Aymptotic freedom

Symmetries have played an important role in high energy physics. Many of them are descibed by invariance under certain transformations. Here we discuss a special transformation of changin the scale. To simplify the discussion we use the so called natural units where we set $\hbar = c = 1$ so that all physical quantities have dimensions either in positive or negative powers of masses. Of course there are physical parameters which are deimesionless. For example, in electrodynamics the coupling constant e is dimensionless (in natural unit). Another example is the coupling constant in the $\lambda\phi^4$ theory is also dimensionless. Under the scale transformation for the masses all physical quanities change their magnitudes according to their dimensions except those parameters which are dimensionless. The most important physical parmeters which carries dimension are energies and momenta. The magnitudes of the these parameters depend on the states of the motion. Thus we can vary the magnitudes of energies and momenta by scaling and leave the dimensionless parameters unchanged. This will give us a handle in the studies of high energy behavior of physical processes. For example, the cross section of scattering of 2 particles has the dimension of area or inverse energy square in natural unit. Hence in the high energy limit it should decrease as the inverse second power of the energy if all dimenionless parameters are kept fixed. However,in this case of 2 particle scattering there is a complication in that the projectile only see the transverse size of the target which is constant and independent of energy(I thank TM Yan for pointing this out to me)

The situation looks fairly simple until it was realized that the process of reormalization might introduce an additional scale dependence into the original dimensionless physical coupling constant. This is due to the fact in the relativistic field theory there are many divergences coming from integration over infinite range of momenta. One needs to redefine some physical quautities by subtracting out these inifitities at some momentm scale before one can compare them to the experimental meaurements. If this can be done for a limited set of physical quantities, then we call it renormalizable theory. Thus this subtraction point introduce a new dimensional parameter and makes physical paramter a scale dependent quantity. If we need to substract the infinities from an infinite number of different physical quantities, we call it unrernomalizable theory and we do not know how to make sense out of it. It turns out that there only very few type of renormalizable theory. This puts a severe restriction on the possible candidates for a sensible theory.

Thus the renomalization makes the coupling constant scale dependent or energy dependent. For example, the strength of the electric charge in electrodynamic will depend on the energy involved in the physical processes. As we will discusse later this concept of enegy dependent coupling constant will lead to the formulation of the theory of strong inteaction in the form of Quantumchromodynamics (QCD). This scale or energy dependent coupling constant is sometime called **effective coupling constant**. Recall that the coupling constant characterizes the strength of interaction between particles. One can ask what happens to this effective coupling constant when the energy is very large. As was studied by Wilson, Callan and Symanski that the aymptotic behavior of the effective coupling is determined by a function of coupling constant called β-function which depends only on the coupling constant. In fact it is the zeros of the β-function and the slope at this zero determine the aymptotic behavior of the coupling constant. For example, if β-function vanishes at g_0, i.e. $\beta(g_0) = 0$ then g_0 is called the critical point, or fixed point. Furthermore if $d\beta/dg|_{g=g_0} < 0$, at $g = g_0$ is called a ultraviolet stable fixed point. This means that the coupling constant approaches g_0 as energy approaches ∞. On the other hand, if $d\beta/dg|_{g=g_0} > 0$, at $g = g_0$, then $g \to g_0$, as energy approaches 0 and g_0 is called the infared stable fiexed point. The $\beta-$function $\beta(g)$ which controls the aymptotic behavior is in general a complicate function of coupling constant and is very difficult to compute. But it has a simple property that in power series expansion it starts with term in some power of coupling constant, g^n , where n is some positve integer. This means that all β functions have a natural zero at origin $g = 0$. So if the first non-zero derivative is negative then the effective coupling constant will approach zero at large energy.

The data from the deep inelastic scattering indicates that the strong interaction beween the constituents inside the proton seems to be descibed by a free field theory. So it is natural to look for renormalizbe field theory which behaves like a free field theory i.e. the effective coupling constant approaches zero at high energies. Antony Zee was the first to investigate this question systematically in all renormalizbe field theory except the non-ablian gauge theory where the computation is quite complicate and concluded that the origin in coupling constant space is not a ultraviolet fixed point. Later D. Politzer find that first non-zero derivative is negative at the origin in the non-abelian gauge theory. Since the computation is all about the sign of the first non-zero derivative at the origin he was not too sure about the result. So he went to talk to his thesis advisor, Sideney Coleman who was on sabatical leave at Princeton about this result. Somehow, Dave Gross and his student Frank Wilczek heard about the result and quickly did their own calculation and verify Politzer's result. Thus the non-Ablelian gauge theory stands out as the only possible class of strong interaction theory which can explain the sacling phenomenena observed in deep inelastic ep scattering. To formulate the theory more precisely we need to specify the detailed nature of this non-abelian symmetry. Soon people recall that in the formulation of quark model in the 60's, $SU(3)$ color symmetry was introducded to explain the quark wave function of the resonance Δ^{++} and other aspect of the quark model. Quickly, people makes this non-ablian color $SU(3)$ symmetry a local gauge symmetry and formulate QCD theory where quarks carry the local $SU(3)$ color symmtry.

This is how the QCD theory develops and had become the only theory for the strong interactions. Eventhogh there are still physical quantities we do not know how to calculate, we seem to strongly believe that it is the right theory.