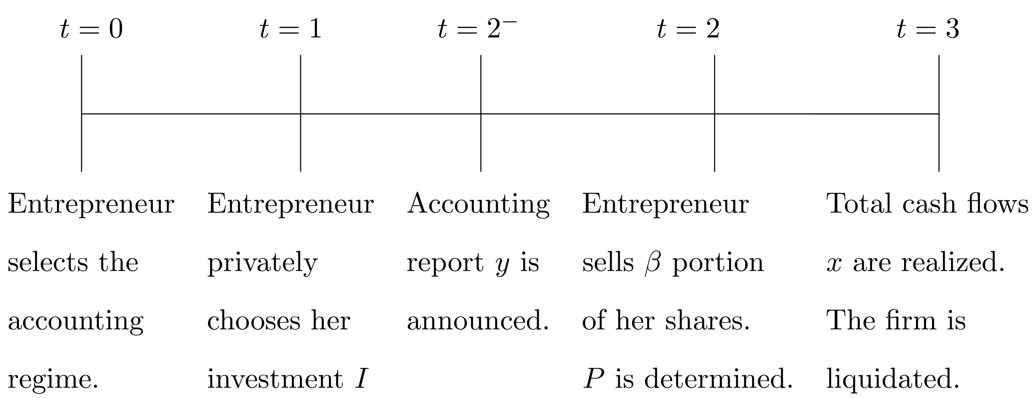

Figure 1. The time line of events

We next provide more details on the model.

2.1 Cash flows

Following prior literature (e.g., Dye 2002; Dye and Sridhar 2004), the date-1 investment I is assumed to be privately chosen by the entrepreneur and is not observable to outside investors.[8] The investment generates stochastic future cash flows, governed by the size of the investment I and the random state of nature captured by a random variable θ. We can interpret θ as the underlying profitability of the investment. For tractability, we assume the realized cash flows x are

$$x(I,\theta) = \theta I. \tag{1}$$

The entrepreneur does not observe θ before making the investment I but knows its prior distribution. The detail of the prior distribution is specified below.

2.2 Two accounting regimes

On date-0, two accounting measurement systems are available: *Partial accounting* and *Full accounting*. The selected accounting system generates a public accounting signal on date-2^-.

2.2.1 Partial accounting

The scope of the Partial accounting system is limited: it excludes from its measurement cash flows with hard-to-measure characteristics such as low precision, lack of evidence, and/or being associated with future activities. There is much detail here, but we capture the measurement-dimension distinction as follows. Suppose there are two components of the profitability variable θ:

$$\theta \equiv \theta^p + \theta^f$$

where θ^p (θ^f) is the profitability variable underlying the cash flows that are easy to measure (hard to measure), and

$$\begin{bmatrix} \theta^p \\ \theta^f \end{bmatrix} \sim N\left(\begin{bmatrix} \theta_0 \\ k\theta_0 \end{bmatrix}, \begin{bmatrix} V_\theta^p & 0 \\ 0 & V_\theta^f \end{bmatrix} \right). \tag{2}$$

[8] This can be a scenario where the entrepreneur takes her personal efforts to develop the project.

Therefore, the prior distribution of θ is normal with mean $(1+k)\theta_0$ and variance $V_\theta \equiv V_\theta^p + V_\theta^f$. The cash flow specification in (2) is designed to capture several important features.

- Growth Prospect: we use $k \in \mathbb{R}^+$, a commonly known parameter, to capture the relative size of the expected future cash flows which are hard to measure and excluded from the measurement consideration under Partial accounting. Because the hard-to-measure property can be caused by the association with future events, we will interpret the parameter k as the firm's growth prospect, although k may capture more.[9] In essence, a higher k reflects that a higher portion of the expected profitability/value of the firm is generated from the hard-to-measure/future-growth cash flows.

- Future Profitability Risk: we use V_θ^f to capture the volatility of the hard-to-measure/future-growth cash flows' profitability. For ease of exposition, we interpret this parameter V_θ^f as the firm's future profitability risk. It is intuitive that V_θ^f increases in k, $V_\theta^f \to +\infty$ as $k \to +\infty$, and the "per capita" V_θ^f is larger than V_θ^p (i.e., the volatility of the easy-to-measure cash flows' profitability). As such, we make the following assumption throughout our study.[10]

  ASSUMPTION: $V_\theta^f > (3/2 + k)V_\theta^p$.

- Independence: we assume the correlation between the two types of cash flows is zero, which reflects the underlying economic and measurement logic. Economically, one important reason for certain cash flows to be hard-to-measure is that they are subject to (future) economy-wide or industry-wide shocks (or systematic risk) which should be less correlated with the firm-specific factors underlying the easy-to-measure cash flows. Further, a strong (e.g., nearly perfect) correlation between the two types of cash flows would defeat the modeling purpose of differentiating the hard-to-measure cash flows from the easy-to-measure cash flows, because a strong correlation would make an accounting signal informative about θ^p become highly informative about θ^f, and thus the hard-to-measure cash flows would become the easy-to-measure cash flows.[11]

[9] For example, two firms from different industries can have the same growth prospect but one has more cash flows that are hard-to-measure, resulting in different k's in our model. Nevertheless, we use the growth prospect interpretation for the ease-of-exposition purpose.

[10] This particular parametric assumption focuses our analysis on the interesting parameter regions to highlight the key economic trade-offs. Relaxing this assumption will not change the main results of the paper qualitatively.

[11] While there are strong reasons supporting a model specification of zero-correlation, some correlation between the

Based on the above cash-flow structure, the Partial accounting signal, denoted by y^p, is a noisy measure of the easy-to-measure cash flows, that is

$$y^p = \theta^p I + \varepsilon_p,$$

where the accounting measurement noise $\varepsilon_p \backsim N[0, V_p]$ is independent of the profitability shock θ. Because accountants are not asked to measure $\theta^f I$, cash flows known to exist but very hard to measure, the Partial accounting measurement can be quite precise. That is, ε_p has a small variance V_p. This modeling choice is designed to be a reduced-form representation of the traditional accounting recognition rules that focus on assets-in-place and ignore measuring any future benefits that are too imprecise and/or from future yet-to-occur firm activities. The idea is that if accountants were, counterfactually, asked to measure the component of x related to θ^f, the variance of that measurement would have been too high or too unreliable to justify including it in the formal financial statements. In other words, the economic events, driven by random shocks captured by θ^f, are so unpredictable that they failed the accounting tests of "probable," "reasonable precision," or "more than likely," terms which can be discovered in many of the current standards. Therefore, they are excluded from the accounting recognition. Recall the loan-loss example from the introduction. Partial accounting shares the spirit of the current incurred loss model for loan losses, which deliberately ignores measuring losses to be expected beyond the short-term and those driven by business- or industry-cycle factors.

The potential problem of Partial accounting is, of course, the measurement is less comprehensive and less aligned with the economic benefit of the investment, which is what investors are interested in. This problem is more pronounced when the portion of cash flows excluded from the measurement are large (i.e., a large k in our model). It is important to note that, on date-2 when the firm price is determined, even though Partial accounting does not recognize the hard-to-measure cash flows captured by the variable θ^f, market participants still take such anticipated future cash flows into consideration in pricing the firm.

cash flows to be measured (θ^p) and the cash flows excluded from the measurement (θ^f) may exist in practice (e.g., there are some foreseeable future events). Technically, our results are robust to a generalization to a non-zero but small correlation between θ^p and θ^f.

2.2.2 Full accounting

Alternatively, if the entrepreneur selects Full accounting, the accounting signal, denoted by y^f, is a noisy measure of the total cash flows x, that is

$$y^f = x + \varepsilon_f = \theta I + \varepsilon_f = (\theta^p + \theta^f)I + \varepsilon_f,$$

where the accounting measurement noise $\varepsilon_f \backsim N[0, V_f]$ is independent of the profitability shock θ. The expression for y^f is meant to represent, in a reduced-form, the informational property of a comprehensive accounting signal, but not the measurement process *per se*.

In order to distinguish Full accounting from Partial accounting to capture the essence of the expansion in accounting recognition, we model Full accounting as separate and distinct from Partial accounting in two important ways. First, the Full accounting signal is centered around the total expected cash flows (i.e., $(\theta^p + \theta^f)I$), making the signal more aligned with the investment return than the Partial accounting signal. Second, we assume that the variance of the signal, V_f, has the following structure. With a slight abuse of notation, we will rewrite V_f as a function $V_f(k, v)$. In other words, the variance depends on the size of the growth prospect k and a generic parameter v. We visualize the construction of $V_f(k, v)$ as follows. Imagine the firm has a project whose future cash flows $\theta^p I$ are easy to measure. The Partial accounting would measure this project with y^p. In addition, the firm also has k additional projects whose future cash flows $\theta_i^f I$ $(i \in \{1, 2, ..., k\})$ are hard to measure. Each of the k projects can be measured with an accounting signal y_i: $y_i = \theta_i^f I + \varepsilon_i$, $i \in \{1, 2, ..., k\}$, with $E[\theta_i^f] = \theta_0$ and $\varepsilon_i \sim N(0, v)$. Intuitively, the parameter v reflects the "per capita" measurement noise of the hard-to-measure projects. Hence, if we can think of the Full accounting signal as an aggregate measure of the easy-to-measure project and the k hard-to-measure projects: $y^f = y^p + \sum y_i$, then the variance of y^f would become a function of k and v (i.e., $V_f(k, v)$).

Further, we assume the variance function V_f satisfies the following properties:

(i) V_f increases in both k and v;

(ii) $\lim_{k \to 0} V_f(k, v) \geq V_p$, $\lim_{v \to 0} V_f(k, v) \geq V_p$, and $\lim_{k \to +\infty} V_f(k, v) = \lim_{v \to +\infty} V_f(k, v) = +\infty$; and

(iii) $\frac{\partial (V_f(k,v)/(1+k)^2)}{\partial k} < 0$ (i.e., $\frac{\frac{\partial}{\partial k} V_f(k,v)}{V_f(k,v)} < \frac{2}{1+k}$).

The first property assumes that the measurement noise in y^f increases with the size of the hard-to-measure projects (i.e., k) and the "per capita" measurement noise of the hard-to-measure projects (i.e., v), consistent with our common intuition. The second property describes the intuitive boundary conditions on V_f. For example, the measurement noise in y^f is at least the same as that in the Partial accounting signal y^p. The third property reflects the idea of economies of scale in the sense that the "per capita" measurement noise decreases with the size of the hard-to-measure projects. In our constructive example, it simply means that as more hard-to-measure projects are measured, the firm's accounting system gets better at measuring each project and the variance of the aggregate measure does not increase proportionately (e.g., due to synergy effects).

Returning to the same loan-loss example, the total estimated credit losses under the proposed CECL model is likely to be constructed based on those obtained under the current incurred loss model with adjustments to include both the expected losses from the added time horizon and those below the "probable" threshold. Naturally, the resulting estimated loan losses would contain more noise when compared with those from the incurred loss model. In fact, the exposure draft acknowledges the demand the new method imposes. While our model abstracts away from the underlying detailed measurement process, the properties of any such summary accounting measures (such as the estimated loan losses from the CECL model) would be fairly represented by the y^f-specification above.

2.3 Entrepreneur's objective function and interim share price

Following prior literature (e.g., Stein 1989; Liang and Wen 2007; Einhorn and Ziv 2007), we assume the entrepreneur is interested in both the firm's current market price and the future cash flows. In particular, we assume, on date-2, after the accounting report (Partial or Full) is released, the entrepreneur must sell $\beta > 0$ portion of her shares in the firm (i.e., β portion of the claims on the total future cash flows x) to outside investors in the secondary market due to exogenous liquidity reasons, and keep the remaining $(1-\beta)$ ownership. Accordingly, the entrepreneur's objective is to maximize a weighted average of the date-2 market price and the firm's total future cash flows, net of the initial private investment cost $\frac{I^2}{2}$.[12] That is, the objective function on date-1 is (assuming a

[12] We model the stock-price incentive as coming from the liquidity needs of the current owners for simplicity. Another potential stock-price incentive in the literature may come from the presence of managerial stock-based compensation, which is mute in our model.

zero discount rate)

$$-\frac{I^2}{2}+\beta P+(1-\beta)x. \tag{3}$$

Here, β measures the extent to which the entrepreneur's investment is share-price motivated. Accordingly, we can interpret β as the managerial myopia. The larger the β, the more myopic the entrepreneur.

The firm shares are priced in a competitive rational capital market. Investors in the market are risk-averse and have a CARA utility function with risk-averse coefficient τ, that is,

$$U(W_i)=-\exp\left(-\tau W_i\right),$$

where W_i denotes the investor's wealth or consumption. Given the CARA utility function, following standard results in the literature, the market price P is the expected future cash flows minus a risk premium that is determined by the investors' perceived cash-flow volatility. We can express the market price in the following mean-variance form:

$$P=E[x|\Omega]-\beta\tau Var\left[x|\Omega\right], \tag{4}$$

where Ω is the publicly available information set to investors on date-2.[13] The first term in (4) represents the market's expected total future cash flows conditional on all available information, and the second term is the risk premium, which depends on the conditional variance (i.e., the unresolved cash-flow uncertainty), the risk-averse coefficient (τ), and the managerial myopia (β). Similar to that of Beyer (2009), a risk-neutral entrepreneur faces a risk-averse pricing of her firm. In Section 4, we provide some discussion on a setting where both the entrepreneur and investors are risk-averse.

[13] Consider a perfectly competitive market. The wealth of a typical investor i is $W_i=(x-P)D_i$, where D_i is the investor's demand of the firm's shares given price P. With the CARA utility function, the investor maximizes $E[W_i|\Omega]-\frac{\tau}{2}Var[W_i|\Omega]=(E[x|\Omega]-P)D_i-\frac{\tau}{2}D_i^2Var[x|\Omega]$. Taking the first order condition, we have $D_i=\frac{E[x|\Omega]-P}{\tau Var[x|\Omega]}$. Since β portion of the firm's shares is available for sale, the market clearing condition gives $\beta=\int_0^1 D_i di=\frac{E[x|\Omega]-P}{\tau Var[x|\Omega]}$. Thus, we have the market price $P=E[x|\Omega]-\beta\tau Var\left[x|\Omega\right]$.

3 Main Analysis

3.1 Equilibrium characterization

Before proceeding to the detailed analysis, we first define the equilibrium:

Definition 1 *An equilibrium relative to Ω consists of an investment decision $I(\cdot)$ and a perfectly competitive market pricing function $P(\cdot)$ such that,*

(i) given the pricing function $P(\cdot)$, the optimal investment $I(\cdot)$ maximizes $E[-\frac{I^2}{2}+\beta P+(1-\beta)x]$;

(ii) given the market's conjecture $\hat{I}(\cdot)$ on the entrepreneur's investment, the market pricing function $P(\cdot)$ satisfies $P = E[x|\Omega, \hat{I}(\cdot)] - \beta\tau Var\left[x|\Omega, \hat{I}(\cdot)\right]$;

(iii) the market's conjecture is correct. That is, $\hat{I}(\cdot) = I(\cdot)$.

The following proposition characterizes a linear equilibrium (i.e., the price is linear in the accounting signal) for both Partial accounting and Full accounting. We denote the equilibrium investment under Partial accounting and Full accounting by I_p and I_f, respectively.

Proposition 1 *There exists a linear equilibrium relative to $y \in \{y^p, y^f\}$, which is given by*

1. *for Partial accounting ($y^p = \theta^p I + \varepsilon_p$), the linear pricing function $P = E[x|y^p] - \beta\tau Var\left[x|y^p\right]$, where*

$$E[x|y^p] = b_p \cdot y^p + \left(\frac{V_p}{I_p^2 V_\theta^p + V_p} + k\right)\theta_0 I_p \text{ and } Var[x|y^p] = I_p^2 V_\theta^f + \frac{I_p^2 V_\theta^p V_p}{I_p^2 V_\theta^p + V_p}, \tag{5}$$

   and the equilibrium investment

$$I_p = \left[\beta b_p + (1-\beta)(1+k)\right]\theta_0, \tag{6}$$

   where $b_p = \frac{I_p^2 V_\theta^p}{I_p^2 V_\theta^p + V_p}$; and

2. *for Full accounting ($y^f = x + \varepsilon_f$), the linear pricing function $P = E[x|y^f] - \beta\tau Var\left[x|y^f\right]$, where*

$$E[x|y^f] = b_f \cdot y^f + \frac{V_f}{I_f^2 V_\theta + V_f}(1+k)\theta_0 I_f \text{ and } Var[x|y^f] = \frac{I_f^2 V_\theta V_f}{I_f^2 V_\theta + V_f}, \tag{7}$$

and the equilibrium investment

$$I_f = [\beta b_f + (1-\beta)]\,(1+k)\,\theta_0, \tag{8}$$

where $b_f = \frac{I_f^2 V_\theta}{I_f^2 V_\theta + V_f}$.

Proof. All proofs are in the Appendix. ■

For a given accounting measure $y \in \{y^p, y^f\}$, the entrepreneur's ex ante payoff or welfare on date-0, denoted by W as a function of her equilibrium investment $I \in \{I_p, I_f\}$, can be expressed as

$$\begin{aligned} W(I) &= E[-\frac{I^2}{2} + \beta P + (1-\beta)x] \\ &= E[-\frac{I^2}{2} + \beta(E[x|y] - \beta\tau Var\,[x|y]) + (1-\beta)x] \\ &= E[x - \frac{I^2}{2}] - \beta^2 \tau Var\,[x|y]\,. \end{aligned} \tag{9}$$

The first term in (9) is the expected total future cash flows net of the investment cost. It depends on the efficiency of the equilibrium investment. The second term in (9) measures the risk premium for which the entrepreneur needs to compensate outside investors. It depends on the conditional variance of the future cash flows (i.e., the unresolved cash-flow uncertainty). Therefore, the entrepreneur's accounting-choice decision depends on both the investment efficiency and the risk premium induced by the two accounting regimes, which leads us to focus our analysis below on these two primary economic factors. Proposition 1 shows that the equilibrium investment (I_p in (6) or I_f in (8)) depends on the managerial myopia (β), the market response coefficient (b_p or b_f), and the growth prospect k. In addition, this proposition also characterizes the conditional variance or the risk premium under the two accounting regimes, as shown in (5) and (7).

In the following, we first analyze how the accounting measures affect the equilibrium investment or investment efficiency. Then, we analyze how the accounting measures affect the conditional variance or the risk premium. In particular, we show that the accounting measures affect the conditional variance both directly (through uncertainty resolution) and indirectly (through the endogenous investment). Some counter-intuitive results arise when the indirect effect dominates the direct effect.

3.2 Investment analysis

In this section, we focus on the investment decision and analyze the induced investment efficiency under the two accounting regimes. Given the equilibrium in Proposition 1, the following lemma presents some comparative statics results regarding the equilibrium investment. As a benchmark, we label $I^* = (1+k)\theta_0$ as the first-best investment, as this would have been the optimal investment if the entrepreneur had no short-term share price incentive (i.e., $\beta = 0$). More efficient investment, in our context, means the investment is closer to I^*.

Lemma 1

1. *The equilibrium investment I_p (I_f) approaches $(1-\beta)(1+k)\theta_0$ as $V_p(V_f) \to +\infty$.*

2. *The equilibrium investment I_p approaches $[1+(1-\beta)k]\theta_0$ as $V_p \to 0$, and I_f approaches the first-best investment level $(1+k)\theta_0$ as $V_f \to 0$.*

3. *Under both accounting regimes, the equilibrium investment is higher when the accounting signal is less noisy and when the entrepreneur is less myopic (i.e., I_p (I_f) decreases in V_p (V_f or v) and β).*

Under both accounting regimes, the equilibrium investment is lower than the first best level. In other words, the entrepreneur under-invests in equilibrium. This is a standard result in the literature.[14] Due to the noise in the accounting signal, investors discount the accounting signal in pricing the firm, leading to the under-investment results. The under-investment problem is alleviated as the accounting signal becomes less noisy. Similarly, lower β indicates that the entrepreneur focuses less on the interim stock price and more on the future cash flows, reducing the under-investment incentive.

The structural differences between the two accounting regimes lead to the different equilibrium investment levels. Under Partial accounting, the accounting signal is only a noisy measure of the easy-to-measure cash flows but not the hard-to-measure cash flows. Therefore, the price does not respond to the incremental hard-to-measure cash flows for any incremental investment I. As a

[14] For example, in Dye and Sridhar (2004), the equilibrium investment level is always below the first best. Liang and Wen (2007) find a similar result that output-based accounting (similar to the two accounting measures in our paper) always induces under-investment by the firm.

result, the hard-to-measure cash flows do not provide any investment incentives through the price. Instead, they provide investment incentives only through the $1-\beta$ portion of the total cash flows belonging to the entrepreneur on date-3. Accordingly, even with no measurement noise, the investment under Partial accounting is still lower than the first best (i.e., $[1+(1-\beta)k]\,\theta_0 < (1+k)\,\theta_0$). When the growth prospect k gets higher, this structural disadvantage of Partial accounting becomes more severe, which further reduces the investment.

However, under Full accounting, the accounting signal measures both the easy-to-measure and hard-to-measure cash flows. Including the hard-to-measure cash flows into the accounting measure makes the Full accounting signal become more "congruent" with the investment return (i.e. the object investors are pricing), which provides more incentives for investment.[15] Further, the Full accounting signal becomes more value-relevant with a higher growth prospect k than the Partial accounting signal, because the higher k increases the weight of the hard-to-measure cash flows in the total cash flows investors are pricing, and thus it becomes more important to measure the hard-to-measure cash flows. As a result, if k is high enough, Full accounting would induce more efficient investment than Partial accounting, although there is more measurement noise in the Full accounting signal. The following proposition summarizes the result.

Proposition 2 *There exists a cutoff point k^* such that Full accounting induces more efficient investment for firms with $k \geq k^*$ (i.e., $I_f \geq I_p$ if $k \geq k^*$), where k^* is the solution to the equation $[1+(1-\beta)k]^2 k = \frac{V_f}{V_\theta \theta_0^2}$. Further, when θ_0 is sufficiently larger than τ, the entrepreneur prefers Full accounting for any $k \geq k^*$.*

Intuitively, Proposition 2 further presents that, when the risk premium concern is relatively less important or the entrepreneur cares much more about the investment efficiency (i.e., θ_0 is sufficiently larger than τ), she prefers Full accounting if the growth prospect is high.

3.3 Risk premium and entrepreneur's welfare analysis

In this section, we focus on the risk premium concern as well as the entrepreneur's accounting choice problem. Proposition 1 characterizes the risk premium/conditional variance under the two

[15] This idea is similar to the goal congruency idea in the design of managerial performance measures (e.g., Feltham and Xie 1994; Dikolli 2001; Dutta and Reichelstein 2003).

accounting regimes. Under Partial accounting, the conditional variance consists of two components as shown in (5). The first component, $I_p^2 V_\theta^f$, is the unconditional variance of the hard-to-measure cash flows $\theta_f I_p$, because the accounting signal y^p provides no information about θ_f at all. The second component, $\frac{I_p^2 V_\theta^p V_p}{I_p^2 V_\theta^p + V_p}$, is the perceived volatility of the easy-to-measure cash flows $\theta_p I_p$, which is smaller than the unconditional variance $I_p^2 V_\theta^p$ due to the informative accounting signal y^p. Under Full accounting, the accounting signal y^f provides information about and thus helps resolve some uncertainty from both the easy-to-measure and hard-to-measure cash flows, leading to a smaller conditional variance than the total unconditional variance $I_f^2 V_\theta$, as shown in (7).

If the investment is exogenously given, it can be easily seen from (5) and (7) that, under both accounting regimes, the noisier the accounting signal, the less cash-flow uncertainty is resolved, and the higher the conditional variance. This is the underlying reason for the common intuition that higher measurement noise normally makes the measure less preferable (because it typically leads to a higher risk premium/conditional variance). However, our model with endogenous investment would cast some doubts on this common intuition. Particularly, with endogenous investment, the relationship between the accounting noise (e.g., V_p or v) and the conditional variance would become more subtle, because the accounting noise not only affects the conditional variance directly through the cash-flow uncertainty resolution, but also indirectly through its impact on the endogenous investment (i.e., I_p or I_f). To see this, differentiating the conditional variance with respect to the accounting noise gives

$$\frac{dVar[x|y]}{dn} = \underbrace{\frac{\partial Var[x|y]}{\partial n}}_{\text{direct effect (+)}} + \underbrace{\frac{\partial Var[x|y]}{\partial I}\frac{\partial I}{\partial n}}_{\text{indirect effect (-)}}, \tag{10}$$

where $(n, y, I) \in \{(V_p, y^p, I_p), (v, y^f, I_f)\}$. The first term on the right hand side of (10) represents the *direct effect* of the accounting noise, which is intuitively a positive effect. The second term represents the *indirect effect* of the accounting noise through the endogenous investment. First, as shown in Lemma 1, higher noise leads to lower investment (i.e., $\frac{\partial I}{\partial n} < 0$). Second, lower investment further reduces the conditional variance because it reduces the total (ex ante) cash-flow uncertainty (i.e., $\frac{\partial Var[x|y]}{\partial I} > 0$). As a result, this indirect effect through the endogenous investment works against the direct effect. Given such countervailing direct and indirect effects, the net impact of the accounting noise on the conditional variance is not unambiguous and depends on the dominating effect. Interestingly, if the indirect effect is dominant, the conditional variance would decrease in the accounting noise, contrary to our common intuition. As a result, some counter-intuitive results emerge, as shown below.

Lemma 2 *With endogenous investment, we have the following relationship between the conditional variance and the accounting noise (V_p or v).*

a) *Under the Partial accounting regime,*

- **i)** *if the managerial myopia is small ($\beta \leq \frac{1+k}{3+k+2V_\theta^f/V_\theta^p}$), then the conditional variance $Var[x|y^p]$ increases in V_p,*
- **ii)** *if the managerial myopia is intermediate ($\frac{1+k}{3+k+2V_\theta^f/V_\theta^p} < \beta < \frac{1+k}{k+2V_\theta^f/V_\theta^p}$), then the conditional variance $Var[x|y^p]$ increases (decreases) in V_p when $V_p \leq V_p^*$ ($V_p > V_p^*$), where $V_p^* = \frac{-I_p^2 V_\theta^p (I_p - (1+(1-\beta)k)\theta_0)}{I_p - (1-\beta)(1+k)\theta_0}\Big|_{I_p = \frac{2(1+(1-\beta)k+\beta V_\theta^f/V_\theta^p)\theta_0}{3}}$,*
- **iii)** *if the managerial myopia is high ($\beta \geq \frac{1+k}{k+2V_\theta^f/V_\theta^p}$), then the conditional variance $Var[x|y^p]$ decreases in V_p, and*
- **iv)** *the conditional variance $Var[x|y^p] \to (1+(1-\beta)k)^2\theta_0^2 V_\theta^f$ as $V_p \to 0$ and $\to L \equiv (1-\beta)^2(1+k)^2\theta_0^2 V_\theta$ as $V_p \to +\infty$.*

b) *Under the Full accounting regime,*

- **i)** *if the managerial myopia is small ($\beta \leq \frac{1}{3}$), then the conditional variance $Var[x|y^f]$ increases in v,*
- **ii)** *if the managerial myopia is intermediate ($\frac{1}{3} < \beta < \beta^*$), then the conditional variance $Var[x|y^f]$ increases (decreases) in v when $v \leq v^*$ ($v > v^*$), where $v^* > 0$ is the value such that $V_f(k, v^*) = \frac{-I_f^2 V_\theta (I_f - (1+k)\theta_0)}{I_f - (1-\beta)(1+k)\theta_0}\Big|_{I_f = \frac{2(1+k)\theta_0}{3}}$, and $\beta^* > \frac{1}{3}$ is the value such that, when $\beta = \beta^*$, $I_f(v=0) = \frac{2}{3}(1+k)\theta_0$.*
- **iii)** *if the managerial myopia is high ($\beta \geq \beta^*$), then the conditional variance $Var[x|y^f]$ decreases in v, and*
- **iv)** *the conditional variance $Var[x|y^f] \to L \equiv (1-\beta)^2(1+k)^2\theta_0^2 V_\theta$ as $v \to +\infty$.*[16]

[16] Notice that, in the results regarding Partial accounting, the cutoff point for the cases (a)-(ii) and (iii), $\frac{1+k}{k+2V_\theta^f/V_\theta^p}$,

[Figures 2 and 3 are inserted here]

Lemma 2 characterizes the impact of the accounting noise on the conditional variance under both accounting regimes. Figures 2 and 3 visualize the results for Partial accounting and Full accounting, respectively. Under Partial accounting (see Figure 2), we can see the conditional variance may increase or decrease in the accounting noise depending on the values of β and V_p. To understand the intuition, we can simplify the expression (10) for Partial accounting as follows,

$$\frac{dVar[x|y^p]}{dV_p} = \frac{3I_p^3 V_\theta^{p2}[I_p - \frac{2}{3}(1 + (1-\beta)k + \beta\frac{V_\theta^f}{V_\theta^p})\theta_0)]}{(I_p^2 V_\theta^p + V_p)f'(I_p)}, \tag{11}$$

where $f'(I_p) > 0$. From (11), one can see the conditional variance decreases in the accounting noise when the equilibrium investment I_p is small (i.e., $I_p < \frac{2}{3}(1 + (1-\beta)k + \beta\frac{V_\theta^f}{V_\theta^p})\theta_0)$). This is because the indirect (direct) effect is dominant when I_p is small (large).[17] As a result, in Figure 2_P3 where the entrepreneur is significantly myopic (i.e., $\beta \geq \frac{1+k}{k+2V_\theta^f/V_\theta^p}$), the equilibrium investment I_p is small enough that the indirect effect is always dominant, leading to the conditional variance monotonically decreasing in the accounting noise. In Figure 2_P2, the entrepreneur is a little less myopic, and thus the indirect effect is a little smaller (but still relatively large). The indirect effect is dominant only when the accounting noise V_p is relatively larger (i.e., $V_p > V_p^*$). The reason is that the larger accounting noise V_p further reduces the investment, leading to a dominant indirect effect. When the entrepreneur is not much myopic as in Figure 2_P1 (i.e., $\beta \leq \frac{1+k}{3+k+2V_\theta^f/V_\theta^p}$), the equilibrium investment I_p is large, and the direct effect is always dominant, leading to the conditional variance increasing in the accounting noise. Similar arguments also apply to the Full accounting regime (see Figure 3).[18]

From Figure 2, one can also see that, under Partial accounting, the cutoff points for the different

is less than one given our previous assumption that $V_\theta^f > (3/2+k)V_\theta^p$. However, if this assumption does not hold, this cutoff point may be greater than or equal to one (i.e., when $V_\theta^f/V_\theta^p \leq 1/2$). If this is the case, then we only have two cases (a)-(i) and (ii), because $\beta < 1$. Similarly, for Full accounting, it is likely that $\beta^* = \frac{1}{3} + \frac{4(1+k)^2\theta_0^2 V_\theta}{27V_f(k,v=0)} \geq 1$ (e.g., when $V_f(k, v=0)$ is relatively smaller than V_θ). If this is the case, we only have two cases (b)-(i) and (ii) as well. For completeness, we present all possible cases in the lemma.

[17] Notice that, treating all parameters independently, differentiating $Var[x|y^p]$ with respect to I_p^2 gives $\frac{\partial Var[x|y^p]}{\partial I_p^2} = \frac{V_\theta^p V_p^2}{(I_p^2 V_\theta^p + V_p)^2} + V_\theta^f$, which decreases in I_p^2. In other words, the (independent) indirect effect from I_p decreases in I_p.

[18] For Full accounting, $\frac{dVar[x|y^f]}{dv} = \frac{\partial V_f}{\partial v} \cdot \frac{3I_f^3 V_\theta^2 (I_f - \frac{2}{3}(1+k)\theta_0)}{(I_f^2 V_\theta + V_f)h'(I_f)}$, where $h'(I_f) > 0$. In terms of the format, the expression of $\frac{dVar[x|y^f]}{dv}$ is similar to that of $\frac{dVar[x|y^p]}{dV_p}$. Therefore, Figure 3 is similar to Figure 2.

cases depend on the relative future profitability risk V_θ^f/V_θ^p (given k). In particular, the indirect effect is more likely to be dominant with a higher relative future profitability risk (i.e., the β range for Figure 2_P1 decreases and that for Figure 2_P3 increases in V_θ^f/V_θ^p). The reason of this result is that, because the entire volatility from the future hard-to-measure cash flows adds to the conditional variance under Partial accounting, the higher the relative future profitability risk, the larger the magnifying effect from any incremental investment, and accordingly the larger the indirect effect. In contrast, under Full accounting, the cutoff points are not related to the relative future profitability risk, because the Full accounting signal measures the total cash flows, and thus the relative volatility between the two cash-flow components has no impact on the indirect effect.

Under both accounting regimes, when the accounting noise goes to infinity, the accounting signal is no longer informative about the cash flows, and therefore the conditional variance becomes the unconditional variance L, as defined in Lemma 2. The fact that the conditional variance sometimes exceeds the unconditional variance under both accounting regimes (e.g., Figures 2_P3 and 3_F3) is due to the indirect effect. That is, when the noise in the accounting signal reduces from infinity to a finite value, the equilibrium investment increases, and the conditional variance increases as well when the indirect effect is dominant as argued above.

Below we consider the consequences of moving from Partial accounting to Full accounting to capture the consequences from the expansion of accounting recognition. One common and convenient criticism of fair value accounting is on the high variance it introduces to the accounting measurements. The high variance serves as a cost against the benefit of bringing about a more comprehensive measure. Our model quantifies the trade-off and, perhaps surprisingly, shows that sometimes there is no trade-off at all. To better appreciate the results, below consider the possible disadvantages of Partial accounting. As shown in our model, under Partial accounting, the risk-premium may get elevated because of the unresolved uncertainty from the future hard-to-measure cash flows, punishing the stock price. At the same time, the price does not reward the investment efficiency because of the incongruent accounting signal. These disadvantages are particularly true for industries where most future cash flows are hard-to-measure (i.e., high k). The results shown in the next two subsections further illustrate the idea.

3.3.1 Full accounting could be preferable even with a high noise.

Using the above results, below we compare the conditional variances under the two accounting regimes. Given the assumption $V_\theta^f > (3/2+k)V_\theta^p$ (i.e., the future profitability risk is relatively large), the cutoff point for Figure 2_P3 does not exceed $\frac{1}{3}$ (i.e., $\frac{1+k}{k+2V_\theta^f/V_\theta^p} < \frac{1}{3}$), indicating that there is a common β region for any graph in Figure 2 and Figure 3_F1. The following lemma presents the relevant result on the conditional variance comparison.

Lemma 3 *If $\beta \in (\frac{1+k}{3+k+2V_\theta^f/V_\theta^p}, \frac{1}{3}]$, the conditional variance under Full accounting is smaller than that under Partial accounting for any v and $V_p \geq \bar{V}_p$, where $\bar{V}_p > 0$ is the value of V_p such that $Var[x|y^p] = L$ when $\beta \in (\frac{1+k}{3+k+2V_\theta^f/V_\theta^p}, \frac{1+k}{k+2V_\theta^f/V_\theta^p})$, and $\bar{V}_p = 0$ when $\beta \in [\frac{1+k}{k+2V_\theta^f/V_\theta^p}, \frac{1}{3}]$.*

Proof. The formal proof is omitted. See below for intuition. ■

The result in Lemma 3 directly comes from the comparison of the related graphs in Figures 2 and 3. By comparing Figure 2_P3 with Figure 3_F1, one can see that, for *any* accounting noise levels (i.e., any v and V_p), the conditional variance is always higher than L under Partial accounting, whereas it is always lower than L under Full accounting. Therefore, Full accounting *always* produces a lower conditional variance than Partial accounting. Similarly, by comparing Figure 2_P2 with Figure 3_F1, one can also see that, for any given v and $V_p \geq \bar{V}_p$, the conditional variance under Partial accounting is larger than that under Full accounting. The intuition is as follows. Under Partial accounting, the entire uncertainty from the hard-to-measure cash flows adds to the conditional variance, while under Full accounting, only a reduced portion adds to the conditional variance due to the comprehensive Full accounting signal. When the volatility from the hard-to-measure cash flows (or the future profitability risk) is relatively high (i.e., $V_\theta^f > (3/2+k)V_\theta^p$), Full accounting, even with an extremely high noise, could lead to a lower conditional variance. Further, the higher the relative future profitability risk, the more likely this case happens (i.e., the range $\beta \in (\frac{1+k}{3+k+2V_\theta^f/V_\theta^p}, \frac{1}{3}]$ in Lemma 3 becomes larger as V_θ^f/V_θ^p increases).

A couple of other observations also emerge from the lemma. First, Lemma 3 is a *robust* result in the sense that it does not rely on any specific (functional) relationship between the accounting noises under the two accounting regimes (i.e., the lemma is valid for any pair of v and $V_p \geq \bar{V}_p$).

Second, the lemma is also a *strong* result in the sense that it holds for any noise levels under Full accounting, including any large noise levels.

The above counter-intuitive result regarding the conditional-variance comparison can lead to a corresponding counter-intuitive result on the entrepreneur's welfare comparison, as shown in the following proposition.

Proposition 3 *When θ_0 is sufficiently smaller than τ, then for any $V_p \geq \bar{V}_p$, if $\beta \in (\frac{1+k}{3+k+2V_\theta^f/V_\theta^p}, \frac{1}{3}]$, the entrepreneur prefers Full accounting for any noise level v.*

Proof. The proof is omitted because the proposition immediately follows from Lemma 3. ■

Proposition 3 shows that, when the investment efficiency concern is relatively less important or the entrepreneur cares much more about the risk premium (i.e., θ_0 is sufficiently smaller than τ), she may prefer Full accounting even if it contains (extremely) large noise. This result is somewhat surprising because, in the current accounting debate regarding fair value accounting vs. historical cost accounting, one major criticism on fair value accounting is that fair value measures (particularly Level III measures) contain much higher measurement noise, which would lead to higher risk premium and render fair value accounting less preferable. Proposition 3 shows that this criticism may not be well grounded when the real effect (i.e., endogenous investment) is considered.

3.3.2 Full accounting may benefit both investment efficiency and risk premium.

The results in Proposition 2 and Proposition 3 isolate the tension in the accounting choice problem by focusing on the sole impact of the investment efficiency and risk premium on the entrepreneur's welfare (i.e., θ_0 is sufficiently larger or smaller than τ), respectively. The following proposition presents a more general comparison of the two accounting regimes by considering the combined impacts of both the investment efficiency and risk premium on the entrepreneur's welfare (i.e., for any parameter-pair θ_0 and τ).

Proposition 4 *For any θ_0, τ, v, and $V_p > \bar{V}_p$, if $k \geq k^*$ and $\beta \in (\frac{1+k}{3+k+2V_\theta^f/V_\theta^p}, \frac{1}{3}]$, then the entrepreneur prefers Full accounting, because both the initial investment is more efficient and the conditional variance is smaller under Full accounting.*

Proof. The formal proof is omitted. See below for intuition. ■

Proposition 4 is a *strong* result in the sense that it shows, regardless of the values of θ_0 and τ (or the relative weights of the investment-efficiency and risk-premium concerns in the entrepreneur's accounting choice problem), Full accounting could perform better on both of these welfare components even if it contains a large noise, as long as the given conditions are satisfied. The intuition is as follows. First, as Proposition 2 shows, for any given other parameters, when the growth prospect is relatively large (i.e., $k \geq k^*$), the initial investment is more efficient under Full accounting. Second, as Lemma 3 implies, for any v and $V_p > \bar{V}_p$, if the entrepreneur has an intermediate level of managerial myopia (i.e., $\beta \in (\frac{1+k}{3+k+2V_\theta^f/V_\theta^p}, \frac{1}{3}])$, the conditional variance under Full accounting is smaller. Combining both results, one can see that the entrepreneur would prefer Full accounting (even with a large noise) if the growth prospect k is relatively large and the entrepreneur's myopia is not too extreme, because Full accounting is better on both fronts.

In the accounting policy debates, the higher measurement noise in fair value measures (relative to historical cost measures) is the major criticism on fair value accounting. To the extent Full accounting in the model captures the key features of fair value accounting, Proposition 4 shows that firms with great growth prospects may still prefer fair value accounting regardless of the larger noise in fair value measures, because fair value accounting not only induces more efficient investment but also leads to a lower risk premium. This is an instance where the trade-off regarding fair value is not between the investment efficiency and risk premium.

4 Discussion

4.1 Risk-averse entrepreneur

In our model, since the entrepreneur is risk-neutral, her accounting preference only depends on the investment efficiency and the risk premium she needs to compensate investors for. However, if the entrepreneur is risk-averse, she has to further consider her own risk premium. Suppose the entrepreneur has a CARA utility function with risk-averse coefficient τ_e. We can express the entrepreneur's certainty-equivalent welfare on date-0, denoted by CE, as follows,

$$
\begin{aligned}
CE \quad &= \quad W - \frac{\tau_e}{2} Var[-\frac{I^2}{2} + \beta P + (1-\beta)x], \qquad (12)\\
&\quad\quad \text{where } W = E[-\frac{I^2}{2} + \beta P + (1-\beta)x], \text{ and}\\
&\quad\quad Var[-\frac{I^2}{2} + \beta P + (1-\beta)x] = \beta^2 b^2 Var(y) + (1-\beta)^2 I^2 V_\theta, \qquad (13)
\end{aligned}
$$

where $(y, I, b) \in \{(y^p, I_p, b_p), (y^f, I_f, b_f)\}$.

From (12), we can see the entrepreneur's certainty-equivalent welfare equals her corresponding welfare in the risk-neutral setting (i.e., W as defined in (9)) minus her own risk premium. Equation (13) indicates that the additional entrepreneur's risk premium depends on the volatility of the share price or the accounting signal and the overall cash-flow volatility, both of which increase in the investment (i.e., the entrepreneur's risk premium increases in the investment). As a result, the equilibrium investment (i.e., I_p and I_f) is lowered in the current risk-averse setting because, given the entrepreneur's risk premium increases in the investment, it further reduces the investment incentive. In addition, under-investment is more severe with higher risk-aversion (i.e., higher τ_e).

In terms of the accounting choice problem, intuitively, if the entrepreneur's risk-averse coefficient τ_e is sufficiently small compared to the investor's risk-averse coefficient τ, all prior qualitative results on the entrepreneur's accounting preference remain the same in the current risk-averse setting, because the *investors'* risk premium will be the dominant determinant of the accounting choice problem.[19] However, if the entrepreneur's risk-averse coefficient τ_e is sufficiently large compared to the investor's risk-averse coefficient τ, the *entrepreneur's* risk premium will be the dominant determinant and some results from the risk-neutral setting may change.

For example, part of the entrepreneur's risk premium arises from the volatility in the share price P, which comes from both the noise in the accounting measurements and the volatility in the cash flows the signal measures. Because the Full accounting signal measures both the easy-to-measure and hard-to-measure cash flows, whereas the Partial accounting signal measures only the easy-to-measure cash flows, the accounting signal is more volatile under Full accounting. When the growth prospect k is high enough (i.e., the volatility from the hard-to-measure cash flows is large

[19] Some economics literature (e.g., Puri and Robinson 2009) has shown that typical entrepreneurs are less risk-averse or more risk-loving.

enough), the higher share-price volatility under Full accounting may dominate other factors, and the entrepreneur may prefer Partial accounting to Full accounting. This result is in contrast to that in the risk-neutral setting, where when the growth prospect is high enough, the entrepreneur may prefer Full accounting to Partial accounting, because Full accounting induces not only more efficient investment but also lower investors' risk premium.

4.2 Link to the empirical cost-of-capital studies

Our results on the relation between the accounting noise and conditional variance have empirical implications on the study regarding the relation between accounting quality and (firm-equity) cost of capital. We define the cost of capital of the firm as follows.[20]

Definition 2 *The ex post cost of capital of the firm on date-2 is defined as*

$$COC = \frac{E[x|y] - P}{E[x]} = \frac{\beta\tau Var\,[x|y]}{(1+k)\theta_0 I},\ where\ (y, I) \in \{(y^p, I_p), (y^f, I_f)\}. \tag{14}$$

The simplification in the definition (14) results from the equation (4). The numerator, $E[x|y]-P$, is the ex post stock return on date-2 after the accounting report $y \in \{y^p, y^f\}$ is released. We further deflate this return by the ex ante expected total future cash flows $E[x]$ to express the cost of capital as a percentage, which neutralizes the impact of different investment levels on the cost of capital. The following lemma characterizes the relationship between the cost of capital and the accounting noise for both accounting regimes.

Lemma 4 *We have the following relationship between the cost of capital and the accounting noise (V_p or v).*

a) *Under the Partial accounting regime,*

i) *if* $\beta \le \frac{1+k}{2+k+V_\theta^f/V_\theta^p}$, COC_p *increases in* V_p,

ii) *if* $\frac{1+k}{2+k+V_\theta^f/V_\theta^p} < \beta < \frac{1+k}{k+V_\theta^f/V_\theta^p}$, COC_p *increases (decreases) in* V_p *when* $V_p \le V_p^{**}$ *(*$V_p > V_p^{**}$*), where* $V_p^{**} = \frac{-I_p^2 V_\theta^p (I_p-(1+(1-\beta)k)\theta_0)}{I_p-(1-\beta)(1+k)\theta_0}\Big|_{I_p=\frac{(1+(1-\beta)k+\beta V_\theta^f/V_\theta^p)\theta_0}{2}}$,

[20] The definition of the cost of capital is for analytic ease. Our qualitative results remain the same if we define the cost of capital as $\frac{E[x|y]-P}{E[P]}$, following prior literature (e.g., Lambert et al. 2007; Gao 2010).

iii) *if* $\beta \geq \frac{1+k}{k+V_\theta^f/V_\theta^p}$, COC_p *decreases in* V_p, *and*

iv) $COC_p \to (1+(1-\beta)k)V_\theta^f/(1+k)$ *as* $V_p \to 0$ *and* $\to M \equiv (1-\beta)(V_\theta^p + V_\theta^f) = (1-\beta)V_\theta$ *as* $V_p \to +\infty$.

b) *Under the Full accounting regime,*

i) *if* $\beta \leq \frac{1}{2}$, *then* COC_f *increases in* v,

ii) *if* $\frac{1}{2} < \beta < \beta^{**}$, *then* COC_f *increases (decreases) in* v *when* $v \leq v^{**}$ $(v > v^{**})$, *where* $v^{**} > 0$ *is the value such that* $V_f(k, v^{**}) = \frac{-I_f^2 V_\theta (I_f - (1+k)\theta_0)}{I_f - (1-\beta)(1+k)\theta_0}\Big|_{I_f = \frac{(1+k)\theta_0}{2}}$, *and* $\beta^{**} > \frac{1}{2}$ *is the value such that, when* $\beta = \beta^{**}$, $I_f(v=0) = \frac{(1+k)\theta_0}{2}$.

iii) *if* $\beta \geq \beta^{**}$, *then* COC_f *decreases in* v, *and*

iv) $COC_f \to M \equiv (1-\beta)V_\theta$ *as* $v \to +\infty$.

Proof. The proof is similar to that of Lemma 2 and thus omitted. ∎

The results in this lemma are similar to those in Lemma 2, which presents the relationship between the conditional variance and the accounting noise. Therefore, Figures 2 and 3 also graph the relation between the cost of capital and the accounting noise. More interestingly, the results in the lemma have implications on how the accounting information quality and investment factors affect the cost of capital under both accounting regimes. The results indicate that the nature and direction of the relation between the cost of capital and accounting information quality (i.e., V_p or v) depends on factors such as the managerial myopia in investment decisions (β), the future profitability risk, the intensity of the use of market-based accounting measures (i.e., Partial accounting or Full accounting), and the firms' growth prospects (k), which seem to be less explored in the empirical literature on cost of capital.

Empirical work has documented mixed evidence on the association between earnings quality and cost of capital, which Beyer et al. (2010) attribute to some empirical challenges, such as the self-selection problem, the existence of possible mechanical relationship, and the measurement errors in the proxies of cost of capital and disclosure quality. Given the non-deterministic relation (between the cost of capital and the accounting noise) shown in Figures 2 and 3, our study provides potential factors (as pointed out above) that may help explain the mixed evidence. For example, based on both Figures 2 and 3, we can see, if the entrepreneur's myopia is not too extreme (i.e., medium β, see Figures 2_P2 and 3_F2) and the accounting information quality is not too high (i.e., relatively large V_p or v), then the cost of capital *increases* in the accounting quality, contrary to our common intuition. Also, if the manager is very myopic (i.e., large β, see Figures 2_P3 and 3_F3), the cost of capital *increases* in the accounting quality regardless of the accounting quality. Otherwise, the relation between the cost of capital and accounting quality is negative, as normally expected.

The different cutoff points for the different cases under the two accounting regimes also have empirical implications. Since the intensity of the use of market-based accounting measures differs across industries, industry could be a good proxy for the accounting regimes. For some industries (e.g., financial or banking industry), market-based accounting measures are widely used, while for other industries (e.g., manufacturing industry), market-based accounting measures may be rarely used. Given the cutoff points for cases (a)-(i) (Partial accounting) and (b)-(i) (Full accounting) are $\frac{1+k}{2+k+V_\theta^f/V_\theta^p}$ and $\frac{1}{2}$, respectively, we predict that, when the growth prospect is high (i.e., k is large) and/or the volatility of the hard-to-measure cash flows is not too large relative to that of the easy-to-measure cash flows (i.e., V_θ^f/V_θ^p is not too large), it is less likely to have a positive relation between cost of capital and accounting quality for firms in the industries with less use of market-based accounting measures (i.e., Partial accounting) than in the industries with more use of market-based accounting measures (i.e., Full accounting).

4.3 Policy implications

The results in our paper also shed some light on the policy discussions about the design of the accounting measurement system. This is relevant to the on-going FASB/IASB efforts to promote fair value accounting as the guiding principle moving forward. Generally, our results show that the accounting measurement choice leading to the highest investment efficiency and/or the lowest risk premium (or cost of capital) depends on firm characteristics (e.g., growth prospects) as well as environmental factors (e.g., measurement noise). In this regard, our results may also justify the existing mixed-attributes model in GAAP as well as the use of the fair value option under SFAS 159 (or Accounting Standards Codification 825).

The result that higher noise in fair value measurements may not necessarily be a disadvantage comes from the *indirect effect* highlighted in our model. At a deeper level, this points to the need to take into account the endogenous responses from reporting firms when establishing accounting standards. At the same time, the lack of apparent trade-off between investment efficiency and risk premium in some cases may be due to the partial equilibrium nature of our model. The more pressing trade-off in the fair-value debate may reside elsewhere.

5 Conclusion

In this paper, we provide an economic model where the conceptual scope issue with every accounting measurement has an economic meaning. In particular, we build an accounting model to highlight one important scope dimension: inclusion or exclusion of hard-to-measure events and/or future actions. We embed the accounting model into a standard economic model in which the accounting measurement choice affects both distributional and allocational efficiency. We conclude that expanding recognition scope in accounting measurements brings complexity into the accounting choice problem at the firm level, because it may impact both real variables such as investment efficiency and financial variables such as risk premium in share prices. Specifically, we show that, in the process of expanding accounting recognition, firms' internal investment efficiency and external share-price risk premium may *not* necessarily be a trade-off in that an expanded recognition may lead to both a higher investment efficiency and a lower risk-premium at the same time.

While we believe the paper opens the question on a key scope issue in accounting measurements, we view the paper is limited on a few fronts. We have limited our attention at the formal accounting measurements and abstracted away from other forms of disclosure such as corporate voluntary disclosure and information production by other market participants. These other forms of disclosure also have impacts on investment efficiency and risk premium. Similarly, we have not considered the issue of accounting or disclosure manipulation by the firm and its welfare implications. Further, outside investors are silent in collecting their own information in our model. These are all fruitful avenues to explore in future studies.

Appendix

Proof. (of Proposition 1 and Lemma 1) Given the market's conjecture $\hat{I}_p$ on the investment decision under Partial accounting, we have

$$\begin{bmatrix} \theta\hat{I}_p \\ \theta^p\hat{I}_p+\varepsilon_p \end{bmatrix} \sim N\left(\begin{bmatrix} (1+k)\,\theta_0\hat{I}_p \\ \theta_0\hat{I}_p \end{bmatrix}, \begin{bmatrix} \hat{I}_p^2\left(V_\theta^p+V_\theta^f\right) & \hat{I}_p^2V_\theta^p \\ \hat{I}_p^2V_\theta^p & \hat{I}_p^2V_\theta^p+V_p \end{bmatrix}\right).$$

From the pricing function (4),

$$\begin{aligned} P &= E[x|y^p]-\tau\beta VAR[x|y^p] \\ &= E[\theta\hat{I}_p|\theta^p\hat{I}_p+\varepsilon_p]-\tau\beta\cdot VAR[\theta\hat{I}_p|\theta^p\hat{I}_p+\varepsilon_p] \\ &= \left(\frac{V_p}{\hat{I}_p^2V_\theta^p+V_p}+k\right)\theta_0\hat{I}_p+\frac{\hat{I}_p^2V_\theta^p}{\hat{I}_p^2V_\theta^p+V_p}y^p-\tau\beta\left(V_\theta^f+\frac{V_p}{I_p^2V_\theta^p+V_p}V_\theta^p\right)\hat{I}_p^2. \end{aligned}$$

The entrepreneur's ex ante expected payoff is $E[-\frac{1}{2}I^2+\beta P+(1-\beta)x]$. Given the above linear pricing conjecture, the entrepreneur selects the optimal investment I_p to maximize her expected payoff. Thus, FOC gives

$$\begin{aligned} I_p &= [\beta b_p+(1-\beta)\,(1+k)]\,\theta_0, \\ where\ b_p &= \frac{\hat{I}_p^2V_\theta^p}{\hat{I}_p^2V_\theta^p+V_p} \end{aligned}$$

Because the market's conjecture is correct in equilibrium $\hat{I}_p=I_p$, we have

$$\begin{aligned} b_p &= \frac{\hat{I}_p^2V_\theta^p}{\hat{I}_p^2V_\theta^p+V_p}=\frac{\frac{I_p}{\theta_0}-(1-\beta)\,(1+k)}{\beta},\ \text{or} \\ f(I_p) &\equiv I_p^2V_\theta^p\,[I_p-(1+(1-\beta)k)\,\theta_0]+V_p\,[I_p-(1-\beta)\,(1+k)\,\theta_0]=0. \end{aligned}$$

It is easy to see that

$$\begin{aligned} f(I_p) &< 0 \text{ if } I_p\leq(1-\beta)\,(1+k)\,\theta_0,\ \text{and} \\ f(I_p) &> 0 \text{ if } I_p\geq(1+(1-\beta)k)\,\theta_0. \end{aligned}$$

From the property of continuity, there exists at least one root of I_p between $(1-\beta)(1+k)\theta_0$ and $(1+(1-\beta)k)\theta_0$. We always pick the root closest to $(1+(1-\beta)k)\theta_0$ if there are multiple roots.

We below show the comparative statics. From the expression of $f(I_p)$, it is easy to see that the root $I_p \to (1+(1-\beta)k)\theta_0$ as $V_p \to 0$, $I_p \to (1-\beta)(1+k)\theta_0$ as $V_p \to +\infty$, and V_θ^f has no impact on I_p. By Implicit Function Theorem, we have $\frac{\partial I_p}{\partial V_p} = \frac{(1-\beta)(1+k)\theta_0 - I_p}{f'(I_p)}$ where $f'(I_p) = 3I_p^2 V_\theta^p - 2I_p V_\theta^p (1+(1-\beta)k)\theta_0 + V_p$. Now we show $f'(I_p) > 0$ where I_p is the root closest to $(1+(1-\beta)k)\theta_0$. If $f'(I_p) < 0$, since $f(I_p) = 0$, there exists an $I_p^* \in (I_p, (1+(1-\beta)k)\theta_0)$ such that $f(I_p^*) < 0$. Given $f(I_p = (1+(1-\beta)k)\theta_0) > 0$, there exists an $I_p^{**} \in (I_p^*, (1+(1-\beta)k)\theta_0)$ such that $f(I_p^{**}) = 0$, which is a contradiction with the assumption that I_p is the root closest to $(1+(1-\beta)k)\theta_0$. Thus, $f'(I_p) > 0$. Therefore, $\frac{\partial I_p}{\partial V_p} = \frac{(1-\beta)(1+k)\theta_0 - I_p}{f'(I_p)} < 0$. Similarly, $\frac{\partial I_p}{\partial \beta} = \frac{-\left(I_p^2 V_\theta^p k\theta_0 + V_p\theta_0(1+k)\right)}{f'(I_p)} < 0$, and $\frac{\partial I_p}{\partial V_\theta^p} = \frac{-I_p^2[I_p - (1+(1-\beta)k)\theta_0]}{f'(I_p)} > 0$.

With similar arguments, given the market's conjecture $\hat{I}_f$ on the investment decision under Full accounting, we have

$$\begin{bmatrix} \theta\hat{I}_f \\ \theta\hat{I}_f + \varepsilon_f \end{bmatrix} \sim N\left(\begin{bmatrix} (1+k)\theta_0\hat{I}_f \\ (1+k)\theta_0\hat{I}_f \end{bmatrix}, \begin{bmatrix} \hat{I}_f^2 V_\theta & \hat{I}_f^2 V_\theta \\ \hat{I}_f^2 V_\theta & \hat{I}_f^2 V_\theta + V_f \end{bmatrix}\right).$$

From the pricing function (4),

$$\begin{aligned} P &= E[x|y^f] - \tau\beta VAR[x|y^f] \\ &= E[\theta\hat{I}_f|\theta\hat{I}_f + \varepsilon_f] - \tau\beta \cdot VAR[\theta\hat{I}_f|\theta\hat{I}_f + \varepsilon_f] \\ &= \frac{V_f}{\hat{I}_f^2 V_\theta + V_f}(1+k)\theta_0\hat{I}_f + \frac{\hat{I}_f^2 V_\theta}{\hat{I}_f^2 V_\theta + V_f} y^f - \tau\beta\frac{\hat{I}_f^2 V_\theta V_f}{\hat{I}_f^2 V_\theta + V_f}. \end{aligned}$$

The entrepreneur's ex ante expected payoff is $E[-\frac{1}{2}I^2 + \beta P + (1-\beta)x]$. Given the above linear pricing conjecture, the entrepreneur selects the optimal investment I_f to maximize her expected payoff. Thus, FOC gives

$$\begin{aligned} I_f &= [\beta b_f + (1-\beta)](1+k)\theta_0, \\ where\ b_f &= \frac{\hat{I}_f^2 V_\theta}{\hat{I}_f^2 V_\theta + V_f} \end{aligned}$$

Because the market's conjecture is correct in equilibrium $\hat{I}_f = I_f$, we have

$$\begin{aligned} b_f &= \frac{\hat{I}_f^2 V_\theta}{\hat{I}_f^2 V_\theta + V_f} = \frac{\frac{I_f}{(1+k)\theta_0} - (1-\beta)}{\beta}, \text{ or} \\ h(I_f) &\equiv I_f^2 V_\theta \left[I_f - (1+k)\,\theta_0\right] + V_f (I_f - (1-\beta)\,(1+k)\,\theta_0) = 0. \end{aligned}$$

It is easy to see that

$$\begin{aligned} h(I_f) &< 0 \text{ if } I_f \leq (1-\beta)\,(1+k)\,\theta_0, \text{ and} \\ h(I_f) &> 0 \text{ if } I_f \geq (1+k)\,\theta_0. \end{aligned}$$

From the property of continuity, there exists at least one root of I_f between $(1-\beta)\,(1+k)\,\theta_0$ and $(1+k)\,\theta_0$. We always pick the root closest to $(1+k)\,\theta_0$ if there are multiple roots.

Similarly, from the expression of $h(I_f)$, it is easy to see that the root $I_f \to (1+k)\,\theta_0$ as $V_f \to 0$, $I_f \to (1-\beta)\,(1+k)\,\theta_0$ as $V_f \to +\infty$. Now we show $h'(I_f) > 0$ where I_f is the root closest to $(1+k)\,\theta_0$. If $h'(I_f) < 0$, since $h(I_f) = 0$, there exists an $I_f^* \in (I_f, (1+k)\,\theta_0)$ such that $h(I_f^*) < 0$. Given $h(I_f = (1+k)\,\theta_0) > 0$, there exists an $I_f^{**} \in (I_f^*, (1+k)\,\theta_0)$ such that $h(I_f^{**}) = 0$, which is a contradiction with the assumption that I_f is the root closest to $(1+k)\,\theta_0$. Thus, $h'(I_f) > 0$. By Implicit Function Theorem, we have $\frac{\partial I_f}{\partial V_f} = \frac{(1-\beta)(1+k)\theta_0 - I_f}{h'(I_f)} < 0$, $\frac{\partial I_f}{\partial v} = \frac{\partial I_f}{\partial V_f}\frac{\partial V_f}{\partial v} < 0$, $\frac{\partial I_f}{\partial V_\theta} = \frac{I_f^2\left[(1+k)\theta_0 - I_f\right]}{h'(I_f)} > 0$, and $\frac{\partial I_f}{\partial \beta} = \frac{-V_f(1+k)\theta_0}{h'(I_f)} < 0$. ■

Proof. (of Proposition 2) From the analysis of Proposition 1, we show that $I_p \in ((1-\beta)\,(1+k)\,\theta_0,\ (1+(1-\beta)k$ and $I_f \in ((1-\beta)\,(1+k)\,\theta_0, (1+k)\,\theta_0)$. That is, $\frac{I_p}{(1+k)\theta_0} \in \left(1-\beta, \frac{1+(1-\beta)k}{1+k}\right)$ and $\frac{I_f}{(1+k)\theta_0} \in (1-\beta, 1)$. One can easily see that the upper bound of $\frac{I_p}{(1+k)\theta_0}$ deceases in k (i.e., $\left(\frac{1+(1-\beta)k}{1+k}\right)'_k = -\frac{\beta}{(1+k)^2} < 0$) and approaches $(1-\beta)$ as $k \to +\infty$.

Under Full accounting, $\frac{I_f}{(1+k)\theta_0} \in (1-\beta, 1)$, where I_f solves $h(I_f) \equiv I_f^2 V_\theta \left[I_f - (1+k)\,\theta_0\right] + V_f(I_f - (1-\beta)\,(1+k)\,\theta_0) = 0$. Then, we have

$$H(I_f') \equiv \left(I_f'\right)^2 (1+k)^2\, \theta_0^2 V_\theta \left[I_f' - 1\right] + V_f \left[I_f' - (1-\beta)\right] = 0 \tag{15}$$

where $I_f' = \frac{I_f}{(1+k)\theta_0}$. Taking the derivative of $H(I_f')$ defined in (15) with respect to k, we have

$$\frac{\partial H(I_f')}{\partial I_f'}\frac{\partial I_f'}{\partial k}+2\left(1+k\right)\left(I_f'\right)^2\theta_0^2 V_\theta\left[I_f'-1\right]+\frac{\partial V_\theta^f}{\partial k}\left(I_f'\right)^2\left(1+k\right)^2\theta_0^2\left[I_f'-1\right]+\frac{\partial V_f}{\partial k}\left[I_f'-\left(1-\beta\right)\right]=0 \tag{16}$$

From (15), we have $2\left(1+k\right)\left(I_f'\right)^2\theta_0^2 V_\theta\left[I_f'-1\right] = -\frac{2V_f\left[I_f'-(1-\beta)\right]}{1+k}$. Then, we can rewrite (16) as

$$\frac{\partial H(I_f')}{\partial I_f'}\frac{\partial I_f'}{\partial k}+\frac{\partial V_\theta^f}{\partial k}\left(I_f'\right)^2\left(1+k\right)^2\theta_0^2\left[I_f'-1\right]+\left(\frac{\partial V_f}{\partial k}-\frac{2V_f}{1+k}\right)\left[I_f'-\left(1-\beta\right)\right]=0 \tag{17}$$

In (17), $\left(\frac{\partial V_f}{\partial k}-\frac{2V_f}{1+k}\right)\left[I_f'-(1-\beta)\right]<0$ since $\frac{\frac{\partial}{\partial k}V_f(k,v)}{V_f(k,v)}<\frac{2}{1+k}$ and $I_f'>(1-\beta)$, $\frac{\partial V_\theta^f}{\partial k}\left(I_f'\right)^2\left(1+k\right)^2\theta_0^2[I_f'-1]<0$ since $\frac{\partial V_\theta^f}{\partial k}>0$ and $I_f'<1$, and $\frac{\partial H(I_f')}{\partial I_f'}>0$ since we show $h'(I_f)>0$ where I_f is the root closest to $(1+k)\,\theta_0$. Thus, from (17), we have $\frac{\partial I_f'}{\partial k}>0$. One can also see from (15) that I_f' approaches 1 as $k\to+\infty$ (notice that $\frac{V_f}{(1+k)^2}<+\infty$). Thus, from continuity, there always exists a k^* such that $\frac{I_f}{(1+k^*)\theta_0}=\frac{1+(1-\beta)k^*}{1+k^*}>\frac{I_p}{(1+k^*)\theta_0}$. Notice that, k^* is the solution to the equation $[1+(1-\beta)k]^2k-\frac{V_f}{V_\theta\theta_0^2}=0$. Accordingly, for any $k\geq k^*$, Full accounting induces more efficient investment than Partial accounting, i.e., $I_f\geq I_p$.

Proof. (of Lemma 2) For Partial accounting, differentiating $Var[x|y^p]$ with respect to V_p, we have

$$\begin{aligned}\frac{dVar[x|y^p]}{dV_p} &= \frac{\partial Var[x|y^p]}{\partial V_p}+\frac{\partial Var[x|y^p]}{\partial I_p}\frac{\partial I_p}{\partial V_p}\\ &= \frac{I_p^4V_\theta^{p2}}{(I_p^2V_\theta^p+V_p)^2}+\left(\frac{2I_pV_\theta^pV_p^2}{(I_p^2V_\theta^p+V_p)^2}+2I_pV_\theta^f\right)\frac{-(I_p-(1-\beta)(1+k)\theta_0)}{f'(I_p)},\end{aligned}$$

where $f'(I_p)=3V_\theta^pI_p^2-2V_\theta^p(1+(1-\beta)k)\theta_0I_p+V_p>0$ as shown in the proof of Proposition 1 and Lemma 1. From $f(I_p)=0$, we have $-(I_p-(1-\beta)(1+k)\theta_0)=V_\theta^pI_p^2(I_p-r)/V_p$, where $r\equiv(1+(1-\beta)k)\theta_0$. By substitution and simplification, we have

$$\begin{aligned}\frac{dVar[x|y^p]}{dV_p} &= \frac{I_p^3V_\theta^{p2}(3I_p-2(1+(1-\beta)k)\theta_0)}{(I_p^2V_\theta^p+V_p)f'(I_p)}+\frac{2I_p^3V_\theta^fV_\theta^p(I_p-r)}{f'(I_p)V_p}\\ &= \frac{I_p^3V_\theta^p[3V_\theta^pV_pI_p-2V_\theta^pV_pr+2V_\theta^fV_p(I_p-r)+2V_\theta^f\cdot V_\theta^pI_p^2(I_p-r)]}{(I_p^2V_\theta^p+V_p)f'(I_p)V_p}\\ &= \frac{3I_p^3V_\theta^{p2}[I_p-\frac{2}{3}(r+\beta\theta_0\frac{V_\theta^f}{V_\theta^p})]}{(I_p^2V_\theta^p+V_p)f'(I_p)}.\end{aligned}$$

The third equality results from the substitution $V_\theta^p I_p^2(I_p - r) = -(I_p - (1-\beta)(1+k)\theta_0)V_p$ (from $f(I_p) = 0$) and simplification. Therefore, if $(1-\beta)(1+k)\theta_0 \geq \frac{2}{3}(r + \beta\theta_0 \frac{V_\theta^f}{V_\theta^p})$ or $\beta \leq \frac{1+k}{3+k+2V_\theta^f/V_\theta^p}$, then $I_p > (1-\beta)(1+k)\theta_0 \geq \frac{2}{3}(r + \beta\theta_0 \frac{V_\theta^f}{V_\theta^p})$ and $\frac{dVar[x|y^p]}{dV_p} > 0$. However, when $(1-\beta)(1+k)\theta_0 < \frac{2}{3}(r + \beta\theta_0 \frac{V_\theta^f}{V_\theta^p})$ or $\beta > \frac{1+k}{3+k+2V_\theta^f/V_\theta^p}$, given $I_p < r$, I_p decreases in V_p, and $I_p \to r$ as $V_p \to 0$, then if $\frac{2}{3}(r + \beta\theta_0 \frac{V_\theta^f}{V_\theta^p}) < r$ or $\beta < \frac{1+k}{k+2V_\theta^f/V_\theta^p}$, there exists a $V_p^* = \frac{-V_\theta^p I_p^2(I_p - r)}{I_p - (1-\beta)(1+k)\theta_0}|_{I_p = \frac{2}{3}(r+\beta\theta_0 \frac{V_\theta^f}{V_\theta^p})}$ such that when $V_p \gtreqless V_p^*$, $I_p \lesseqgtr \frac{2}{3}(r + \beta\theta_0 \frac{V_\theta^f}{V_\theta^p})$ and $\frac{dVar[x|y^p]}{dV_p} \lesseqgtr 0$. On the other hand, if $\frac{2}{3}(r + \beta\theta_0 \frac{V_\theta^f}{V_\theta^p}) \geq r$ or $\beta \geq \frac{1+k}{k+2V_\theta^f/V_\theta^p}$, then $I_p < \frac{2}{3}(r + \beta\theta_0 \frac{V_\theta^f}{V_\theta^p})$ and $\frac{dVar[x|y^p]}{dV_p} < 0$. ■

Given $I_p \to r$ ($(1-\beta)(1+k)\theta_0$) as $V_p \to 0$ ($+\infty$) from Lemma 1, we can see $Var[x|y^p] \to (1 + (1-\beta)k)^2\theta_0^2 V_\theta^f$ as $V_p \to 0$ and $\to L \equiv (1-\beta)^2(1+k)^2\theta_0^2(V_\theta^p + V_\theta^f) = (1-\beta)^2(1+k)^2\theta_0^2 V_\theta$ as $V_p \to +\infty$.

For Full accounting, differentiating $Var[x|y^f]$ with respect to v, we have

$$
\begin{aligned}
\frac{dVar[x|y^f]}{dv} &= \frac{\partial Var[x|y^f]}{\partial v} + \frac{\partial Var[x|y^f]}{\partial I_f}\frac{\partial I_f}{\partial v} \\
&= \frac{I_f^4 V_\theta^2}{(I_f^2 V_\theta + V_f)^2} \cdot \frac{\partial V_f}{\partial v} + \frac{2 I_f V_\theta V_f^2}{(I_f^2 V_\theta + V_f)^2} \cdot \frac{-(I_f - (1-\beta)(1+k)\theta_0)\frac{\partial V_f}{\partial v}}{h'(I_f)} \\
&= \frac{\partial V_f}{\partial v} \cdot \frac{3 I_f^3 V_\theta^2 (I_f - \frac{2}{3}(1+k)\theta_0)}{(I_f^2 V_\theta + V_f) h'(I_f)},
\end{aligned}
$$

where $h'(I_f) = 3V_\theta I_f^2 - 2V_\theta(1+k)\theta_0 I_f + V_f > 0$ as shown in the proof of Proposition 1 and Lemma 1. The third equality results from the substitution $-(I_f - (1-\beta)(1+k)\theta_0) = I_f^2 V_\theta (I_f - (1+k)\theta_0)/V_f$ (from $h(I_f) = 0$) and simplification. Notice $\frac{\partial V_f}{\partial v} > 0$. Therefore, if $(1-\beta)(1+k)\theta_0 \geq \frac{2}{3}(1+k)\theta_0$ or $\beta \leq \frac{1}{3}$, then $I_f > (1-\beta)(1+k)\theta_0 \geq \frac{2}{3}(1+k)\theta_0$ and $\frac{dVar[x|y^f]}{dv} > 0$.

Below consider the other case where $(1-\beta)(1+k)\theta_0 < \frac{2}{3}(1+k)\theta_0$ or $\beta > \frac{1}{3}$. Notice that $I_f < (1+k)\theta_0$ and I_f decreases in v. If $I_f(v=0) > \frac{2}{3}(1+k)\theta_0$, then there exists a $v^* > 0$ such that $V_f(k, v^*) = \frac{-V_\theta I_f^2(I_f - (1+k)\theta_0)}{I_f - (1-\beta)(1+k)\theta_0}|_{I_f = \frac{2}{3}(1+k)\theta_0}$. In other words, when $v \gtreqless v^*$, $I_f \lesseqgtr \frac{2}{3}(1+k)\theta_0$ and $\frac{dVar[x|y^f]}{dv} \lesseqgtr 0$. However, if $I_f(v=0) \leq \frac{2}{3}(1+k)\theta_0$, then $I_f \leq \frac{2}{3}(1+k)\theta_0$ and $\frac{dVar[x|y^f]}{dv} \leq 0$. Now we show the existence of a $\beta^* > \frac{1}{3}$ such that, when $\beta \gtreqless \beta^*$, $I_f(v=0) \lesseqgtr \frac{2}{3}(1+k)\theta_0$. Notice that $I_f \in ((1-\beta)(1+k)\theta_0, (1+k)\theta_0)$ for any v, I_f decreases in β, and $I_f \to -\infty$ as $\beta \to +\infty$ (from $h(I_f) = 0$). When $\beta = \frac{1}{3}$, $I_f(v=0) > \frac{2}{3}(1+k)\theta_0$. Thus, there exists a $\beta^* = \frac{1}{3} + \frac{4(1+k)^2\theta_0^2 V_\theta}{27 V_f(k, v=0)} > \frac{1}{3}$ such that, when $\beta \gtreqless \beta^*$, $I_f(v=0) \lesseqgtr \frac{2}{3}(1+k)\theta_0$. Notice it is likely that $\beta^* \geq 1$ (for example, when

$V_f(k, v=0)$ is relatively smaller than V_θ).

Given $I_f \to (1-\beta)(1+k)\theta_0$ as $v \to +\infty$ from Lemma 1, we can see $Var[x|y^p] \to L \equiv (1-\beta)^2(1+k)^2\theta_0^2 V_\theta$ as $v \to +\infty$. ■

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Figure 2. The impact of the accounting noise on the conditional variance under Partial accounting

P1: $\beta \leq \frac{1+\mathrm{k}}{3+\mathrm{k}+2V_\theta^f/V_\theta^p}$	P2: $\frac{1+\mathrm{k}}{3+\mathrm{k}+2V_\theta^f/V_\theta^p} < \beta < \frac{1+\mathrm{k}}{\mathrm{k}+2V_\theta^f/V_\theta^p}$	P3: $\beta \geq \frac{1+\mathrm{k}}{\mathrm{k}+2V_\theta^f/V_\theta^p}$

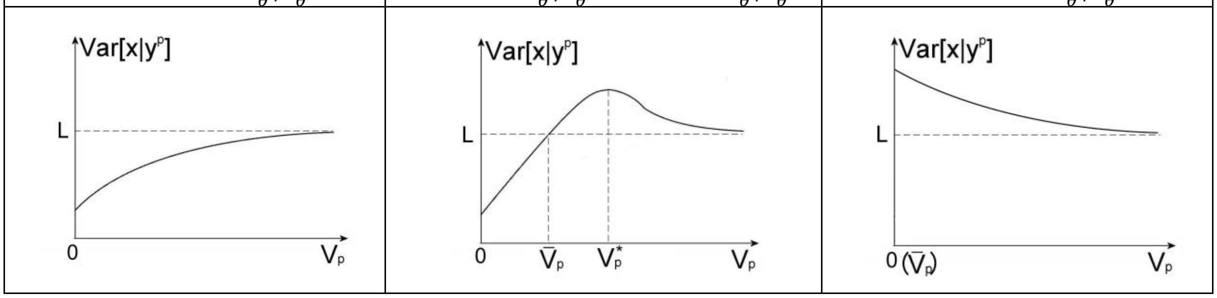
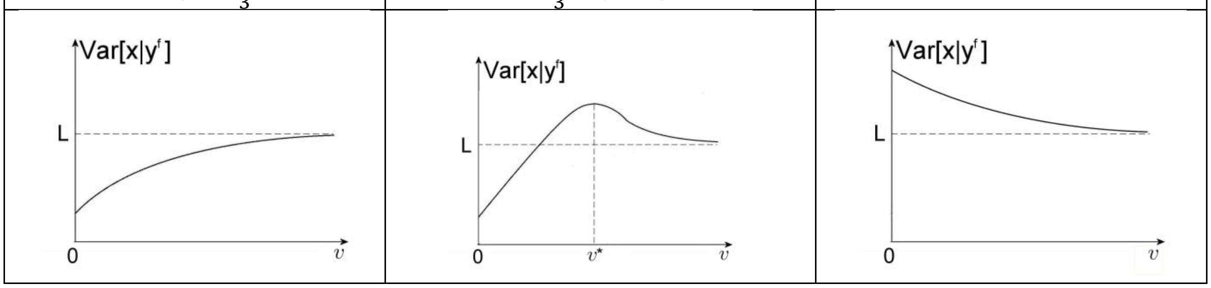


Figure 3. The impact of the accounting noise on the conditional variance under Full accounting

F1: $\beta \leq \frac{1}{3}$	F2: $\frac{1}{3} < \beta < \beta^*$	F3: $\beta \geq \beta^*$



L is the unconditional cash flow volatility, and $L = (1-\beta)^2(1+k)^2\theta_0^2 V_\theta$.