Necessitism, Contingentism and Plural Quantification

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Abstract: Necessitism is the view that necessarily everything is necessarily something; contingentism is the negation of necessitism. The dispute between them is reminiscent of, but clearer than, the more familiar one between possibilism and actualism. A mapping often used to 'translate' actualist discourse into possibilist discourse is adapted to map every sentence of a first-order modal language to a sentence the contingentist (but not the necessitist) may regard as equivalent to it but which is neutral in the dispute. This mapping enables the necessitist to extract a 'cash value' from what the contingentist says. Similarly, a mapping often used to 'translate' possibilist discourse into actualist discourse is adapted to map every sentence of the language to a sentence the necessitist (but not the contingentist) may regard as equivalent to it but which is neutral in the dispute. This mapping enables the contingentist to extract a 'cash value' from what the necessitist says. Neither mapping is a translation in the usual sense, since necessitists and contingentists use the same language with the same meanings. The former mapping is extended to a second-order modal language under a plural interpretation of the second-order variables. It is proved that the latter mapping cannot be. Thus although the necessitist can extract a 'cash value' from what the contingentist says in the second-order language, the contingentist cannot extract a 'cash value' from some of what the necessitist says, even when it raises significant questions. This poses contingentism a serious challenge.

1. Incompossibility

Often, some things seem individually possible yet not jointly compossible: although there could be any one of them, there could not be all of them. The sort of possibility at issue here and throughout is the most unrestricted metaphysical possibility, the dual of metaphysical necessity; something is metaphysically necessary if and only if it would have been the case whatever had been the case. Likewise, the sort of being at issue is being identical with something in the most unrestricted sense of the quantifier.[1]

Here are three putative examples of incompossible possibles:

(i) A human H could grow from a sperm S and an egg E. A human H* could grow from the sperm S and an egg E* distinct from E. But there could not be both H and H*. For given the essentiality to humans of their origins (Kripke 1980), H could be only by growing from S and E, while H* could be only by growing from S and E*. Given the nature of the entities, H cannot grow from S and E while H* grows from S and E*. Necessarily, therefore, H is only if H* is not. Thus H and H*, although both individually possible, are not jointly compossible.[2]

(ii) Let 'WWI' name the concrete, unrepeatable token event of the First World War, in all its terrible detail. WWI is possible, because it happened. In its stead, a concrete, unrepeatable token event GEP of a golden era of world peace from 1914 to 1918 could just about have happened, if Princip had missed at Sarajevo. GEP is possible too. But there could not have been both WWI and GEP. For concrete, unrepeatable, token events can be only by happening, and their main features are not completely accidental to them. WWI could not have happened without a major war in the period 1914–18, while GEP could not have happened with a major war in 1914–18. Even if an event similar to

WWI had happened in a period other than 1914–18, it would not have been WWI, and if an event similar to GEP had happened in a period other than 1914–18, it would not have been GEP. Necessarily, therefore, WWI happened only if GEP did not. Thus WWI and GEP, although both individually possible, were not jointly compossible.[3]

(iii) Suppose that contingently true propositions have truthmakers, things that make them true in at least the sense that if T is a truthmaker for the proposition P then necessarily there is T only if P is true.[4] Since the proposition that this computer is on (at time t) is true, it has a truthmaker T. Necessarily, there is T only if the proposition that this computer is on is true, in which case this computer *is* on. Since the proposition that this computer is not on (at t) could be true, it could have a truthmaker T*. Necessarily, there is T* only if the proposition that this computer is not on is true, in which case this computer *isn't* on. Necessarily, therefore, there is T only if there is not T*. Thus T and T*, although both individually possible, are not jointly compossible.

Given the necessity of identity (Kripke 1971), individually possible but jointly incompossible things are never identical. For necessarily, if there is x and x is identical with y then there is y. By the necessity of identity, if x is identical with y then necessarily x is identical with y if there is x. Therefore, if x is identical with y, if there could be x then there could be both x and y. Thus, in the examples, H is not identical with H*, WWI is not identical with GEP and T is not identical with T*. A similar argument applies whenever $x_1, \ldots, x_n$ are individually possible but jointly incompossible, even if they are pairwise compossible: $x_1, \ldots, x_n$ are not all identical.

None of (i)-(iii) is uncontroversial. Our present interest is less in the particular examples themselves than in the general phenomenon which any of them illustrates, if genuine. On further reflection, it is puzzling. For suppose that x and y are individually possible but jointly incompossible. Thus there could not be both x and y. Consequently, there are not both x and y. Hence either there is no such thing as x or there is no such thing as y after all. Either way, there is no example of shared possibility without compossibility, no pair of individually possible, jointly incompossible things. Indeed, the argument has modal force: it implies that there could not be individually possible, jointly incompossible things.

That last piece of reasoning has an air of sophistry, for it does nothing to explain away the plausibility of the examples. Rather, it suggests that we should speak more carefully. We can do that by using the formal language of quantified modal logic. We formalize 'x and y are individually possible but jointly incompossible' by this open formula:

$$\Diamond\exists z\, x{=}z \;\&\; \Diamond\exists z\, y{=}z \;\&\; \neg\Diamond(\exists z\, x{=}z \;\&\; \exists z\, y{=}z)$$

The formalization generalizes naturally to claims of more than two individually possible, jointly incompossible things. By the obvious principle that whatever is could be (the T schema $A \rightarrow \Diamond A$ of modal logic) and the first-order logic of identity, we can show that everything is compossible with everything, and even apply the rule of necessitation to conclude that necessarily everything is compossible with everything. There could not be jointly incompossible things; this formula is inconsistent:

(1) $\Diamond\exists x\, \exists y\, \neg\Diamond(\exists z\, x{=}z \;\&\; \exists z\, y{=}z)$

However, the inconsistency of (1) in elementary first-order modal logic does not imply that the open formula $\neg\Diamond(\exists z\, x=z \;\&\; \exists z\, y=z)$ itself is inconsistent. In many familiar systems of quantified modal logic with identity, (2) is consistent:

(2) $\Diamond\exists x\, \Diamond\exists y\, \neg\Diamond(\exists z\, x=z \;\&\; \exists z\, y=z)$

Although there could not be incompossible things, there could be something with which there could be something incompossible. Under reasonable assumptions, from (2) one can derive the individual possibility clauses too, so that (2) and (3) are interderivable:[5]

(3) $\Diamond\exists x\, \Diamond\exists y\, (\Diamond\exists z\, x=z \;\&\; \Diamond\exists z\, y=z \;\&\; \neg\Diamond(\exists z\, x=z \;\&\; \exists z\, y=z))$

In many familiar systems of quantified modal logic, (3) is consistent. When we spoke loosely of examples of individually possible but jointly incompossible things, we were gesturing towards arguments for (3) from specific illustrations.

Formally, (3) is consistent in a system of quantified modal logic with varying domains (Kripke 1963), for it has a model with just two possible worlds, each with a domain of just one object (the only object there is in that world), the two domains being disjoint from each other. Informally, in explaining the crucial semantic difference between (1) and (3), it is tempting to say that the absence of a possibility operator between the initial quantifiers in (1) constrains the second of the verifying objects to be in the same possible world as the first, automatically making them compossible, whereas the presence of such an operator between the initial quantifiers in (3) allows the second of the verifying objects to be in a different possible world from the first. But putting it like that presupposes that we can quantify here and now over whatever there would be if any of those possible circumstances obtained: in other words, that necessarily whatever could be is already something, on the unrestricted reading of ‘something’. So we lose our examples of incompossibility again, for a different but still perfectly general reason. Although we can still use the Kripke model to establish the consistency of (3) in some formal systems, we cannot treat it as literally *exemplifying* a way for (3) to hold, since there is such a model only if there are the elements of both domains, in which case they are not really incompossible. At best, the model *represents* a way for (3) to hold by using representatives which are not themselves incompossible. The model is not even isomorphic to the situation it represents, since the situation but not the model has the relevant structural feature of being a case of incompossibility. For some purposes such representations may be quite useful. But we cannot assume that studying them will cast much light on the metaphysics. Although it is uncontroversial that some Kripke models represent incompossibility in the intended sense, it by no means follows that incompossibility is genuinely possible.

From one perspective, examples (i)-(iii) provide at least superficially plausible arguments for (3), and so for the genuineness of incompossibility. From another, we seem to have a bird’s eye view of such cases, on which we can see them as involving two things each of which can be something, even though they cannot both be something: but then the incompossibility seems to be an illusion, since otherwise there would not be *two* things to count. Of course, some of those same issues arise whenever there could have been something which in fact there is not, even if incompossibility is not at issue.

However, the phenomenon of apparent incompossibility not only raises them in an acute form: it will play a crucial role in the later argument of this paper, and so makes a convenient starting-point.

2. Necessitism and Contingentism

To understand the conflicting perspectives on incompossibility, it is useful to take a step back and contrast two general views of the modal status of ontology. I will call them 'necessitism' and 'contingentism'. Necessitism partially resembles views associated with the term 'possibilism'; contingentism partially resembles views associated with the term 'actualism'. However, the use of the words 'actualism' and 'possibilism' has become badly confused.

One might expect the difference to be that the actualist holds that everything is actual while the possibilist holds that not everything is actual, but everything is possible. If so, what is it for an object to be actual, or to be possible? 'To be actual is to be in the actual world' is no better than a pseudo-explanation, for 'in the actual world' is more obscure than 'actual'.

Modal realists such as David Lewis claim to hear a reading on which 'the actual world' refers just to our spatiotemporal system, even though there are other spatiotemporal systems spatiotemporally unrelated to ours. However, most participants in the actualism-possibilism debate reject modal realism, and hold that if there are such other spatiotemporal systems, they are just as actual as our own. The debate is not about whether there are other spatiotemporal systems. One might believe that there are many other such systems and still count as an actualist, because one holds that their inhabitants are just as straightforwardly real and actual as the inhabitants of our own, like the inhabitants of other countries. If we presuppose modal realism, we cannot explain what is at stake in the actualism-possibilism debate.

On a less loaded account, what is in the actual world is simply what there actually is. The reference to a world was a digression; the point is just that what is actual is what there actually is. Analogously, what is possible is what there could be. But, on standard accounts of the logic of 'actually', its insertion makes a difference in truth-value only when in the scope of a modal operator. If there is a talking donkey then there actually is a talking donkey, even though there *could have been* a talking donkey without there actually being a talking donkey.[6] Therefore, since whatever is is, whatever is actually is: if there is something, then there actually is such a thing. So on this understanding, actualism is trivially true and possibilism trivially false. Thus being actual had better be actually doing something more exclusive than just being, otherwise the supposed dispute is silly. But what is that more demanding thing, if a dispute about whether everything does it is fundamental to the interpretation of quantified modal logic, as the dispute between actualism and possibilism is supposed to be? And why should the alternative to the view that everything actually does the more exclusive thing be a view on which everything *could* do the more exclusive thing? Why cannot something be impossible, in the sense that it could not do the more exclusive thing? Although we might complicate the definitions of 'actualism' and 'possibilism' in attempts to construct a more sensible dispute, it is better to make a fresh start with fresh terminology and clearer distinctions.[7]

Contingentism asserts that ontology is contingent: what there is is an at least partly contingent matter. *Necessitism* denies that ontology is contingent: what there is is a wholly necessary matter.

For instance, a contingentist typically holds that it is contingent that there is the Thames: there could have been no such river, and in those circumstances there would have been no Thames. By contrast, a necessitist typically holds that it is necessary that there is the Thames: there could have been no such river, but in those circumstances there would still have been the Thames, a non-river located nowhere that could have been a river located in England. Thus the contingentist will insist that necessarily if there is the Thames it is a river, while the necessitist allows at most that necessarily if the Thames is located somewhere it is a river.

In the definitions of 'contingentism' and 'necessitism', 'what there is' should not be understood as 'what kinds are instantiated'. The necessitist may agree with the contingentist that it is contingent whether there are rivers. The dispute concerns whether it is contingent or necessary which particular things there are, never mind what kinds they instantiate (where 'thing' is unrestricted). According to the necessitist, it is necessary that there is the Thames. It does not follow that necessarily the Thames is a river; at most it follows that necessarily the Thames is a possible river.[8] By contrast, the contingentist denies that it is necessary that there is the Thames: there could have been no such thing (river or non-river) as the Thames at all. In those circumstances, there would have been no Thames for even the most unrestricted quantifier to range over.

Of course, given the austerity of the two opposing general claims, neither of them will entail a unique specific account of particular examples. For example, neither of them by itself entails that there are rivers, or that there are not, or that it is contingent what rivers there are, and contingentism by itself does not even entail that rivers have contingent being: the general doctrine says that there are or could be counterexamples to necessitism, but not what they are. Nevertheless, for purposes of illustration, it is sometimes (as here) convenient to describe the contingentist or the necessitist as saying things about an example that follow not from the general doctrine by itself but only from its conjunction with independently plausible auxiliary assumptions that are neutral between the two views.

At first sight, contingentism seems by far the more plausible of the two views. However, some logical considerations turn out to favour necessitism.[9] The purpose of this paper is not to go over that old ground again but to analyse the dispute within a more abstract and systematic framework. Some clarifications are needed first.

Necessitists can simulate contingentist talk, by tacitly restricting their quantifiers to what is concrete or in space-time, and may even hold that most ordinary talk employs such tacit restrictions. Modal realists such as David Lewis constitute an interesting special case. They regard quantifiers over worlds and their inhabitants as metaphysically more perspicuous than modal operators. They often restrict their quantifiers to the inhabitants of the world at issue. When they condescend to use modal operators, which they regard as implicit quantifiers over worlds, they typically apply the corresponding tacit restriction to explicit quantifiers, and sound like contingentists. But sometimes they quantify without any such restriction, for example when they articulate their modal realism. In natural languages, we can meaningfully apply a modal operator to a sentence with unrestricted quantifiers, and doing so does not automatically enforce such a restriction on those quantifiers, otherwise the elementary principle $A \rightarrow \Diamond A$ would fail, for example when A expresses the modal realist claim that there are two things which do not cohabit any world. Since necessitism is intended to be understood with an unrestricted reading of the quantifiers, modal realists had better interpret it as the claim that for any worlds w and w*, if in w there (unrestrictedly) is something o then in w* there (unrestrictedly) is o too. Since the absence of a restriction on the quantifiers makes the qualifications 'in w' and 'in w*' redundant here, modal realism reduces necessitism to a triviality. But one can also be a necessitist without being a modal realist. If one does not treat modal operators as reducible to quantifiers over worlds, necessitism looks like a much less trivial proposition. In this paper, the focus will be more on necessitists who are not modal realists than on those who are.

In order not to beg the question against contingentism, we can continue to work within a framework of quantified modal logic, without prejudice to the question of reducibility. For simplicity, I will assume as part of the background modal logic the attractive principles of the modal system S5, on which what is possible is necessarily possible and what is necessary is necessarily necessary. Thus if it is contingent whether there is the Thames, it is necessarily contingent whether there is the Thames, and if it is necessary that there is the Thames, it is necessarily necessary that there is the Thames. Although one could reformulate all the main points of this paper to avoid reliance on S5, doing so would involve complicating many of the formulations in distracting ways, and is left as an exercise for the reader sceptical about S5. For example, in S5 it is redundant for the necessitist to say that matters of ontology are not just necessary, but necessarily necessary, and for the contingentist to say that they are not just contingent but necessarily contingent. Without S5, those amplifications would be needed.[10]

Given S5, we can capture necessitism in a single formula of first-order modal logic with identity, saying that necessarily everything is necessarily something:

(NNE) $\Box \forall x \Box \exists y\, x{=}y$

Correspondingly, we can capture contingentism in the negation of (NNE): possibly something is possibly nothing. Using the principles of S5, we can derive from (NNE) that necessarily necessarily everything is necessarily necessarily something, and so on.

From (NNE), we can derive two further principles of first-order modal logic which the necessitist will also endorse, the Barcan formula (BF) and its converse (CBF):

(BF) $\Diamond \exists x\, \mathrm{A} \rightarrow \exists x \Diamond \mathrm{A}$

(CBF) $\exists x \Diamond \mathrm{A} \rightarrow \Diamond \exists x\, \mathrm{A}$

The schematic letter 'A' in (BF) and (CBF) can be replaced by any open formula; typically the variable '*x*' is free in A. Roughly, (BF) says that if there could have been something which met a condition, there is something which could have met that condition; (CBF) says that if there is something which could have met a condition, there could have been something which met the condition. The necessitist endorses (BF) because if there could have been something that met a condition, by (NNE) there could have been something which met the condition and was necessarily something, so in the spirit of S5 there could have been something which met the condition and is actually something, so there *is* something which could have met the condition. (BF) holds if and only if there could not have been something which is not already something; there could not have been more than there is. The necessitist endorses (CBF) because if there is something which could have met a condition, by (NNE) it would still have been something, so there could have been something which met the condition. (CBF) holds if and only if everything is necessarily something; there could not have been less than there is.

Since the reasoning from (NNE) to (BF) and (CBF) appeals to nothing contingent about how things actually are, and is valid even if the condition is specified using individual parameters or free variables, it generalizes to schemas whose instances comprise the results of prefixing (BF) and (CBF) by any string of necessity operators and universal quantifiers in any order. Consequently, the necessitist endorses those strengthenings too. The strengthened versions of (BF) and (CBF) turn out to be equivalent to each other in S5. Informally, if (BF) fails, then there could have been more than there is, so it could have been that there could have been less than there was, so (CBF) could have failed; if (CBF) fails, then there could have been less than there is, so it could have been that there could have been more than there was, so (BF) could have failed. A generalization of that argument shows that strengthened (BF) is equivalent to strengthened (CBF).

Conversely, still given S5, an instance of unstrengthened (BF) and an instance of unstrengthened (CBF) jointly entail (NNE), and so necessitism. Informally, if there could not have been more or less than there actually is, then there could only have been what there actually is, so ontology is not contingent.[11] Since each of strengthened (BF) and strengthened (CBF) by itself entails both unstrengthened schemas, each strengthened schema by itself entails (NNE), and so necessitism.

Thus contingentists must reject each strengthened schema and the conjunction of the unstrengthened ones. Indeed, they will typically reject each unstrengthened schema. If we read A in (BF) as 'x is actually nothing' (where being nothing is just not being something), they will normally accept the antecedent 'There could have been something which is actually nothing', for example on the grounds that there could have been but actually is not something such as H*, or GEP, or T* in examples (i)-(iii) respectively in section 1. But they will reject the consequent 'There is something which could have been actually nothing', for evidently there is not actually something which could have been actually nothing: whether in the relevant sense it is *actually* (as it were, in *this* world) something is not contingent. Thus contingentists will normally reject unstrengthened (BF). Similarly, if we read A in (CBF) as 'x is nothing', they will normally accept the antecedent 'There is something which could have been nothing', with any ordinary material object as an example. But they will reject the consequent 'There could have been something which was nothing', since it attributes possibility to something inconsistent in first-order logic ($\exists x \neg\exists y\, x=y$). Thus contingentists will normally reject unstrengthened (CBF) too.

How should the necessitist and the contingentist handle putative examples of individually possible, jointly incompossible things?

For the necessitist, since the non-modal subformulas of (3) only concern what there is, they are all non-contingent, so all the modal operators in (3) are redundant; thus (3) is equivalent to a contradiction:

(4) $\exists x\ \exists y\ (\exists z\ x=z\ \&\ \exists z\ y=z\ \&\ \neg(\exists z\ x=z\ \&\ \exists z\ y=z))$

On this view, the appearance of incompossible possibles is an illusion. The necessitist can explain the appearance by appealing to alternative readings that make (3) true by implicitly restricting its quantifiers to a domain, for instance to concrete objects. Even though it is not contingent what objects there are, it may still be contingent what *concrete* objects there are, because it is contingent which of all the objects are concrete. Recall GEP, the possible concrete token event of a golden age of peace in the period 1914–18. For the necessitist, although GEP is something on the unrestricted reading, it was never something on a reading restricted to concrete objects, because it never happened. If GEP had happened, it would have been concrete, and so would have been something on the restricted as well as the unrestricted reading.[12]

In crude terms, for the necessitist we can generalize over an all-inclusive 'outer domain'; necessarily, the concrete objects constitute a varying 'inner domain' properly included in the constant 'outer domain'. Formally, we can make the restricted reading of (3) explicit thus:

(3R) $\Diamond\exists x\ (Cx\ \&\ \Diamond\exists y\ (Cy\ \&\ (\Diamond\exists z\ (Cz\ \&\ x=z)\ \&\ \Diamond\exists z\ (Cy\ \&\ y=z)\ \&$
$\neg\Diamond(\exists z\ (Cz\ \&\ x=z)\ \&\ \exists z\ (Cz\ \&\ y=z)))))$

The restricting predicate C ('is concrete') is treated as atomic. Then (3R) is consistent, even given (NNE), (BF) and (CBF), since it has a model with a constant outer domain that contains two objects o and o*, where o is in the extension of C at some worlds and o* is in the extension of C at other worlds, but at no world are both o and o* in the extension of C. The same model falsifies the correspondingly restricted readings of (NNE), (BF) and (CBF).

The necessitist can treat putative cases (i) and (ii) of incompossible possibles in section 1 as true cases of (3R). Putative case (iii) is much harder to deal with in that way, since it is the truthmaker itself, not its being concrete, that is supposed to make the proposition at issue true. In that case, the necessitist must follow a more drastic eliminativist strategy, but the arguments for postulating truthmakers in the first place were never strong (Williamson 1999a).

Of course, (NNE) by itself does not entail (3R). (NNE) is logically consistent with the claim that necessarily everything is concrete, which collapses (3R) back into (3). A necessitist who accepted that claim would have to find some other means of explaining away apparent cases of incompossible possibles. One can imagine such a necessitist, perhaps one who holds that necessarily everything is composed from a fixed stock of mereological atoms. However, such a metaphysical theory involves an implausibly cramped conception of the concrete. For simplicity, this paper will focus on a more plausible form of necessitism, one which involves no unusual commitments within the domain of the concrete, and makes the boundaries of that domain as contingent as others conceive them to be, while preserving necessitism by adding a domain of the contingently non-concrete, with matchingly contingent boundaries so that the boundaries of the union of the two domains are not contingent. Thus the main focus of the dispute between such necessitists and contingentists will be the domain of the contingently non-concrete.[13]

By contrast with necessitists, contingentists have no special reason to consider restricted readings of the quantifiers in (3), since for them it is consistent even on the unrestricted reading, as are the negations of (NNE) and many instances of (BF) and (CBF). Strictly speaking, contingentism by itself does not entail (3), since it is consistent, although bizarre, to suppose that it is contingent whether a principle of ontological plenitude holds, according to which whatever could have been something is something. If ontological plenitude could indeed hold contingently, then contingentism is true and (3) false. Apparent counterexamples such as (i)-(iii) in section 1 would have to be explained away somehow. But the contingentist is also free to take the examples at face value, as arguments for (3) on the unrestricted reading, and in practice may well do so. The contingentist can simply accept the phenomenon of incompossible possibles in the sense of (3) as veridical.

We can deepen our understanding of the comparative advantages and disadvantages of the two views by considering how their proponents may try to communicate with each other. The next section introduces this issue.

3. Mappings between necessitist and contingentist discourse

The dispute between necessitism and contingentism is genuine, not verbal. It does not depend on mutual misunderstanding; a given formula means in the contingentist's mouth just what it means in the necessitist's mouth. When the necessitist asserts and the contingentist denies (NNE), each correctly understands the other as using unrestricted quantification; they disagree about what specifically there is or could have been to be unrestrictedly quantified over. For example, those contingentists who assert 'Nothing is contingently non-concrete' do not restrict the quantifier to exclude the contingently non-concrete, for that would trivialize their claim; by leaving the quantifier unrestricted, they enable the sentence to express exactly the metaphysical generality they intend. They hold that reality, not a verbal stipulation, excludes the contingently non-concrete. Necessitism and contingentism are no mere notational variants of each other. The dispute cannot be resolved by clever non-homophonic translation schemes between the necessitist's talk and the contingentist's.[14]

By contrast, discussion of actualism and possibilism made much of so-called 'translations' between actualist and possibilist discourse. Such schemes mapped the actualist quantifiers to the restrictions of the corresponding possibilist quantifiers by an 'existence' predicate, and conversely the possibilist quantifiers to combinations of the corresponding actualist quantifiers with appropriate modal operators. The idea was that each side could accept the translations of the sentences the other side accepted, so that any remaining dispute concerned the relative explanatory priority, not the truth, of the actualist's and the possibilist's sentences.[15] That made it even more obscure what the metaphysical difference between actualism and possibilism was supposed to be.

Our present concern is with the clear dispute between contingentism and necessitism. Since (NNE) is unequivocal, and contingentists accept nothing synonymous with it, no correct translation scheme maps all sentences necessitists accept to sentences contingentists accept. Similarly, since necessitists accept nothing synonymous with the negation of (NNE), no correct translation scheme maps all sentences contingentists accept to sentences necessitists accept. Whereas translation schemes between actualist and possibilist languages were used to compare them in expressive power, contingentism and necessitism use the very same language unequivocally.

Nevertheless, we can define mappings between contingentist and necessitist discourse which are formally similar to those between actualist and possibilist discourse. We can refine our understanding of the dispute between contingentism and necessitism by asking what those mappings achieve. The next sections begin that task.

4. Mapping contingentist to necessitist discourse

An initial obstacle to mapping contingentist talk to necessitist talk on the model of the mapping from actualist talk to possibilist talk is that the latter mapped the actualist quantifiers to the restrictions of the corresponding possibilist quantifiers by an 'existence' predicate. But if 'x exists' is equivalent to 'x is something', where 'something' is unrestricted, then the restriction makes no logical difference: the restricted quantifiers are logically equivalent to the corresponding unrestricted ones; we might as well have mapped each formula to itself. For the mapping to have any point, we need a more exclusive restriction.

Fortunately, we have an appropriate precedent in the mapping from (3), the contingentist claim of incompossible possibles, to the corresponding necessitist claim (3R), in which the quantifiers are restricted by the predicate C, read 'is concrete' rather than 'exists'. The effect is to map a sentence in dispute between contingentism and necessitism (since the contingentist accepts (3) while the necessitist rejects it) to a sentence neutral between contingentism and necessitism (since both a contingentist and a necessitist can accept (3R)). For example, the latter holds on both their competing accounts of the original purported cases of incompossible possibles. Indeed, a contingentist who holds that necessarily everything is concrete is committed to the equivalence of (3) and (3R), for a quantifier restricted by a condition that necessarily everything meets is necessarily equivalent to the corresponding unrestricted quantifier. By contrast, necessitists are typically committed to the non-equivalence of (3) and (3R), since they reject the former and accept the latter. They deny that necessarily everything is concrete.

The assumption that the contingentist holds that necessarily everything is concrete is unnecessarily restrictive. The contingentist may acknowledge sets of concrete things, sets of such sets, and so on. It is a stretch to call all these sets 'concrete'. The contingentist may even acknowledge abstract objects such as numbers (to be a number is not to be a merely possible something else). If numbers have necessary being, that is still consistent with contingentism, which says that some things have contingent being, not that all do. Nevertheless, we may hope that a fully developed contingentist metaphysics will involve *some* putatively necessary condition on being that clearly excludes at least some of the necessitist's contingently non-concrete objects. In that case, the dispute may shift to whether that condition really is necessary for being. Although the contingentist may be tempted to call the condition 'existence', that will not help explain what the contingentist objects to in any of the necessitist's claims of contingent non-concreteness, because 'existing' may be naturally understood as 'being something', and it is trivial that being something is a necessary condition for being something. By contrast, it is not at all trivial that being concrete is a necessary condition for being something. A more liberal version of contingentism replaces the contentious metaphysical claim that necessarily everything is concrete with the weaker claim that necessarily nothing is contingently non-concrete.

We may subsume a range of contingentist ways of fleshing out the denial of (NNE) under the vague slogan that necessarily everything is *grounded in* the concrete. Any concrete thing is grounded in the concrete, and any set of things which are grounded in the concrete is itself grounded in the concrete, although not itself concrete. Thus even the empty set counts as grounded in the concrete. Similarly, numbers may count as grounded in the concrete, perhaps through one or more stages of logicist abstraction. But, we may suppose, at least some of the necessitist's purported contingently non-concrete things would not count as grounded in the concrete, because the things which would be required to ground them in the concrete would themselves be (contingently) non-concrete. The notion of groundedness in the concrete will be left in this vague, highly schematic form here, in order to achieve generality over many different more specific forms of contingentism. Nor do we require the notion to be reducible to more basic or precise terms. We do assume it intelligible to non-contingentists. For convenience, let us use the word 'chunky' as short for 'grounded in the concrete', and reinterpret 'C' in (3R) accordingly as a predicate for chunkiness rather than concreteness.[16] Call any form of contingentism which can be regimented into this form 'chunky-style contingentism'. It adds to the denial of (NNE) the assertion that necessarily everything is chunky. The correspondingly specific form of necessitism adds to the assertion of (NNE) the denial that necessarily everything is chunky.

Chunky-style contingentism provides one satisfying explanation of the contingentist's denial of (NNE). For it is presumably common ground between the contingentist and the necessitist that it is contingent what chunky things there are. In particular, for any material object o, it is presumably contingent whether there is such a chunky thing as o. Given that premise, the claim that necessarily the only things are chunky things entails that it is contingent what things there are, which is contingentism itself, the negation of (NNE). Thus the chunky-style contingentist denies (NNE) and, by contraposition, the corresponding necessitist denies that necessarily the only things are chunky things.

However, that is not the only conceivable form for the dispute between contingentism and necessitism to take. No notion of chunkiness even occurs in (NNE). Contingentism as such (the negation of (NNE)) does not entail chunky-style contingentism, and necessitism as such ((NNE)) does not entail chunky-style necessitism. Someone who denies that necessarily everything is grounded in the concrete might also deny (NNE), perhaps on the basis of liberalism about possibility. For example, they might hold that there are no necessary connections between disjoint concrete objects, so that any of them could have been without the others. What such a non-chunky-style contingentist objects to in necessitism is not its postulation of non-chunky objects but its restriction on possibility. A fuller treatment of the topic would need to say much more about such forms of contingentism and corresponding forms of necessitism. However, this paper will concentrate on the dispute between chunky-style contingentism and the corresponding form of necessitism. Chunky-style contingentism is the closest form of contingentism to common sense. Moreover, this form of the dispute is tractable in a special way, as we shall see. For brevity, the qualification 'chunky-style' will usually be left tacit below.

The underlying idea is that the contingentist and necessitist are not in dispute over the domain of the chunky. Even if an individual contingentist and an individual necessitist happen to disagree about something internal to the domain of the chunky, that is as it were a personal matter between them; it is not a disagreement to which they are committed by their adherence to the relevant metaphysical theories. The contingentist's view that necessarily everything is chunky entails the equivalence of the disputed metaphysical sentences with corresponding undisputed sentences about the domain of the chunky, but the necessitist disputes those equivalences. In effect, what the contingentist and the necessitist disagree on is how to project outwards from a neutral inner domain of sentences to a contentious outer domain. Even so, the disputed equivalences enable the necessitist to gain information from the contingentist's use of disputed sentences. For suppose that the contingentist asserts a disputed sentence A. The necessitist rejects A. Nevertheless, the contingentist's view that necessarily everything is chunky entails a biconditional of the form $(A)^{Con} \leftrightarrow A$, where $(A)^{Con}$ is a neutral sentence about the domain of the chunky. Thus the contingentist is also committed to $(A)^{Con}$, since it follows from A and the biconditional. But then, assuming that the contingentist has made no incidental slips, the necessitist can accept $(A)^{Con}$ too, since it is neutral. The mapping from A to $(A)^{Con}$ gives a systematic way of extracting from the contingentist's assertions what the necessitist can regard as a kernel of truth independent of the metaphysical dispute between them. Instead of treating contingentists as hopelessly mistaken, necessitists can see them as cognitively reasonable people, modulo one pervasive theoretical mistake, tracking genuine distinctions but misdescribing them.

The use of the mapping from A to $(A)^{Con}$ is not confined to cases in which the contingentist makes a positive assertion. If the contingentist simply questions A, the necessitist can take questioning $(A)^{Con}$ as the 'cash value' of the contingentist's speech act, even if they are both agnostic over the truth-value of the neutral sentence $(A)^{Con}$. More generally, the mapping gives a systematic way of extracting from the contingentist's speech acts what the necessitist can regard as a kernel of truth or falsity independent of the metaphysical dispute between them.

It would be a fundamental error to suppose that the neutral sentence $(A)^{Con}$ expresses what the contingentist 'really means' by the disputed sentence A, or what the necessitist believes the contingentist really means by A. For A is already a perfectly meaningful sentence, different in meaning from $(A)^{Con}$ because it lacks the latter's restriction to the domain of the chunky, and the necessitist knows that. There is no reason to patronize contingentists by treating them as linguistically incompetent, unable to express their views accurately in their own language. Contingentists know what they are doing when they speak. In particular, when they deny (NNE) in response to necessitists' assertion of it, that should be taken at face value. They are denying (NNE) itself, not irrelevantly denying the neutral $((NNE))^{Con}$, whose falsity is not in dispute. It is just that the mapping gives necessitists a way to factorize the consequences of A given the contingentist theory into those (such as $(A)^{Con} \leftrightarrow A$) which merely reflect the commitment to a contested general metaphysical principle and those (such as $(A)^{Con}$)

which are neutral. Think of sentences as goods, and neutral sentences as forming a common currency. Then $(A)^{Con}$ is the cash value of A to contingentists. Although A may not be worth exactly $(A)^{Con}$ to necessitists, they can use their knowledge of its cash value to contingentists in making sense of contingentist behaviour, and thereby gain useful information.

The equivalence of $(A)^{Con}$ and A depends on the chunky-style contingentist claim that necessarily everything is chunky; it does not follow from the bare theoretical claim of contingentism, the negation of (NNE). By itself, the mere negation of (NNE) is too undeveloped a metaphysical theory to provide a useful mapping to necessitist discourse. The extended theory, chunky-style contingentism, is needed for the equivalence of A and $(A)^{Con}$.

The sketch so far has glossed over some crucial details. It is time to fill them in. We must first be clear as to what language is in question. Working with a formal language will enable us to define the needed terms with precision and establish the needed results with rigour. The obvious choice is a standard first-order modal language, with identity and a variety of non-logical atomic predicates of various numbers of argument places, including the 1-place predicate C discussed above. Here we focus on matters of philosophical motivation; the appendix gives the formal details, including the proofs of the required results.

For a formula to be neutral, it does not suffice that the quantifiers in it are restricted by C wherever they occur. Consider this formula:

(5) $\exists x\,(Cx \,\&\, \Diamond(x{=}x \,\&\, \neg Cx))$

It says that something chunky could have been self-identical without being chunky. The only quantifier in (5) is restricted by C. Nevertheless, (5) is not neutral. A necessitist will normally accept (5), for example on the grounds that although there could have failed to be any such chunky thing as the Thames, in that case the Thames would still have been something (although not a river) and so would still have been self-identical. By contrast, a contingentist may well reject (5), on the grounds that something chunky could only have failed to be chunky by there failing to be any such thing as it at all, in which case there would have been no such thing as it to be self-identical. The problem is that in (5) $x=x$ occurs separated from Cx by the possibility operator $\Diamond$, so that evaluating (5) involves evaluating $x=x$ with respect to counterfactual circumstances for which Cx is not given: the modal operator has taken us outside the domain of the chunky. The natural solution is to make the initial basis of neutral formulas consist of conjunctions of the form $Fv_1 \ldots v_n \ \& \ Cv_1 \ \& \ \ldots \ Cv_n$, where F is an n-place atomic predicate, logical or non-logical.[17] For example, $x=y \ \& \ Cx \ \& \ Cy$ is neutral, but $x=y$ is not. Thus we need only consider predications of chunkiness and of anything else under the assumption that all the relevant objects are chunky. Evaluations of such formulas do not prejudge the issue between contingentism and necessitism. This treatment of atomic formulas turns out to make only a marginal difference to the arguments below, most of which could be reconstructed given the opposite treatment, which denies $\Box \forall x \ \Box(x=x \rightarrow \exists y \ x=y)$ the status of a logical truth. However, the present policy has the advantage of yielding arguably both the purest version of contingentism and the most perspicuous contrast between it and necessitism.

As for more complex expressions, the results of combining neutral formulas with truth-functional operators are in turn neutral formulas. For instance, if A is neutral then so is $\neg$A; if A and B are neutral then so is A & B. Similarly, the results of applying modal operators to neutral formulas are in turn neutral formulas. For instance, if A is neutral then so is $\Diamond$A: the dispute between contingentism and necessitism does not concern possibility or necessity *per se*, but rather what there would be in various possible circumstances. The results of applying quantifiers restricted by C to neutral formulas are in turn neutral formulas. For instance, if A is neutral and x an individual variable then $\exists x$ (Cx & A) is neutral too. Every formula logically equivalent to a neutral formula is also neutral.

The mapping from A to $(A)^{Con}$ is defined recursively. For instance, $(Fv_1\ldots v_n)^{Con}$ is $Fv_1\ldots v_n$ & Cv_1 & … & Cv_n, where F is an n-place atomic predicate. Similarly, $(\exists x\, A)^{Con}$ is $\exists x\, (Cx \,\&\, (A)^{Con})$. In other respects the mapping simply preserves the syntactic structure of the input formula. Since the mapping merely introduces the restrictions required for neutrality, a formula A is neutral if and only if it is logically equivalent to $(B)^{Con}$ for some formula B. Trivially, $(A)^{Con}$ itself is always neutral.

The main task is to show that each formula A is equivalent to $(A)^{Con}$, and therefore to a neutral formula, in the relevant contingentist theory, which can simply be axiomatized by the chunky-style contingentist claim that necessarily everything is chunky:

Aux[Con] $\quad \Box \forall x\, Cx$

Aux[Con] is not by itself inconsistent with (NNE), since both hold on the bizarre but consistent view that necessarily everything is necessarily something chunky. Thus, strictly speaking, the theory axiomatized by Aux[Con] does not entail contingentism. Contingentism itself turns out not to be needed for the main results about the mapping, but when it comes to applications we shall of course be primarily interested in the combination of Aux[Con] with contingentism.

The talk of logically equivalent formulas or of formulas equivalent in a theory presupposes a background logic for the first-order modal language. That logic should itself be neutral between contingentism and necessitism, for the sake of perspicuity. The natural course is to specify such a logic in terms of a possible worlds model theory in which the domain of quantification is permitted but not required to vary from world to world. Thus (NNE) is true in some models and false in others. A formula is valid if and only if it is true at every world in every model. Similarly, a formula is a valid consequence of a set of formulas if and only if in every model it is true at every world at which every member of the set is true. Thus the logic validates neither contingentism nor necessitism; it leaves each as a consistent metaphysical thesis. That by itself is not yet sufficient for full neutrality between contingentism and necessitism, since a logic might leave both (NNE) and its negation consistent while nevertheless drawing implausible (yet still consistent) consequences from one of them but not from the other. However, none of the constraints to be imposed on models will embody such a bias.

In specifying the logic in terms of the possible worlds model theory, we do not treat the latter as explanatorily or metaphysically basic. It is not, for none of the models is faithful to the intended unrestricted reading of the quantifiers, since each model restricts them to a set domain that contains less than everything. For example, no domain contains itself, and in models that invalidate (NNE) the domains of some worlds omit members of the domains of other worlds. Rather, both contingentists and necessitists should regard the model theory as a convenient algebraic device for encoding a large natural class of arguments which they do in fact agree to be valid. As a further check, we might formulate an axiomatic system of first-order modal logic with identity, prove it complete with respect to the model theory, and then look at the axioms and rules of the system case by case to see whether they were metaphysically uncontentious. We will not do that here. One limitation of such an approach is that it can be hard to tell by looking whether a principle of quantified modal logic is metaphysically uncontentious. Another limitation is that the approach does not generalize properly to second-order modal logic, with which we shall be concerned later, since no formal axiomatic system is sound and complete for second-order modal logic under the relevant interpretation.

Some specific features of the models deserve comment.

First, the values of the individual variables are drawn from a non-empty outer domain of which the domains of all worlds are subsets. Since the semantics is bivalent, an open formula is evaluated as true or false even at a world with respect to an assignment on which the value of a free variable is not in the domain of that world.[18] The value of the variable may even be omitted from the domains of all worlds: the union of the inner domains may be a proper subset of the outer domain. The domain of a world may be empty; indeed, the domains of all worlds in a given model may be empty. Thus not even $\Diamond \exists x\, x=x$ is valid. The point is not whether these are all legitimate metaphysical possibilities but that the background logic is weak enough not to exclude any of them.

Allowing members of the outer domain which belong to no inner domain conveniently enables us to avoid the technical inconvenience of lacking values for the variables without forcing us to make at least one inner domain non-empty (Williamson 1999b).

Second, the extension of an n-place non-logical atomic predicate at a given world is required to contain only n-tuples of members of the domain of that world. The point of the requirement is that for any object o, this combination is impossible: o has a property or relation, but there is (unrestrictedly) no such thing as o. For otherwise a property or relation could be had without there being anything at all to have it. Thus being something (on the unrestricted reading) is a necessary precondition for having a property or relation. But a predicate expresses a property or relation.[19] Thus both the contingentist and the necessitist should accept the formula $\Box \forall x \, \Box(\mathrm{F}x \rightarrow \exists y \; x{=}y)$, where F is a 1-place predicate and the quantifiers are unrestricted; likewise for many-place predicates. On the intended understanding of the possible worlds model theory, the domain of a world represents what there unrestrictedly is at that world. Within that framework, the way to validate $\Box \forall x \, \Box(\mathrm{F}x \rightarrow \exists y \; x{=}y)$ is by making the extension of F at a world be a subset of the domain of that world; likewise for other predicates. Similarly, $x{=}x$ is evaluated as true at a world only if the value of the variable x is in the domain of that world. For since the identity symbol is a logical constant, the model theory should stipulate its behaviour, rather than leaving it open to different interpretations in different models. The only natural alternative to the uniformly restricted treatment adopted here is a uniformly unrestricted one, on which $x{=}x$ is evaluated as true at every world, whether or not the value of x is in the domain of that world. But the uniformly unrestricted treatment, unlike the uniformly restricted one, is liable to be accused of begging the question against the contingentist, by not taking contingent non-being seriously enough: how could o have been self-identical if there had been no such thing as o? One may suspect contingentists who reject $\Box\forall x\, \Box(x=x \rightarrow \exists y\, x=y)$ of having allowed a modal realist picture to cloud their contingentist vision, as if the missing object were merely elsewhere, and so still self-identical.[20] The uniformly restricted treatment of identity is dialectically more appropriate. Once we evaluate $x=x$ as false at a world whose domain does not contain the value of x, we should evaluate all other atomic formulas F…x… as false at that world too, for none of them has a better right than $x=x$ to be evaluated as true. We can still evaluate some more complex formulas in which x occurs free as true at the world, by not parsing them as predications. For example, $\Diamond x=x$ is true at every world provided that the domain of some world contains the value of x; $\Diamond x=x$ is not a predication because $\Diamond$ operates on formulas, not on predicates. Similarly, both $\neg x=x$ and $\neg\exists y\, x=y$ are true at every world whose domain does not contain the value of x; they are not predications because $\neg$ and $\exists y$ operate on formulas, not on predicates. Again, both $\Diamond Fx$ and $\neg Fx$ may be true at a world whose domain does not contain the value of x. We should not regard such complex open formulas as themselves substitution instances of Fx, otherwise we forget that the predicational structure of Fx imposes its own restriction.[21] The model theory does not provide for distinctions at a world amongst the non-members of its domain that cannot be reduced to complex formulas in such ways, but neither necessitism nor contingentism requires such distinctions. In any case, as already noted, many of the results in this paper hold even on a different treatment of atomic formulas, but the present policy is the most perspicuous.

Third, the models have no relation of accessibility or relative possibility. Thus necessity at any world is truth at all worlds in the model, and possibility at any world is truth at one or more worlds. Consequently, every model validates all the principles of the system S5 of propositional modal logic. If one wanted to, one could easily add an accessibility relation, interpret necessity as truth at every accessible world and possibility as truth at some accessible world, and validate only a weaker modal logic than S5. Although such complications raise interesting issues, they will be ignored here. Both contingentism and necessitism are compatible with S5.

With respect to this background logic, $(A)^{Con} \leftrightarrow A$ follows from Aux[Con] for every formula A, so every formula is equivalent to a neutral formula (appendix 1.8). Moreover, the choice of $(A)^{Con}$ is unique up to logical equivalence, in the sense that any neutral formula equivalent given Aux[Con] to A is logically equivalent to $(A)^{Con}$ independently of Aux[Con] (appendix 1.10). In particular, if A itself is neutral, it is logically equivalent to $(A)^{Con}$ independently of Aux[Con]. Nothing weaker than Aux[Con] suffices to derive all the biconditionals $(A)^{Con} \leftrightarrow A$, for Aux[Con] follows from $(\text{Aux[Con]})^{Con} \leftrightarrow$ Aux[Con] (appendix 1.6), and so from any theory from which all such biconditionals follow. Indeed, $(\text{Aux[Con]})^{Con}$ is just a trivial logical truth; it says that necessarily everything chunky is chunky.

The assumption Aux[Con] that necessarily everything is chunky turns out to be independent of neutral formulas in the sense that its addition to a set of neutral formulas always constitutes a conservative extension: a neutral formula follows from the set with Aux[Con] only if it already followed from the set without Aux[Con] (appendix 1.9). Any theory T which entails Aux[Con] can be factorized into Aux[Con] and the set of its neutral consequences: the consequences of T are exactly the consequences of the combination of Aux[Con] and the neutral consequences of T (appendix 1.12). The logical relations between theories which entail Aux[Con] simply reflect the logical relations between their neutral parts (appendix 1.11). Once one has chosen Aux[Con] and which neutral formulas to accept, nothing else remains to be decided.

5. Mapping necessitist discourse to contingentist discourse

We now turn to mappings in the reverse direction. The point of such a mapping is to give the contingentist a systematic way of calculating the cash value of non-neutral sentences to the necessitist in the common currency of neutral sentences. Less metaphorically, it will enable the contingentist to extract useful information in terms independent of the metaphysical dispute from the necessitist's utterances.

For the contingentist, the necessitist is liable to track the truth that there could be an F, but misstate it as the falsehood that there is something that could be an F. That is not to impute a gross scope confusion to the necessitist; a more plausible accusation is that the necessitist has adopted an over-simple, over-strong theory which implies (BF) and so permits the quantifier 'there is' to be moved outside the scope of the modal operator 'could'. This diagnosis of necessitism is the starting point for the mappings considered below.

Sometimes the contingentist might simply prefix the necessitist's quantifiers with appropriate modal operators ($\Diamond$ for $\exists$, $\Box$ for $\forall$). Consider, for instance, the necessitist's claim that there is something which could have been a concrete event of an era of global peace in 1914–18:

(6) $\exists x \Diamond Px$

One can extract a kernel of contingentistically acceptable truth from (6) by mapping it to the claim that there could have been something which could have been a concrete event of global peace in 1914–18:

(7) $\Diamond \exists x \Diamond Px$

The contingentist accepts (7) because it follows from the clear truth that there could have been something which was a concrete event of peace in 1914–18:

(8) $\Diamond \exists x\, Px$

In other cases, however, that simple recipe works poorly. For example, the necessitist typically asserts that there is something contingently non-chunky:

(9) $\exists x\, (\neg Cx \;\&\; \Diamond Cx)$

The result of applying the simple recipe to (9) is (10):

(10) $\Diamond \exists x\, (\neg Cx \;\&\; \Diamond Cx)$

This says that there could have been something contingently non-chunky. But the contingentist who accepts Aux[Con] denies that there could have been something non-chunky and so cannot regard (10) as a kernel of truth in (9). Moreover, for the necessitist (10) makes a weaker claim than (9), even granted (NNE), since (9) entails that something is non-chunky while (10) does not.[22]

For the contingentist, the necessitist's mistake is to take into account mere general possibilities at the point of quantification; it is not a mistake at the point of first-level predication. The contingentist prefixes the necessitist's quantifiers with modal operators to mediate the effect of those quantifiers, not of the subsequent open formula. For instance, the initial possibility operator in (10) was put there to modify the quantifier, not to modify the matrix $\neg Cx \mathbin{\&} \Diamond Cx$. The contingentist needs to achieve the effect of removing the matrix from the scope of the modal operator. In (10), that can be achieved by inserting an 'actually' operator @ after the quantifier:

(11) $\Diamond \exists x\, @(\neg Cx \mathbin{\&} \Diamond Cx)$

This says that there could have been something which is *actually* contingently non-chunky. That should be acceptable to most contingentists and most necessitists. A contingentist who holds that there could have been something chunky which is actually nothing will accept (11) as a consequence, since being chunky uncontentiously requires being something and it is not contingent what is *actually* (back here in this world, as it were) the case. A necessitist who accepts the actual truth of (9) will accept (11) as an obvious consequence. The same technique works for the previous example: the necessitist's (6) ('There is something which could have been a concrete event of peace in 1914–18') is replaced by (12) ('There could have been something which actually could have been a concrete event of peace in 1914–18'):

(12) $\Diamond\exists x\, @\Diamond Px$

For the contingentist, (6) is literally false, whereas (12) is a needlessly complex truth: the contingentist accepts (8), from which $\Diamond\exists x\, \Box\Diamond Px$ and so (12) follows in S5 supplemented with the operator @.

However, the 'actually' operator @ is still not exactly what the contingentist requires for the mapping when the quantifier occurs within the scope of modal operators in the necessitist's original sentence. For example, the necessitist uncontentiously holds that there could have been no snowy mountains—not even nonchunky ones:

(13) $\Diamond\neg\exists x\, (Sx \,\&\, Mx)$

If we insert possibility and actuality operators on either side of the existential quantifier in (13) on the pattern of (12), the result is (14):

(14) $\Diamond\neg\Diamond\exists x\, @(Sx \,\&\, Mx)$

Since S5 makes the initial possibility operator in (14) redundant, in effect (14) falsely denies that there could be an *actually* snowy mountain. Since there is an actually snowy mountain, there could be one. For the contingentist, the necessitist is getting at the point that in the possible circumstances in which there are no snowy mountains, it is true to say 'There could not be an actually snowy mountain'. The metalinguistic element here is inessential. What the contingentist needs is an operator in place of @ in (14) which has the effect of exempting the matrix S*x* & M*x* from the scope of the immediately preceding modal operator but not from the scope of further modal operators in which it may be less directly embedded.

A standard solution in actualist 'translations' of possibilist discourse is to use a pair of operators, ↑ and ↓, where an occurrence of ↓ has the effect of exempting what follows it from the scope of modal operators within the scope of the previous occurrence of ↑, if any.[23] The same technique is applicable here. The appendix provides technical details. Such operators seem to make sense; we may charitably assume that contingentists can understand them without violating their own principles. Thus (15) replaces (14):

(15) $\Diamond\neg\uparrow\Diamond\exists x \downarrow(Sx \,\&\, Mx)$

Kit Fine compares the modal operators ↑ and ↓ to 'once' and 'then' in tense logic. Using 'then' modally rather than temporally, we might even English (15) as 'It could have been impossible for there to be something that would then have been a snowy mountain', which is true on the relevant reading.

Even with these refinements, the idea of mapping necessitist discourse to contingentist discourse by prefixing modal operators to quantifiers depends on major assumptions about the necessitist's views. For consider the claim that there could be something which could not be chunky:

(16) $\Diamond \exists x \neg \Diamond Cx$

Just as (13) is mapped to (15), so (16) is mapped to (17):

(17) $\Diamond \uparrow \Diamond \exists x \downarrow \neg \Diamond Cx$

Given S5, (17) still entails (16), for in (17) the third occurrence of $\Diamond$ makes the occurrence of $\downarrow$ redundant, which makes the occurrence of $\uparrow$ redundant, which makes the first occurrence of $\Diamond$ redundant. But the contingentist considered above accepts the auxiliary principle Aux[Con], that necessarily everything is chunky, which is incompatible with (16). Given that (17) entails (16), such a contingentist cannot regard (17) as a kernel of truth in (16). For the time being, we will therefore assume that the necessitist denies (16), holding that necessarily everything is possibly chunky, which for the necessitist is equivalent in S5 to the simpler claim that everything is possibly chunky (necessitism makes $\Box \forall x \Diamond Cx$ equivalent to $\forall x \Box \Diamond Cx$, which is equivalent in S5 to $\forall x \Diamond Cx$).

If the mapping from necessitist talk to contingentist talk is to emulate the achievement of the previous mapping from contingentist talk to necessitist talk, it should map each formula A to a neutral formula $(A)^{Nec}$ whose equivalence to A follows from necessitism combined with auxiliary principles which a necessitist may find plausible. A formula such as (15) is not neutral, since it is not equivalent in the background logic to one in which the quantifiers and atomic predications are restricted by the predicate C. The result of adding those restrictions to (15) is this neutral formula:

(18) $\Diamond\neg\uparrow\Diamond\exists x\,(Cx\ \&\ \downarrow((Sx\ \&\ Mx)\ \&\ Cx))$

This says in effect that it could be impossible for something chunky to be in the previous possible circumstances a chunky snowy mountain. The necessitist who accepts the auxiliary principle that everything is possibly chunky is committed to the equivalence of (15) and (18). We can use 'then' dependent on 'once' modally to explain the point. Suppose that once there could have been something that was then a snowy mountain. By the auxiliary principle, it follows that once there could have been something which could have been chunky and was then a snowy mountain. It follows by S5 that once there could have been something chunky which was then a snowy mountain. Since necessarily every snowy mountain is chunky, it follows that once there could have been something chunky which was then a chunky snowy mountain. Thus, for the necessitist in question, what (15) says could fail strictly implies what (18) says could fail, so (18) entails (15). Conversely, suppose that once there could have been something chunky which was then a chunky snowy mountain. Trivially, therefore, once there could have been something which was then a snowy mountain. Thus, for the necessitist, what (18) says could fail strictly implies what (15) says could fail, so (15) entails (18). Consequently, (15) and (18) are equivalent for the necessitist in question. The reasoning can be generalized.

The argument just given from (18) to (15) required the obvious assumption that necessarily every snowy mountain is chunky. For simplicity, we will add to the necessitist's auxiliary principles the assumption for each non-logical atomic predicate that it requires chunkiness with respect to each argument place, in the sense that $\mathrm{F}v_1 \ldots v_n$ strictly and universally implies $\mathrm{C}v_1$ & …& $\mathrm{C}v_n$. The mapping can then take $\mathrm{F}v_1 \ldots v_n$ to $\mathrm{F}v_1 \ldots v_n$ & $\mathrm{C}v_1$ & …& $\mathrm{C}v_n$.

However, we cannot apply exactly the same treatment to identity. For example, the necessitist of course holds that everything is self-identical:

(19) $\forall x\, x=x$

Treating '=' like any other 2-place atomic predicate involves mapping (19) to (20):

(20) $\uparrow\Box\forall x\, (\mathrm{C}x \rightarrow \downarrow(x=x\ \&\ \mathrm{C}x\ \&\ \mathrm{C}x))$

But (20) says in effect that necessarily everything chunky is *actually* self-identical and chunky. The contingentist who holds that necessarily everything is chunky therefore denies (20) on the grounds that there could have been something chunky which is actually nothing, and so is not self-identical, let alone chunky. Such a contingentist cannot regard (20) as a kernel of truth in the necessitist's (19). The solution is to map the formula $v_1=v_2$ not to $v_1=v_2$ & $\mathrm{C}v_1$ & $\mathrm{C}v_2$ but to $\Diamond(v_1=v_2\ \&\ \mathrm{C}v_1\ \&\ \mathrm{C}v_2)$. Thus (19) is mapped not to (20) but to (21):

(21) $\uparrow\Box\forall x\, (\mathrm{C}x \rightarrow \downarrow\Diamond(x=x\ \&\ \mathrm{C}x\ \&\ \mathrm{C}x))$

This says in effect that necessarily everything chunky is actually possibly self-identical and chunky, which is trivial in S5: necessarily, whatever is the case is possibly the case and therefore actually possibly the case. In a slight variant of the present approach, atomic sentences involving other special atomic predicates (such as one for set-membership) would be treated like '=' rather than the non-logical atomic predicates above. It will do no harm to ignore such complications for the time being.[24]

The formal development can now be sketched, motivated by the foregoing considerations. The appendix provides more details. The language, background logic and definition of 'neutral formula' are as before. The mapping from A to $(A)^{Nec}$ is defined recursively. For an n-place non-logical atomic predicate F, $(Fv_1 \ldots v_n)^{Nec}$ is $Fv_1 \ldots v_n \,\&\, Cv_1 \,\&\, \ldots \,\&\, Cv_n$, which is the same as $(Fv_1 \ldots v_n)^{Con}$, whereas $(v_1{=}v_2)^{Nec}$ is $\Diamond(v_1{=}v_2 \,\&\, Cv_1 \,\&\, Cv_2)$, unlike $(v_1{=}v_2)^{Con}$. $(\exists x\, A)^{Nec}$ is $\uparrow\Diamond\exists x\, (Cx \,\&\, \downarrow(A)^{Nec})$. In other respects the mapping simply preserves the syntactic structure of the input formula. It can easily be checked that $(A)^{Nec}$ is neutral for all formulas A of the language (appendix 1.15).

The main task is to show that every formula A is equivalent to $(A)^{Nec}$, and therefore to a neutral formula, in the relevant necessitist theory. That theory can be axiomatized by the following formulas: (NNE), $\forall x\, \Diamond Cx$ and $\Box\forall z_1 \ldots \forall z_n\, (Fz_1 \ldots z_n \rightarrow (Cz_1 \,\&\, \ldots \,\&\, Cz_n))$ for each n-place non-logical atomic predicate F. For simplicity, we assume that there are only finitely many non-logical atomic predicates in the language, so that the theory is equivalent to the conjunction of its axioms:

Aux[Nec] $\quad$ (NNE) & $\forall x \Diamond Cx$ & … $\Box \forall z_1 \ldots \forall z_n (Fz_1 \ldots z_n \rightarrow (Cz_1 \mathbin{\&} \ldots \mathbin{\&} Cz_n))$ …

Only minor technical details depend on the finiteness assumption.

Since (NNE) is a conjunct of Aux[Nec], the auxiliary necessitist theory entails necessitism itself. By contrast, the auxiliary contingentist theory axiomatized by Aux[Con] does not entail contingentism. The reason for the difference is simply that necessitism, a 'positive' claim, turns out to be needed as a premise in the derivation of the equivalences $(A)^{Nec} \leftrightarrow A$, whereas contingentism, a 'negative' claim, was not needed as a premise in the derivation of the corresponding equivalences $(A)^{Con} \leftrightarrow A$. Of course, in the technical sense of 'neutral', Aux[Con] is no more neutral than Aux[Nec] is, since the quantifier in Aux[Con], like those in Aux[Nec], is unrestricted by 'C' (otherwise Aux[Con] would be trivial). In practice, necessitists will typically deny Aux[Con], since they regard material objects as only contingently chunky; thus in a non-technical sense too Aux[Con] is not neutral between contingentism and necessitism.

With respect to the background logic, $(A)^{Nec} \leftrightarrow A$ follows from Aux[Nec] for every formula A, so every formula is equivalent to a neutral formula (appendix 1.21). Moreover, the choice of $(A)^{Nec}$ is unique up to logical equivalence, in the sense that any neutral formula equivalent given Aux[Nec] to A is logically equivalent to $(A)^{Nec}$ independently of Aux[Nec] (appendix 1.23).[25] In particular, if A itself is neutral, it is logically equivalent to $(A)^{Nec}$ independently of Aux[Nec]. Nothing weaker than Aux[Nec] suffices to derive all the biconditionals $(A)^{Nec} \leftrightarrow A$, for Aux[Nec] follows from $(\text{Aux[Nec]})^{Nec} \leftrightarrow$ Aux[Nec] (appendix 1.19), and so from any theory from which all such biconditionals follow. Indeed, $(\text{Aux[Nec]})^{Nec}$ is just a trivial logical truth. Thus a theory which, unlike Aux[Nec], did not entail (NNE) would be too weak to derive all the biconditionals.

The extended necessitist theory Aux[Nec] turns out to be independent of neutral formulas in the sense that its addition to a set of neutral formulas always constitutes a conservative extension: a neutral formula follows from the set with Aux[Nec] only if it already followed from the set without Aux[Nec] (appendix 1.22). Any theory T which entails Aux[Nec] can be factorized into Aux[Nec] and the set of its neutral consequences: the consequences of T are exactly the consequences of the combination of Aux[Nec] and the neutral consequences of T (appendix 1.25). The logical relations between theories which entail Aux[Nec] simply reflect the logical relations between their neutral parts (appendix 1.24). Once one has chosen Aux[Nec] and which neutral formulas to accept, nothing else remains to be decided.

The mapping from A to $(A)^{Nec}$ gives contingentists a systematic way of seeing necessitists not as hopelessly mistaken but as cognitively reasonable, modulo one pervasive theoretical mistake, tracking genuine distinctions even though misdescribing them. It would be just as fundamental an error to suppose that the neutral sentence $(A)^{Nec}$ expresses what the necessitist 'really means' by a disputed sentence A as it was to suppose that $(A)^{Con}$ is what the contingentist 'really means' by A. For A is already a perfectly meaningful sentence, different in meaning from $(A)^{Nec}$ because it lacks the latter's restriction to the domain of the chunky, and the contingentist knows that. There is no reason to patronize necessitists by treating them as linguistically incompetent, unable to express their views accurately in their own language. Necessitists know what they are doing when they speak. In particular, when they reassert (NNE) in response to contingentists' denial of it, that should be taken at face value. They are asserting (NNE) itself, not irrelevantly asserting the neutral $((\text{NNE}))^{\text{Nec}}$, whose truth is not in dispute. It is just that the mapping gives contingentists a way to factorize the consequences of A given the necessitist theory into those (such as $(A)^{\text{Nec}} \leftrightarrow A$) which merely reflect the commitment to the general metaphysical claim Aux[Nec] and those (such as $(A)^{\text{Nec}}$) which are neutral. $(A)^{\text{Nec}}$ is the cash value of A to necessitists in neutral terms. Although A may not be worth exactly $(A)^{\text{Nec}}$ to contingentists, they can use their knowledge of its cash value to necessitists in making sense of necessitist behaviour, and thereby gain useful information.

Having found appropriate mappings in both directions, we can say more about the relation between contingentism and necessitism. At first sight we face a puzzle. We may assume that the contingentist and the necessitist agree on all neutral formulas.[26] We may also assume that the contingentist accepts the auxiliary assumption Aux[Con] while the necessitist accepts the auxiliary assumption Aux[Nec]. By what has already been established, Aux[Con] and the agreed neutral formulas together fully determine the contingentist's commitments over all non-neutral formulas, while Aux[Nec] and the same neutral formulas fully determine the necessitist's commitments over the non-neutral formulas. But Aux[Con] and Aux[Nec] are mutually consistent; their conjunction is true if and only if it is both necessary what there is and necessary that everything is chunky; the latter conjunction is the conjunction of Aux[Con] and (NNE). Models can easily be provided for that conjunction. So how can the contingentist and the necessitist disagree over (NNE)?

The answer, of course, is that Aux[Con], Aux[Nec] and the set of neutral formulas on which the two sides agree form an inconsistent triad, any two of the three being consistent. In particular, consider the claim that contingent chunkiness is possible:

(22) $\Diamond \exists x\, (Cx \,\&\, \Diamond \neg Cx)$

This formula is neutral because it is equivalent to the explicitly restricted (23):

(23) $\Diamond \exists x\, (Cx \,\&\, \Diamond \neg \exists y\, (Cy \,\&\, (x{=}y \,\&\, Cx \,\&\, Cy)))$

Contingentists and necessitists typically agree on (22) and (23). For instance, they agree that a given material object is chunky but possibly not chunky, even though they disagree on whether it would be anything at all in the latter counterfactual circumstances. By contrast, this non-neutral formula is disputed:

(24) $\Diamond \exists x\, \Diamond \neg \exists y\, x{=}y$

Contingentists typically accept (24), regarding every material object as a verifying instance. Indeed, $(24)^{\mathrm{Con}}$ is (23). Thus, by the fundamental result about the mapping, Aux[Con] entails (23) $\leftrightarrow$ (24) and therefore (22) $\leftrightarrow$ (24), so Aux[Con] and the agreed neutral formula (22) entail (24). By contrast, necessitists must all reject (24), because it is inconsistent with (NNE). Since Aux[Nec] entails (NNE), Aux[Nec] entails $\neg$(24). Thus Aux[Con], Aux[Nec] and (22) form an inconsistent triad, each two members of which are mutually consistent.

Indeed, (22) is the weakest neutral formula (up to logical equivalence) with the property of forming an inconsistent set with Aux[Con] and Aux[Nec]. For suppose that Aux[Con] and Aux[Nec] are jointly inconsistent with the neutral formula A, so Aux[Con] and Aux[Nec] jointly entail ¬A. From that, one can show that ¬(22) entails ¬A (by appendix 1.28), so A entails (22). Contrapositively, the claim that necessarily everything chunky is necessarily chunky (equivalent to ¬(22)) axiomatizes the neutral consequences of the conjunction of Aux[Con] and Aux[Nec]:

(25) $\Box \forall x\, (Cx \rightarrow \Box Cx)$

But contingentists and necessitists typically agree in denying (25). If your parents had never met, you would not still have been chunky.

Although the different principles by which contingentists and necessitists move beyond the domain of the chunky can be consistently combined, their combination depends on assumptions about the domain of the chunky which both contingentists and necessitists typically, and rightly, reject.

Incidentally, the idea that the contingentist and the necessitist might take a fictionalist view of each other's discourse provides no serious alternative to the mappings just discussed. Those mappings enable each side to extract useful information from many of the other's claims. When the contingentist says A, the necessitist may indeed conclude 'A is true in the contingentist fiction', which says something about contingentism, but—in frequent contrast with $(A)^{Con}$—not yet anything useful about the subject matter of A. Similarly, when the necessitist says B, the contingentist may conclude 'B is true in the necessitist fiction', which says something about necessitism, but—in frequent contrast with $(B)^{Nec}$—not yet anything useful about the subject matter of B. After all, it would trivialize the view that we can learn from fiction to add that what we learn from it are facts such as that in the novel Anna Karenina kills herself, not least because we can learn just as much of that sort from bad fiction as from good. The mappings avoid such trivialization.

6. Possible worlds

Much of the debate between actualists and possibilists revolved around the questionable legitimacy of quantification over possible worlds. Such quantification was seen as far more problematic for actualists than for possibilists, since it was unclear that the former could acknowledge non-actual worlds, especially those supposed to contain non-actual individuals. In response, actualists tried to show how they could simulate possibilist discourse, and thereby gain the advantages of quantification over worlds without the unwanted theoretical commitments. The use of less standard modal operators such as ↑ and ↓ was one focus of controversy; they were suspected of being not just the scope-indicating devices they were presented as, but a Trojan horse for quantification over possible worlds. They look like devices of cross-reference; what is the cross-reference to if not worlds?[27]

The actualism-possibilism debate overlapped and interacted in complex ways with a debate between modalism and anti-modalism. According to modalism, quantification over worlds can be reductively explained in terms of modal operators. Anti-modalism says the reverse: modal operators can be reductively explained in terms of quantification over worlds. If a modal operator is itself a quantifier over worlds, or a device of cross-reference between variables for worlds, it cannot be used in a modalist explanation of quantification over worlds.

This paper concerns the dispute between contingentism and necessitism, which does not map straightforwardly onto either the dispute between actualism and possibilism (whatever they are) or that between modalism and anti-modalism. The use of possible worlds semantics to characterize the background logic in the appendix is not especially problematic, since its role there is algebraic and instrumental. However, the need for the additional operators ↑ and ↓ in formulas such as $(\exists x \text{ A})^{\text{Nec}}$ raises the question whether the contingentist can understand them without invoking possible worlds in a more than instrumentalist capacity, and more generally how the contingentism-necessitism dispute interacts with the dispute between modalism and anti-modalism. Some brief remarks on those questions may be helpful.

By itself, necessitism is consistent with modalism, with anti-modalism, and with the conjunction of the negation of modalism and the negation of anti-modalism; the necessary being of objects entails nothing about the relative explanatory priority of modal operators and quantification over worlds.

The combination of contingentism with anti-modalism is arguably inconsistent. Contingentism says that there could have been something which could have been nothing. Anti-modalism typically reduces that claim to the claim that some possible world has a domain which is not a subset of the domain of some other world. However, such an explanation involves restricting the quantifiers to the domains of the worlds of evaluation. But the issue between contingentism and necessitism concerns (NNE) on an *unrestricted* reading of the quantifiers. Consequently, the denial of domain inclusion seems to be irrelevant to the point at issue. To demote the worlds to the status of mere 'representational devices', and claim that a restricted quantifier can *represent* an unrestricted one, is to retreat from anti-modalism, for merely representing purported truths involving modal operators (such as the negation of (NNE)) in terms of quantification over worlds does not amount to reductively explaining them in those terms, even if one happens to represent the right set of logical truths.

The tension between contingentism and anti-modalism means that a first-order language with quantification over worlds but no modal operators is a hopelessly misleading medium for the debate between contingentism and necessitism. One cannot fairly capture what is at stake in (NNE) by asking whether all worlds have the same domain. It is far safer to pose the issues in the pretheoretically clearly understood language of quantified modal logic, without prejudging whether it is ultimately reducible to a language which lacks modal operators.

Even if contingentism excludes anti-modalism, it does not obviously follow that contingentism requires modalism. For it may be consistent with contingentism that quantification over worlds permits genuine questions about the nature of worlds which cannot be answered in terms of modal operators, so that quantification over worlds as well as modal operators must be taken as primitive. What sort of quantification over worlds, if any, the contingentist can permit is a delicate question which can be left open here.

If contingentists are not anti-modalists, whether or not they are modalists, they are not obliged to explain failures of unnecessitated (BF) in terms of possible worlds semantics or any other semantic framework which eschews modal operators in the meta-language. They may prefer a more homophonic style of semantics, in which a semantic theory for a modal object-language is formulated in a modal meta-language.[28] The question remains whether an independent argument for anti-modalism could make trouble for contingentists. However, the overall aim of this paper is to argue against contingentism, but not by arguing for anti-modalism. It is therefore dialectically appropriate to allow contingentists the use of their modal operators, including the less standard operators ↑ and ↓. Although one may well find the extant arguments for anti-modalism quite unconvincing, it is unnecessary to debate their merits here. In what follows, possible worlds will continue to play a merely instrumental role.

7. Sets of incompossibles

In section 5, we saw that a necessitist theory implies the equivalence of each formula of the language with a neutral formula about the domain of the chunky, which enabled the contingentist to calculate the cash value to the necessitist of the latter's utterances. However, the necessitist theory Aux[Nec] in section 5 was not necessitism by itself ((NNE)), but rather its conjunction with the auxiliary claims that everything is possibly chunky and, for each non-logical n-place atomic predicate, that it is satisfied only by n-tuples of chunky objects. Those auxiliary claims do not follow from necessitism in the background logic; they require examination.

The claim that the non-logical n-place atomic predicates are satisfied only by n-tuples of chunky objects is quite plausible with respect to many natural candidates for such predicates, even from a necessitist point of view. For instance, a necessitist might happily agree that necessarily whenever c causes e, both c and e are chunky (grounded in the concrete). However, some candidates are more problematic; as we shall see shortly, the set membership predicate $\in$ is an example.

Consider the other auxiliary claim, that everything is possibly chunky:

(26) $\forall x \Diamond Cx$

Why should the necessitist accept (26)? If the contingentist insists that everything *does* meet a condition, why should the necessitist concede that everything *could* meet that condition? A necessitist might make that concession, if they simply used the mapping from A to $(A)^{Nec}$ to determine which non-neutral formulas to accept on the basis of the agreed neutral formulas, since $(Aux[Nec])^{Nec}$ is an uncontentious logical truth and (26) is a conjunct of Aux[Nec] (appendix, proof of 1.19). But there are other theoretical constraints for the necessitist to respect, as we shall see.

A necessitist typically reconstrues the contingentist's putative cases of individually possible but joint incompossibility, as in (3), along the lines of (3R), that is, as cases where two or more things can each be chunky but cannot all be chunky. Indeed, (3R) is easily seen to be neutral, so we expect the contingentist and the necessitist to agree on (3R). Moreover, since (3R) is equivalent to (3) given Aux[Con], we expect contingentists who accept (3) to accept (3R) too. Of course, (3R) is not equivalent to (3) given Aux[Nec], since (3) is inconsistent with (NNE), whereas (3R) and Aux[Nec] are jointly consistent with (NNE). Suppose that although c can be chunky and c* can be chunky, they cannot both be chunky (necessitism permits us to instantiate a possibility claim such as (3R) in this way). By the Pairing Axiom in Zermelo-Fraenkel set theory, some set {c, c*} contains just c and c* as members. The mere fact that c and c* cannot both be chunky is no reason for the necessitist not to apply Pairing here; at the level of generality at which principles of set theory are formulated, chunkiness is of no special significance. Thus, by (26), {c, c*} is possibly chunky. Since it is impossible for c and c* both to be chunky, {c, c*} can be chunky even though at least one of its members is not. But how is that possible?[29] Surely a set is grounded in the concrete only if all its members are.

Let us develop the problem more carefully. Sets are made up of their members. On the most plausible modal metaphysics for sets, this means that it is not contingent whether one thing is a member of another, at least when there are those things. Necessitism makes the latter condition redundant. If this is a member of that, this could not have failed to be a member of that; if this is not a member of that, this could not have been a member of that. For the necessitist, therefore, the non-contingency of membership (Membership Rigidity) can be expressed very simply:[30]

(MR) $\forall x\ \forall y\ (\Diamond x \in y \rightarrow \Box x \in y)$

Another plausible principle (Chunky Membership) is that, necessarily, a set is chunky only if its members are chunky:

(CM) $\forall x\ \forall y\ \Box(x \in y \rightarrow (\mathrm{C}y \rightarrow \mathrm{C}x))$

Indeed, (CM) follows in the background logic from the conjunct of Aux[Nec] for the '$\in$' predicate, according to which it is satisfied only by ordered pairs of chunky objects:

(27) $\quad \Box \forall x \ \forall y\ (x \in y \rightarrow (\mathrm{C}x \ \& \ \mathrm{C}y))$

For (CBF), which follows from necessitism, permits the relative order of the quantifiers and the modal operator in (27) to be reversed in (CM). From (MR) and (CM), (28) follows easily in S5:

(28) $\quad \forall x \ \forall y\ (x \in y \rightarrow \Box(\mathrm{C}y \rightarrow \mathrm{C}x))$

For $x \in y$ yields $\Diamond x \in y$ and therefore $\Box x \in y$ by (MR); (CM) gives $\Box x \in y \rightarrow \Box(\mathrm{C}y \rightarrow \mathrm{C}x)$. Since c $\in$ {c, c*} and c* $\in$ {c, c*}, {c, c*} is such that necessarily it is chunky only if both c and c* are chunky. Since it is uncontentiously impossible for c and c* to be both chunky, {c, c*} is such that it is impossible for it to be chunky. But that violates (26), the principle that everything is possibly chunky. Since (MR) is a compelling principle for the metaphysics of sets in a necessitist setting, and both (26) and (27) follow from Aux[Nec], a necessitist who accepts incompossible possibles in the sense of (3R) and elementary set theory must reject Aux[Nec].

The argument suggests that such a necessitist must reject (26), whether or not they reject (27). For even without (27), (CM) is independently plausible. If we understand 'chunky' as 'grounded in the concrete', (CM) should hold because a set will be grounded in the concrete only if its members are grounded in the concrete; indeed, that is the natural recursive explanation of what it is for a set to be grounded in the concrete. More generally, if we understand ‘chunky’ as ‘meeting the contingentist’s standard for being something’, (CM) should hold because a set will presumably meet the contingentist’s standard for being something only if its members meet the contingentist’s standard for being something. In the presence of (CM), (27) is not needed to generate the problem. For, given the independently compelling (MR), (CM) itself gives the necessitist a plausible argument against (26), and therefore against Aux[Nec].

Necessitists who want to accept both incompossible possibles in the sense of (3R) and elementary set theory will want to reject Aux[Nec]. That renders inapplicable the method described in section 5 for contingentists to calculate the cash value to the necessitist of the latter’s utterances. Modifying the necessitist’s quantifiers with modal operators and restrictions to the chunky as proposed does not yield a formula $(A)^{Nec}$ which necessitists will generally regard as equivalent to the original formula A given their background metaphysics. For example, such necessitists will reject $(\neg\text{Aux[Nec]})^{Nec}$, since it is inconsistent in the background logic, but accept ¬Aux[Nec], for the reasons just explained.

8. Plural incompossibles

At first sight, the crux of the problem in section 7 might appear to be the necessitist’s acceptance of an ontology of sets. One might therefore suppose that it would not arise if the necessitist achieved the same practical results by speaking plurally of the Fs rather than singularly of the corresponding set.[31] In that case, the contingentist could try to find the cash value to the necessitist of the latter’s plural utterances in plural terms, for the necessitist has no exclusive right to the plural. Although it is not uncontroversial that plural quantification really does avoid ontological commitment to sets, let us assume for the sake of argument that it does: if not, the problem about sets remains, which is all the better for the overall argument of this paper. However, granted that plural quantification avoids commitment to sets, related difficulties arise even within that setting. They concern not whether Aux[Nec] is acceptable to the necessitist but instead whether the mapping of every formula A to a neutral formula $(A)^{Nec}$ equivalent to A given Aux[Nec] can be extended from the first-order language to its enhancement by plural quantifiers.

That the problem generalizes is hardly surprising. It is raised by c and c*, each of which can be chunky, although they cannot both be chunky. From the point of view of necessitist set theory, the true claim that there is a set of which c and c* are members is not equivalent given modal set theory to the false claim that there could have been a chunky set of which c and c* were members. But equally, from the point of view of necessitist plural theory, the true plural claim that c and c* are some things is not equivalent to the false plural claim that c and c* could be some chunky things. For if c and c* were some chunky things, they would both be chunky, which is impossible. The mere fact that c and c* cannot both be chunky is no reason for the necessitist not to allow that there are some things of which c and c* are each one; at the level of generality at which principles of plural logic are formulated, chunkiness is of no special significance.

We can develop the point formally by using Boolos's proposed plural reading of second-order logic, as usual with an extra clause to permit the empty 'plurality' (Boolos 1984, 1985).[32] In the set-theoretic case, the principles underlying the problem were membership rigidity ((MR)) and chunky membership ((CM)). We must first consider what they correspond to for plurals.

The plural analogue of (MR) is compelling. It is not contingent whether something is one of some things, at least when there are those things. Necessitism makes the latter condition redundant. If something is one of some things, it could not have failed to be one of them; if it is not one of them, it could not have been one of them. Thus the necessitist has this simple plural analogue of (MR), where X is a plural variable:[33, 34]

(PR) $\forall x \; \forall X \, (\Diamond Xx \rightarrow \Box Xx)$

In words: if something could have been one of some things, it could not have failed to be one of them.

Someone might object to (PR) that since it is contingent whether 7 is a number named on this page, it is contingent whether 7 is one of the numbers named on this page. However, that objection rests on a scope fallacy concerning the plural definite description 'the numbers named on this page', analogous to the fallacy involved in the objection to the necessity of identity that since it is contingent whether 7 is the number named on that page, it is contingent whether 7 is identical with the number named on that page, which concerns the singular definite description 'the number named on that page'. For ease of comparison, we rehearse the diagnosis of the fallacy in the singular case. On the reading on which 'the number named on that page' takes narrow scope with respect to the modal operator 'it is contingent whether', it is indeed contingent whether 7 is identical with the number named on that page, but it does not follow that some number is such that it is contingent whether 7 is identical with it. By contrast, on the reading on which 'the number named on that page' takes wide scope with respect to 'it is contingent whether', the inference is valid but the premise false: it is not contingent whether 7 is identical with the number named on that page. In the plural case the diagnosis is similar. On the reading on which 'the numbers named on this page' takes narrow scope with respect to 'it is contingent whether', it is indeed contingent whether 7 is one of the numbers named on this page, but it does not follow that some numbers are such that it is contingent whether 7 is one of them. By contrast, on the reading on which 'the numbers named on this page' takes wide scope with respect to the modal operator, the inference is valid but the premise false: it is not contingent whether 7 is one of the numbers named on this page. Either way, the objection fails.

Here is an argument to confirm (PR), under the assumption of necessitism. If these *are* those, then nothing could be one of these without being one of those or vice versa. But to say that these are those is just to say that every one of these is one of those and vice versa; coextensiveness is the plural analogue of identity. Thus a principle of extensionality holds for plurals:[35]

(29) $\forall X \, \forall Y \, (\forall x \, (Xx \leftrightarrow Yx) \rightarrow \forall x \, \Box(Xx \leftrightarrow Yx))$

Obviously (29) would fail on a more intensional reading of the second-order variables: if Tom, Dick and Harry are slaves and no one else is, it does not follow that someone could not have been a slave without being Tom, Dick or Harry. Now for any things X, there are some things Y such that every one of X is one of Y and vice versa, and it is not contingent whether anything is one of Y. For example, if X are the slaves, Y may be

Tom, Dick and Harry; it is not contingent whether anything is one of Tom, Dick and Harry (necessitism has no contingency in what there is to worry about). Consequently:

(30) $\forall X \exists Y (\forall x (Xx \leftrightarrow Yx) \& \forall x (\Diamond Yx \rightarrow \Box Yx))$

But (PR) is an easy consequence of (29) and (30).[36] Contingentists will presumably want to qualify these principles to take account of contingency in what there is, but that is not our immediate concern.

We have no direct plural analogue of the principle (CM) that chunky sets have chunky members, since the singular predicate '… is chunky' cannot as such be grammatically predicated of a plural subject. We can use the plural predicate '… are chunky', but presumably to say that some things are chunky is just to say that each of them is chunky: the analogue of (CM) comes for free. There does not seem to be any point in introducing a primitive plural analogue of 'chunky' to be applied collectively rather than distributively. Some things are collectively grounded in the concrete if and only if each of them is grounded in the concrete. More generally, some things collectively meet the contingentist standard for being some things if and only if each of them meets the contingentist standard for being something. Thus, on the natural extension of the term 'neutral' to sentences with plural quantifiers, a sentence is neutral only if it is equivalent to one all of whose plural quantifiers are restricted by the condition that each of the things in question is chunky. In the formal language, sentences with such restricted second-order quantifiers take forms like $\forall X (\forall x (Xx \rightarrow Cx) \rightarrow A)$ and $\exists X (\forall x (Xx \rightarrow Cx) \& A)$. Correspondingly, the natural plural analogue of (26) is this:

(26P) $\forall X \Diamond \forall x (Xx \rightarrow \mathrm{C}x)$

For any things, possibly whatever is one of them is chunky.

Unfortunately for (26P), it is vulnerable to a plural analogue of the argument from (CM) and (MR) against (26). The plural analogue of the Pairing Axiom is (31):

(31) $\forall x \forall y \exists X (Xx \,\&\, Xy)$

Both necessitists and contingentists regard (31) as an elementary logical truth. But from (26P), (31), (PR) and (NNE) we can easily conclude that any two things could have been co-chunky:

(32) $\forall x \forall y \Diamond(\mathrm{C}x \,\&\, \mathrm{C}y)$

But two things c and c* as above which are not possibly co-chunky falsify (32). Thus necessitists should reject (26P), even if they can hang on to (26).

Given the failure of (26P), we cannot extend the mapping from A to $(\mathrm{A})^{\mathrm{Nec}}$ to sentences with plural quantifiers in the obvious 'possibilist' way, by stipulating $(\exists X\, \mathrm{A})^{\mathrm{Nec}}$ to be $\uparrow\Diamond\exists X (\forall x (Xx \rightarrow \mathrm{C}x) \,\&\, \downarrow(\mathrm{A})^{\mathrm{Nec}})$. If the contingentist applies such a mapping to (31), in an attempt to find a neutral truth '$(31)^{\mathrm{Nec}}$' which the necessitist will regard as equivalent to (31), on necessitist assumptions the complicated formula '$(31)^{\mathrm{Nec}}$' will entail that there can be some chunky things of which c and c* can be two, and therefore that c and c* can be co-chunky. Thus necessitists will deny this neutral '$(31)^{Nec}$'. Since necessitists regard (31) as a logical truth, they will regard '$(31)^{Nec}$' as not equivalent to (31).[37]

That is not the end of the matter, for since necessitists regard (31) as logically true, they will regard it as equivalent to various trivial neutral logical truths. We have not yet eliminated the supposition that some other mapping takes each formula A of the second-order modal language to a neutral formula $(A)^{Nec}$ which necessitists regard as equivalent to A given Aux[Nec]. We therefore turn to a more systematic investigation of the matter.

9. Mappings between second-order contingentist and necessitist discourse

The first task is to extend the background logic to the second-order language under the plural interpretation. As before, we use set-theoretic possible worlds models in a purely instrumental capacity.

Our second-order language will include n-place 'plural' variables for n > 1, which have no straightforward English reading on the plural interpretation of higher-order logic. Usually, one simulates polyadic 'plural' quantification by closing the individual domain under the formation of n-tuples and using monadic plural quantification over n-tuples. However, such n-tuples, and in particular ordered pairs, create the same sort of problem for contingentism that unordered pairs did in section 7, so in fairness to contingentists, it is best to add the polyadic 'plural' variables directly instead.[38] In the instrumentally conceived formal semantics, quantification with an n-place second-order variable is still equivalent to plural quantification over n-tuples of members of the relevant domain (where the constituents of an n-tuple are taken to be constant across worlds in the obvious way). This polyadic second-order quantification is not crucial; we allow it only to check that the results below do not depend on an expressive limitation of the language which excludes it. The informal exposition will concentrate on the monadic case. For technical details see section 2 of the appendix.

The same models will do as before, for the first-order domains already determine the corresponding second-order domains and for present purposes we do not need non-logical second-order predicate constants whose interpretations would have to be specified.

Since the intended interpretation is plural rather than intensional, the value of a second-order variable is independent of the world of evaluation, in accordance with the plural rigidity principle (PR) defended above. The value is a subset of the domain of the model rather than a function from worlds to sets. The assigned subset may contain members which do not belong to the domain of the world of evaluation. However, in fairness to contingentism, the formula Xx is evaluated as true at a world only if, in addition to the value of the first-order variable x belonging to the value of the second-order variable X, the latter is a subset of the domain of the world, since something is one of some things only if there are those things for it to be one of.[39] It follows that Xx is true at the world only if the value of x belongs to the domain of that world, which is similar to the constraint that (in set-theoretic terms) the extension of an n-place predicate constant at a world should contain only n-tuples of members of the domain of that world (appendix 1).[40] Correspondingly, second-order quantifiers are treated as ranging at a world only over what there are in that world: in set-theoretic terms, $\exists X$ A is true at the world on an assignment only if A is true at that world on an assignment which differs from the former only in assigning some subset of the domain of the world to X. This corresponds to the restriction on the first-order quantifier to members of the domain of the world (appendix, section 1).

The assignment of sets as values of the second-order variables is unfaithful to their intended plural interpretation. On the latter, for instance, X might apply to all and only *sets*, even though there is no set of all sets. A faithful interpretation would employ plural quantifiers in the meta-language to interpret the plural quantifiers in the object-language (Boolos 1985; Bricker 1989, p. 389; Rayo and Uzquiano 1999). The arguments of this paper could be carried out in such a plural meta-language. For familiarity and ease, however, a set-theoretic meta-language has been employed in the instrumental role.[41]

A neutral second-order formula should be equivalent to one in which an atomic subformula Xx occurs only when restricted by the condition that the things in question are all chunky ($\forall y\ (Xy \rightarrow \mathrm{C}y)$, abbreviated to $X \leq \mathrm{C}$) and a second-order quantifier occurs only when restricted by the same condition applied to its bound variable. As before, the restrictions by C parallel those by the domain of the world of evaluation in the clauses of the definition of truth in a model. Of course, when the value of the variable x is not in the domain of the world, it is not covered by the condition $X \leq \mathrm{C}$, but then Xx is false anyway by the restriction on its semantic clause.

It is straightforward to extend the mapping from contingentist discourse to necessitist discourse in section 4 to the second-order language accordingly. Since the new clauses in the recursive definition of $(\mathrm{A})^{\mathrm{Con}}$ straightforwardly express the desiderata on neutrality just explained, as before a formula A is neutral if and only if it is logically equivalent to $(B)^{Con}$ for some formula B. Trivially, $(A)^{Con}$ itself is always neutral.

The results about the mapping for the first-order case (appendix 1.1-1.12) also extend smoothly to the second-order case (appendix 2.1-2.9). Each formula A of the second-order language is equivalent to the neutral formula $(A)^{Con}$ given the auxiliary assumption Aux[Con] congenial to contingentism. Thus the necessitist can calculate the cash value to the contingentist of each sentence the latter utters.

The difficulty is in the reverse process. We can prove that some second-order formulas are not equivalent to *any* neutral formula, even given the strong, necessitism-entailing auxiliary assumption Aux[Nec]. Thus under no extension of the mapping from A to $(A)^{Nec}$ to the second-order language does the crucial result about the first-order case (appendix 1.21) extend to the second-order case. Aux[Nec] does not make each sentence *have* a cash value to the necessitist. Consequently, the contingentist cannot always extract information in neutral terms from the necessitist's utterances.

Strictly speaking, the phenomenon already arises for atomic formulas, since even Xx is not equivalent under Aux[Nec] to any neutral formula (appendix 2.17).[42] Philosophically, this is not very significant, since an open formula (understood as such) is unsuitable for independent use in a speech act. But some closed formulas are also not equivalent under Aux[Nec] to any neutral formula. Here is an example, for any one-place atomic predicate F and two-place atomic predicate R:

(33) $\exists X (\exists x (\mathrm{F}x \,\&\, Xx) \,\&\, \exists x (\mathrm{F}x \,\&\, \neg Xx) \,\&\, \forall x \forall y (\Diamond \mathrm{R}xy \rightarrow (Xx \rightarrow Xy)))$

This says that there are some things, of which some F but not every F is one, which can have R only to one of themselves. That is equivalent to the claim that there are two F things not linked by any finite sequence each member of which (but the last) could have R to the next; in effect, (33) uses the resources of second-order logic to define the ancestral of the possibly-R relation.

The contingentist and the necessitist agree that necessarily only chunky things stand in the R relation. For instance, R may be a causal or spatial relation. But the necessitist allows that non-chunky things may stand in the possibly-R relation; the contingentist disagrees, on the grounds that there are only chunky things anyway. Whether one chunky thing has the ancestral of the possibly-R relation to another may depend on whether non-chunky intermediate links are allowed.

Here is an example. Imagine a world with two populations of humanoids who reproduce as we do. By chance, no interbreeding ever occurs between the two populations, although it easily could have done. Let F be the property of being humanoid and R the relation of sharing at least one parent. Assume that ancestry is an essential property of humanoids. Thus no actual member of one population could have had R to any actual member of the other population. According to contingentists, therefore, (33) is true; we can take *X* to refer to the members of just one population. According to necessitists, however, (33) is false. For any two humanoids c and c*, the father of c could in principle have had a child d with the mother of c* (perhaps incestuously); thus c could have shared a parent with d and d could have shared a parent with c*.

Here is another sort of example. Imagine six lights numbered 0-5. Consider the following six events, all at a fixed time t:

e_0: lights 0 and 1 flash red; lights 3 and 4 flash green

e_1: lights 1 and 2 flash red; lights 4 and 5 flash green

e_2: lights 2 and 3 flash red; lights 5 and 0 flash green

e_3: lights 3 and 4 flash red; lights 0 and 1 flash green

e_4: lights 4 and 5 flash red; lights 1 and 2 flash green

e_5: lights 5 and 0 flash red; lights 2 and 3 flash green

Each event involves only four of the six lights; for example, events e_0 and e_1 co-occur when lights 0, 1 and 2 flash red while lights 3, 4 and 5 flash green. But no given light can flash both red and green at t, so e_0 and e_2 cannot co-occur, because e_0 occurs only if light 0 flashes red while e_2 occurs only if light 0 flashes green. For similar reasons, each event can co-occur only with its immediate predecessor and successor in the series, where the successor of e_i is e_{i+1}, '+' being understood modulo 6 (so 5 + 1 = 0). Let F be the property of being one of e_0-e_5 and R the relation which each e_i has to e_{i+1}, if both occur, and to nothing else. Thus e_0 can have R to e_1, which can have R to e_2, which can have R to e_3, which can have R to e_4, which can have R to e_5, which can have R to e_0; no shorter sequence of possible R-links runs from e_0 to e_5. According to typical contingentists, for an event to be is for it to occur; thus if e_0 and e_5 co-occur, (33) is true; we can then take *X* to refer to just e_0, for e_1 is not there to provide a possible R-link from e_0 to any other event. According to necessitists, however, (33) is false, since each of the six events is connected to each of the others by possible R-links. In this example, any sequence of possible R-links from e_0 to e_5 involves at least three events no two of which can be chunky together: for instance, e_0, e_2 and e_4. For any natural number n, one can easily construct a similar example in which for some pair of events any sequence of possible links of the relation from the first to the second involves at least n events no two of which can be chunky together.[43]

Since (33) has a neutral equivalent under Aux[Con], necessitists can calculate the cash value of (33) to contingentists, but the reverse process does not work. As already stated, under Aux[Nec], (33) is provably not equivalent to any neutral formula in the language (appendix 2.16). The idea of the proof is that any neutral formula is equivalent given Aux[Nec] to a first-order formula under the hypothesis that it is impossible for more than two things to be chunky together, for then a second-order quantifier restricted to things all of which are chunky is equivalent to a pair of first-order quantifiers both restricted to chunky things. But one can show that (33) is not equivalent given Aux[Nec] to a first-order formula even under that hypothesis, using a variant of David Kaplan's proof that the Kaplan-Geach sentence 'Some critics admire only one another' is not equivalent to a first-order sentence, since it can be 'interpreted' as a sentence of (non-modal) arithmetic which is false in all standard models and true in all non-standard ones.[44]

Obviously, the hypothesis that it is impossible for more than two things to be chunky together is quite implausible, but that does not affect the soundness of the proof, since it uses the hypothesis in a purely instrumental role, as a diagnostic, not a premise. Of course, the question arises whether (33) is equivalent to a neutral formula under some strengthening of Aux[Nec] which remains congenial to the necessitist. Such a strengthening might include as a conjunct that it is possible for more than two things to be chunky together. Handling that would require further complications in the proof. For example, if the hypothesis that it is impossible for more than two things to be chunky together is replaced by the hypothesis for some large natural number n that it is impossible for more than n things to be chunky together, a variant of the proof still goes through, for under the latter hypothesis a second-order quantifier restricted to things all of which are chunky is equivalent to a sequence of n first-order quantifiers all restricted to chunky things. Aux[Nec] might then be strengthened by the conjunct that there is no finite upper bound to the number of things which can be chunky together. However, necessitists need have no commitment to that stronger claim, especially since we are most concerned with those who are using plural quantification in place of an ontology of sets (since, as seen in section 7, such an ontology by itself undermines the conjunct of Aux[Nec] that everything is possibly chunky). Furthermore, we have been given no positive reason to suppose that (33) is equivalent to a neutral formula under such stronger auxiliary assumptions.

One can also show that not even this slightly simpler variant of (33) is equivalent under Aux[Nec] to a neutral formula (appendix 2.15):

(34) $\exists X (\exists x\, Xx \;\&\; \exists x \neg Xx \;\&\; \forall x \forall y (\Diamond \mathrm{R}xy \rightarrow (Xx \rightarrow Xy)))$

This is equivalent to the result of substituting x=x for Fx in (33). For the sake of simplicity, the focus will henceforth be on (34) rather than (33); this makes no difference to the underlying philosophical and logical issues.

One special augmentation of Aux[Nec] under which (33) and (34) have neutral equivalents is as follows. For some non-logical two-place atomic predicate E, read E*xy* ‘x is an essence of y’. Add to Aux[Nec] the claims that being chunky universally strictly implies having an essence, that being an essence universally strictly implies being necessarily chunky, and that nothing can be an essence of more than one thing:

(35) $\forall x \, \Box(\mathrm{C}x \rightarrow \exists z \, \mathrm{E}zx)$

(36) $\forall x \, \forall z \, \Box(\mathrm{E}zx \rightarrow \Box \mathrm{C}z)$

(37) $\forall x \, \forall y \, \forall z \, ((\Diamond \mathrm{E}zx \, \& \, \Diamond \mathrm{E}zy) \rightarrow x{=}y)$

Under the auxiliary assumptions Aux[Nec] + (35)-(37), (34) is equivalent to a neutral claim about essences (recall that by Aux[Nec] standing in R entails chunkiness):

(38) $\exists X \, (X{\leq}\mathrm{C} \, \&$

$\Diamond \exists x \, (\mathrm{C}x \, \& \, \exists z \, (\mathrm{C}z \, \& \, Xz \, \& \, \mathrm{E}zx)) \, \& \, \Diamond \exists x \, (\mathrm{C}x \, \& \, \neg \exists z \, (\mathrm{C}z \, \& \, Xz \, \& \, \mathrm{E}zx)) \, \&$

$\Box \forall x \, \forall y \, ((\mathrm{C}x \, \& \, \mathrm{C}y \, \& \, \mathrm{R}xy) \rightarrow (\exists z \, (\mathrm{C}z \, \& \, Xz \, \& \, \mathrm{E}zx) \rightarrow \exists z \, (\mathrm{C}z \, \& \, Xz \, \& \, \mathrm{E}zy))))$

The idea is that, given Aux[Nec] + (35)-(37), if some things are verifying values of *X* in (34), then their possible essences are verifying values of *X* in (38); conversely, if some things are verifying values of *X* in (38), the things of which they are possible essences are verifying values of *X* in (34). The technique is loosely modelled on Plantinga’s interpretation of quantified modal logic in terms of quantification over essences, and can be generalized to other closed formulas of the second-order language.[45]

However, the auxiliary assumptions (35)-(37) are a high price to pay for such equivalences. Unlike the original auxiliaries Aux[Nec] and Aux[Con], they have significant first-order consequences for the domain of the chunky itself. For example, (35) and (36) jointly entail this neutral formula, which is clearly not a logical truth:

(39) $\forall x\,(Cx \rightarrow \exists z\,(Ezx \,\&\, \Box Cz))$

By contrast, any neutral first-order formula which follows from Aux[Con] or Aux[Nec] is a logical truth (appendix 1.9, 1.22). Moreover, (39) lacks metaphysical plausibility. It implies that Socrates has an essence which would be chunky even if he were not chunky. Being chunky is supposed to be something like being grounded in the concrete. But if Socrates were not grounded in the concrete, how could his essence be so grounded? It might be grounded in the *possibly* chunky, but that is not enough for being grounded in the concrete (perhaps it is enough for being *possibly* grounded in the concrete). To the claim that Socrates would have been something even if he had never been conceived, the spirit of the contingentist objection is that in those circumstances Socrates would not have been grounded in the concrete, as required for being something; but he would still have been possibly concrete, and so a fortiori grounded in the possibly concrete.[46] More generally, there is no apparent basis for asserting that an essence of Socrates is necessarily something while denying the corresponding claim about Socrates himself. In what follows, we assume that no such ad hoc additions concerning the domain of the chunky have been made to the auxiliary assumptions, and that (34) is therefore not equivalent for necessitists to a neutral formula of the second-order language.

The non-equivalence is robust with respect to the treatment of atomic formulas. The only atomic predicate constant in (34) is R. The proof of non-equivalence would go through even if we built the principle $Rxy \rightarrow (Cx \mathbin{\&} Cy)$ into the background logic, which would make redundant the stipulation that a formula is neutral only if every occurrence of R in it is restricted by C. Nor would anything be gained by adding a new 'identity' symbol $=^*$ subject to the principle $\Box\forall x\, \Box x{=}^*x$, for one can already simulate $=^*$ in the present background logic by substituting $\Diamond v_1{=}v_2$ for $v_1{=}^*v_2$ throughout. As for dropping constraints on the interpretation of atomic predicates in models, it of course cannot generate new equivalence results, since it merely weakens the background logic.

Imagine necessitists discussing whether (34) is true, under assumptions that do not make it equivalent to a neutral formula. What are contingentists to make of their discussion? Of course, (34) is a meaningful formula for them too, but it raises different considerations. Can contingentists plausibly regard the necessitists' discussion as engaging no further question of interest? The examples above strongly suggest that both contingentists and necessitists use (34) to sort some relations R from others in perfectly genuine but different ways. If either side cannot rationalize the other's sorting in its own terms, its theory is inadequate. The asymmetry is that the contingentist sorting can be rationalized in necessitist terms, while the necessitist sorting cannot be rationalized in contingentist terms.

To reinforce the point, consider (40_n) for $n \geq 1$:

(40_n) $\exists z_1 \ldots \exists z_n (\exists x \neg(x{=}z_1 \vee \ldots \vee x{=}z_n))\ \&$

$$\forall y (\Diamond(\mathrm{R}z_1y \vee \ldots \vee \mathrm{R}z_ny) \rightarrow (y{=}z_1 \vee \ldots \vee y{=}z_n)))$$

This says in effect that there are some things, at most n in number, of which something but not everything is one, which can have R only to one of themselves. Thus (40_n) entails (34); it differs from (34) only in adding the requirement 'at most n in number'. But contingentists can see necessitists' discussion of (40_n) as engaging a further question of interest, not settled by Aux[Con], if the necessitists accept the auxiliary assumption Aux[Nec]. For (40_n) is a first-order formula, so it is equivalent given Aux[Nec] to $(40_n)^{\mathrm{Nec}}$, which is equivalent to this simpler formula:[47]

(41_n) $\Diamond\exists z_1 (\mathrm{C}z_1\ \&\ \ldots \Diamond\exists z_n (\mathrm{C}z_n\ \&\ \Diamond\exists x (\mathrm{C}x\ \&\ \neg(x{=}z_1 \vee \ldots \vee x{=}z_n)\ \&$

$$\Box\forall y ((\mathrm{C}y\ \&\ ((\mathrm{C}z_1\ \&\ \mathrm{R}z_1y) \vee \ldots \vee (\mathrm{C}z_n\ \&\ \mathrm{R}z_ny))) \rightarrow (y{=}z_1 \vee \ldots \vee y{=}z_n)\ldots)$$

Contingentists and necessitists need not disagree on (41_n), since it is neutral; it makes a claim about the modal distribution of R over chunky things which cannot be settled positively or negatively without serious inquiry. When necessitists who accept Aux[Nec] discuss whether (40_n) holds, contingentists can see them as in effect engaging with the significant neutral question whether (41_n) holds. Since (34) differs from (40_n) only in not imposing the finite bound n, when necessitists who accept Aux[Nec] discuss whether (34) holds, contingentists should be able to see them as in effect engaging with a significant question which stands in the relevant way to (34) as (41_n) stands to (40_n). But no formula of the language stands in that way to (34) as (41_n) stands to (40_n). For (41_n) is a neutral formula equivalent under Aux[Nec] to (40_n), while no neutral formula is equivalent under Aux[Nec] to (34). Of course, a contingentist might insist that *no* significant question stands in the relevant way to (34) as (41_n) stands to (40_n). But when one reads the formulas carefully, it is hard not to regard that insistence as a dogmatic refusal to see the point.

A contingentist may assert that if (34) as used by the necessitist has no cash-value in neutral terms, so much the worse for (34) as so used: it inextricably involves the necessitist's errors. There may be no means of persuading such a contingentist otherwise. But that is the usual way with expressive impoverishment. It is hard to argue with a claim not to understand. In the heyday and aftermath of logical positivism, claiming not to understand was a standard philosophical tactic, employed with varying degrees of plausibility. These days it is rather less popular. I will proceed on the basis that an intelligent contingentist can see what distinctions a necessitist is getting at in applying (33) and (34): but on this, as on everything else, the reader will have to decide.

In a nutshell, the problem for contingentism in a second-order language is this: necessitists can draw distinctions whose genuineness contingentists can neither plausibly deny nor explain on their own terms. The situation is not symmetrical, for necessitists can explain contingentists' distinctions, since the mapping from contingentist to necessitist discourse extends smoothly to the second-order case.

10. Infinitary languages

The natural move for the contingentist at this point is to resort to an infinitary language, for the problem occurs when the finite bound n in (41_n) is lifted to produce (34). Kit Fine proposed that the actualist do just that when interpreting a higher-order possibilist language (1977a, pp. 146-8; 1977b, pp. 161-2; 2003, pp. 173-4). We might envisage (34) as corresponding to (42) in an infinitary first-order language:

$$(42) \quad \exists z_1 \exists z_2 \ldots (\exists x \neg(x{=}z_1 \vee x{=}z_2 \vee \ldots) \; \& \; \forall y (\Diamond(Rz_1y \vee Rz_2y \vee \ldots) \rightarrow (y{=}z_1 \vee y{=}z_2 \vee \ldots)))$$

Just as (40_n) is equivalent given Aux[Nec] to (41_n), so (42) might be envisaged as equivalent given Aux[Nec] to the 'neutral' formula (43):

$$(43) \quad \Diamond\exists z_1 (Cz_1 \; \& \; \Diamond\exists z_2 (Cz_2 \; \& \; \ldots (\Diamond\exists x (Cx \; \& \; \neg(x{=}z_1 \vee x{=}z_2 \vee \ldots) \; \& \; \Box\forall y ((Cy \; \& \; ((Cz_1 \; \& \; Rz_1y) \vee (Cz_2 \; \& \; Rz_2y) \vee \ldots)) \rightarrow (y{=}z_1 \vee y{=}z_2 \vee \ldots)\ldots)$$

The proposal is that when necessitists who accept Aux[Nec] discuss whether (34) (or (42)) holds, contingentists can see them as in effect engaging with the significant neutral question whether (43) holds. The strategy can be generalized to the necessitist's use of other second-order sentences.

As an attempt to explain how we can understand what is at stake in the necessitist's use of straightforward plural quantification by finding a neutral equivalent, the contingentist's appeal to an infinitary language is prima facie implausible. One might expect finitary languages to precede infinitary ones in the order of human understanding. However, instead of trying to make that vague objection precise, we may examine two more specific problems for the proposal.

The first is this. We might expect the variables in an infinitary formula to form a set. Let its cardinality be k. When we simulate the necessitist's plural quantification with an infinite sequence of modally qualified quantifiers as in (43), it is redundant to use two quantifiers with the same variable, so in effect we use at most k quantifiers. The result therefore corresponds to the cardinally restricted plural quantifier 'some things, at most k in number' rather than to the unrestricted 'some things'. For instance, if the formula has only countably many variables, we are in effect quantifying only over countable pluralities. More generally, we have raised the finite bound n in (41_n) to the infinite bound k implicit in (43), but we have not abolished cardinality bounds altogether. Yet no such bound is implicit in the plural quantifier, when used unrestrictedly by the necessitist. We can truly say 'Some things are sets while all other things are non-sets', quantifying plurally over all sets whatsoever. But there are more than k sets, since by Cantor's theorem a set with k members has more than k subsets. Thus the simulation in the infinitary first-order language is not equivalent to the plural original.[48]

Presenting (43) as a mere 'schema' would not help, for each instance of the schema makes a more specific claim than was intended. Nor would the contingentist be on firm ground in trying to explain (43) as containing too many variables to form a set.

The contingentist might gain some expressive power by using infinite sequences of second-order quantifiers $\exists Z_1 \; \exists Z_2 \ldots$ where the necessitist has a single second-order quantifier. In attempting to construct a neutral infinitary formula equivalent to the original given Aux[Nec], the contingentist would then qualify each second-order quantifier with a modal operator and restrict it to the chunky:

$$\Diamond \exists Z_1 (\forall x (Z_1 x \rightarrow Cx) \,\&\, \Diamond \exists Z_2 (\forall x (Z_2 x \rightarrow Cx) \,\&\, \ldots$$

From the necessitist's perspective, this would have the effect of quantifying over pluralities consisting of the union of k pluralities of things which can be chunky together (where k is now the number of second-order variables in the sequence). This will raise the bound on the size of the pluralities in question (the unions) above k if it is possible for there to be more than k chunky things. Even so, however, the result will not generally be equivalent to the original given the necessitists' auxiliary assumptions, in particular Aux[Nec]. For example, the necessitist need not assume that of any collection of more than k possibly chunky things, at least two can be chunky together. But a plurality of more than k things no two of which can be chunky together is not the union of any k pluralities of things which can be chunky together.

Could the necessitist strengthen Aux[Nec] by adding the assumption that every plurality *is* the union of an infinite sequence of pluralities of possibly co-chunky things?

(44) $\forall X \Diamond \exists Z_1 (\forall x (Z_1 x \rightarrow \mathrm{C}x) \ \& \Diamond \exists Z_2 (\forall x (Z_2 x \rightarrow \mathrm{C}x) \ \& \ \ldots$

$$\forall x (Xx \leftrightarrow (Z_1 x \vee Z_2 x \vee \ \ldots) \ldots)$$

Given (44), one might hope to extend the mapping from A to $(\mathrm{A})^{\mathrm{Nec}}$ to the second-order language, and show that A is always equivalent to $(\mathrm{A})^{\mathrm{Nec}}$ given the strengthened Aux[Nec], by mapping each subformula of the form *Vv* to one of this form:

$$\Diamond ((\forall x (Z_1 x \rightarrow \mathrm{C}x) \ \& \ Z_1 x)) \vee ((\forall x (Z_2 x \rightarrow \mathrm{C}x) \ \& \ Z_2 x)) \vee \ \ldots) \ldots)$$

The problem is that the necessitist has no apparent reason to accept (44). The reason to accept the corresponding conjunct of the unstrengthened Aux[Nec], (26) (everything is possibly chunky), was at least clear, although not conclusive: if the basic mistake in the contingentist's ontology of chunky objects is that it omits the category of the contingently non-chunky, then one can get from contingentism to necessitism by adding that category in order to obtain (NNE); the result will be an ontology of possibly chunky objects. There is no corresponding motivation for (44). What there are for unrestricted plural quantifiers to range over simply depends on what there is for unrestricted singular quantifiers to range over. There is no room for independent determination of the former. In particular, since there was no limitation of size on the singular quantifiers, there was no limitation of size on the plural quantifiers. Thus not even infinite sequences of second-order quantifiers do the trick for the contingentist.

The second problem for the proposed use of infinitary languages is independent of the first. It arises even if we ignore uncountable pluralities. We can simply focus on ω-sequences ordered like the natural numbers with first-order quantifiers. Furthermore, we may assume that the quantifiers occur uniformly, either all $\exists$ or all $\forall$, as in (43). The difficulty is to define what an infinite sequence like $\Diamond\exists z_1 (Cz_1 \,\&\, \Diamond\exists z_2 (Cz_2 \,\&\, \ldots$ could mean to a contingentist. A parallel difficulty arises for a corresponding infinite sequence with plural quantifiers; it is unnecessary to discuss the latter separately.

The difficulty is not general to all infinitary devices. The truth-conditions of infinite conjunctions and disjunctions are clear enough. But they are infinitely *broad*, whereas $\Diamond\exists z_1 (Cz_1 \,\&\, \Diamond\exists z_2 (Cz_2 \,\&\, \ldots$ is infinitely *deep*, in the sense that each quantifier has the next in its scope. Not every ω-sequence of meaningful operators itself constitutes a meaningful operator. The simplest example is negation. Let $\neg\neg\ldots$ be an ω-sequence of negations. Prefixing $\neg$ to such a sequence still gives an ω-sequence of negations. Thus for any sentence A, $\neg(\neg\neg\ldots A) = \neg\neg\neg\ldots A = \neg\neg\ldots A$, so $\neg\neg\ldots A$ is its own negation; but nothing in the meaning of a standard negation operator ¬ provides for such a non-bivalent case. Although the language assigns a meaning to ¬¬…¬A for each finite sequence ¬¬…¬, it assigns no meaning to ¬¬…A for the infinite sequence. Similarly, although the language of arithmetic assigns meaning to each finite product '(−1)(−1)…(−1)', it assigns no meaning to the infinite product '(−1)(−1)…'. To take an example closer to (43), it is quite unclear what is meant by the infinite sequence of modalized quantifiers in 'There cannot be someone S such that there cannot be someone S* such that there cannot be someone S** such that … such that S, S*, S**, … are all friends of each other'. There is no default way to construct an appropriate meaning for such infinitary expressions which will work except when something special goes wrong. The onus is on contingentists to explain what they intend the infinite sequence of modal operators and restricted quantifiers to mean.

For a non-modal language, one can easily extend the semantics of a first-order quantifier $\exists$ to the case in which it binds many variables v_1, …, v_i, …. Just as $\exists\, v$ A is true on assignment <u>a</u> if and only if A is true on some assignment <u>a</u>* that differs from <u>a</u> at most in the value of the variable v, so $\exists\, v_1,\ldots,v_i, \ldots$ A is true on <u>a</u> if and only if A is true on some assignment <u>a</u>* that differs from <u>a</u> at most in the values of the variables v_1, …, v_i, …. This explanation works equally well whether the variables v_1, …, v_i, … are finite or infinite in number. It serves to assign a natural meaning to the sentence $\exists\, v_1 \ldots \exists\, v_i \ldots$ A, whether the sequence of quantifiers is finite or infinite, by equating its meaning with that of $\exists\, v_1,\ldots,v_i, \ldots$ A.

For a modal language with a possible worlds semantics, such an explanation can be extended to a sentence such as $\Diamond\exists\, v_1$ (Cv_1 & $\Diamond\exists\, v_2$ (Cv_2 & … A)…). It is true at a world w on an assignment a if and only if A is true at w on some assignment a* which differs from a at most in the values of the variables v_1, …, v_i, …, where for each natural number i a*(v_i) belongs to the extension of C at some world (here assumed to be a subset of the domain of that world).[49] A necessitist sympathetic to possible worlds semantics may accept such an explanation. But it should not satisfy the contingentist. For it is framed in a non-modal meta-language which treats the domains of all worlds as laid out together, available for their members to be simultaneously quantified over.

Indeed, the special case of the preceding semantic clause for a single variable in effect stipulates that [◊∃]*v* (C*v*, A) is true at w on a if and only if A is true at w on some assignment a* which differs from a at most in the value of the variable *v*, where a*(*v*) belongs to the extension of C at some world. The square brackets indicate that a new operator is being defined, not to be confused with, and not equivalent to, ◊∃ as the contingentist normally uses that ordered pair of operators with their compositional semantics. But the clause for [◊∃] is the semantic clause for a quantifier which works in the way the necessitist postulates (given Aux[Nec]). For contingentists to admit such a quantifier would be to represent their own intra-world quantifier as a restriction of the necessitist's trans-world quantifier, and thereby to surrender their central claim that (NNE) is false on the unrestricted reading of its quantifiers.

In using $(A)^{Nec}$ to simulate the necessitist's use of a sentence A, it is crucial for contingentists to insist on the compositional semantics of $(A)^{Nec}$, on which the quantifiers are governed by modal operators, in order not to undermine their own claim that the quantifiers themselves are already unrestricted. They must make a corresponding claim about infinitary formulas such as (43), for the same reason. A semantic clause like the one above which gives no independent role to the modal operators in (43) thereby assimilates it to a sentence with quantification less restricted than standard contingentist quantification, and so abandons contingentism. Thus the contingentist cannot rely on such possible worlds semantics for infinitary sentences, on pain of defeat.

Of course, contingentists can still use the possible worlds model theory in an instrumental role, the one it plays in this paper. They can use it as a convenient mathematical device for establishing consistency or independence results. For example, they might use it to show that (NNE) is not a theorem of some proof system for infinitary formulas like (43). As David Lewis recognized, when we use possible worlds model theory as such an instrument of metalogic, 'We are doing mathematics, not metaphysics' (1986, p. 17). What contingentists cannot do is treat the possible worlds clause as the distinguishing semantic mark of the infinitary construction, for doing so would play into the necessitist's hands, by giving it the same meaning as quantification less restricted than standard contingentist quantification. This point applies whether the contingentist treats the possible worlds semantics as explanatorily basic, or more modestly as a convenient device for representing meanings. It depends not on interpreting the possible worlds semantics literally or at face-value, but simply on its capacity to distinguish distinct meanings. What contingentists need is some way of semantically distinguishing the infinitary construction in (43) from a less restricted sort of quantification. Since a semantic clause like the one above provides no such way, contingentists need an independent semantic account.

Is the problem of infinitely deep embedding solved by construing 'sentences' of the form $\Diamond\exists v_1 (Cv_1 \& \Diamond\exists v_2 (Cv_2 \& \ldots A)\ldots)$ as involving infinitely branching modalized quantification, in which no modalized quantifier is in the scope of any other? We can diagram the proposed structure in something like this form:

$$
\begin{array}{ll}
\Diamond\exists\, v_1\, Cv_1 & \\
\Diamond\exists\, v_2\, Cv_2 & \\
\Diamond\exists\, v_3\, Cv_3 & \qquad\qquad A \\
\Diamond\exists\, v_4\, Cv_4 & \\
\ldots &
\end{array}
$$

The problem is that, in general, applying a finite or infinite sequence of sentential operators to a sentence 'simultaneously' has no clear meaning. For example, consider this tree:

$$
\begin{array}{ll}
\Diamond & \\
 & \qquad A \\
\Box &
\end{array}
$$

It is unclear whether this should be understood as $\Diamond A \;\&\; \Box A$, or as $\Diamond\Box A$ (which are equivalent in S5 to $\Box A$), or as $\Box\Diamond A$ (which is equivalent in S5 to $\Diamond A$), or as something else. Again, writing the ω-sequence of negations in $\neg\neg\ldots A$ vertically rather than horizontally does not clarify its meaning. By using the method of Skolem functions, Henkin (1961) showed how to interpret some branching structures of quantifiers. His interpretation can be applied to partial orderings of quantifiers of the forms $\exists v$ and $\forall v$, even to infinite examples such as this:

$$\begin{array}{ll} \exists v_1\, Cv_1 & \\ \exists v_2\, Cv_2 & \\ \exists v_3\, Cv_3 & \qquad\qquad A \\ \exists v_4\, Cv_4 & \\ \ldots & \end{array}$$

Barwise (1978) gave a semantics for further sorts of branching generalized quantifiers in natural languages (see also Peters and Westerståhl 2006, pp. 66-72, 363-4). However, the only apparent way of extending the available semantic treatments of branching quantifiers to modalized examples such as those postulated for sentences like (43) is by treating the modal operators as quantifiers, as in possible worlds semantics. Thus the previous problems have not yet been avoided. Contingentists still need an independent semantic account.

Presumably, the independent account must be framed in other terms more appropriate to contingentism. Whence are those other terms to come, if not from the contingentist's modal language? But that takes us round in a circle, for the problem was that on the very point at issue contingentists had found reason to regard their finitary modal language as expressively inadequate, while the sentences of their infinitary modal language are still waiting to be assigned meaning. It is hard to avoid the suspicion that if contingentists think they understand infinitary modal sentences such as (43), they do so because, unwittingly, they look at them from a necessitist perspective.

A more inferentialist strategy for the contingentist avoids such cheating. If the infinite sequence of quantifiers in (43) is just an ω-sequence, then we might regard (43) as something like the limit of the infinite sequence (41_1), (41_2), (41_3), …, since (41_n) has a sequence of n terms where (43) has an ω-sequence. Moreover, it is easy to see that (41_m) entails (41_n) if and only if $m \leq n$, so (41_n) becomes logically weaker as n increases. It is therefore natural to regard the limit of the series as making the strongest claim that every member of the series entails, just as the limit of a monotonically decreasing series of real numbers is their greatest lower bound. The strongest claim that (41_n) entails for each n is equivalent to the disjunction of (41_n) over all n, for each disjunct entails the disjunction (by disjunction introduction), and the disjunction entails whatever every disjunct entails (by disjunction elimination). Clearly, this conception avoids illicit use of necessitist resources.

Unfortunately for the contingentist, however, the inferentialist account generates the wrong truth-conditions. We can show this formally using the possible worlds semantics, as always in an instrumental capacity, in this case simply to check consequence relations in the background logic. Consider a non-standard model of first-order Peano arithmetic with domain N and successor operation ′. Since it is non-standard, some members of N cannot be reached from zero by finitely many applications of ′. Define a possible worlds model in which the set of worlds is N and the domain of any world is also N; in each world n, the extension of C is {n, n′} and R*xy* is equivalent to

Cx & Cy (such a model is used in appendix 2.11). Aux[Nec] holds in this model; the domain is constant and every number n is in the extension of C at world n. In effect, (41_n) requires a nonempty proper subset of N with at most n members of which each number m is a member if and only if m′ is a member. But any such subset with a member m would contain infinitely many members m, m′, m″, … and so not have at most n members. Thus (41_n) is false for each (genuine!) natural number n. Consequently, the disjunction of (41_n) over all such n is also false. But (43) is true, for what it in effect requires is a nonempty proper subset of N of which each number m is a member if and only if m′ is a member. The set of standard members of N (those reachable from the zero by finitely many applications of ′) is such a subset. Unlike the disjunction of (41_n) over n, (43) does not require the subset in question to be finite. Thus (43) is not the inferential limit of (41_n) as n goes to infinity.

Since each world in the model contains just two 'chunky' things, the contingentist cannot avoid appeals to infinitary expressions by using a sequence of modal operators and second-order quantifiers, each restricted to pluralities of 'chunky' things (as required for neutrality), for the union of finitely many two-membered pluralities is finite; since an infinite plurality is needed, only an infinite sequence of such quantifiers will do.

It would be pointless for the contingentist simply to stipulate that the infinitary formula (43) is to have the truth-conditions of the disjunction of (41_n) over n. For the point of (43) was to capture in neutral terms the significant question with which necessitists engage when they discuss the finitary second-order formula (34). But (34) is clearly true in the model above. Thus (43) cannot do the job which the contingentist introduced it to do if it is equivalent to the disjunction of (41_n) over n. So that inferentialist strategy fails.

Could one give an inferentialist account of (43) without treating it as the limit of the infinite sequence (41_1), (41_2), (41_3), …? Such an account might use an infinitary consequence relation. But it is quite unclear how it would distinguish the intended meaning of (43) from all other meanings, or even from all logically inequivalent meanings. Merely accepting the rule that each of (41_1), (41_2), (41_3), … entails (43) while rejecting the rule that (43) entails whatever each of (41_1), (41_2), (41_3), … entails does not uniquely characterize an inferential role for (43). It does not even differentiate (43) from a tautology, or from the infinite disjunction of (41_1), (41_2), (41_3), … and another sentence logically independent of them all. Appealing to inferentialism here is whistling in the dark.

By more or less ad hoc means, the contingentist can sometimes find a neutral equivalent under Aux[Nec] which uses only the less problematic infinitary devices. In particular, we can define the ancestral of a relation by an infinite disjunction. Thus the technique works for (34). Let $(\Diamond\mathrm{R})^n(x,y)$ formalize the claim that there is a chain from x to y of n links of the possibility of being chunky and standing in R. Technically, $(\Diamond\mathrm{R})^0(x,y)$ is $x=y$ and $(\Diamond\mathrm{R})^{n+1}(x,y)$ is $\Diamond\exists z\,(\mathrm{C}x \,\&\, \mathrm{C}z \,\&\, \mathrm{R}xz \,\&\, (\Diamond\mathrm{R})^n(z,y))$, where the bound variables are chosen to avoid clashes. Then (34) is equivalent under Aux[Nec] to this neutral infinitary formula, where n ranges over the natural numbers:

(45) $\Diamond\exists x\,(\mathrm{C}x \,\&\, \Diamond\exists y\,(\mathrm{C}y \,\&\, \neg\vee_n (\Diamond\mathrm{R})^n(x,y)))$

Of course, it is not especially plausible that contingentists were all along using the equivalence of (34) to (45) under Aux[Nec] to grasp what significant question necessitists were in effect engaging with when they uttered (34), since no such thought may have crossed their minds. Fortunately, we need not rely on slippery considerations about conditions for understanding. For the appeal to (45) does not adequately generalize. Many sentences of finitary second-order non-modal logic have no equivalents in infinitary first-order logic, even when the quantifiers can bind arbitrarily large sets of variables and arbitrarily large sets of formulas can be conjoined and disjoined. In particular, many standard notions in algebra and topology cannot be characterized in infinitary first-order logic.[50] By giving any such example a modal twist, one can construct a sentence of second-order modal logic which has no neutral equivalent under Aux[Nec] in infinitary modal logic (appendix 3.5-6). One need only prefix each occurrence of a non-logical atomic predicate constant with ◊. The reason for choosing (34) as an example above rather than one of those other formulas was only its comparative simplicity. There is no difference of principle.

Necessitists use the modalized formulas at issue to draw genuine distinctions. Contingentists cannot simulate those distinctions with the less problematic infinitary devices. If they attempt to simulate them with infinitely embedded combinations of quantifiers and modal operators, as in (43), the problem is that they have failed to provide a meaning for such constructions without exceeding the limits of contingentist resources. Only the necessitist can explain what the contingentist wants the infinitary formula to mean.

11. Conclusion

Necessitists have no structural problem in seeing what contingentists are getting at. Even in a language with plural quantification, each sentence is equivalent by contingentist's lights to a neutral sentence on which the two sides need not disagree. But contingentists have a structural problem in seeing what necessitists are getting at. In a language with plural quantification, some sentences are not equivalent by necessitist lights to any neutral sentence. That asymmetry would not harm contingentists if they could dismiss the necessitists' use of the recalcitrant sentences as tracking only theoretical confusion. But contingentists have found no plausible way to do that. Even from a contingentist perspective, there are strong indications that necessitists are getting at, even if mischaracterizing, a significant question about the patterns of modal distribution of various relations. The trouble for contingentism is that, given the expressive resources considered in this paper, it leaves no room for anything to be the question necessitists are getting at. The asymmetry favours necessitism. Every distinction contingentists can draw has a working equivalent in neutral terms, but the extra commitments of necessitism allow one to draw genuine distinctions which have no working equivalents in neutral terms. If one wants to draw those distinctions, one may have to be a necessitist.

Appendix

1. Consider a standard language $L1_{\Box}$ for first-order modal logic with countably many individual variables *x*, *y*, *z*, …, a finite number of atomic non-logical predicate constants each with a fixed finite number of argument places, including the 1-place predicate constant C, the atomic 2-place logical predicate identity constant = (in practice, it is always clear whether it is being used as a symbol of the object-language or the meta-language), the usual truth-functors ($\neg$, &, $\vee$, $\rightarrow$, $\leftrightarrow$), modal operators ($\Diamond$, $\Box$, $\uparrow$, $\downarrow$) and first-order quantifiers ($\exists$, $\forall$). Of those operators, $\neg$, &, $\Diamond$, $\uparrow$, $\downarrow$ and $\exists$ are treated as primitive, and the others as metalinguistic abbreviations in the usual way. Only slight adjustments to the formal development below would be needed for a language with infinitely many non-logical atomic predicates.

For purposes of this appendix, a model is a quadruple <W, D, dom, int> where W and D are nonempty sets, dom is a function mapping each $w \in W$ to $dom(w) \subseteq D$, and int is a function mapping each non-logical n-place atomic predicate constant F to a function int(F) mapping each $w \in W$ to $int(F)(w) \subseteq dom(w)^n$ (S^n is the set of n-tuples of members of set S). Informally, imagine W as the set of possible worlds, dom(w) as the set of objects in world w, int(F) as the intension of F and int(F)(w) as the extension of F in w; these informal glosses play no official role in the model theory itself.

In order to accommodate the modal operators $\uparrow$ and $\downarrow$, the model theory relativizes the evaluation of a formula in a model at a world on an assignment to a finite sequence of worlds; the sequence parameter acts as a sort of memory. Consider a model

$M = \langle W, D, dom, int\rangle$. An assignment is a function $\underline{a}$ mapping each variable v to a member $\underline{a}(v)$ of D. Let $W^{<\omega}$ be the set of finite sequences of members of W, $\langle\,\rangle$ be the empty sequence and s^w the sequence that results from appending $w \in W$ to s. For a set S, diag(S) is $\{\langle o, o\rangle\colon\ o \in S\}$. $M, w, s, \underline{a} \models A$ if and only if formula A is true in model M at $w \in W$ relative to $s \in W^{<\omega}$ on assignment $\underline{a}$. We define this relation recursively. Below, F is any non-logical n-place atomic predicate constant; $v, v_1, \ldots, v_n$ are any variables; $\underline{a}[v/o]$ is the assignment like $\underline{a}$ except that it assigns o to v; $w \in W$; $s \in W^{<\omega}$.

$M, w, s, \underline{a} \models Fv_1 \ldots v_n$	iff $\langle \underline{a}(v_1), \ldots, \underline{a}(v_n)\rangle \in int(F)(w)$
$M, w, s, \underline{a} \models v_1 = v_2$	iff $\langle \underline{a}(v_1), \underline{a}(v_2)\rangle \in diag(dom(w))$
$M, w, s, \underline{a} \models \neg A$	iff not $M, w, s, \underline{a} \models A$
$M, w, s, \underline{a} \models A \,\&\, B$	iff $M, w, s, \underline{a} \models A$ and $M, w, s, \underline{a} \models B$
$M, w, s, \underline{a} \models \exists\, v\, A$	iff for some $o \in dom(w)$: $M, w, s, \underline{a}[v/o] \models A$
$M, w, s, \underline{a} \models \Diamond A$	iff for some $w^* \in W$: $M, w^*, s, \underline{a} \models A$
$M, w, s, \underline{a} \models \uparrow A$	iff $M, w, s^\wedge w, \underline{a} \models A$
$M, w, s^\wedge x, \underline{a} \models \downarrow A$	iff $M, x, s, \underline{a} \models A$
$M, w, \langle\,\rangle, \underline{a} \models \downarrow A$	iff $M, w, \langle\,\rangle, \underline{a} \models A$

As an example, we check that the formula $\uparrow\Diamond\exists\, v \downarrow A$ has the intended effect of 'possibilist' quantification: $M, w, s, \underline{a} \models \uparrow\Diamond\exists\, v \downarrow A$ iff $M, w, s^\wedge w, \underline{a} \models \Diamond\exists\, v \downarrow A$ iff for some $w^* \in W$, $M, w^*, s^\wedge w, \underline{a} \models \exists\, v \downarrow A$ iff for some $w^* \in W$ and $o \in dom(w^*)$, $M, w^*, s^\wedge w, \underline{a}(v/o) \models \downarrow A$ iff for some $w^* \in W$ and $o \in dom(w^*)$, $M, w, s, \underline{a}(v/o) \models A$, as required. Note that the world parameters have been restored to their original values (w and s) by the end of this computation. The alternative semantics for ↑ and ↓ which uses just two world parameters (as in Forbes 1989, pp. 28-9) is insufficiently general because it lacks this feature: the clause for ↑ 'forgets' the original value of one parameter, which therefore cannot be recovered later in the evaluation. The use of finite sequences of worlds solves the problem by enhancing memory power; it goes back to the semantics for the 'backspace' operator ↓ in Hodes 1984, although some details of the present treatment are different.

For a set of formulas S: M, w, s, $\underline{a}$ |= S iff M, w, $\underline{a}$ |= A for every A ∈S. If S and T are single formulas or sets of such, S |= T iff M, w, s, $\underline{a}$ |= T whenever M, w, s, $\underline{a}$ |= S. M |= X iff M, w, s, $\underline{a}$ |= S for all w, s, $\underline{a}$ in M; |= S iff M |= S for all models M.

We recursively define the restriction of a formula A by the predicate C, $(A)^{Con}$ for $L1_{\Box}$ as follows, where F is any n-place atomic predicate (logical or non-logical) and v, v_1, …, v_n any variables:

$$(Fv_1 \ldots v_n)^{Con} = Fv_1 \ldots v_n \mathbin{\&} Cv_1 \mathbin{\&} \ldots \mathbin{\&} Cv_n$$

$$(\neg A)^{Con} = \neg(A)^{Con}$$

$$(A \mathbin{\&} B)^{Con} = (A)^{Con} \mathbin{\&} (B)^{Con}$$

$$(\Diamond A)^{Con} = \Diamond(A)^{Con}$$

$$(\uparrow A)^{Con} = \uparrow(A)^{Con}$$

$$(\downarrow A)^{Con} = \downarrow(A)^{Con}$$

$$(\exists\, v\, A)^{Con} = \exists\, v\, (Cv \mathbin{\&} (A)^{Con}).$$

A formula A is *neutral* iff for some B, |= A ↔ $(B)^{Con}$. Trivially, $(A)^{Con}$ is always neutral.

For economy in the statement of the results below, A is any formula of $L1_{\Box}$, M = <W, D, dom, int> is any model, $w \in W$, $s \in W^{<\omega}$ and $\underline{a}$ is any assignment. We define an operation on models corresponding to restriction by C. For M = <W, D, dom, int>, let M^{Con} = <W, D, dom^C, int^C>, where for $w \in W$ $dom^C(w) = int(C)(w)$ and $int^C(F)(w) = int(F)(w) \cap int(C)(w)^n$. Thus M^{Con} is the restriction of M by C; it too is a model. Note that M and M^{Con} have the same worlds and assignments. The first result connects the operation on models with the operation on formulas.

1.1. ($A \in L1_{\Box}$) M, w, s, $\underline{a}$ |= $(A)^{Con}$ iff M^{Con}, w, s, $\underline{a}$ |= A.

Proof: By induction on the complexity of A.

Basis: M, w, s, $\underline{a}$ |= $(Fv_1 \ldots v_n)^{Con}$

iff M, w, s, $\underline{a}$ |= $Fv_1 \ldots v_n$ & Cv_1 & … & Cv_n

iff $<\underline{a}(v_1), \ldots, \underline{a}(v_n)> \in int(F)(w) \cap int(C)(w)^n = int^C(F)(w)$

iff M^{Con}, w, s, $\underline{a}$ |= $Fv_1 \ldots v_n$.

The argument for $v_1=v_2$ is similar.

Induction step: The cases for ¬, &, ◊, ↑ and ↓ are routine; we consider only that for ∃. By the induction hypothesis: M, w, s, $\underline{b}$ |= $(A)^{Con}$ iff M^{Con}, w, s, $\underline{b}$ |= A for each assignment $\underline{b}$.

M, w, s, $\underline{a}$ |= $(\exists v\, A)^{Con}$

iff M, w, s, $\underline{a}$ |= $\exists v\, (Cv \,\&\, (A)^{Con})$

iff for some $o \in dom(w)$: M, w, s, $\underline{a}[v/o]$ |= $Cv \,\&\, (A)^{Con}$

iff for some $o \in dom(w) \cap int(C)(w)$: M, w, s, $\underline{a}[v/o]$ |= $(A)^{Con}$

iff for some $o \in dom^C(w)$: M, w, s, $\underline{a}[v/o]$ |= $(A)^{Con}$ (since $int(C)(w) \subseteq dom(w)$)

iff for some $o \in dom^C(w)$: M^{Con}, w, s, $\underline{a}[v/o]$ |= A (by induction hypothesis) iff $M^{Con}, w, s, \underline{a} \models \exists v\, A$.

1.2 ($A,B \in L1_{\Box}$) If $\models A \leftrightarrow B$ then $\models (A)^{Con} \leftrightarrow (B)^{Con}$.

Proof: If not $\models (A)^{Con} \leftrightarrow (B)^{Con}$, in other words not $\models (A \leftrightarrow B)^{Con}$, then in some model not $M, w, s, \underline{a} \models (A \leftrightarrow B)^{Con}$, so by 1.1 not $M^{Con}, w, s, \underline{a} \models A \leftrightarrow B$, so not $\models A \leftrightarrow B$.

1.3. ($A \in L1_{\Box}$) $\models ((A)^{Con})^{Con} \leftrightarrow (A)^{Con}$.

Proof: By induction on the complexity of A, eliminating redundant conjuncts.

1.4. ($A \in L1_{\Box}$) If A is neutral, $\models (A)^{Con} \leftrightarrow A$.

Proof: If A is neutral, for some B $\models A \leftrightarrow (B)^{Con}$, so by 1.2 $\models (A)^{Con} \leftrightarrow ((B)^{Con})^{Con}$, but by 1.3 $\models ((B)^{Con})^{Con} \leftrightarrow (B)^{Con}$, so $\models (A)^{Con} \leftrightarrow A$.

Let Aux[Con] be $\Box \forall x\, Cx$.

1.5. $M^{C} \models$ Aux[Con].

Proof: Routine.

1.6. $(\text{Aux[Con]})^{Con} \leftrightarrow$ Aux[Con] $\models$ Aux[Con].

Proof: For any model M, by 1.5 and 1.1, $M \models (\text{Aux[Con]})^{Con}$. Thus $\models (\text{Aux[Con]})^{Con}$.

1.7. If $M, w, s, \underline{a} \models$ Aux[Con] then $M^{Con} = M$.

Proof: If M, w, s, $\underline{a}$ |= $\Box\forall x\, Cx$ then for all $w^* \in W$ $dom(w^*) \subseteq int(C)(w^*)$; the converse inclusion holds by definition of a model, since C is atomic. Thus $dom^C(w^*) = dom(w^*)$ and $int^C(F)(w^*) = int(F)(w^*) \cap int(C)(w^*)^n = int(F)(w^*) \cap dom(w^*)^n = int(F)(w^*)$. Hence $M^{Con} = \langle W, D, dom^C, int^C \rangle = \langle W, D, dom, int \rangle = M$.

1.8. ($A \in L1_\Box$) Aux[Con] |= $(A)^{Con} \leftrightarrow A$.

Proof: From 1.1 and 1.7.

1.9. ($A \in L1_\Box$, $S \subseteq L1_\Box$) If A and all members of S are neutral, then S, Aux[Con] |= A only if S |= A.

Proof: Suppose that not S |= A. Then for some model M = $\langle W, D, dom, int \rangle$, $w \in W$, $s \in W^{<\omega}$ and assignment $\underline{a}$: M, w, s, $\underline{a}$ |= S but not M, w, s, $\underline{a}$ |= A. By 1.4, M, w, s, $\underline{a}$ |= $(S)^{Con}$ but not M, w, s, $\underline{a}$ |= $(A)^{Con}$. Hence by 1.1, M^{Con}, w, s, $\underline{a}$ |= S but not M^{Con}, w, s, $\underline{a}$ |= A. By 1.5, M^{Con}, w, s, $\underline{a}$ |= Aux[Con]. Hence not S, Aux[Con] |= A.

1.10. ($A,B \in L1_\Box$) If B is neutral and Aux[Con] |= $A \leftrightarrow B$ then |= $(A)^{Con} \leftrightarrow B$.

Proof: Assume the antecedent. Hence, by 1.8, Aux[Con] |= $(A)^{Con} \leftrightarrow B$. But $(A)^{Con} \leftrightarrow B$ is neutral, so by 1.9 |= $(A)^{Con} \leftrightarrow B$.

Let Neutral(S) = {A: A is neutral and S |= A}.

1.11. ($S,T \subseteq L1_\Box$) If S |= Aux[Con] and T |= Aux[Con] then S |= T iff Neutral(S) |= Neutral(T).

Proof: If S |= T, Neutral(T) $\subseteq$ Neutral(S) so Neutral(S) |= Neutral(T). Conversely, suppose that Neutral(S) |= Neutral(T). Let A $\in$ T. By 1.8, Aux[Con] |= $(A)^{Con} \leftrightarrow A$. Since T |= Aux[Con], T |= $(A)^{Con} \leftrightarrow A$; thus T |= $(A)^{Con}$. Since $(A)^{Con}$ is neutral, $(A)^{Con} \in$ Neutral(T). Hence, by hypothesis, Neutral(S) |= $(A)^{Con}$, so S |= $(A)^{Con}$. But S |= Aux[Con], so S |= $(A)^{Con} \leftrightarrow A$; thus S |= A. Consequently, S |= T.

1.12. (T $\subseteq L1_{\Box}$) If T |= Aux[Con] then T =| |= Neutral(T), Aux[Con].

Proof: Only the right-to-left direction is non-trivial. But Neutral(T) $\subseteq$ Neutral(Neutral(T)), so Neutral(Neutral(T), Aux[Con])) |= Neutral(T), so Neutral(T), Aux[Con] |= T by 1.11.

Next we define the possibilification of a formula A $(A)^{Nec}$ as follows, where F is any non-logical n-place atomic predicate:

$(Fv_1 \ldots v_n)^{Nec} = Fv_1 \ldots v_n \,\&\, Cv_1 \,\&\, \ldots \,\&\, Cv_n$

$(v_1 = v_2)^{Nec} = \Diamond(v_1 = v_2 \,\&\, Cv_1 \,\&\, Cv_2)$

$(\neg A)^{Nec} = \neg(A)^{Nec}$

$(A \,\&\, B)^{Nec} = (A)^{Nec} \,\&\, (B)^{Nec}$

$(\Diamond A)^{Nec} = \Diamond(A)^{Nec}$

$(\uparrow A)^{Nec} = \uparrow(A)^{Nec}$

$(\downarrow A)^{Nec} = \downarrow(A)^{Nec}$

$(\exists\, v\, A)^{Nec} = \uparrow\Diamond\exists\, v\, (Cv \,\&\, \downarrow(A)^{Nec})$

Again there is a corresponding operation on models. For M = <W, D, dom, int>, let M^{Nec} = <W, D, dom^N, int^C>, where for $w \in W$ $dom^N(w) = \cup_{w^* \in W}$ int(C)(w*). Since for $w \in W$ $dom^C(w) \subseteq dom^N(w) \subseteq D$, M^{Nec} is a model just as M^{Con} is. M and M^{Nec} have the same worlds and assignments. 1.13 connects the operation on models and the operation on formulas.

1.13. ($A \in L1_{\Box}$) M, w, s, $\underline{a}$ |= $(A)^{Nec}$ iff M^{Nec}, w, s, $\underline{a}$ |= A.

Proof: By induction on the complexity of A.

Basis: For non-logical predicates, similar to the basis of 1.1. For identity:

M, w, s, $\underline{a}$ |= $(v_1=v_2)^{Nec}$

iff M, w, s, $\underline{a}$ |= $\Diamond(v_1=v_2 \ \& \ Cv_1 \ \& \ Cv_2)$

iff for some $w^* \in W$: M, w*, s, $\underline{a}$ |= $v_1=v_2 \ \& \ Cv_1 \ \& \ Cv_2$

iff for some $w^* \in W$: $<\underline{a}(v_1), \underline{a}(v_2)> \in \mathrm{diag}(\mathrm{dom}(w^*)) \cap \mathrm{int}(C)(w^*)^2$

iff $<\underline{a}(v_1), \underline{a}(v_2)> \in \mathrm{diag}(\cup_{w^* \in W} \mathrm{int}(C)(w^*)) = \mathrm{diag}(\mathrm{dom}^N(w))$

iff M^{Nec}, w, s, $\underline{a}$ |= $v_1=v_2$.

Induction step: The cases for ¬, &, ◊, ↑, and ↓ are routine; we consider only that for ∃.

By induction hypothesis, M, w, s, $\underline{b}$ |= $(A)^{Nec}$ iff M^{Nec}, w, s, $\underline{b}$ |= A for each assignment $\underline{b}$.

M, w, s, $\underline{a}$ |= $(\exists v \ A)^{Nec}$

iff M, w, s, $\underline{a}$ |= $\uparrow\Diamond\exists v \ (Cv \ \& \downarrow(A)^{Nec})$

iff M, w, s^w, $\underline{a}$ |= $\Diamond\exists v \ (Cv \ \& \downarrow(A)^{Nec})$

iff for some $w^* \in W$: M, w*, s^w, $\underline{a}$ |= $\exists v \ (Cv \ \& \downarrow(A)^{Nec})$

iff for some $w^* \in W$, $o \in \mathrm{dom}(w^*)$: M, w*, s^w, $\underline{a}[v/o]$ |= $Cv \ \& \downarrow(A)^{Nec}$

iff for some $w^* \in W$, $o \in \mathrm{dom}(w^*) \cap \mathrm{int}(C)(w^*) = \mathrm{int}(C)(w^*)$:

$M, w^*, s\hat{}w, \underline{a}[v/o] \models \downarrow(A)^{Nec}$

iff for some $w^* \in W$, $o \in int(C)(w^*)$: $M, w, s, \underline{a}[v/o] \models (A)^{Nec}$

iff for some $o \in \cup_{w^* \in W} int(C)(w^*) = dom^N(w)$: $M, w, s, \underline{a}[v/o] \models (A)^{Nec}$

iff for some $o \in dom^N(w)$: $M^{Nec}, w, s, \underline{a}[v/o] \models A$ (by induction hypothesis)

iff $M^{Nec}, w, s, \underline{a} \models \exists v\, A$.

1.14. ($A,B \in L1_\Box$) If $\models A \leftrightarrow B$ then $\models (A)^{Nec} \leftrightarrow (B)^{Nec}$.

Proof: From 1.13, as 1.2 was derived from 1.1.

1.15. ($A \in L1_\Box$) $(A)^{Nec}$ is neutral.

Proof: By induction on the complexity of A. For the basis, note that $(Fv_1\ldots v_n)^{Nec} = (Fv_1\ldots v_n)^{Con}$ and $(v_1=v_2)^{Nec} = (\Diamond v_1=v_2)^{Con}$. The only non-trivial part of the induction step is the case for $\exists$. The induction hypothesis is that $(A)^{Nec}$ is neutral, so $\models (A)^{Nec} \leftrightarrow (B)^{Con}$ for some B. Hence $\models \uparrow\Diamond\exists v\, (Cv \,\&\, \downarrow(A)^{Nec}) \leftrightarrow \uparrow\Diamond\exists v\, (Cv \,\&\, \downarrow(B)^{Con})$, in other words $\models (\exists v\, A)^{Nec} \leftrightarrow (\uparrow\Diamond\exists v \downarrow B)^{Con}$, so $(\exists v\, A)^{Nec}$ is neutral.

1.16. ($A \in L1_\Box$) $\models ((A)^{Con})^{Nec} \leftrightarrow (A)^{Con}$.

Proof: By induction on the complexity of A. Basis: If A is $Fv_1\ldots v_n$, where F is a non-logical n-place predicate, $((A)^{Con})^{Nec} = ((A)^{Con})^{Con}$, so $\models ((A)^{Con})^{Nec} \leftrightarrow (A)^{Con}$ by 1.3. If A is $v_1=v_2$:

$M, w, s, \underline{a} \models ((v_1=v_2)^{Con})^{Nec}$

iff $M, w, s, \underline{a} \models \Diamond(v_1=v_2 \,\&\, Cv_1 \,\&\, Cv_2) \,\&\, Cv_1 \,\&\, Cv_2$

iff for some $w^* \in W$: $\langle\underline{a}(v_1), \underline{a}(v_2)\rangle \in diag(dom(w^*)) \cap int(C)(w^*)^2 \cap int(C)(w)^2$

iff $\langle \underline{a}(v_1), \underline{a}(v_2)\rangle \in \text{diag}(\text{dom}(w)) \cap \text{int}(C)(w)^2$ (since $\text{int}(C)(w) \subseteq \text{dom}(w)$)

iff M, w, s, $\underline{a} \models (v_1=v_2)^{\text{Con}}$.

Induction step: The cases for ¬, &, ◊, ↑ and ↓ are routine; we consider only that for $\exists v$ A. By induction hypothesis, $((A)^{\text{Con}})^{\text{Nec}} \leftrightarrow (A)^{\text{Con}}$.

M, w, s, $\underline{a} \models ((\exists v\, A)^{\text{Con}})^{\text{Nec}}$

iff M, w, s, $\underline{a} \models \uparrow\Diamond\exists v\, (Cv\ \&\ \downarrow(Cv\ \&\ ((A)^{\text{Con}})^{\text{Nec}}))$

iff M, w, s, $\underline{a} \models \uparrow\Diamond\exists v\, (Cv\ \&\ \downarrow(Cv\ \&\ (A)^{\text{Con}}))$ (by induction hypothesis)

iff for some $o \in \text{dom}^{N}(w)$: M, w, s, $\underline{a}[v/o] \models Cv\ \&\ (A)^{\text{Con}}$ (as in proof of 1.13)

iff M, w, s, $\underline{a} \models (\exists v\, A)^{\text{Con}}$ ($\text{dom}^{N}(w) \cap \text{int}(C)(w) = \text{int}(C)(w) = \text{dom}(w) \cap \text{int}(C)(w)$).

1.17. ($A \in L1_{\Box}$) If A is neutral, $(A)^{\text{Nec}} \leftrightarrow A$.

Proof: If A is neutral, for some B $\models A \leftrightarrow (B)^{\text{Con}}$, so by 1.14 $\models (A)^{\text{Nec}} \leftrightarrow ((B)^{\text{Con}})^{\text{Nec}}$, but by 1.16 $\models ((B)^{\text{Con}})^{\text{Nec}} \leftrightarrow (B)^{\text{Con}}$, so $\models (A)^{\text{Nec}} \leftrightarrow A$.

Let F≤C be the formula $\forall z_1 \ldots \forall z_n\, (Fz_1 \ldots z_n \rightarrow (Cz_1\ \&\ \ldots\ \&\ Cz_n))$, where $z_1, \ldots, z_n$ are the first n distinct variables and F any non-logical n-place atomic predicate. Let Aux[Nec] be the conjunction of $\Box\forall x\, \Box\exists y\, x=y$, $\forall x\, \Diamond Cx$ and $\Box$F≤C for each such predicate F.

1.18. $M^{\text{Nec}} \models$ Aux[Nec].

Proof: Routine.

1.19. $(\text{Aux[Nec]})^{\text{Nec}} \leftrightarrow \text{Aux[Nec]} \models \text{Aux[Nec]}$.

Proof. For any model M, by 1.18 and 1.13, M |= $(\text{Aux[Nec]})^{\text{Nec}}$. Thus |= $(\text{Aux[Nec]})^{\text{Nec}}$.

1.20. If M, w, s, $\underline{a}$ |= Aux[Nec] then M^{Nec} = M.

Proof. Let M = <W, D, dom, int>. Suppose that M, w, s, $\underline{a}$ |= Aux[Nec]. Thus M, w, s, $\underline{a}$ |= $\Box\forall x\,\Box\exists y\; x{=}y$, so dom is constant, so $\text{dom}^{N}(w^*) = \cup_{w^{**}\in W}\text{int}(C)(w^{**}) \subseteq \cup_{w^{**}\in W}\text{dom}(w^{**}) = \text{dom}(w^*)$. Moreover M, w, s, $\underline{a}$ |= $\forall x\,\Diamond Cx$, so for any $w^* \in W$ $\text{dom}(w^*) = \text{dom}(w) \subseteq \cup_{w^{**}\in W}\text{int}(C)(w^{**}) = \text{dom}^{N}(w^*)$. Thus $\text{dom}^{N}(w^*) = \text{dom}(w^*)$. For any n-place non-logical atomic predicate F: M, w, s, $\underline{a}$ |= $\Box F\leq C$, so $\text{int}(F)(w^*) \subseteq \text{int}(C)(w^*)^{n}$, so $\text{int}^{C}(F)(w^*) = \text{int}(F)(w^*)\cap\text{int}(C)(w^*)^{n} = \text{int}(F)(w^*)$. Hence M^{Nec} = <W, D, dom^{N}, int^{C}> = <W, D, dom, int> = M.

1.21. ($A \in L1_{\Box}$) Aux[Nec] |= $(A)^{\text{Nec}} \leftrightarrow A$.

Proof: From 1.13 and 1.20.

1.22. ($A \in L1_{\Box}$, $X \subseteq L1_{\Box}$) If A and all members of S are neutral, then S, Aux[Nec] |= A only if S |= A.

Proof: Suppose that not S |= A. Then for some model M = <W, D, dom, int>, $w \in W$, $s \in W^{<\omega}$ and assignment $\underline{a}$: M, w, s, $\underline{a}$ |= S but not M, w, s, $\underline{a}$ |= A. By 1.17, M, w, s, $\underline{a}$ |= $(X)^{\text{Nec}}$ but not M, w, s, $\underline{a}$ |= $(A)^{\text{Nec}}$. Hence by 1.13, M^{Nec}, w, s, $\underline{a}$ |= S but not M^{Nec}, w, s, $\underline{a}$ |= A. By 1.18, M^{Nec}, w, s, $\underline{a}$ |= Aux[Nec]. Hence not S, Aux[Nec] |= A.

1.23. ($A,B \in L1_{\Box}$) If B is neutral and Aux[Nec] |= $A \leftrightarrow B$ then |= $(A)^{\text{Nec}} \leftrightarrow B$.

Proof: Assume the antecedent. Hence, by 1.21, Aux[Nec] $\models (A)^{Nec} \leftrightarrow B$. But by 1.15 $(A)^{Nec} \leftrightarrow B$ is neutral, so by 1.22 $\models (A)^{Nec} \leftrightarrow B$.

1.24. ($S,T \subseteq L1_{\Box}$) If S $\models$ Aux[Nec] and T $\models$ Aux[Nec], then S $\models$ T iff Neutral(S) $\models$ Neutral(T).

Proof: From 1.21, as 1.11 was derived from 1.8.

1.25. ($T \subseteq L1_{\Box}$) If T $\models$ Aux[Nec] then T $=\mid\,\models$ Neutral(T), Aux[Nec].

Proof: From 1.24 as 1.12 was derived from 1.11.

1.26. $\models (\text{Aux[Con]})^{Nec} \leftrightarrow \Box \forall x\, (Cx \rightarrow \Box Cx)$.

Proof: By a routine computation of truth-conditions.

1.27. $\models (\text{Aux[Nec]})^{Con} \leftrightarrow \Box \forall x\, (Cx \rightarrow \Box Cx)$.

Proof: As easily checked, $\models (\Box F \leq C)^{Con}$ and $\models (\forall x\, \Diamond Cx)^{Con}$. Now $(\Box \forall x\, \Box \exists y\, x{=}y)^{Con}$ is $\Box \forall x\, (Cx \rightarrow \Box(\exists y\, (Cx\ \&\ (x{=}y\ Cx\ \&\ Cy))))$, and $\models \exists y\, (Cx\ \&\ (x{=}y\ Cx\ \&\ Cy)) \leftrightarrow Cx$ by the semantics of = and the constraint on models that $\text{int}(C)(w) \subseteq \text{dom}(w)$. Hence $\models (\Box \forall x\, \Box \exists y\, x{=}y)^{Con} \leftrightarrow \Box \forall x\, (Cx \rightarrow \Box Cx)$. Hence $\models (\text{Aux[Nec]})^{Con} \leftrightarrow \Box \forall x\, (Cx \rightarrow \Box Cx)$.

1.28. ($A \in L1_{\Box}$) If A is neutral, Aux[Con] & Aux[Nec] $\models$ A iff $\Box \forall x (Cx \rightarrow \Box Cx) \models$ A.

Proof: Aux[Con] & Aux[Nec] $\models \Box \forall x\, Cx\ \&\ \Box \forall x\, \Box \exists y\, x{=}y$

so Aux[Con] & Aux[Nec] $\models \Box \forall x\, (Cx \rightarrow \Box \exists y\, (Cy\ \&\ x{=}y))$

so $\quad$ Aux[Con] & Aux[Nec] $\models \Box \forall x\, (Cx \rightarrow \Box Cx)$

That gives the right-to-left direction. For the converse, suppose that

Aux[Con] & Aux[Nec] $\models$ A where A is neutral. By 1.8,

Aux[Con] $\models$ $(\text{Aux[Nec]})^{\text{Con}} \leftrightarrow$ Aux[Nec]. Thus Aux[Con] $\models$ $(\text{Aux[Nec]})^{\text{Con}} \rightarrow$ A. Since $(\text{Aux[Nec]})^{\text{Con}} \rightarrow$ A is neutral, $\models (\text{Aux[Nec]})^{\text{Con}} \rightarrow$ A by 1.9, so by 1.27

$\models \Box \forall x\, (Cx \rightarrow \Box Cx) \rightarrow$ A.

2. The second part of the appendix extends the mapping from A to $(A)^{\text{Con}}$ to a second-order modal language and verifies that the results in the first part about it extend to that language, including the key result (1.8, 2.5) that every formula A is equivalent given Aux[Con] to a neutral formula $(A)^{\text{Con}}$. But it shows that no extension of the mapping from A to $(A)^{\text{Nec}}$ to the second-order language permits the extension to that language of the corresponding key result (1.21) that every formula A is equivalent given Aux[Nec] to a neutral formula $(A)^{\text{Nec}}$, because the second-order formula (33) is not equivalent given Aux[Nec] to any neutral formula.

To extend the first-order modal language $L1_{\Box}$ to a second-order modal language $L2_{\Box}$, add for each natural number n a countable infinity of n-place second-order variables and a second-order quantifier $\exists$ (in practice no confusion with the first-order quantifier results). The models are as before. An assignment $\underline{a}$ for a model <W, D, dom, int> maps each first-order variable v to $\underline{a}(v) \in D$ and each second-order n-place variable V to $\underline{a}(V) \subseteq D^n$. The assignment of extensions rather than intensions to second-order variables reflects the account of second-order plural logic in section 8. The new semantic clauses are these:

$M, w, s, \underline{a} \models Vv_1 \ldots v_n$ iff $<\underline{a}(v_1), \ldots, \underline{a}(v_n)> \in \underline{a}(V) \subseteq \mathrm{dom}(w)^n$

$M, w, s, \underline{a} \models \exists V A$ iff for some $S \subseteq \mathrm{dom}(w)^n$: $M, w, s, \underline{a}[V/S] \models A$

In both cases the restriction to subsets of dom(w) or $\mathrm{dom}(w)^n$ is motivated by respect for contingentist scruples. We can check that it has the intended effect by verifying that it invalidates the second-order Barcan principle $\Diamond \exists V A \rightarrow \exists V \Diamond A$ and its converse. Let M = <W, D, dom, int> where W = $\{w_1, w_2\}$, $\mathrm{dom}(w_1) = \{0\}$, $\mathrm{dom}(w_2) = D = \{0, 1\}$. Thus:

$M, w_1, s, \underline{a} \models \Diamond \exists X \exists x \exists y (Xx \,\&\, Xy \,\&\, \neg x{=}y)$

For $M, w_2, s, \underline{a} \models \exists X \exists x \exists y (Xx \,\&\, Xy \,\&\, \neg x{=}y)$. But we do not have:

$M, w_1, s, \underline{a} \models \exists X \Diamond \exists x \exists y (Xx \,\&\, Xy \,\&\, \neg x{=}y)$

For at w_1 the second-order quantifier is restricted to subsets of $\{0\}$. Conversely, we have:

$M, w_2, s, \underline{a} \models \exists X \Diamond \exists x (\Diamond Xx \,\&\, \neg Xx)$

For $M, w_1, s, \underline{a}[X/\{0, 1\}][x/0] \models \Diamond Xx \,\&\, \neg Xx$. But we do not have:

$M, w_2, s, \underline{a} \models \Diamond \exists X \exists x (\Diamond Xx \,\&\, \neg Xx)$

For if $M, w, s, \underline{a} \models \neg Xx$ where $\underline{a}(x) \in \mathrm{dom}(w)$ and $\underline{a}(X) \subseteq \mathrm{dom}(w)$ then $\underline{a}(x) \notin \underline{a}(X)$ so not $M, w, s, \underline{a} \models \Diamond Xx$.

We extend the definition of $(A)^{Con}$ by these clauses, where $V \leq C$ is $\forall y (Vy \rightarrow Cy)$:

$(Vv_1 \ldots v_n)^{Con} = Vv_1 \ldots v_n \,\&\, V \leq C$

$(\exists V A)^{Con} = \exists V (V \leq C \,\&\, (A)^{Con})$

In the context of $L2_\Box$, A is neutral iff $\models A \leftrightarrow (B)^{Con}$ for some formula B of $L2_\Box$.

2.1. ($A \in L2_\square$) M, w, s, $\underline{a}$ |= $(A)^{Con}$ iff M^{Con}, w, s, $\underline{a}$ |= A.

Proof: Similar to that of 1.1, with an additional clause in the basis for second-order variables and another in the induction step for second order quantifiers.

Basis: M, w, s, $\underline{a}$ |= $(Vv_1\ldots v_n)^{Con}$

iff M, w, s, $\underline{a}$ |= $Vv_1\ldots v_n$ & $V \leq C$

iff $<\underline{a}(v_1), \ldots, \underline{a}(v_n)> \in \underline{a}(V) \subseteq \mathrm{dom}(w)^n \cap \mathrm{int}(C)(w)^n = \mathrm{dom}^C(w)^n$

iff M^{Con}, w, s, $\underline{a}$ |= $Vv_1\ldots v_n$.

Induction step: By induction hypothesis: M, w, s, $\underline{b}$ |= $(A)^{Con}$ iff M^{Con}, w, s, $\underline{b}$ |= A for each assignment $\underline{b}$.

M, w, s, $\underline{a}$ |= $(\exists V A)^{Con}$

iff M, w, s, $\underline{a}$ |= $\exists V (V \leq C$ & $(A)^{Con})$

iff for some $S \subseteq \mathrm{dom}(w)^n$: M, w, s, $\underline{a}[V/S]$ |= $V \leq C$ & $(A)^{Con}$

iff for some $S \subseteq \mathrm{dom}(w)^n$: $S \cap \mathrm{dom}(w)^n \subseteq \mathrm{int}(C)(w)^n$ and M, w, s, $\underline{a}[V/S]$ |= $(A)^{Con}$

iff for some $S \subseteq \mathrm{int}(C)(w)^n = \mathrm{dom}^C(w)^n$: M, w, s, $\underline{a}[V/S]$ |= $(A)^{Con}$

iff for some $S \subseteq \mathrm{dom}^C(w)^n$: M^{Con}, w, s, $\underline{a}[V/S]$ |= A (by induction hypothesis)

iff M^{Con}, w, s, $\underline{a}$ |= $\exists V$ A.

2.2 ($A,B \in L2_\square$) If |= $A \leftrightarrow B$ then |= $(A)^{Con} \leftrightarrow (B)^{Con}$.

Proof: From 2.1 as 1.2 was derived from 1.1.

2.3. ($A \in L2_\square$) |= $((A)^{Con})^{Con} \leftrightarrow (A)^{Con}$.

Proof: Similar to that of 1.3, with routine extra clauses. Note that |= $(V \leq C)^{Con}$.

2.4. ($A \in L2_{\Box}$) If A is neutral, $\models (A)^{Con} \leftrightarrow A$.

Proof: From 2.2 and 2.3 as 1.4 was derived from 1.2 and 1.3.

2.5. ($A \in L2_{\Box}$) Aux[Con] $\models (A)^{Con} \leftrightarrow A$.

Proof: From 2.1 and 1.7.

2.6. ($A \in L2_{\Box}$, $S \subseteq L2_{\Box}$) If A and all members of S are neutral, then S, Aux[Con] $\models$ A only if S $\models$ A.

Proof: From 2.4, 2.1 and 1.5 as 1.9 was derived from 1.4, 1.1 and 1.5.

2.7. ($A,B \in L2_{\Box}$) If B is neutral and Aux[Con] $\models A \leftrightarrow B$ then $\models (A)^{Con} \leftrightarrow B$.

Proof: Assume the antecedent. Hence, by 2.5, Aux[Con] $\models (A)^{Con} \leftrightarrow B$. But $(A)^{Con} \leftrightarrow B$ is neutral, so by 2.6 $\models (A)^{Con} \leftrightarrow B$.

2.8. ($S,T \subseteq L2_{\Box}$) If S $\models$ Aux[Con] and T $\models$ Aux[Con], then S $\models$ T iff Neutral(S) $\models$ Neutral(T).

Proof: From 2.4 as 1.11 was derived from 1.8.

2.9. ($S \subseteq L2_{\Box}$) If S $\models$ Aux[Con] then S $=\mid\ \models$ Neutral(S), Aux[Con].

Proof: From 2.8 as 1.12 was derived from 1.11.

We will now show a particular formula of $L2_{\Box}$ not to be equivalent to any neutral formula, even given Aux[Nec]. We consider 'interpretations' of $L2_{\Box}$ in the (non-modal language) $L2_{PA}$ of second-order arithmetic. We require a model for $L2_{PA}$ to be standard in the sense of second-order logic (second-order quantifiers range over all subsets of the domain of individuals); it need not be standard in the sense of arithmetic (it may contain members not reachable from 'zero' by a finite number of applications of the 'successor' operation). By a 'first-order' formula of $L2_{PA}$ we mean one not containing second-order quantifiers, even if it contains second-order variables. PA1 is the set of first-order formulas of $L2_{PA}$ true on the intended interpretation. Thus PA1 |= A iff A is true in all models in which the same first-order sentences are true as in standard (intended) models of arithmetic.

We define a family of mappings from $L2_{\Box}$ to $L2_{PA}$. We may assume that $L2_{PA}$ contains all the variables of $L2_{\Box}$ and in addition a countable infinity of new first-order variables, called 'world variables', which syntactically and semantically behave just like other first-order variables. For each world variable *u* and sequence of world variables *r*, we map each formula A of $L2_{\Box}$ to a formula $[A]_{u,r}$ of $L2_{PA}$. Here is the recursive definition, where *v* is a first-order variable, *V* an n-place second-order variable, *r#* is the first world variable on some fixed ordering not in *r*, and $u \leq v \leq u+1$ is $v=u \vee v=u+1$:

$[Fv_1 \ldots v_n]_{u,r} = u \leq v_1 \leq u+1 \;\&\; \ldots \;\&\; u \leq v_n \leq u+1$ for any non-logical n-place predicate constant F

If A is an atomic formula of any other form, $[A]_{u,r} = A$

$[\neg A]_{u,r} = \neg [A]_{u,r}$

$[A \mathbin{\&} B]_{u,r} = [A]_{u,r} \mathbin{\&} [B]_{u,r}$

$[\exists\, v\, A]_{u,r} = \exists\, v\, [A]_{u,r}$

$[\exists\, V\, A]_{u,r} = \exists\, V\, [A]_{u,r}$

$[\Diamond A]_{u,r} = \exists\, r\#\, [A]_{r\#,r}$

$[\uparrow A]_{u,r} = [A]_{u,r\hat{}u}$

$[\downarrow A]_{u,r\hat{}u*} = [A]_{u*,r}$

$[\downarrow A]_{u,<>} = [A]_{u,<>}$

Some more abbreviations will be convenient. Subscripted $\wedge$ and $\vee$ express finite conjunctions and disjunctions respectively. If $z_1, \ldots, z_n$ are the first n distinct first-order variables other than u, and V is n-place, V:u is $\forall\, z_1 \ldots \forall\, z_n\, (Vz_1 \ldots z_n \rightarrow \wedge_{1 \leq i \leq n}\, u \leq z_i \leq u+1)$. #n is the power set of $\{u, u+1\}^n$, the set of all sets of n-tuples of terms from amongst u and u+1. Henceforth, $Y \in$ #n; $\underline{y}[i]$ is the ith member of the n-tuple $\underline{y}$. A[V/Y] is the result of replacing each subformula of the form $Vt_1 \ldots t_n$ in A by $\vee_{\underline{y} \in Y} \wedge_{1 \leq i \leq n}\, t_i = \underline{y}[i]$, if necessary changing bound variables in A so that u (the only variable in $\underline{y}[i]$) never becomes bound by a quantifier in A (if Y is {}, count the disjunction as a contradiction); thus V is replaced by a formula whose extension is the set of n-tuples of values of the corresponding variables in n-tuples in Y. $V \approx Y$ is $\forall\, z_1 \ldots \forall\, z_n\, (Vz_1 \ldots z_n \leftrightarrow Vz_1 \ldots z_n[V/Y])$; it implies that A and A[V/Y] always coincide in extension.

2.10. ($A \in L2_\Box$) $\models [(A)^{Con}]_{u,r} \leftrightarrow A^*$ for some first-order formula A*.

Proof: By induction on the complexity of A. If A is first-order, $[(A)^{Con}]_{u,r}$ is first-order and so will serve as A*. The basis and the induction step for truth-functors, modal operators and first-order quantifiers are trivial. In the interesting case, A is $\exists V$ B (V is n-place). Thus $[(A)^{Con}]_{u,r} = [\exists V (V \leq C \;\&\; (B)^{Con}]_{u,r} = \exists V (V{:}u \;\&\; [(B)^{Con}]_{u,r})$. By induction hypothesis, $\models [(B)^{Con}]_{u,r} \leftrightarrow B^*$ for some first-order formula B*. We show that $\vee_{Y \in \#n} B^*[V/Y]$ is a first-order formula equivalent to $[(A)^{Con}]_{u,r}$. It is first-order because B* is first-order, so $B^*[V/Y]$ is first-order for each $Y \in \#n$. Since V occurs in B* only in atomic formulas of the form $Vv_1 \ldots v_n$, V does not occur in $B^*[V/Y]$. We prove the equivalence via the following chain of validities:

(1)	$[(B)^{Con}]_{u,r} \leftrightarrow B^*$	induction hypothesis
(2)	$\forall V ([(B)^{Con}]_{u,r} \leftrightarrow B^*)$	1, generalization
(3)	$\forall V (V{:}u \rightarrow \vee_{Y \in \#n} V \approx Y))$	1st-order logic
(4)	$\forall V \wedge_{Y \in \#n} (V \approx Y \rightarrow (B^* \leftrightarrow B^*[V/Y]))$	1st-order logic
(5)	$\forall V \wedge_{Y \in \#n} (V \approx Y \rightarrow ([(B)^{Con}]_{u,r} \leftrightarrow B^*[V/Y]))$	2, 4
(6)	$\forall V (V{:}u \rightarrow \vee_{Y \in \#n} ([(B)^{Con}]_{u,r} \leftrightarrow B^*[V/Y]))$	3, 5
(7)	$\forall V (V{:}u \rightarrow ([(B)^{Con}]_{u,r} \rightarrow \vee_{Y \in \#n} B^*[V/Y]))$	6
(8)	$\exists V (V{:}u \;\&\; [(B)^{Con}]_{u,r}) \rightarrow \vee_{Y \in \#n} B^*[V/Y]$	7, V is not in $B^*[V/Y]$
(9)	$\wedge_{Y \in \#n} \forall V (V \approx Y \rightarrow V{:}u)$	1st-order logic
(10)	$\wedge_{Y \in \#n} \exists V\, V \approx Y$	2nd-order comprehension
(11)	$\wedge_{Y \in \#n} \exists V (V{:}u \wedge ([(B)^{Con}]_{u,r} \leftrightarrow B^*[V/Y]))$	5, 9, 10
(12)	$\wedge_{Y \in \#n} (B^*[V/Y] \rightarrow \exists V (V{:}u \;\&\; [(B)^{Con}]_{u,r})$	11, V is not in $B^*[V/Y]$
(13)	$\vee_{Y \in \#n} B^*[V/Y] \rightarrow \exists V (V{:}u \;\&\; [(B)^{Con}]_{u,r})$	12
(14)	$\exists V (V{:}u \;\&\; [(B)^{Con}]_{u,r}) \leftrightarrow \vee_{Y \in \#n} B^*[V/Y]$	8, 13

This completes the induction.

For any model M for $L2_{PA}$ with domain N and successor operation $'$, let M* be the model <N, N, dom, int> for $L2_{\Box}$, where for all $m \in N$ dom(m) = N, and int(F)(m) = $\{m, m'\}^n$ for each non-logical n-place atomic predicate F. For any sequence of world variables r, let $\underline{a}(r)$ be the corresponding sequence of values assigned by the assignment $\underline{a}$. For any assignment $\underline{a}$ of values to variables in $L2_{PA}$, let $\underline{a}|$ be the restriction of $\underline{a}$ to variables in $L2_{\Box}$ (non-world variables).

2.11. ($A \in L2_{\Box}$) For any model M of PA1, M, $\underline{a} \models [A]_{u,r}$ iff M*, $\underline{a}(u)$, $\underline{a}(r)$, $\underline{a}| \models A$.

Proof: By induction on the complexity of A. We give only a few representative cases. The others are similar. Basis for $Fv_1 \ldots v_n$:

M, $\underline{a} \models [Fv_1 \ldots v_n]_{u,r}$

iff M, $\underline{a} \models u \leq v_1 \leq u+1$ & … & $u \leq v_n \leq u+1$

iff for $1 \leq i \leq n$, $\underline{a}(v_i) = \underline{a}(u)$ or $\underline{a}(v_i) = \underline{a}(u)'$

iff $\langle \underline{a}(v_1), \ldots, \underline{a}(v_n)\rangle \in$ int(F)($\underline{a}(u)$)

iff $\langle \underline{a}|(v_1), \ldots, \underline{a}|(v_n)\rangle \in$ int(F)($\underline{a}(u)$) ($\underline{a}|(v_i) = \underline{a}(v_i)$ because v_i is no world variable)

iff M*, $\underline{a}(u)$, $\underline{a}(r)$, $\underline{a}| \models Fv_1 \ldots v_n$

Induction step for a second-order quantifier:

M, $\underline{a} \models [\exists V\, A]_{u,r}$

iff M, $\underline{a} \models \exists V\, [A]_{u,r}$

iff for some $S \subseteq N^n$: M, $\underline{a}[V/S] \models [A]_{u,r}$

iff for some $S \subseteq N^n$: M*, $\underline{a}[V/S](u)$, $\underline{a}[V/S](r)$, $\underline{a}[V/S]| \models A$ (by induction hypothesis)

iff for some $S \subseteq N^n$: M*, $\underline{a}(u)$, $\underline{a}(r)$, $\underline{a}|[V/S] \models A$ ($\underline{a}[V/S](u) = \underline{a}(u)$ and $\underline{a}[V/S](r) = \underline{a}(r)$ because V is no world variable; $\underline{a}[V/S]| = \underline{a}|[V/S]$)

iff $\quad$ M*, $\underline{a}(u)$, $\underline{a}(r)$, $\underline{a}|$ |= $\exists\, V$ A

Induction step for ◊:

$\quad$ M, $\underline{a}$ |= $[\Diamond A]_{u,r}$

iff $\quad$ M, $\underline{a}$ |= $\exists\, r\#\, [A]_{r\#,r}$

iff $\quad$ for some n $\in$N: M, $\underline{a}[r\#/n]$ |= $[A]_{r\#,r}$

iff $\quad$ for some n $\in$N: M*, $\underline{a}[r\#/n](r\#)$, $\underline{a}[r\#/n](r)$, $\underline{a}[r\#/n]|$ |= A (by induction hypothesis)

iff $\quad$ for some n $\in$N: M*, n, $\underline{a}(r)$, $\underline{a}|$ |= A $\quad$ ($\underline{a}[r\#/n](r\#)$ = n; $\underline{a}[r\#/n](r) = \underline{a}(r)$ because $r\#$ is not in r; $\underline{a}[r\#/n]| = \underline{a}|$ because $r\#$ is no world variable)

iff $\quad$ M*, $\underline{a}(u)$, $\underline{a}(r)$, $\underline{a}|$ |= ◊A

2.12. (A $\in$ $L2_{\Box}$) $\quad$ If |= A then PA1 |= $[A]_{u,r}$.

Proof: If not PA1 |= $[A]_{u,r}$, then for some such model M and assignment $\underline{a}$, not M, $\underline{a}$ |= $[A]_{u,r}$, so by 2.11 not M*, $\underline{a}(u)$, $\underline{a}(r)$, $\underline{a}|$ |= A, so not |= A.

2.13. PA1 |= $[\mathrm{Aux}[\mathrm{Nec}]]_{u,r}$.

Proof: $[\Box\forall x\, \Box\exists y\; x{=}y]_{u,r} = \forall r\#\; \forall x\; \forall r\#\; \exists y\; x{=}y$

$[\forall x\, \Diamond Cx]_{u,r} = \forall x\; \exists r\#\, (x{=}r\# \vee x{=}r\#{+}1)$

$[\Box F{\le}C]_{u,r} = \forall r\#\; \forall z_1 \ldots \forall z_n\, ((r\#{\le}z_1{\le}r\#{+}1\ \&\ \ldots\ \&\ r\#{\le}z_n{\le}r\#{+}1)$
$\rightarrow (r\#{\le}z_1{\le}r\#{+}1\ \&\ \ldots\ \&\ r\#{\le}z_n{\le}r\#{+}1))$

All these formulas are truths of first-order logic.

2.14. (A,B $\in$ $L2_{\Box}$) $\quad$ If $[A]_{u,r}$ is closed, B is neutral and Aux[Nec] |= A ↔ B, then either PA1 |= $[A]_{u,r}$ or PA1 |= $\neg[A]_{u,r}$.

Proof: Suppose that $[A]_{u,r}$ is closed, B is neutral and Aux[Nec] |= A ↔ B. Since B is neutral, |= B ↔ $(D)^{Con}$ for some D, so Aux[Nec] |= A ↔ $(D)^{Con}$. By 2.12, PA1 |= $[\mathrm{Aux[Nec]}]_{u,r} \rightarrow ([A]_{u,r} \leftrightarrow [(D)^{Con}]_{u,r})$ ($[.]_{u,r}$ commutes with → and ↔). Thus by 2.13, PA1 |= $[A]_{u,r} \leftrightarrow [(D)^{Con}]_{u,r}$. By 2.10, |= $[(D)^{Con}]_{u,r} \leftrightarrow$ B* for some first-order formula B*, so PA1 |= $[A]_{u,r} \leftrightarrow$ B*. Let B** be the result of substituting 0 for any first-order variables free in B* and 0=1 for any subformulas $\mathrm{V}t_1 \ldots t_n$ of B* where the occurrence of V is free in B*. Since $[A]_{u,<>}$ is closed, PA1 |= $[A]_{u,r} \leftrightarrow$ B**. But B** is closed and first-order, so either PA1 |= B** or PA1 |= ¬B**, so either PA1 |= $[A]_{u,r}$ or PA1 |= $\neg[A]_{u,r}$.

2.15. (B ∈ $L2_{\Box}$) If B is neutral, then not

Aux[Nec] |= $\exists X (\exists x\, Xx \,\&\, \exists x\, \neg Xx \,\&\, \forall x \forall y\, (\Diamond Rxy \rightarrow (Xx \rightarrow Xy))) \leftrightarrow$ B.

Proof: Suppose otherwise. Abbreviate the biconditional as A ↔ B. If w is <>#, $[A]_{u,<>}$ is:

$\exists X (\exists x\, Xx \,\&\, \exists x\, \neg Xx \,\&\, \forall x \forall y\, (\exists w\, (w \leq x \leq w+1 \,\&\, w \leq y \leq w+1) \rightarrow (Xx \rightarrow Xy)))$.

But PA1 |= $\exists w\, (w \leq x \leq w+1 \,\&\, w \leq y \leq w+1) \leftrightarrow x=y \vee x=y+1 \vee x+1=y$. Thus

PA1 |= $[A]_{u,<>} \leftrightarrow \exists X (\exists x\, Xx \,\&\, \exists x\, \neg Xx \,\&\, \forall x\, (Xx \leftrightarrow Xx+1))$. Hence $[A]_{u,<>}$ is false in standard models of arithmetic (a nonempty set of natural numbers closed under successor and predecessor contains all natural numbers). But $[A]_{u,<>}$ is true in all non-standard models of arithmetic (let the extension of X be just the standard numbers). Thus neither PA1 |= $[A]_{u,<>}$ nor PA1 |= $\neg[A]_{u,<>}$. Since $[A]_{u,<>}$ is closed, by 2.14 B is not neutral.

2.16. (B ∈ $L2_{\Box}$) If B is neutral, then not

Aux[Nec] |= $\exists X (\exists x\, (Fx \,\&\, Xx) \,\&\, \exists x\, (Fx \,\&\, \neg Xx) \,\&\, \forall x \forall y\, (\Diamond Rxy \rightarrow (Xx \rightarrow Xy))) \leftrightarrow$ B.

Proof: Otherwise the equivalence would hold under the substitution of x=x for Fx, giving a formula equivalent to that in 2.15.

2.17. (B ∈ $L2_{\Box}$) If B is neutral, then not Aux[Nec] |= $Xx \leftrightarrow$ B.

Proof: Consider a model M = <{0, 1}, {0, 1}, dom, int>, where dom(0) = dom(1) = {0, 1}, int(C)(0) = {0}, int(C)(1) = {1}, and int(F)(0) = int(F)(1) = {} for every other non-logical atomic predicate F. Let <u>a</u> be an assignment such that <u>a</u>(X) = {0, 1}, <u>a</u>(x) = 0. One easily shows by induction on the complexity of A ∈ $L2_{\Box}$ that M, w, s, <u>a</u> |= $(A)^{Con}$ iff M, w, s, <u>a</u>(X/{}) |= $(A)^{Con}$ for all w and s in M. The only interesting case is when A is Xv for some variable v. $(Xv)^{Con}$ is Xv & X≤C; for any w and s in M, neither M, w, s, <u>a</u> |= X≤C (since A(X) = {0, 1}) nor M, w, s, <u>a</u>(X/{}) |= Xv, we have the required equivalence. Suppose that B is neutral and Aux[Nec] |= $Xx \leftrightarrow$ B. So for some A, Aux[Nec] |= $Xx \leftrightarrow (A)^{Con}$. Clearly M |= Aux[Nec]. Hence M, w, s, <u>a</u> |= $Xx \leftrightarrow (A)^{Con}$ for all w and s in M. Thus, as just established, M, w, s, <u>a</u> |= Xx iff M, w, s, <u>a</u>(X/{}) |= Xx. But M, w, s, <u>a</u> |= Xx and not M, w, s, <u>a</u>(X/{}) |= Xx.

3. Finally, we extend the negative results of section 2 to infinitary languages. In what follows, L2 is the non-modal fragment of $L2_{\Box}$, excluding the predicate constant C. $L1_{\infty\infty}$ is an infinitary first-order non-modal language with the same predicate constants as L2 and an unlimited supply of variables; it is closed under ¬, ∃ binding any set of first-order variables and ∧ over any set of formulas (Dickmann 1985). Imagine each variable of $L1_{\infty\infty}$ as one of the two symbols 'x' and 'y' with an arbitrary subscript; subscripted 'y' variables are called 'special'; they do not belong to L2 or $L2_{\Box}$. The infinitary second-

order modal language $L2_{\infty\infty\Box}$ is the result of omitting the special variables from $L1_{\infty\infty}$ and adding an unlimited supply of second-order variables, the predicate constant C, $\exists$ binding any set of second-order variables, and the operators $\Box$, $\uparrow$ and $\downarrow$.

The model theory for the infinitary languages is standard; the quantifier clause is briefly discussed in section 10. In particular, the models themselves for $L1_{\infty\infty}$ and $L2_{\infty\infty\Box}$ are just those for the corresponding finitary non-modal and modal languages respectively. For convenience, formulas of all the non-modal languages are evaluated in the non-modal models relative to assignments to all variables, first-order and second-order, special and non-special (*full* assignments); formulas of all languages are evaluated in the modal models relative to assignments only to all non-special variables, first-order and second-order (*restricted* assignments). If $\underline{a}$ is a full assignment, $\underline{a}|$ is the corresponding restricted assignment, the restriction of $\underline{a}$ to non-special variables. The assignment of values to currently irrelevant variables is harmless. However, since the infinitary languages have more than set-many variables, both full and restricted assignments are more than set-sized. Strictly speaking, to avoid paradox one should replace the notion of an assignment by a higher-order notion or work with partial assignments (Rayo and Uzquiano 1999). The complications involved in doing so are routine. For convenience, however, we continue talking of assignments.

For any model M = <D, ext> for L2 with domain D and extension function ext, and any natural number n, M^n is the model <W, D, dom, int> for $L2_{\Box}$ such that W is the set of subsets w of D with at least one and at most n members, dom(w) = D, $int(F)(w) = ext(F) \cap w^m$ for each m-place predicate constant F of L2, and int(C)(w) = w. For $A \in L2$, $(A)^{\Diamond}$ is the result of prefixing each non-logical predicate constant in A with $\Diamond$.

3.1. ($A \in L2$) If A contains no predicate constant of more than n places, and $\underline{a}$ is a full assignment, then $M^n, w, s, \underline{a}| \models (A)^\lozenge$ iff $M, \underline{a} \models A$.

Proof: By induction on the complexity of A. Let F be a non-logical m-place predicate constant. Since $A \in L2$, A contains no special variables. Thus:

$M^n, w, s, \underline{a}| \models (Fv_1 \ldots v_m)^\lozenge$

iff $M^n, w, s, \underline{a}| \models \lozenge Fv_1 \ldots v_m$

iff for some $w^* \in W$, $M^n, w^*, s, \underline{a}| \models Fv_1 \ldots v_m$

iff for some $w^* \in W$, $<\underline{a}|(v_1), \ldots, \underline{a}|(v_m)> \in \mathrm{int}(F)(w^*) = \mathrm{ext}(F) \cap w^{*m}$

iff for some $w^* \in W$, $<\underline{a}(v_1), \ldots, \underline{a}(v_m)> \in \mathrm{ext}(F) \cap w^{*m}$ (v_i is not special)

iff $<\underline{a}(v_1), \ldots, \underline{a}(v_m)> \in \mathrm{int}(F)(\{\underline{a}(v_1), \ldots, \underline{a}(v_m)\} = \mathrm{ext}(F) \cap \{\underline{a}(v_1), \ldots, \underline{a}(v_m)\}^m$

($m \leq n$, so $\{\underline{a}(v_1), \ldots, \underline{a}(v_m)\} \in W$)

iff $<\underline{a}(v_1), \ldots, \underline{a}(v_m)> \in \mathrm{ext}(F)$

iff $M, \underline{a} \models Fv_1 \ldots v_m$

The rest of the induction is routine.

In what follows, u is a set of n special variables $\{u_1, \ldots, u_n\}$, r a finite sequence of such sets, and $r\#$ such a set disjoint from all such sets in r. If $\underline{a}$ is a full assignment, $\underline{a}(u)$ is $\{\underline{a}(u_1), \ldots, \underline{a}(u_n)\}$; if r is $<u^1, \ldots, u^k>$, where each u^i is a set of n special variables, then $\underline{a}(r)$ is the sequence $<\underline{a}(u^1), \ldots, \underline{a}(u^k)>$. We define a family of mappings from formulas of $L2_{\infty\infty\Box}$ to non-modal formulas of $L2_{\infty\infty\Box}$ thus:

$$[Cv]_{u,r,n} = \vee_{1 \leq j \leq n} v{=}u_j$$

$[Fv_1\ldots v_m]_{u,r,n} = Fv_1\ldots v_n \;\&\; \wedge_{1\leq i\leq m} \vee_{1\leq j\leq n}\, v_i{=}u_j$

If A is an atomic formula of any other form, $[A]_{u,r} = A$

$[\neg A]_{u,r,n} = \neg[A]_{u,r,n}$

$[\wedge_{i\in I} A_i]_{u,r,n} = \wedge_{i\in I} [A_i]_{u,r,n}$

$[\exists \{v_i\}_{i\in I} A]_{u,r,n} = \exists \{v_i\}_{i\in I} [A]_{u,r,n}$

$[\exists \{V_i\}_{i\in I} A]_{u,r,n} = \exists \{V_i\}_{i\in I} [A]_{u,r,n}$

$[\Diamond A]_{u,r,n} = \exists \{r\#_1, \ldots, r\#_n\} [A]_{r\#,r,n}$

$[\uparrow A]_{u,r,n} = [A]_{u,r^\wedge u,n}$

$[\downarrow A]_{u,r^\wedge u^*,n} = [A]_{u^*,r,n}$

$[\downarrow A]_{u,<>,n} = [A]_{u,<>,n}$

We extend the mapping from A to $(A)^{Con}$ to $L2_{\infty\infty\Box}$ in the obvious way; in particular:

$(\wedge_{i\in I} A_i)^{Con} = \wedge_{i\in I} (A_i)^{Con}$

$(\exists \{v_i\}_{i\in I} A)^{Con} = \exists \{v_i\}_{i\in I} ((\wedge_{i\in I} Cv_i) \;\&\; (A)^{Con})$

$(\exists \{V_i\}_{i\in I} A)^{Con} = \exists \{V_i\}_{i\in I} ((\wedge_{i\in I} V_i{\leq}C) \;\&\; (A)^{Con})$

As usual, $A \in L2_{\infty\infty\Box}$ is neutral iff for some $B \in L2_{\infty\infty\Box}$, $\models A \leftrightarrow (B)^{Con}$.

3.2. ($A \in L2_{\infty\infty\Box}$) $\qquad \models [(A)^{Con}]_{u,r,n} \leftrightarrow A^*$ for some first-order formula A^* of $\in L2_{\infty\infty\Box}$.

Proof: By induction on the complexity of A. As in the proof of 2.10, the non-trivial part is the induction step for the second-order quantifier, but now it binds a possibly infinite set of variables: $[(A)^{Con}]_{u,r,n} = [(\exists \{V_i\}_{i \in I} B)^{Con}]_{u,r,n} = [\exists \{V_i\}_{i \in I} ((\wedge_{i \in I} V_i \leq C) \& (B)^{Con})]_{u,r,n}$

$= \exists \{V_i\}_{i \in I} [(\wedge_{i \in I} V_i \leq C) \& (B)^{Con}]_{u,r,n} = \exists \{V_i\}_{i \in I} [(\wedge_{i \in I} V_i \leq C) \& (B)^{Con}]_{u,r,n} =$

$\exists \{V_i\}_{i \in I} ((\wedge_{i \in I} \forall z_1 \ldots \forall z_{m(i)} (V_i z_1 \ldots z_{m(i)} \rightarrow \wedge_{1 \leq i \leq m} \vee_{1 \leq j \leq n} v_i = u_j)) \& [(B)^{Con}]_{u,r,n})$.

By induction hypothesis, $\models [(B)^{Con}]_{u,r} \leftrightarrow B^*$ for some first-order formula B^*. Hence

$\models [(A)^{Con}]_{u,r,n} \leftrightarrow$

$$\exists \{V_i\}_{i \in I} ((\wedge_{i \in I} \forall z_1 \ldots \forall z_{m(i)} (V_i z_1 \ldots z_{m(i)} \rightarrow \wedge_{1 \leq i \leq m} \vee_{1 \leq j \leq n} v_i = u_j)) \& B^*).$$

By an argument like that in the proof of 2.10, but slightly more complicated, we then show: $\models [(A)^{Con}]_{u,r,n} \leftrightarrow \vee_{i \in I} \vee_{Y \in \#m(i)} B^*[V_i/Y]$. Thus $\vee_{i \in I} \vee_{Y \in \#m(i)} B^*[V_i/Y]$ is a first-order equivalent of $[(A)^{Con}]_{u,r,n}$, as required.

3.3. ($A \in L2_\Box$) For any model M for L2 and full assignment $\underline{a}$,

$M, \underline{a} \models [A]_{u,r,n}$ iff $M^n, \underline{a}(u), \underline{a}(r), \underline{a}| \models A$.

Proof: By induction on the complexity of A. Basis for $Fv_1 \ldots v_m$:

$M, \underline{a} \models [Fv_1 \ldots v_m]_{u,r,n}$

iff $M, \underline{a} \models Fv_1 \ldots v_m \& \wedge_{1 \leq i \leq m} \vee_{1 \leq j \leq n} v_i = u_j$

iff $\langle \underline{a}(v_1), \ldots, \underline{a}(v_m) \rangle \in ext(F) \cap \underline{a}(u)^m = int(F)(\underline{a}(u))$

iff $\langle \underline{a}|(v_1), \ldots, \underline{a}|(v_m) \rangle \in int(F)(\underline{a}(u))$ ($\underline{a}|(v_i) = \underline{a}(v_i)$ because v_i is not special)

iff $M^n, \underline{a}(u), \underline{a}(r), \underline{a}| \models Fv_1 \ldots v_m$

The rest of the induction is similar to that in the proof of 2.11.

3.4. $M^n \models Aux[Nec]$.

Proof: Routine.

3.5. ($A \in L2$, $B \in L2_{\infty\infty\Box}$) If A is closed, B is neutral and Aux[Nec] $\models (A)^{\Diamond} \leftrightarrow B$, then for some $A^* \in L1_{\infty\infty}$, $\models A \leftrightarrow A^*$.

Proof: Suppose that A is closed, B is neutral and Aux[Nec] $\models (A)^{\Diamond} \leftrightarrow B$. Since B is neutral, $\models B \leftrightarrow (D)^{Con}$ for some D, so Aux[Nec] $\models (A)^{\Diamond} \leftrightarrow (D)^{Con}$. Pick n so A contains no m-place predicate constant for m > n. Let M be a model for L2 and $\underline{a}$ a full assignment. By 3.4, $M^n \models$ Aux[Nec]. Hence for all u, r in M^n:

$M^n, \underline{a}(u), \underline{a}(r), \underline{a}| \models (A)^{\Diamond} \leftrightarrow (D)^{Con}$. By 3.1, $M^n, \underline{a}(u), \underline{a}(r), \underline{a}| \models (A)^{\Diamond}$ iff $M, \underline{a} \models A$. By 3.3, $M^n, \underline{a}(u), \underline{a}(r), \underline{a}| \models (D)^{Con}$ iff $M, \underline{a} \models [(D)^{Con}]_{u,r,n}$. Hence $M, \underline{a} \models A \leftrightarrow [(D)^{Con}]_{u,r,n}$. Since M and $\underline{a}$ were arbitrary, $\models A \leftrightarrow [(D)^{Con}]_{u,r,n}$. But by 3.2, $\models [(D)^{Con}]_{u,r,n} \leftrightarrow D^*$, where D^* is a first-order formula of $L2_{\infty\infty}$. Thus $\models A \leftrightarrow D^*$. Let A^* be the result of substituting a formula of L1 for any atomic formulas with a free second-order variable in D^*; thus $A^* \in L1_{\infty\infty}$. But A is closed, so $\models A \leftrightarrow A^*$.

3.6. For some closed $D \in L2_{\Box}$, Aux[Nec] $\models D \leftrightarrow B$ for no neutral $B \in L2_{\infty\infty\Box}$.

Proof: For some closed $A \in L2$, $\models A \leftrightarrow A^*$ for no $A^* \in L1_{\infty\infty}$ (Dickmann 1985, p. 323). Hence by 3.5, Aux[Nec] $\models (A)^{\Diamond} \leftrightarrow B$ for no neutral $B \in L2_{\infty\infty\Box}$. $(A)^{\Diamond} \in L2_{\Box}$ is closed.[51]

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Notes

1 On the legitimacy of the unrestricted readings of the quantifiers see Williamson 2003 and Rayo and Uzquiano 2006; for a characterization of metaphysical possibility in terms of counterfactual conditionals see Williamson 2007, pp. 155-61. The arguments of this paper are robust in both respects: they appeal neither to the sort of reasoning which has been thought to discredit unrestricted generality by generating versions of Russell's paradox nor to 'science fiction' metaphysical possibilities which outrun physical possibilities. In this paper the word 'exist' is nowhere used (rather than mentioned), since it is a prime cause of philosophical confusion in the area (as argued in Williamson 1998, 2000a). Although 'exist' can be used synonymously with 'be something' on the unrestricted reading, and the arguments of the paper can be rewritten accordingly, restricted readings tend to generate noise which interferes with the intended reading.

2 For this example see Salmon 1987, pp. 47-48.

3 Davidson 1970 supplies a suitable metaphysics of events for this example.

4 See Fox 1987 for such a principle, and Rami 2009, pp. 16-17, for more references. Many truthmaker theorists regard the modal condition as too coarse-grained to be sufficient for truthmaking, but it would of course not follow that it is not necessary. For similar examples involving facts see Fine 1981a, p. 302, and Williamson 1988.

5 The relevant assumptions are the (necessitated) T schema and the (necessitated) B schema $A \rightarrow \Box\Diamond A$ of modal logic, which implies that (necessarily) if x is something then necessarily x can be something. It is valid in the modal system S5.

6 Some logics of 'actually' lack the principle $A \leftrightarrow @A$ (@ for 'actually'), for example the system S5A in Crossley and Humberstone 1977. However, the reason is that they are intended to axiomatize those principles which are true at every world of every model (this is general validity, in the terminology of Davies and Humberstone 1980) rather than those which are true at the actual world of every model (real world validity). Systems such as S5A do not distinguish @ from an operator that rigidly takes the evaluation of a formula to an arbitrary fixed world in the model, which is permitted to be non-actual. Thus they fail to capture the intended nature of @. $A \leftrightarrow @A$ is true as uttered in any context, even though it may be false at some circumstances of evaluation independent of the context of utterance. Although 'actually' is capable of other readings, they do not help actualism. In particular, $A \leftrightarrow @A$ is still real world valid if @ is made non-rigid; likewise if @ is treated as a mere scope-indicating device. Such readings make a difference only when @ is embedded within modal operators. A reading on which @ restricts quantifiers in its scope to individuals 'in' the actual world might deprive the principle $A \leftrightarrow @A$ of its real world validity on a modal realist view (Lewis 1986), but the actualist is not supposed to be a modal realist, and if modal realism is not assumed the restriction still seems trivial (when @ is not in the scope of further modal operators).

7 For example, Forbes 1989 takes the question to be: whose sense of 'exist' is more basic? But a question about the relative basicness of senses looks to be a confusingly indirect proxy for a fundamentally metaphysical question. Bennett 2005 is a more recent example of an attempt to clarify the issues (see also Bennett 2006). Despite the relevance of the wide range of philosophical considerations the paper deploys, it fails to assign clear contents to the opposing 'views'. One source of the trouble in much of the recent literature is a tendency even on the part of those who officially reject Lewis's modal realism (Lewis 1986) to use its language to characterize both actualism and possibilism, with unmodalized talk of 'domains' of worlds and their 'inclusion' relations, even though such a way of speaking is far from neutral (it favours possibilism). One cannot restore neutrality afterwards by adding that one is, of course, not assuming modal realism; the damage has already been done. The non-neutrality of such formulations is clarified below. Modal realism as such presents no special challenge to the conclusions of this paper. However, it is closely associated with counterpart theory, most versions of which generate weaker and less tidy quantified modal logics than the background logic used here (see appendix). For criticisms of counterpart theory and related semantic theories see Fara and Williamson 2005 and Williamson 2006. The present paper is not concerned to argue against such 'unclassical' conceptions of quantified modal logic.

8 It follows by the B schema.

9 See Williamson 1990b, 1998, 2000a, 2000b, 2002 and Linsky and Zalta 1994, 1996. Most other accounts of modal ontology seem to be contingentist.

10 Salmon (1989, 1993, 2005) is an example of a sceptic about S5; contrast Williamson 1990a, pp. 126-43. Salmon's scepticism is directed specifically at the characteristic S4 axiom $\Box A \rightarrow \Box\Box A$, which is derivable in S5. Without such a principle, one must independently add the necessitations of the claims in the text, and the necessitations of the latter, and so on ad infinitum.

11 Here is a formal derivation in the weakest normal propositional modal logic, K:

(1)	$\exists x \Diamond \neg \exists y\, x=y \rightarrow \Diamond \exists x \neg \exists y\, x=y$	(CBF)
(2)	$\neg \exists x \neg \exists y\, x=y$	first-order logic with identity
(3)	$\neg \Diamond \exists x \neg \exists y\, x=y$	2, necessitation, K
(4)	$\neg \exists x \Diamond \neg \exists y\, x=y$	1, 3
(5)	$\neg \exists x \Diamond\Diamond \neg \exists y\, x=y$	4, S5
(6)	$\Diamond \exists x \Diamond \neg \exists y\, x=y \rightarrow \exists x \Diamond\Diamond \neg \exists y\, x=y$	(BF)
(7)	$\neg \Diamond \exists x \Diamond \neg \exists y\, x=y$	5, 6
(8)	$\Box \forall x \Box \exists y\, x=y$	7, K

12 To say that GEP is contingently non-concrete is not to say that it is contingently abstract, any more than to say that Socrates has become non-concrete is to say that he has become abstract. 'Abstract' is no synonym of 'non-concrete'. Rather, 'abstract' has its own positive prototypes, such as numbers. The necessitist can classify things like GEP as neither abstract nor concrete, and insist that nothing is both possibly abstract and possibly

concrete. See Williamson 1998, p. 266, and contrast Linsky and Zalta 1996, p. 293, with Linsky and Zalta 1994, p. 446.

13 The talk of domains is mildly metaphorical; there is no commitment to the claim that the relevant entities form a set.

14 The intelligibility of unrestricted quantification and metaphysical modality are assumed here (see n. 1). For the requisite shared understanding of unrestricted quantification see Williamson 2000, 2003. I will not discuss here recent attempts to argue that ontological disputes are in some sense merely verbal (for some recent contributions to the debate see Chalmers, Manley and Wasserman 2009), except to say that I do not regard them as having established even a *prima facie* case to answer and that results in the appendix (2.15-16, 3.6) would be a technical obstacle to attempts to interpret contingentism and necessitism as in some sense 'intertranslatable', if any such technical obstacle were needed (presenting one is not a primary aim of this paper). (NNE) clearly and unambiguously expresses a proposition; there is a non-verbal dispute to be had over whether that proposition is true. Of course, it may sometimes be unclear whether a given philosopher is committed to a given side in that dispute. Considerations of semantic externalism also favour the view that the opposing sides in logical and philosophical disputes conducted in a natural language such as English are using the key terms with the same meaning; discussion of those disputes often neglects the significance of the fact that both sides are intentionally using the same words of a public language as such. See further Williamson 2007, pp. 88-98, 117-30. The claim that apparent disputes over non-

contingent matters really depend on unclarity over the truth-conditions of the disputed sentences is often associated with extremely coarse-grained theories of content, on which the only impossible proposition is the null set (Stalnaker 1984). In Williamson forthcoming (a), I argue in response to Stalnaker (forthcoming) that, whether or not such a coarse-grained theory of content is granted, he has not provided good reason to regard apparent disputes over non-contingent matters as more meta-linguistic than apparent disputes over mundane contingent matters.

15 According to Kit Fine, the actualist should hold that possibilist talk 'is legitimate, but not basic; it stands in need of analysis'; he describes the actualist as providing a 'translation' from actualist talk into possibilist talk (1977a, pp. 118-19), although he informs me (p.c.) that he no longer requires the mappings to provide synonyms or even equivalents: a ground which is sufficient but not necessary for the original may do for his purposes, although not for those of this paper. Forbes (1985, p. 243) and Pollock (1985, pp. 130-2) write in terms similar to those of Fine 1977a, although Forbes later questioned and now rejects the claim that the 'translation' preserves meaning (1989, pp. 34; 2008, p. 283). For a vigorous defence of the view that the actualist can understand possibilist claims and still disagree with them in the absence of any such 'translation' see Plantinga 1985, pp. 330-1.

16 On another interpretation of 'chunky' in a similar spirit, something is chunky if and only if it is either necessarily not something non-concrete or necessarily not something concrete. The main arguments of the paper would still apply on such a reading

of ‘chunky’, even though the paraphrase ‘grounded in the concrete’ would be rather misleading.

17 With respect to this treatment of non-logical predicates see the notion of a restricted formula in Fine 1981, p. 296.

18 Forms of contingentism on which truth-value gaps occur when we speak of an individual with respect to possible circumstances in which it is nothing are beyond the scope of this paper (compare the system Q in Prior 1957).

19 The reification of properties and relations here is not what matters; the point could be articulated using quantification into predicate position. For reasons concerned with Russell’s paradox and explained in Williamson 2003, the latter approach is preferable in the setting of unrestricted quantification. In any case, the usual objections to formulas such as $\Box \forall x \Box(\mathrm{F}x \rightarrow \exists y\, x=y)$ have nothing to do with Russell’s Paradox.

20 Kit Fine (2005, p. 324) argues that ‘Socrates is self-identical’ is true regardless of the circumstances, rather than however the circumstances turn out, so that its truth does not depend on the being of Socrates. I doubt the robustness of this distinction.

21 Stalnaker 1994 follows a similar policy on predication in modal contexts, but permits the formation of overtly complex predicates. They could easily be added to the

present language. The criticisms in Williamson 2006 do not tell against this aspect of his account.

22 Although inserting ◊ before as well as after ∃ would help with (9), it does not in general produce a formula which the necessitist regards as equivalent to the original, since it maps $\exists x\, Px$ (which the historically informed necessitist rejects) to (7) (which the necessitist accepts).

23 See Fine 1977a, pp. 143-4, and Forbes 1989, pp. 27-9, also Bricker 1989, pp. 384-5. Fine credits earlier work by Frank Vlach on tense logic from 1970; see Vlach 1973. The argument of the present paper can be generalized to tense logic; the generalized argument constitutes an objection to the (usual) version of presentism on which ontology is mutable, in the sense that sometimes something is such that it is not always something. The generalization is left as an exercise for the reader. Of course, if the most perspicuous framework for thinking about the metaphysics of time turns out to be four-dimensionalist rather than tense-logical, then permanentism (the view that ontology is constant) is true anyway, because an unrestricted quantifier ranges over at least the entire contents of space-time.

24 For similar issues in relation to actualist 'translations' of possibilist discourse see Fine 1977a, pp. 132-5, and 1981a and Forbes 1989, pp. 45-77.

25 The uniqueness of the neutral equivalent would fail if neutrality only required restrictions on the quantifiers, not on the atomic formulas. For example, both $\Box\forall x\,(Cx \rightarrow x{=}x)$ and $\Box\forall x\,(Cx \rightarrow \Box x{=}x)$ would count as neutral, and they are equivalent to each other given Aux[Nec], but not equivalent to each other without Aux[Nec]. The uniqueness of neutral equivalents would also fail if the extension of an atomic formula at a world were not restricted to what there is at that world, even on the more restricted definition of neutrality in the text. For example, both $\Box\forall x\,(Cx \rightarrow Cx)$ and $\Box\forall x\,(Cx \rightarrow \Box(Cx \rightarrow \exists y\,(Cy\ \&\ x{=}y)))$ are neutral, and equivalent to each other given Aux[Nec], but they would not be logically equivalent to each other without Aux[Nec] if $Cx \rightarrow \exists y\, x{=}y$ were invalid. If uniqueness fails, the contingentist cannot use $(A)^{Nec}$ to measure in neutral terms what A is worth to the necessitist.

26 Our concern here is with chunky-style contingentism.

27 For an exchange on this issue see Melia 1992 and Forbes 1992.

28 See Davies 1978, Gupta 1978 and 1980, Peacocke 1978, Rumfitt 2001.

29 The problem that sets of incompossible possibles are not even possible is raised at Fine 1977b, p. 141; Fine 1981b, p. 183; Salmon 1987, p. 48. Salmon raises a parallel problem for singular Russellian propositions about such individuals (ibid.), as does Peacocke for the sequences of such individuals required by a naïve Tarskian truth theory for a first-order modal language (1978, pp. 481-2).

30 The rigidity of set membership is defended by Fine 1981b, pp. 179-80, also Forbes 1985, p. 109, and Bricker 1989, p. 387. Unlike Fine and Forbes, Bricker refuses to qualify the principle for possible worlds which do not contain the member or set; no such restriction is needed for the necessitist's version of the principle, as here.

31 Obviously, much more than this is needed to simulate the resources of the whole cumulative hierarchy of sets. However, for the applications considered in this paper, that hierarchy is irrelevant, and plural quantified logic is an adequate substitute for set theory.

32 The use of second-order notation and terminology for plural logic serves a heuristic process, because we shall be concerned with issues about the expressive limitations of first-order (singular) logic which arise most familiarly in relation to the comparison with second-order logic.

33 Rumfitt 2005, pp. 113-17, defends a similar principle, (M): $Xx \rightarrow \Box(\mathrm{E}^2X \rightarrow Xx)$, where E^2X is defined as equivalent to $\forall y\,(Xy \rightarrow \mathrm{E}y)$ and may be read 'X exist'; Ey may be read 'y exists'. However, Rumfitt countenances a definition on which Ey is equivalent to $\exists y\; x{=}y$, which would make the condition E^2X trivial and redundant, in which case (M) is equivalent to (PR). Alternatively, since Rumfitt is not a necessitist, E^2X could instead be defined 'possibilistically' by $\uparrow\Box\forall y\,(Xy \rightarrow \downarrow\mathrm{E}y)$.

34 The plural interpretation of second-order variables is not the only legitimate one. For some purposes a more predicative reading is preferable. In a modal setting, the latter is intensional and falsifies the rigidity principle (PR) (Williamson 2003, pp. 456-7). Second-order quantification with such intensional variables has not been discussed in this paper because it is unclear, for quite different reasons, what restriction on it would be required for neutrality. That raises a separate problem for contingentism. See Williamson forthcoming (b).

35 See also Williamson 2003, p. 457.

36 Of course, (29) and (30) must also be checked for the case when there are no X, but it can be handled on the model of the other cases.

37 The contingentist will also reject ‘(31)$^{\mathrm{Nec}}$’, and therefore its equivalence to (31), on related grounds.

38 On the limitations of monadic second-order logic see Shapiro 1991, pp. 221-6.

39 It is assumed here that necessarily, if something is one of some things, then necessarily there are those things only if there is that thing. Consider Tom, Dick and Harry. How could there be *those* things without there being Harry? There could still have been Tom and Dick, but Harry could not have been one of *them*. Roughly speaking, every plurality is coextensive with a plurality that could not have had another member. Given

the plural extensionality principle (29), it follows that no plurality could have had another member. In S5, it follows in turn that no plurality could have lacked one of its members. Unlike (PR), (29) is plausible as stated even for contingentists. Possible circumstances from which the relevant pluralities are absent do not falsify it: the prefix and antecedent are non-modal, so we are to suppose that there are these and those, and these are those; then if this had been one of these, there would have been these for it to be one of, and they would still have been those, so this would have been one of those. The principle for sets corresponding to 'Necessarily, if something is one of some things, then necessarily there are those things only if there is that thing' is equally plausible: necessarily, if a set has a member, then necessarily there is the set only if there is the member. For a defence of such principles for sets see Fine 1981b, pp. 180-3, also Forbes 1985, p. 118, and Bricker 1989, p. 387.

40 The formalization in Bricker 1989, p. 389, of the plural *de re* reading of 'Every F might be G' as (in present notation) $\exists X (\forall y (Xy \leftrightarrow Fy) \ \& \ \Diamond\langle\forall x\rangle (Xx \rightarrow Gx))$ (where the angle brackets indicate an 'outer', 'possibilist' reading of the quantifier) depends on the assumption that Xx can be true even at a world of whose domain the value of X is not a subset, or, to put it in terms of a meta-linguistic plural, at a world which does not contain all the objects assigned to X. For otherwise, although the possibilist quantifier sweeps up all the relevant objects, $Xx \rightarrow Gx$ will be vacuously true for each of them at that world even if Gx is false, and the formalization may receive the wrong truth-value. This is related to Bricker's assumption that a member belongs to a set even at a world which does not contain the set (1989, p. 387). The assumptions are plausible on a

necessitist view, on which the ‘outer’ quantifiers are simply unrestricted quantifiers, but dubious on a contingentist view if the ‘outer’ quantifiers are eliminated in favour of modal operators and ‘inner’ quantifiers, as Bricker contemplates (1989, p. 394). How can Tom be one of Tom, Dick and Harry unless there are Tom, Dick and Harry for him to be one of? To formulate the concern without reference to worlds: Bricker’s paraphrases of the plural *de re* are correct only if (PR) is true, but what right has the contingentist to assert (PR)?

41 Shapiro 1991, pp. 141-7, discusses principles which imply that the restriction to set-sized domains makes no difference to which formulas are valid in (non-modal) second-order logic. The arguments of this paper are robust with respect to such issues; in particular, the proof of the central result (appendix 2.15) works even under a restriction to countable models.

42 The problem does not arise for an atomic formula Fx where F is an atomic predicate constant, because F$x \rightarrow$ Cx is a consequence of Aux[Nec] given the background logic. We cannot add the principle $Xx \rightarrow$ Cx to Aux[Nec], for if we do we obtain $\Box \forall X\ \forall x\ (Xx \rightarrow Cx)$; since the background logic provides $\Box \exists X \forall x\ Xx$ we can derive $\Box \forall x$ Cx, which the necessitist does not want.

43 The use of an ontology of sequences here is for ease of exposition only; the point can be made without it.

44 Boolos 1984, pp. 333-4, explains Kaplan's proof.

45 The account is presented in Plantinga 1974, 1976; a similar account is presented but rejected in Fine 1977.

46 The problem is closely related to some objections to Plantinga's account in Fine 1985. In his response to the relevant objections, Plantinga (1985, pp. 332-40) succeeds in showing that they were misleadingly expressed, but fails to make the metaphysical view at issue plausible. For related discussion see Williamson forthcoming (b).

47 The resetting with ↑ and ↓ is redundant when the embedded quantifiers and atomic formulas are all confined within the scope of □ or ◊; once ↑ and ↓ have been eliminated, various further simplifications can be made using the semantics of identity and the fact that $\text{int}(C)(w) \subseteq \text{dom}(w)$ at any world w in any model.

48 Fine raises such a problem for proxy reductions of possibilia (2003, p. 169), but does not discuss its implications for his own non-proxy reduction. He informs me (p.c.) that he may have been presupposing a view of 'indefinitely extensible' quantification over set domains. That contrasts with the absolutist conception of unrestricted quantification assumed in this paper. For present purposes (which are different from Fine's), it also fails to provide any particular neutral infinitary formula equivalent to (34) given Aux[Nec]. The additional resources in Fine (2005) may also be relevant.

49 That A is to be evaluated at w rather than some other world really requires the operators ↑ and ↓ and an interpretation of their role in the infinite case. They have been omitted for simplicity.

50 See Dickmann 1985, p. 323, and Shapiro 1991, p. 242. Dickmann lists the following examples of classes that can be characterized in a finitary second-order language but not in an infinitary first-order language: topological spaces; compact spaces; discrete spaces; T_i spaces ($i = 0, \ldots, 5$); regular, completely regular, normal, completely normal spaces; metrizable spaces; Stone spaces, extremally disconnected spaces; complete uniform spaces; topological groups, rings and modules; complete partial and linear orderings; complete lattices and complete distributive lattices; complete boolean algebras and complete atomic boolean algebras; completely distributive boolean algebras.

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