A “Position Paradox” in Sponsored Search Auctions

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Abstract

In this paper, we study the bidding strategies of vertically differentiated firms that bid for sponsored search advertisement positions for a keyword at a search engine. We study two popular auction mechanisms: pay-per-impression and pay-per-click. Our model yields several interesting insights and one main counter-intuitive result we focus on is the “position paradox.” The paradox is that a superior firm may bid lower than an inferior firm and obtain a position below it, but still obtain more clicks than the inferior firm. Our results are in contrast to the extant literature on position auctions in sponsored search advertising which has shown, or often simply assumed, that higher positions obtain more clicks and firms will bid so as to order themselves in decreasing order of quality.

Under a pay-per-impression mechanism, the inferior firm wants to be at the top where more consumers click on its link, even though it has to pay a higher advertising fee. At the same time, the superior firm is better off by displaying its link at a lower position as it pays less but some consumers will still reach it in the search of a higher-quality firm. We also find that, surprisingly, as the quality premium for the superior firm increases, the incentive for the inferior firm to be at the top may increase. Furthermore, under the pay-per-click mechanism, we find that the inferior firm has a stronger incentive to be at the top since now it only has to pay for the consumers who do not know the firms’ reputations and, therefore, click on its link. We also find that if more consumers know the identity of the superior firm, the incentive for the inferior firm to be at the top may increase. We test our basic results empirically using a unique dataset from a major search engine firm in Korea and find strong empirical support for the position paradox.

Keywords: Sponsored search advertising, search cost, vertical differentiation, bidding strategy, pay-per-impression, pay-per-click
1 Introduction

Sponsored search advertising has grown into one of the major forms of online advertising in the past decade and is expected to grow at an annual compound rate of more than 12% in the near future (Riley et al. 2007). Firms — global and local, big and small — now actively advertise in the “sponsored links” sections of search engines such as Google, Yahoo!, Bing (MSN Live Search) and Ask. When a consumer searches for a specific keyword on the search engine, she is presented with two lists of clickable links: one a list of organic search results and the other a list of sponsored links. The list of sponsored links is determined by auctioning the positions to firms which want to advertise in response to the searched keyword. Advertised links are typically ordered in decreasing order of the firms’ bids and some search engines, such as Google, also augment bids by a quality metric for the associated link. Two widely used payment mechanisms are pay-per-impression, where all firms are charged whenever a consumer searches the keyword and their links are displayed, and pay-per-click, where a firm is charged only when a consumer clicks on its link. Typically, search engines use a second-price auction, i.e., a firm has to pay the bid of the firm directly below it or, if it is in the last position, it has to pay a prespecified minimum bid amount.

The spectacular commercial success of sponsored search has motivated several recent academic studies on it. While this literature is in its nascent stage, a main take-away from both empirical and theoretical work is that advertisements at higher positions attract more clicks from consumers (e.g., Feng et al. 2007, Misra et al. 2006) and higher-quality firms will be placed at higher positions among the sponsored links (e.g., Athey and Ellison 2008). In this paper, we analytically model the bidding strategies of firms when quality differentiation exists and some consumers know the firms’ qualities beforehand (the “informed” consumers). We analyze both pay-per-impression and pay-per-click mechanisms and obtain a host of results on the optimal bidding strategies of high-quality (“superior”) firms and low-quality (“inferior”) firms and the outcomes of these bidding strategies.

An intriguing finding revealed in our study is the “position paradox” — we find that,
under certain conditions, a superior firm may bid lower than an inferior firm and obtain a position below it, but still obtain more clicks than the inferior firm does. We start by showing this in the pay-per-impression mechanism. Beyond this basic effect, we also find that, surprisingly, as the overall quality level in the market increases and/or the quality premium for the superior firm increases, the inferior firm is more likely to be at the top. Next, we examine how a change of the auction mechanism from pay-per-impression to pay-per-click influences bidding strategies. We find that the position paradox is actually strengthened, i.e., the inferior firm is more likely to be at the top under pay-per-click than under pay-per-impression. Moreover, we also find that while under the pay-per-impression mechanism the number of informed consumers does not influence which firm bids higher, under the pay-per-click mechanism, as this number increases, the position paradox again gets strengthened. We also confirm that, qualitatively, our results hold even when the search engine weights each firm’s bid by a quality score for the firm. Finally, we derive implications for search engine profit and find that the search engine should prefer the mechanism which leads to a more aggressive lower bid, and this can be the pay-per-impression or the pay-per-click mechanism under different conditions.

Our position paradox result is not only counterintuitive, it is also in contrast with the prevailing view in the literature, which almost takes it as a given that the higher the link position, the more clicks it will attract from consumers and, consequently, superior firms will also emerge at the top. (We review the literature in detail later.) However, we show in our analytical modeling that this position paradox is a natural result of three key factors present in sponsored search auctions — “residual demand,” “incremental value,” and “differential cost” — and discuss these factors in detail in the paper. Furthermore, to empirically test this result, we obtain a rich dataset of firms’ advertisement link positions and consumer clicks at the daily level for several keywords from a major search engine firm in Korea. This search engine firm uses a pay-per-impression mechanism to conduct its auctions for sponsored links. In this dataset, we frequently observe that superior firms are above inferior
firms and superior firms obtain more clicks. But we also observe for several keywords that lower-ranked links belong to firms with higher quality as compared to higher-ranked links. Moreover, these lower-ranked links also receive more clicks than the higher-ranked ones. This is a direct confirmation of our basic position paradox result and a formal empirical analysis further lends strong support to the same.

Why is it that, in some cases, firms ranked lower receive more clicks than firms ranked higher? And more importantly, why would such an outcome be optimal for all firms, i.e., why would the top-ranked firm pay the highest bid even though it receives fewer clicks than lower-ranked ones? We explain this puzzle using an analytical model which accounts for differentiation in firms’ product qualities, and differentiation in consumers, where some consumers know the firms’ qualities beforehand.

Our analysis uncovers three key factors behind our results: “residual demand,” “incremental value,” and “differential cost.” Intuitively, the “residual demand” effect means that a consumer may continue searching through sponsored links if she could not find a product of her choice at the firms she visited earlier. Thus, even if she is processing links starting from the top, lower-ranked links can still benefit from the “residual” consumers from above and generate positive sales. The “incremental value” effect refers to the fact that, from a firm’s point of view, a higher-ranked link is worth only the additional revenue it generates over the lower-ranked links, instead of the absolute revenue it generates. The “differential cost” effect, relevant to the pay-per-click mechanism, refers to the fact when a firm pays if a consumer clicks on its link, informed and uninformed consumers have different relative cost implications to the competing firms. The first two factors influence the revenues of firms, while the third factor influences the advertisement costs of firms.

With the interaction of these three factors, a superior firm may prefer a lower-ranked position over a higher-ranked one if it can receive only slightly fewer number of clicks at the lower-ranked position but can substantially reduce its cost there. An inferior firm, on the other hand, may want to take the higher-ranked position because it would receive
substantially fewer clicks at the lower-ranked positions. Furthermore, under the pay-per-click scheme, if some consumers (the informed consumers) already know the identity of the superior firm and will click on that firm’s link regardless of its position, then the superior firm will be even more unwilling to be placed at higher positions with higher per-click costs, making the inferior firm relatively more competitive.

Numerous empirical studies on sponsored search have found that, when aggregated across keywords, the number of clicks on a link decreases approximately exponentially as one proceeds down a list of sponsored links (e.g., Feng et al. 2007, Misra et al. 2006). In contrast to these widely accepted notions, we find in our data that, when analyzed at the keyword level, the number of clicks is often greater at lower positions. Other studies have looked at how position influences purchase conversion rates after a consumer has clicked on a link, and Agarwal et al. (2009) find that, interestingly, the conversion probability is often highest at the second or third position and not always the first position. Other empirical studies have focused on how keyword-level characteristics (e.g., whether the keyword being bid on is a generic keyword, a brand-specific keyword or a retailer-specific keyword) influence clicks and purchase conversion (e.g., Ghose and Yang 2009, Rutz and Bucklin 2007, Rutz and Bucklin 2008). Yao and Mela (2008) develop a dynamic structural model to jointly study the strategic behavior of searchers, advertisers and the search engine firm. Goldfarb and Tucker (2008) empirically find that, because sponsored search is typically well targeted, firms bid overall higher in markets where it is harder for them to find matching consumers on their own. Another line of research focuses on developing computational methods to determine, using past data, keywords that firms should bid on and the corresponding bids they should make (e.g., Abhishek and Hosanagar 2007, Kitts and Leblanc 2004). Note that most of these studies use data from search engine intermediaries (intermediaries that bid for client firms in sponsored search auctions) and have click data only for the keywords they themselves bid on. Our dataset is from the search engine itself and is richer — for all keywords, we have the number of clicks for links at every position.
Several analytical studies in this area focus on the optimal design of the auction mechanism and its implications. Edelman et al. (2007) and Varian (2007) study auction formats that closely resemble the auction scheme of Google and discuss the properties of the equilibria arising from this auction. Both papers assume a priori that higher positions obtain more links. Liu et al. (2006) study an auction mechanism where the search engine determines link ranking by weighting firms’ bids by the quality of each link, and show that incorporating quality both increases efficiency and boosts the revenue of the search engine. Weber and Zheng (2007) argue that the optimal scheme for a search engine to rank links may be to weight bids with relevant performance metrics and a low-quality bidder will lose in such an auction. Balachander and Kannan (2008), on the other hand, find that using more information to weight links might lead to a reduction in profits for the search engine. Katona and Sarvary (2008) develop a normative model to suggest how search engines should adjust advertisers’ bids taking into account consumers’ clicking behavior for both organic and sponsored links. Chen and He (2006) model bidding behavior of sellers with products of varying degrees of relevance to the keyword. They find that when search cost is present, higher-relevance sellers bid higher and get placed higher. Athey and Ellison (2008) study an equilibrium model which accounts for both firms’ bids and consumer search at the same time, and describes the equilibria under two different scenarios, one where consumers believe that firms are sorted randomly, the other where consumers believe that firms are sorted on decreasing quality. Zhu and Wilbur (2009) analyze auctions in which advertisers decide to bid on a pay-per-impression or pay-per-click basis. Shin (2009) studies the incentives firms have to purchase their own and their competitors’ branded keywords. Desai and Shin (2009) find that employing advertiser-specific minimum bids can increase search engine profit.

Our research finds that neither is it always true that higher positions obtain more clicks, nor is it always true that firms are sorted in descending order of quality. These observations and an associated explanation are missing from the literature, and our attempt in this paper is to fill this gap.
The rest of this paper is organized as follows. In Section 2 we describe our analytical model. In Section 3, we derive the equilibrium when consumers have homogeneous search costs, first for the pay-per-impression scheme, and then for the pay-per-click scheme. In Section 4, we extend our results to the case of heterogeneous search costs for consumers. In Section 5, we validate our basic analytical results using data obtained from a search engine firm in Korea. Section 6 concludes and suggests directions for future research.

2 Model

We model two firms, $S$ and $I$, competing for sponsored search advertisement positions for a specific keyword at a search engine firm. Firm $S$, the “superior” firm, has a higher-quality product than firm $I$, the “inferior” firm. The product of the inferior firm provides a consumer a net product utility (i.e., the utility of the product minus its price) of $V$, while that of the superior firm provides a consumer a net utility of $V + Q$. Here, $Q > 0$ represents the quality premium of firm $S$ over firm $I$. The per-unit margin of firm $i$ is denoted by $m_i, i \in \{S, I\}$. We assume that the superior firm has a higher per-unit margin, i.e., $m_S > m_I$.\footnote{This assumption is only made to highlight the position paradox, as we will show that even with higher margin, the superior firm may not desire the top position, and bid to be placed at the lower position. Relaxing this assumption will only make this more likely to happen, as we elaborate later.} We use the term “product” throughout the paper, but this could also be a “service,” including a retailing service. For example, GoodRetailer.com could provide better product descriptions, easier search functions, a more secure online payment mechanism, better customer service, etc., as compared to BadRetailer.com, even if both are selling products produced by other manufacturers.

A group of consumers, with mass normalized to 1, search the keyword in question at the search engine. In response, the search engine returns an ordered list of sponsored links.\footnote{Search engines typically return both sponsored and organic links in response to a query. In this paper, we study only the sponsored links.} We assume that there are two types of consumers. The first type is the “informed” consumers, who can tell whether a firm is the superior firm or the inferior firm upon viewing its adver-
tisement link in the search results. The second type is the “uninformed” consumers, who know that there is a superior firm offering quality \( V + Q \) and an inferior firm offering quality \( V \), but cannot tell a firm’s quality from the advertisement link itself. These consumers have to “search” for this information by clicking on a firm’s link and obtaining information about the product, e.g., find out the product specifications and price, read consumer reviews, etc. After this exercise, the uninformed consumers can also determine the quality of the firm. We assume that the size of the informed consumers is \( \phi \in (0, 1) \), so the size of the uninformed consumers is \( 1 - \phi \). The parameter \( \phi \) can be interpreted as a measure of how widespread the reputation of the superior firm is in the market.

However, knowing the qualities of firms alone is not sufficient for a consumer to make the purchase decision — she also has to assess her “match” with a product. This match can only be assessed by clicking on a firm’s link and obtaining information about the product. Hence, before purchasing a product, both informed and uninformed consumers have to click on a firm’s link to assess their match with the product being offered. In any consumer’s purchase decision, product quality can be interpreted as an objective dimension and match can be interpreted as a subjective dimension.

We assume that the informed consumers always start their search with the superior firm irrespective of its position and may go to the inferior firm if they do not obtain a match with the superior firm. For the uninformed consumers, we assume that they are boundedly rational (Simon 1955) and start their search with the firm at the top, which could be the superior or the inferior firm, and based on the quality and the match they may then choose to search further or stop.\(^3\)\(^4\)

Given that the population of consumers using the Internet is

\(^3\)As an illustrative example, suppose the query “running shoes” returns the links BadShoes.com (the inferior firm) at the top and GoodShoes.com (the superior firm) at the bottom. An informed consumer first clicks on GoodShoes.com but her choice of color may not match and she may, therefore, also click on BadShoes.com. An uninformed consumer first clicks on BadShoes.com, realizes that it is the low quality product and, even if her color choice matches, she may choose to explore GoodShoes.com and click on it.

\(^4\)The bounded rationality of uninformed consumers can be interpreted in the following manner. They are trading off between two “costs”: (1) the cost of thinking through the bidding strategies of the firms to figure out how they will be ordered in equilibrium, and (2) the expected cost of the search effort they have to expend in searching through the links of the firms using the heuristic of starting from the top and going downwards. Given their bounded rationality, they find the first option to be more costly than the second
very diverse and heterogeneous, and considering the evidence from Behavioral Economics that shows that consumers often do not have perfect strategic foresight (e.g., Camerer et al. 2004, Ho et al. 2006), the assumption that some consumers are boundedly rational is a reasonable one. Similar assumptions on bounded rationality of a fraction of consumers have been used in other studies (e.g., Gabaix and Laibson 2006). Furthermore, the assumption that these consumers start searching from the top position is in accordance with literature on how online consumers process ordered lists (e.g., Granka et al. 2004, Hoque and Lohse 1999).

Consistent with the existing literature on search, we assume that the first search is free (this assumption does not qualitatively impact our results), while a search cost of $s > 0$ applies for subsequent searches. As several studies have shown, this search cost can be substantial and has a significant impact on how much an online consumer searches (Brynjolfsson et al. 2009, Johnson et al. 2004). We also assume that once a link has been clicked on by a consumer, she can always go back to this link without incurring any further search cost. We study both the case of homogeneous search cost, where every consumer has the same search cost, and that of heterogeneous search costs, where different consumers may have different search costs. We assume that the match probability is the same for both firms and is equal to $p$. (Assuming this to be different for different firms does not alter our results qualitatively.) We assume that every consumer makes her purchase or subsequent search decisions to maximize her expected utility.

The search engine can auction the position either through a pay-per-impression or a pay-per-click mechanism. We study both mechanisms in this paper. Consistent with industry practice, we assume that both firms submit their bids simultaneously, the winning firm pays...
the amount of the losing firm’s bid and the losing firm pays the minimum bid $b$. (Ties are broken randomly with equal probability.) In both mechanisms, the firm which bids higher is placed on top, while the other is placed at the bottom. In the pay-per-impression mechanism, both firms pay their respective fees whenever their links are displayed, i.e., whenever a consumer searches the keyword. In the pay-per-click mechanism, a firm pays only when its link is actually clicked. Note that in both cases, we assume the search engine imposes a minimum bid $b$, which is the minimum bid amount required for either firm to participate in the auction. In real-world cases, firms regularly impose this restriction. An alternative interpretation of this minimum bid is that it is the minimum amount a firm needs to bid to be placed in the top two positions, when there are more than two firms bidding for more than two positions. We make the assumption that both firms always place their bids, i.e., no firm exits the bidding game. This assumption does not affect any insights from the model, while it makes the analysis cleaner in some cases.

Note that we do not endogenously model the prices charged by the firms for their products and assume that margins on sale are exogenously specified, as discussed earlier. This assumption is a standard assumption in the literature on position auctions and is justified by the fact that sponsored search advertising is typically a small part of any firm’s total budget and does not significantly influence the price of its product.

We model the game in two stages. In the first stage, both firms submit bids simultaneously and the firm that bids higher obtains the top position. In the second stage, each consumer conducts her search. The equilibrium concept we use is subgame perfect Nash equilibrium. Under this solution concept, in the first stage, each firm calculates the profit it will make in the second stage for different outcomes. i.e., each firm considers how consumers will react in all possibilities in the second period based on the positions the firms will obtain according to their bids in the first period. In equilibrium, each firm’s bid in the first period is such that it maximizes the firm’s expected profit given the other firm’s bid.
3 Homogeneous Search Costs

We begin our analysis with the case of homogeneous search cost, where all consumers have the same search cost. This enables us to analyze the workings of the underlying factors in various situations. We extend our result to the case of heterogeneous search costs later.

3.1 Pay-Per-Impression

In this section, we study the pay-per-impression case, in which firms pay their respective advertisement fees whenever a consumer searches the keyword and the link is displayed. Depending on the parameters $s$, $p$, $m_S$, $m_I$, $V$ and $Q$, multiple scenarios exist, each with different optimal search behavior by consumers and, therefore, different optimal bidding behavior by firms. Behind all these scenarios, however, are two key factors that drive the bidding behavior of firms.

The first is that of “residual demand”, which intuitively means that the firm placed on top may not get all the sales. This arises from the uncertainty in finding a matched product. When a consumer goes to a firm’s website, she finds a matched product only with probability $p$. If a match is not found, the consumer may search on. Furthermore, a consumer may decide to search on if the first firm she visits is the inferior one, even if she already finds a match there, provided the quality premium, $Q$, is sufficiently high to outweigh her search cost. This makes it possible for a losing bidder to still make sales. This factor makes a sponsored search auction qualitatively different from a standard winner-take-all auction.

The second factor, partly arising from the first, is that of “incremental value”. Since both the winning and losing bidders may make positive sales, it is the difference in revenue between winning and losing that decides a firm’s bid (not the absolute revenue from winning). For instance, a firm may make higher sales than the other if placed on top, but if it also makes higher sales than the other if placed on bottom, then its desire to be placed on top is not necessarily higher. Taking this argument further, if one firm makes the same sales
regardless of its link position, while the other makes less sales at the lower position, then the second firm is expected to outbid the first even if its absolute sales are lower. Hence, a firm decides its bid based on the additional profit of being at the top position.

We now discuss each scenario in detail and then consolidate the results across these scenarios.

**Scenario I:** \( s \leq p \min\{Q, V\} \)

Scenario I can be described as that of “low search cost.” In this case, the search cost is sufficiently low so that consumers’ decisions are primarily driven by product values.

Consider an uninformed consumer. Being uninformed, she will start from the top link. If the top link is that of the superior firm and she finds a match there, she will buy the product and stop. If she does not find a match, she will continue to click on the bottom link of the inferior firm, because the search cost is lower than the expected product value, \( pV \). If the top link is that of the inferior firm, however, the consumer will continue to click on the bottom link, regardless of whether she finds a match at the top link. This is because the expected incremental value to the consumer, \( pQ \), is higher than the search cost. If she finds a match at the superior firm, she will buy the product and stop. Otherwise, she will either buy the product from the inferior firm if a match had been found when she first searched there, or stop if not.

The case for an informed consumer is simpler, as she always starts from the superior firm, regardless of where it is placed. Such a consumer will buy from the superior firm if a match is found, and searches the inferior firm in case of no match.

Denote the profit of firm \( i \in \{S, I\} \) when it is placed at position \( j \in \{1, 2\} \) by \( \Pi_{i,j} \).\(^5\)

Given the expected search behavior of consumers, if the superior firm is placed on top and

\(^5\)Throughout the paper, we use “position 1,” “above,” and “top” interchangeably with each other, and “position 2,” “below” and “bottom” interchangeably with each other.
pays $b_S$ per search, the expected profits for the two firms are given by

$$E[\Pi_{S,1}] = p m_S - b_S \text{ and } E[\Pi_{I,2}] = (1 - p) p m_I - b.$$ 

If the inferior firm is placed on top and pays $b_I$ per search, the expected profits for the two firms are given by

$$E[\Pi_{S,2}] = p m_S - b \text{ and } E[\Pi_{I,1}] = (1 - p) p m_I - b_I.$$ 

Note that, in the expressions above, $b_S$ and $b_I$ denote the payment per search for firm $S$ and firm $I$, respectively, and not their equilibrium bids. These equilibrium bids, denoted by $b^*_S$ and $b^*_I$, will be derived subsequently.

We can observe from the above that, in this case of low search cost, each firm has the same expected revenue at both positions (equal to $p m_S$ and $(1 - p) p m_I$ for firms $S$ and $I$, respectively). This is a direct outcome of the effect of “residual demand” — because search cost is low, every consumer will search the other firm whenever she fails to find a match; and because the search cost is low relative to the product quality premium, the consumer will search the superior firm even if she first visits the inferior one and finds a match there. Consequently, the effect of “incremental value” suggests that neither firm will compete for the top position. We see from the derivations below for the firms’ bids that this is indeed the case.

We can derive the equilibrium bids for each firm for position 1 by noting that a firm in position 2 pays $\hat{b}$, and will bid an amount such that if it indeed gets the top position and has to pay this full amount, its profit from should be equal to its profit at the bottom position where it pays the minimum bid. Intuitively, a firm will bid an amount equal to the additional benefit of the top position over the bottom one. As noted in Varian (2007), these will be Nash equilibrium bids and this is a natural equilibrium to consider, because it assumes that a firm sets its bid so that it makes a profit if it moves up in the ranking.
Note that these bids also characterize exactly the “locally envy-free” equilibrium defined in Edelman et al. (2007), since another way of looking at this equilibrium is that if a firm in the bottom position is already making this bid then the firm in the top position must be bidding more than this value. However, the firm in the bottom position does not want to bid more than this anyway and, therefore, does not envy the firm above it. We also note that this is a weakly dominant strategy for both firms.

In accordance with this, firm \( S \) will be willing to pay up to \( b^*_S \) for position 1, where \( b^*_S \) equates its profit from position 1 and position 2, i.e.,

\[
pm_S - b^*_S = pm_S - b \implies b^*_S = b
\]

Similarly, firm \( I \) will be willing to pay up to \( b^*_I \) for position 1, where \( b^*_I \) equates its profit from position 1 and position 2, i.e.,

\[
(1 - p)pm_I - b^*_I = (1 - p)pm_I - b \implies b^*_I = b
\]

Hence, in this scenario, both firms will bid \( b^*_S = b^*_I = b \), the minimum required amount, with the tie broken randomly by the search engine (with equal probability).

Note that we already have a case where the inferior firm might be at the top position. However, this is a somewhat uninteresting result because the positions are decided randomly. In the scenarios to follow, we will show situations which leads to the position paradox not in a random manner.

**Scenario II:** \( Q > V \) and \( pV < s \leq pQ \)

Scenario II exists if the quality premium is significant, i.e., \( Q > V \). In this scenario, the search cost is higher than the expected utility of visiting the inferior firm, but lower than the expected quality premium.

An uninformed consumer in this case will still start from the top link. If the superior
firm has the top link and the consumer finds a match there, she will buy and stop, just as in Scenario I. Different than Scenario I, however, is that she will stop even if she does not find a match. This is because after the first search, she already knows that the lower link belongs to the inferior firm, and since the search cost is higher than the expected utility from searching the inferior firm for a match, her best action is to stop searching. If the top link belongs to the inferior firm, the consumer will search on regardless of whether she finds a match, since the expected quality premium outweighs the search cost, the same as in Scenario I.

An informed consumer, in contrast, will always start from the superior firm. And in this case, she will not visit the inferior firm regardless of whether she finds a match at the superior one. Given these, the expected firm profits when the superior firm is placed on top are $E[\Pi_{S,1}] = pm_S - b_S$ and $E[\Pi_{I,2}] = 0 - b$. If the inferior firm is placed on top, the expected firm revenues are $E[\Pi_{S,2}] = pm_S - b$ and $E[\Pi_{I,1}] = (1 - \phi)(1 - p)pm_I - b_I$.

As in Scenario I, the superior firm has the same expected revenue regardless of where its link is placed. The inferior firm, however, makes no sales if it is placed at bottom. From a similar analysis as before, the equilibrium bids for the two firms will be $b_{\ast}^S = b$ and $b_{\ast}^I = (1 - \phi)(1 - p)pm_I + b$. Note that the inferior firm bids higher and, therefore, will have the top link position. Once again, we have a case that exhibits the position paradox, with the superior firm also obtaining more clicks even though it is at the bottom.

**Scenario III:** $Q < V$ and $pQ < s \leq pV$

Note that Scenario III and Scenario II are similar and mutually exclusive, and Scenario III exists if the quality premium is not high, i.e., $Q < V$. In this scenario, if a consumer first visits the superior firm and finds a match, she will still buy and stop. If she first visits the inferior firm and finds a match, she will also buy and stop, because the expected quality premium does not warrant an additional search. If a consumer does not find a match in the

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\footnote{Note that this profit is negative. However, we have assumed that firms do not exit the auction even if they make negative profit. This is a purely technical assumption that keeps the analysis cleaner; it has no effect on the insights from the model.}

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first firm she visits, she will continue to search the other firm. As in the first two scenarios, an uninformed consumer will first search whichever firm that is on top, while an informed one will search the superior firm first.

The expected firm profits when the superior firm is on top are $E[\Pi_{S,1}] = pm_S - b_S$ and $E[\Pi_{I,2}] = (1 - p)pm_I - b$. If the inferior firm is placed on top, the expected firm profits are $E[\Pi_{S,2}] = \phi pm_S + (1 - \phi)(1 - p)pm_S - b$ and $E[\Pi_{I,1}] = \phi(1 - p)pm_I + (1 - \phi)pm_I - b_I$. The equilibrium bids can be derived as $b^*_S = (1 - \phi)p^2m_S + b$ and $b^*_I = (1 - \phi)p^2m_I + b$.

In this scenario, both firms will generate higher revenue when placed on top than when placed on bottom. However, since $m_S > m_I$, the superior firm has “more to lose” at the bottom position than does the inferior firm. Therefore, the superior firm will outbid the inferior firm and will occupy the top position while the inferior firm will be placed at the bottom.

**Scenario IV:** $p \max\{Q, V\} < s \leq p(V + Q)$

In Scenario IV, the search cost is higher than both the expected quality premium and the expected utility of visiting the inferior firm, but lower than the expected utility of visiting the superior firm. In this case, if a consumer finds a match at the first firm she visits, she will buy the product and stop. If she does not find a match there, then if the other firm is the superior firm, she will search on, but if the other firm is the inferior one, she will stop. Again, an uninformed consumer searches whichever firm is on top first, while an informed consumer starts with the superior firm.

The expected firm profits when the superior firm is on top are $E[\Pi_{S,1}] = pm_S - b_S$ and $E[\Pi_{I,2}] = 0 - b$. If the inferior firm is placed on top, the expected firm profits are $E[\Pi_{S,2}] = \phi pm_S + (1 - \phi)(1 - p)pm_S - b$ and $E[\Pi_{I,1}] = (1 - \phi)pm_I - b_I$. The equilibrium bids can be derived as $b^*_S = (1 - \phi)p^2m_S + b$ and $b^*_I = (1 - \phi)p^2m_I + b$.

In this scenario, both firms value the top position more than they value the bottom one. If the margin of the superior firm is significantly higher than that of the inferior firm, i.e.,
$pm_S > m_I$, then the superior firm will have bid higher and be placed on top. Otherwise, the inferior firm will be placed on top. Even if inferior firm is on top, it will obtain more clicks only if $\phi < p/(1 + p)$, otherwise the superior firm will obtain more clicks.

**Scenario V: $s > p(V + Q)$**

In Scenario V, the search cost is higher than even the expected utility of visiting the superior firm. In this case, a consumer will stop after conducting the first search, regardless of which firm is visited or whether a match is found. An uninformed consumer again starts from the top, while an informed one again starts from the superior firm.

The expected firm profits when the superior firm is on top are $E[\Pi_{S,1}] = pm_S - b_S$ and $E[\Pi_{I,2}] = 0 - b_I$. The profits when the inferior firm is on top are $E[\Pi_{S,2}] = \phi pm_S - b$ and $E[\Pi_{I,1}] = (1 - \phi) pm_I - b_I$. The equilibrium bids can be derived as $b^*_S = (1 - \phi) pm_S + b$ and $b^*_I = (1 - \phi) pm_I + b$.

In this case, the superior firm has more to lose if it does not win the top position than does the inferior firm. Hence, it will bid to be placed on top.

**Summarizing the Scenarios**

The results of the five scenarios discussed above are summarized in the following proposition:

**Proposition 1** *In the pay-per-impression bidding game:*

- If the search cost is very low (Scenario I), both firms will bid an equal amount.
- If the search cost is moderately low (Scenarios II and III) then, if the quality premium is high (Scenario II), the inferior firm will bid higher, otherwise (Scenario III) the superior firm will bid higher.
- If the search cost is moderately high (Scenario IV) then, if the superior firm’s margin is significantly higher than that of the inferior firm ($pm_S > m_I$), the superior firm will bid higher, otherwise the inferior firm will bid higher.
• *If the search cost is very high (Scenario V), the superior firm will bid higher.*

For each scenario, the equilibrium bids by the two firms are presented in Table 1.

[Insert Table 1 about here.]

In the above proposition, the cases in which the search cost is moderate are intriguing. When search cost is moderately low, if the quality premium is high, the superior firm will not lose sales when placed at the bottom, but the inferior firm will. Therefore, the inferior firm will bid higher and get the top position. But if the quality premium is low, then the superior firm will also lose sales when placed at the bottom, and more so than the inferior firm. Therefore, the superior firm will bid higher and take the top position.

When search cost is moderately high, both firms always prefer the top position. However, whereas the superior firm gets “a second chance” even if its is placed at the bottom, the inferior firm does not, and thus it needs the top position more “desperately.” The result is that the superior firm will win the bid only when its margin is significantly higher than that of the inferior firm.

Two more points are worth noting. First, as we can see from the equilibrium bids in each of the five scenarios, the size of the portion of informed consumers has no effect on which firm will be the winner. This may not be obvious at first, as informed consumers will always click on the superior firm first, so the more such consumers, the better it is for the superior firm. However, recall that one key intuition is the incremental demand. Since the informed consumers do not change their click behavior in response to different link positioning, it is only the uninformed consumers that the two firms are competing for. Furthermore, for pay-per-impression, both firms will pay for both types of consumers anyway, regardless of whether and which links the consumers click. Therefore, although the size of the informed consumers changes the expected values of the top position to both firms, it changes them in a proportional manner that has no bearing on the outcome of the auction. We state this result in the following proposition.
Proposition 2 In the pay-per-impression auction, the number of informed consumers in the population does not impact which firm wins the auction.

The second point is that a higher quality premium does not make the superior firm more likely to be the winning bidder. Quite to the contrary, comparing Scenarios II and III shows that a higher quality premium may, in fact, make the inferior firm more likely to win the auction. Although surprising at first look, this can again be understood from the key intuitions of incremental value and residual demand — the higher the quality premium, the more likely that the superior firm will be searched even if it is placed at the bottom. This reduces the equilibrium bid of the superior firm, therefore making the inferior firm more competitive. We state this result in the following proposition.

Proposition 3 In the pay-per-impression auction, under certain conditions, a higher quality premium for the superior firm makes the inferior firm more likely to win the auction.

The analysis above also shows that in Scenarios III, IV and V, the per-unit margins of the firms influence which firm wins the auction. But, as we noted earlier, if we relax the assumption that $m_S > m_I$, it will only become more likely for the inferior firm to be placed on top. In fact, if $m_S < m_I$, then the inferior firm intuitively has the characteristic of the high-valuation bidder, given its higher per-unit profit. The assumption $m_S > m_I$ thus serves to raise the bar for the inferior firm to win the auction and highlights the effects discussed above, by showing that they can bring about the position paradox even in the case where other factors work in the opposite direction.

3.2 Pay-Per-Click

We now analyze the case of the pay-per-click auction mechanism, where a firm pays the advertisement fee only when its link is clicked. The two key factors discussed in the pay-per-impression case — residual demand and incremental value — continue to apply here. In addition, there is another key factor, which changes the outcome of the auction compared
to the pay-per-impression case and, in general, makes the superior firm even more likely to end up at the bottom position.

We call this effect the “differential cost” effect. Both firms know that some consumers are informed, and these consumers will start with the superior firm no matter how links are positioned. In the pay-per-click case, the inferior firm will pay for these consumers only if they actually click on its link, i.e., it will pay for them selectively. This is different from the pay-per-impression case, in which both firms pay as long as the links are displayed, i.e., both firms pay for search by every consumer. In other words, under pay-per-click the inferior firm will not unnecessarily pay for the informed consumers who never click on it. This reduces the expected cost per search of the inferior firm, thereby increasing its bidding capacity. In contrast, the superior firm will see the added fee (the additional amount needed to be at the top) paid for these consumers as pure waste, since these consumers will click on the firm’s link anyway. Hence, this effect, on the margin, increases the relative bidding power of the inferior firm.

Similar to the case of pay-per-impression, there are five scenarios depending on the values of parameters $s$, $p$, $m_S$, $m_I$, $V$ and $Q$, each resulting in different optimal bids and link positions. In each scenario, the search behavior of consumers and the expected revenues of firms when either is placed on top are exactly the same as they are in the corresponding scenario in the pay-per-impression case. The only difference is on the cost side — in pay-per-click a firm pays its bid weighted by the probability of click (which is $\leq 1$), whereas in pay-per-impression the firm pays its bid for the full mass of all consumers who search (which is equal to 1).

In the following, we highlight the basic insights by discussing Scenario III in detail. We simply summarize the results for the other scenarios and discuss them in detail in Section A1 in the appendix.

**Scenario I:** $s \leq p \min\{Q, V\}$ Since all consumers click on both firms, this is exactly the same as the pay-per-impression case.
**Scenario II:** \( Q > V \) and \( pV < s \leq pQ \)  

The inferior firm always emerges on top, but the superior firm obtains more clicks.

**Scenario III:** \( Q < V \) and \( pQ < s \leq pV \)

In Scenario III, in which the search cost is higher than the expected quality premium but lower than the expected utility of the inferior firm, both firms prefer the top position. The expected revenues are as in the pay-per-impression mechanism, so we now look at the expected cost. If the superior firm is placed on top, the probability that it will be clicked is 1. If it is placed at the bottom, this probability is \( \phi + (1 - \phi)(1 - p) \), where the first part corresponds to informed consumers and the second part corresponds to the uninformed consumers who do not find a match at the inferior firm. If the inferior firm is placed on top, its probability of being clicked is \( \phi(1 - p) + (1 - \phi) \), where the first part corresponds to informed consumers who do not find a match at the superior firm and the second part corresponds to uninformed consumers. If the inferior firm is placed at the bottom, the probability of it being clicked is \( 1 - p \).

If the superior firm pays \( b_S \) per click and is placed on top (so the inferior firm pays \( b_I \) per click), then the expected profits of the firms are \( E[\Pi_{S,1}] = pm_S - b_S \) and \( E[\Pi_{I,2}] = (1 - p)(pm_I - b_I) \). If the inferior firm pays \( b_I \) per click and is placed on top (so the superior firm pays \( b_S \) per click), then the expected profits of the firms are \( E[\Pi_{S,2}] = (\phi + (1 - \phi)(1 - p))(pm_S - b_S) \) and \( E[\Pi_{I,1}] = (\phi(1 - p) + (1 - \phi))(pm_I - b_I) \).

If the maximum amount the superior firm is willing to pay per click is given by \( b_S^* \), then

\[
pm_S - b_S^* = (\phi + (1 - \phi)(1 - p))(pm_S - b) \implies b_S^* = (1 - \phi)p^2m_S + (1 - (1 - \phi)p)b,
\]

and the maximum amount the inferior firm is willing to pay per click is given by \( b_I^* \), then

\[
(\phi(1 - p) + (1 - \phi))(pm_I - b_I^*) = (1 - p)(pm_I - b) \implies b_I^* = \frac{(1 - \phi)p^2m_I + (1 - p)b}{1 - \phi p}.
\]
Therefore, in equilibrium, the superior firm will bid $b^s_*$ and the inferior firm will bid $b^i_*$. 

The expressions above show that, depending on the parameters, either bid can be higher, i.e., either firm can win the auction. This is different from the pay-per-impression case, in which the superior firm always wins in this scenario. Note that the differential cost effect is driving this difference — since the informed consumers will only click on the superior firm and the inferior firm does not have to pay for these consumers, it can bid higher for the clicks of the uninformed consumers. The expression for $b^i_*$ also shows that, as $\phi$ increases, the inferior firm bids higher, which is consistent with this intuition. Furthermore, if the inferior firm wins in this scenario then, comparing the expressions for clicks, we find that the inferior firm will obtain more clicks than the superior firm only if $\phi < 1/2$, otherwise the superior firm will obtain more clicks even though it is at the bottom position.

**Scenario IV:** $s > p \max\{Q, V\}$ and $s \leq p(V + Q)$  

Depending on the values of the parameters, either firm may end up at the top position and, even if the inferior firm wins the auction, the superior firm may obtain more clicks.

**Scenario V:** $s > p(V + Q)$  

Depending on the values of the parameters, either firm may end up at the top position and, even if the inferior firm wins the auction, the superior firm may obtain more clicks.

**Summarizing the Scenarios**

The results of the five scenarios discussed above are summarized in the following proposition:

**Proposition 4** *In the pay-per-click bidding game:*

- If the search cost is very low (Scenario I), both firms will bid an equal amount.
- If the search cost is moderately low and the quality premium is high (Scenario II), the inferior firm will bid higher.
- In all other scenarios (Scenarios III, IV and V), either firm may bid higher.

*For each scenario, the equilibrium bids by the two firms are presented in Table 2.*
The situation of low search cost and that of moderately low search cost with high quality premium are similar to that in the pay-per-impression case. The other three scenarios are qualitatively different. As we can see, in the pay-per-click case the inferior firm is more likely to be the higher bidder than in the pay-per-impression case: while in pay-per-impression the superior firm is guaranteed to be the winner in Scenarios III and V, in pay-per-click the inferior firm may be the winner in these situations as well. This result can be readily deduced from Propositions 1 and 4; we state it below as a corollary to these propositions.

**Corollary 1** The inferior firm will win the position auction for a larger range of parameter values in the pay-per-click auction as compared to the pay-per-impression auction.

Under the pay-per-click scheme, the existence of informed consumers reduces the willingness to pay of the superior firm but does not affect that of the inferior firm, thereby increasing the relative bidding power of the latter. Furthermore, as we have already discussed above, as the number of informed consumers increases, the inferior firm bids higher and is more likely to win the position auction. Hence, we obtain a somewhat ironical result — the more widespread the reputation of the superior firm, the more likely it is that the inferior firm will win the position auction. We state this result in the following proposition.

**Proposition 5** In the pay-per-click auction, as the fraction of informed consumers in the population increases, the inferior firm will bid higher and is more likely to win the auction and be placed on top.

**Weighted Bids:** We have analyzed two popular auction mechanisms in detail — pay-per-impression and pay-per-click. However, as mentioned before, many search engines such as Google and Yahoo! control for the quality of the bidding firms by weighting firms’ bids with weights that are usually positively correlated with their qualities. The intuition here is that
a higher-quality firm can generate more clicks and, to increase revenues, the search engine should favor it for positions that generate higher revenue per click. In our model, such an adjustment of the bids will make it less likely that the inferior firm is placed at the top. However, we find that the position paradox can still occur, although it occurs for a reduced range of the parameter values. A sketch of the analysis for this case is available in Section A2 in the appendix. A main challenge in modeling this scenario realistically is that the exact quality scores used by search engines are not publically released. As more information on this aspect becomes available, future work can look into this mechanism in more detail.

4 Heterogeneous Search Costs

In the basic model, we assumed that all consumers have the same search cost $s$. In reality, however, we may expect that there is heterogeneity in search costs across consumers. In this section, we extend our study to the case where consumers are differentiated in their search costs.

We assume that both the informed consumers and the uninformed consumers are distributed uniformly in their search costs on the interval $[0, 1]$. Recall that each consumer, informed or uninformed, falls into one of the five scenarios discussed earlier. Hence, to make the search cost scale meaningful and ensure that all the five scenarios in the previous section fall into this scale, we normalize the other parameters accordingly by assuming that $p(V + Q) \leq 1$. In this case, firms will choose the bidding strategy to maximize expected profit across all consumers. The assumption of uniform distribution is made solely for the ease of exposition. Qualitatively, the results in the following sections hold in the case of arbitrary distribution of consumers, as the intuitions remain the same. To demonstrate this, we include in Appendix A3.3 the full details of the pay-per-impression auction when consumers are differentiated according to an arbitrary discrete distribution.
4.1 Pay-Per-Impression

We start with pay-per-impression. To facilitate the discussion, we label each segment of the consumers using the scenario number they belong to (e.g., consumers with search cost \( s \leq p \min\{Q, V\} \) are labeled as Segment I consumers since they fall under Scenario I). In equilibrium, each firm simply bids the incremental value of the top position relative to the bottom one, plus the minimum required bid. The incremental value of the top position is the weighted average of that value for each segment of the consumers. We state the result in the following proposition. The technical details are presented in Appendix A3.1.

**Proposition 6** In the pay-per-impression auction with uniformly distributed search costs for consumers, the superior firm will bid \( b^*_S = p(1 - p(1 - p)V - pQ)(1 - \phi)m_S + b \), while the inferior firm will bid \( b^*_I = p(1 - p(1 - p)V - p^2Q)(1 - \phi)m_I + b \). Depending on the parameter values, either firm’s bid may be higher, so either firm may win the auction and take the top position.

We now discuss some comparative statics results. First, note that the higher \( m_S \) (\( m_I \)) is, the more likely it is that the superior firm (inferior firm) will win the auction. This is easy to understand, as the higher a firm’s profit margin is, the more valuable the top position is to the firm. Second, we can see that the higher the quality premium \( Q \) is, the more likely it is that the inferior firm will win the auction. The reason is the same as explained in the earlier discussion when search cost is not differentiated.

Third, we can also see that the higher \( V \) is, the more likely it is that the inferior firm will win the auction. To understand the intuition behind this, note that changing \( V \) changes the relative size of each consumer segment. If \( V \geq Q \), increasing \( V \) increases the size of the Segment III of consumers while decreasing that of the Segment V. Both firms would like to bid lower for consumers in Segment III than for those in Segment V, but the reduction in bid is higher for the superior firm. Therefore, increasing \( V \) reduces the relative bid of the superior firm and makes it more likely for the inferior firm to win. If \( V < Q \), increasing
increases the size of Segments I and IV while decreasing that of Segments II and V. The superior firm would like to bid lower for Segment IV consumers than for Segment V consumers, while bid the same for Segment I and II consumers. The inferior firm would like to bid lower for Segment I consumers than for Segment II consumers, while bid the same for Segments IV and V consumers. Overall, the reduction in bid is higher for the superior firm, so increasing $V$ again reduces the relative bid of the superior firm and makes it more likely for the inferior firm to win the auction. We state this result in the following proposition.

**Proposition 7** In the pay-per-impression auction with uniformly distributed search costs for consumers, the higher the base product value ($V$) or quality premium ($Q$), the more likely it is that the inferior firm will win the auction.

Finally, we note that the size of the informed consumers does not change the likelihood of either firm winning the auction, as both firms are effectively competing only for the uninformed consumers, while the informed consumers add to both firms’ costs proportionally. This result is similar to the result in Proposition 2. However, it is a stronger result, since it holds for the more general case of heterogeneous search costs for consumers. We highlight this result by stating it below as a remark.

**Remark 1** In the pay-per-impression auction with uniformly distributed search costs for consumers, the number of informed consumers in the population does not impact which firm wins the auction.

### 4.2 Pay-Per-Click

We now analyze the pay-per-click case. Using a similar approach as in the pay-per-impression case, we can derive the optimal bids for both firms. We state the result in the following proposition, while the technical details can be found in Appendix A3.2.

**Proposition 8** In the pay-per-click auction with uniformly distributed search costs for consumers, the superior firm will bid

$$b^*_S = p(1 - p(1 - p)V - pQ)(1 - \phi)m_S + (\phi + p(1 - p)(1 - p)$$

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If $V \geq Q$, the inferior firm will bid $b_I^V = \frac{p(1-p)(1-p)V-p^2Q)(1-\phi)m_1+p(1-p)Vb}{1-\phi+\phi p(1-p)V}$, while if $V < Q$, the inferior firm will bid $b_I^Q = \frac{p(1-p)(1-p)V-p^2Q)(1-\phi)m_1+p(1-p)Vb}{(1-\phi)(1+p^2V-p^2Q)+\phi p(1-p)V}$. Depending on the parameter values, either firm’s bid may be higher, so either firm may win the auction and take the top position.

As before, the case of pay-per-click is, in general, similar to that of pay-per-impression. The significant difference concerns the size of the informed consumers — the larger the size of informed consumers, $\phi$, is, the more likely it is that the inferior firm will win the auction. This is because the superior firm needs to pay for the informed consumers, while the inferior firm does not, unless these consumers click on its link. This increases the relative bidding power of the inferior firm, making it more likely to win. This result is a generalization of Proposition 5 and we highlight it by stating it below as a remark.

**Remark 2** In the pay-per-click auction with uniformly distributed search costs for consumers, as the fraction of informed consumers in the population increases, the inferior firm will bid higher and is more likely to win the auction and be placed on top.

### 4.3 Search Engine Profit Comparison

We now compare the search engine profits under pay-per-impression and under pay-per-click. Under pay-per-impression, the search engine profit is determined only by firms’ bids. With the second-price auction, search engine profit is simply the minimum bid $b$ paid for the bottom link plus the lower one of the bids $b_S^*$ and $b_I^*$, paid for the top link, where $b_S^*$ and $b_I^*$ are as stated in Proposition 6. Under pay-per-click, the probabilities of users clicking on the links also need to be accounted for. The expected profit of the search engine is $\Pi_{SE} = P_T \min\{b_S^*, b_I^*\} + P_B b$, where $P_T$ is the probability that the top link will be clicked by a consumer, $P_B$ is the probability that the bottom link will be clicked by a consumer, and $b_S^*$ and $b_I^*$ are as stated in Proposition 8 ($b_I^*$ corresponds to $b_I^{V*}$ when $V \geq Q$, and $b_I^{Q*}$ otherwise). The click probabilities in each scenario can be calculated as before.
The comparison of search engine profit between pay-per-impression and pay-per-click is stated in the proposition below. (To simplify the analysis when comparing profits, we assume that the exogenous minimum bid is very close to zero, i.e., $b \to 0$.)

**Proposition 9** If $V \geq Q$, then if $m_S(1 - p(1 - p)V - pQ)(1 - \phi + \phi(1 - p)pV) > m_I(1 - p(1 - p)V - p^2Q)$, search engine profit is higher under pay-per-click, otherwise the search engine profit is higher under pay-per-impression.

If $V < Q$, then if $m_S(1 - p(1 - p)V - pQ)((1 - \phi)(1 - p^2Q + p^2V) + \phi(1 - p)pV) > m_I(1 - p(1 - p)V - p^2Q)$, search engine profit is higher under pay-per-click, otherwise the search engine profit is higher under pay-per-impression.

The result can be alternatively stated as: if the condition is such that the superior firm will win the auction under pay-per-click, then pay-per-click brings higher profit to the search engine than pay-per-impression, while if the inferior firm will win the auction under pay-per-click, then pay-per-impression brings higher profit. The basic insight behind this result can be understood as follows. With the second-price auction, search engine profit is determined by the lower bid. Hence, the mechanism that leads to a more aggressive lower bid will yield more profit for the search engine. With the presence of informed consumers, the inferior firm will bid more aggressively under pay-per-click than under pay-per-impression, while the reverse is true for the superior firm. Therefore, when the superior firm will give the lower bid, the profit is higher under pay-per-impression, while when the inferior firm gives the lower bid, the profit is higher under pay-per-click. The search engine is better off with the mechanism that intensifies the competition between the two firms for positions, and either pay-per-impression or pay-per-click can be that mechanism depending on the situation. Again, the difference between the two mechanisms lies in the presence of informed consumers. It can shown that when there are no informed consumers (i.e., $\phi = 0$), search engine profit is the same under both pay-per-impression and pay-per-click.
5 Empirical Support

The theoretical analysis offers insights into the bidding behavior of firms and predicts a surprising outcome, i.e., the position paradox, which is in contrast to the prevailing view in the literature. In this section, we analyze a dataset from a popular Korean search engine and find strong support for this basic outcome of our theory (i.e., a position paradox may be observed in sponsored search auctions), and for our assumptions regarding consumer search.

5.1 Data Description

To empirically validate our model’s predictions, we obtained a unique dataset of sponsored search advertising from a leading search engine in Korea. This search engine employs a pay-per-impression mechanism, and offers five different positions to potential advertisers. Each advertising position is sold through position auctions conducted daily. Note that the data from a pay-per-impression auction provides a conservative test of the position paradox because, as shown by our analytical model, the position paradox is less likely under the pay-per-impression mechanism as compared to the pay-per-click mechanism.

For a given keyword, the data consist of the daily positions of the advertisers and the corresponding daily impressions (i.e., the number of times the keyword was searched) and click counts at each position over a 15-day period in July 2008. We also have the exact URL that each advertiser displays when the keyword is searched. However, we do not have data on quality scores of advertisers. Hence, we decided to use firm’s reputation to impute the quality of a given web link. We hired three independent Korean annotators from the United States and Korea to assess, for each keyword, whether each advertiser is a high-quality or a low-quality firm. Among 246 keywords provided for this research, we excluded keywords if annotators could not recognize the firms by their names, and only considered keywords for which annotators could confidently classify the qualities of the firms. We also excluded keywords when only one advertiser was displayed during the data period. We then
computed the proportion of agreements on the quality of the firms across keywords between the annotators. The inter-rater reliability score ranged from 0.90 to 0.94, indicating a very high level of reliability. From the high-quality firms in a given keyword, we selected the best-ranked firm among them as the superior firm. Likewise, from the low-quality firms, we selected the best-ranked firm among the low-quality firms as the inferior firm. Finally, we ended up with a total of 102 keywords and categorized each into one of three different configurations based on the average number of clicks over the data period.

From our theoretical analysis, we obtain three different configurations for the outcome of the position auction:

**C1** The superior firm is above the inferior firm and the superior firm obtains more clicks.

**C2** The inferior firm is above the superior firm and the inferior firm obtains more clicks.

**C3** The inferior firm is above the superior firm but the superior firm obtains more clicks.

We find that all three configurations above have significant representation in our dataset. Out of 102 keywords, we find 48, 25 and 27 keywords in C1, C2 and C3, respectively. Even without formal empirical analysis, we strongly believe that this is an important observation as the extant literature shows, or often simply assumes, that C1 should be the equilibrium configuration. However, in our model C2 and C3 are also shown to arise in equilibrium, and the above observation supports this analytical result. In a recent empirical study, Animesh et al. (2009) document situations where low-quality firms are placed above high-quality firms in sponsored search results and assume in their study that firms at the top must be obtaining more clicks, which is similar to C2, and formulate this as an adverse selection issue that reduces social welfare. On the basis of our data which include clicks at every advertising

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7 Selecting the worst-ranked firm among the low-quality firms as the inferior firm gives qualitatively similar results.

8 We note two points here: (1) While only two keywords, our database also contained the case in which the superior firm is above the inferior firm and obtains fewer clicks. (2) The classification of each keyword into one of the different configurations is very stable, primarily because advertisers and their ranks do not change significantly during the 15-day data period.
position, we find that the inferior firm could obtain fewer clicks at the top, but still want to be at the top. This is C3, the position paradox situation.

5.2 Data Analysis and Results

We have already shown above examples of the position paradox configuration (C3). In this section, we show that quality, and not just position, is a significant predictor of the number of clicks a web link attracts.

To facilitate our empirical investigation, we included a total of 100 different keywords across three configurations (C1, C2 and C3) noted above. Our model considers two types of advertisers (superior or inferior) in a given keyword. Thus, the total number of observations considered here is 3000 (= 100 keywords × 2 firms × 15 days). The average number of daily impression counts is 849.35 (std. dev. = 1448.97), and the average number of daily clicks is 10.05 (std. dev. = 25.65). Figure 1 shows a histogram of daily click counts across keywords, in which a large variation in the daily click counts is observed, ranging from 0 to 295. Table 3 shows the daily average number of clicks corresponding to the three different configurations. It shows that across the configurations, on average, the superior firm obtains more clicks than the inferior firm (12.68 and 7.42, respectively), and the high-ranked firm tends to obtain more clicks than the low-ranked firm (16.80 and 6.37, respectively, in C1, and 11.58 and 4.18, respectively, in C2). But it also highlights C3 in which the low-ranked superior firm obtains more clicks than the high-ranked inferior firm (13.34 and 5.42, respectively).

[Insert Figure 1 and Table 3 about here.]

To formally analyze the data, we adopt the negative binomial distribution (NBD) as the base model for the daily click counts. As the NBD model is flexible in that it can exhibit a wide range of degrees of overdispersion, it has been commonly used in various marketing applications (e.g., Danaher 2007, Manchanda et al. 2004). An NBD distribution with mean
\( \lambda_{ijt} \) and overdispersion parameter \( \theta \) is given by

\[
\Pr(y_{ijt} = r) = \frac{\Gamma(\theta + r)}{\Gamma(\theta)\Gamma(r + 1)} \left( \frac{\lambda_{ijt}}{\lambda_{ijt} + \theta} \right)^r \left( \frac{\theta}{\lambda_{ijt} + \theta} \right)^\theta,
\]

where \( y_{ijt} \) is the number of clicks on firm \( j \) in keyword \( i \) at day \( t \), \( i \in \{1, \ldots, 100\}, j \in \{1, 2\}, t \in \{1, \ldots, 15\} \). As \( \theta \) approaches infinity, this distribution converges to the Poisson distribution.

We assume a log-linear regression decomposition of the expected value,

\[
\log(\lambda_{ijt}) = \alpha_i + \beta_1 \text{QUALITY}_{ij} + \beta_2 \text{POSITION}_{ijt},
\]

where the \( \alpha_i \) is a fixed effect for keyword \( i \), \( \text{QUALITY}_{ij} \) is a binary variable which is 1 if advertiser \( j \) in keyword \( i \) is the superior firm (and 0 otherwise) and \( \text{POSITION}_{ijt} \) is a binary variable which is 1 if advertiser \( j \) in keyword \( i \) is placed above the other advertiser on day \( t \) (and 0 otherwise).

Table 4 presents the results of the NBD regression with the fixed effects. The aggregate-level regression shows that both position and quality significantly influence the number of daily clicks that a web link receives and these effects are comparable in magnitude. This empirically verifies the position paradox result.\(^9\)

[Insert Table 4 about here.]

Besides confirming the existence of the position paradox for vertically differentiated firms, our empirical analysis also shows that, controlling for quality, higher positions attract more clicks. This indicates that, all else equal, a larger proportion of consumers start their search from the top position and go down sequentially. This is in accordance with our assumption regarding the behavior of uninformed consumers and lends more support to our modeling structure.

\(^9\)We also examined a few other specifications, e.g., adding impression counts and weekend-dummy variables to the covariates. Results of the position paradox are quite robust across alternative specifications in the log-link function.
Here, we empirically tested the position paradox results based on the data of a pay-per-impression scheme. In the pay-per-click scheme, such a position paradox is more likely, as shown by our analytical model. As we provide empirical support for our theoretical result in the “more difficult” case of pay-per-impression, we can expect it to hold in the “easier” case of pay-per-click. This is a promising area for future research, as and when data becomes available.

6 Conclusions and Discussion

Surfers on the World Wide Web typically rely on search engines to direct them to websites that contain content of immediate interest to them. In sponsored search advertising, firms bid for links of their websites to be displayed in response to keywords that consumers search. This provides a highly targeted advertising medium for firms and, therefore, sponsored search has been hugely successful as an online advertising medium.

The unprecedented rise of sponsored search has fueled recent academic study into it. A main result from both empirical and theoretical work is that advertisements at higher positions attract more clicks from consumers (e.g., Feng et al. 2007, Misra et al. 2006) and higher-quality firms will be placed at higher positions (e.g., Athey and Ellison 2008). In this paper, we study position auctions when firms offer products of different quality levels and some consumers know which firm offers the higher-quality product.

In this scenario, we uncover three key insights with respect to sponsored search. The “residual demand” effect implies that consumers may continue searching if they do not find a product to their satisfaction. The “incremental value” effect implies that, for a firm, a higher position is worth only the extra revenue it generates over the position below it, not the absolute revenue it generates. The “differential cost” effect, which is relevant under the pay-per-click mechanism where a firm has to pay only when a consumer clicks on its link, implies that different types of consumers have different relative cost implications to firms. Specifically, the inferior firm only has to pay for a fraction of the population (i.e., only the
uninformed consumers) and can bid higher as compared to the pay-per-impression auction.

Using the three building blocks, we determine how firms will bid in pay-per-impression and pay-per-click auctions. From these bidding profiles, we find the existence of an important paradoxical outcome in which a high-quality firm bids less than a low-quality firm to be placed below it, yet still obtains more clicks than the low-quality firm.

In the pay-per-impression auction, this “position paradox” can occur because the residual demand effect and the incremental value effect together imply that an inferior firm may value a higher position much more than a lower one, while in contrast a superior firm may be confident that even if it is placed lower, it will get a sufficient mass of residual consumers because it has a high-quality product. In other words, if the residual demand effect is sufficiently strong for a superior firm, an inferior firm may have more incremental value for the top position than the superior firm and may outbid it. In the pay-per-click auction, this becomes even more likely because the differential cost effect implies that the inferior firm only has to pay for the uninformed consumers and can, therefore, bid more aggressively and win more often.

We show that the position paradox is more likely when the quality differential between the two firms is larger and when the high-quality firm has a better reputation (i.e., more consumers are informed of its identity). We also show that it persists in the case of quality-based bid-weighting by the search engine. We extend our results to the case of heterogeneous search costs for consumers and also derive implications for search engine profit under the different mechanisms.

To support our theoretical analysis, we show strong empirical support for the basic position paradox result in a rich dataset obtained from a leading Korean search engine. We now discuss some limitations of our analysis and some avenues for future work.10 First, mechanism design for position auctions is not in the scope of our paper. In Section 4.3, for completeness of the current study, we compare search engine profits under the pay-

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10 Several specific questions related to the broad issues we outline below are being currently explored in various working papers; please refer to the literature review part in the introductory section of this paper.
per-impression and pay-per-click mechanisms and find that, under different conditions, either
mechanism can yield higher profit. Mechanism design with firm reputation effects can be
a fruitful area for future work and such studies can also consider other practices in more
detail, such as click weighting discussed earlier.

Second, we consider two firms bidding for two positions, while there are usually multiple
firms bidding for multiple positions in a real sponsored search auction. However, the three
basic effects that drive our results (residual demand, incremental value and differential cost)
are robust effects that should carry over to a multiple-firm setting. The complete analysis
for such a case is extremely complicated due to combinatorial explosion in the number of
cases to consider in consumer search. However, for a three-firm case, we have analytically
confirmed that a position paradox in which superior firms are placed below inferior firms but
obtain more clicks can still exist.11 Future work can address the multiple-firms case with
firm reputations and consumer search in greater detail.

Third, we have not considered the role of organic links. However, it is possible that,
for some keywords, uninformed consumers can get an indication of firm quality from the
organic listings. In our model, it is reasonable to assume that if some consumers consider
both sponsored and organic links, then the proportion of informed consumers increases (i.e.,
φ increases). In this case, Proposition 2 and Remark 1 indicate that this will not impact the
result of the pay-per-impression auction, and Proposition 5 and Remark 2 indicate that this
could strengthen the position paradox in a pay-per-click auction. Future research can model
this explicitly and provide deeper insights.

Fourth, results from position auction models depend, among other things, on assump-
tions regarding consumer search behavior. Our model formulation is supported by the fact
that our results are supported by data from real position auctions. Future work can as-
sess the implications of other assumptions regarding consumer search behavior, and theory
development and empirical work should go hand-in-hand in this area.

11Results are available upon request from the authors.
Fifth, we model the auction as a one-shot game for simplicity, as do almost all other theoretical papers on this subject. In our case, this assumption is reasonable since the search engine we obtain data from conducts its auctions daily. However, many search engines such as Google and Yahoo! conduct position auctions on a continuous basis, i.e., any bidder can change its bid at any time. Modeling a repeated position auction presents an interesting, but challenging, opportunity for future research.

Finally, the position paradox is generated because consumers lack information on the quality of products offered by firms and search sequentially for their desired product. This indicates that we should observe this paradox to be stronger for keywords that are related to markets in which consumers have high uncertainty about firm offerings. Exploring the relationships between keyword and market characteristics and the outcome of sponsored search auctions can be a very fruitful area for future research.

References


### Scenario

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>I: $s \leq p \min{Q, V}$</td>
<td>$b$</td>
<td>$b$</td>
</tr>
<tr>
<td>II: $p \min{Q, V} &lt; s \leq p \max{Q, V}$ and $Q &gt; V$</td>
<td>$b$</td>
<td>$(1 - \phi)(1 - p)pm_I + b$</td>
</tr>
<tr>
<td>III: $p \min{Q, V} &lt; s \leq p \max{Q, V}$ and $Q &lt; V$</td>
<td>$(1 - \phi)p^2m_S + b$</td>
<td>$(1 - \phi)p^2m_I + b$</td>
</tr>
<tr>
<td>IV: $p \max{Q, V} &lt; s \leq p(V + Q)$</td>
<td>$(1 - \phi)p^2m_S + b$</td>
<td>$(1 - \phi)pm_I + b$</td>
</tr>
<tr>
<td>V: $s &gt; p(V + Q)$</td>
<td>$(1 - \phi)pm_S + b$</td>
<td>$(1 - \phi)pm_I + b$</td>
</tr>
</tbody>
</table>

Table 1: Equilibrium Bids under the Pay-Per-Impression Mechanism

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### Scenario

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>I: $s \leq p \min{Q, V}$</td>
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<td>$b$</td>
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<tr>
<td>II: $p \min{Q, V} &lt; s \leq p \max{Q, V}$ and $Q &gt; V$</td>
<td>$b$</td>
<td>$(1 - \phi)(1 - p)pm_I + b$</td>
</tr>
<tr>
<td>III: $p \min{Q, V} &lt; s \leq p \max{Q, V}$ and $Q &lt; V$</td>
<td>$(1 - \phi)p^2m_S + b$</td>
<td>$(1 - \phi)p^2m_I + b$</td>
</tr>
<tr>
<td>IV: $p \max{Q, V} &lt; s \leq p(V + Q)$</td>
<td>$(1 - \phi)p^2m_S + b$</td>
<td>$(1 - \phi)pm_I + b$</td>
</tr>
<tr>
<td>V: $s &gt; p(V + Q)$</td>
<td>$(1 - \phi)pm_S + b$</td>
<td>$(1 - \phi)pm_I + b$</td>
</tr>
</tbody>
</table>

Table 2: Equilibrium Bids under the Pay-Per-Click Mechanism
<table>
<thead>
<tr>
<th>No. of keywords</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clicks</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High-quality firm</td>
<td>48</td>
<td>25</td>
<td>27</td>
<td>100</td>
</tr>
<tr>
<td>Low-quality firm</td>
<td>16.80</td>
<td>4.18</td>
<td>13.34</td>
<td>12.68</td>
</tr>
</tbody>
</table>

Table 3: Daily Average Number of Clicks for the Different Configurations

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>QUALITY</td>
<td>0.312</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>POSITION</td>
<td>0.463</td>
<td>&lt;.0001</td>
</tr>
</tbody>
</table>

Note: The fixed effects are not presented.

Table 4: NBD Regression Results
Figure 1: Histogram of Daily Number of Clicks
Appendix

A1 Pay-Per-Click Auction with Homogeneous Search Costs

In this section, we present the analysis for all search-cost scenarios under the pay-per-click mechanism. The following analysis will focus primarily on the cost side for the firms, since the revenue side is the same as in the pay-per-impression mechanism.

Scenario I: \( s \leq p \min\{Q, V\} \)

In Scenario I, the low-search-cost scenario, both firms will generate the same revenue regardless of the position of their links. Therefore, neither firm wants to pay for the top position, and both will bid the minimum required bid \( b^*_S = b^*_I = b \), with ties broken randomly by the search engine. This is the same as the pay-per-impression case.

Scenario II: \( p \min\{Q, V\} < s \leq p \max\{Q, V\} \) and \( Q > V \) (i.e., \( pV < s \leq pQ \))

In Scenario II, the expected revenue is the same for the superior firm whether it is placed on top or bottom and all consumers click on it. For the inferior firm, however, the expected revenue is higher when it is placed on top. Moreover, when the inferior firm is placed on top, the probability that it is clicked is \( (1 - \phi) \) (the portion of uninformed consumers) and when it is placed on bottom, no consumer clicks on it.

Hence, when the superior firm is placed on top, the firms’ expected profits are \( E[\Pi_{S,1}] = pm_S - b_S \) and \( E[\Pi_{I,2}] = 0 \). When the inferior firm is placed on top, the firms’ expected profits are \( E[\Pi_{S,2}] = pm_S - b \) and \( E[\Pi_{I,1}] = (1 - \phi)(1 - p)pm_I - (1 - \phi)b_I \). The equilibrium bids can be derived as \( b^*_S = \bar{b} \) and \( b^*_I = (1 - p)pm_I \).

The insight here is that since the inferior firm pays only when its link is clicked, it can bid the expected revenue conditional on the link being clicked, instead of the unconditional
expected revenue as in the pay-per-impression case. Hence, we again have a case in which the inferior firm emerges on top, but the superior firm obtains more clicks.

**Scenario III:** \( p \min\{Q, V\} < s \leq p \max\{Q, V\} \) and \( Q < V \) (i.e., \( pQ < s \leq pV \))

Please see Section 3.2 in the main text of the paper.

**Scenario IV:** \( s > p \max\{Q, V\} \) and \( s \leq p(V + Q) \)

In Scenario IV, both firms again prefer the top position. Consider the cost side of the equation. If the superior firm is placed on top, the probability that it is clicked is 1. If it is placed at the bottom, the probability that it is clicked is \( \phi + (1 - \phi)(1 - p) \). If the inferior firm is placed on top, the probability of it being clicked is \( 1 - \phi \). While if it is placed at the bottom, this probability is zero.

If the superior firm pays \( b_S \) per click and is placed on top, then the expected firm profits are \( E[\Pi_{S,1}] = pm_S - b_S \) and \( E[\Pi_{I,2}] = 0 \). If the inferior firm pays \( b_I \) per click and is placed on top, then the expected firm profits are \( E[\Pi_{S,2}] = (\phi + (1 - \phi)(1 - p))(pm_S - b) \) and \( E[\Pi_{I,1}] = (1 - \phi)(pm_I - b_I) \). The equilibrium bids can be derived as \( b_S^* = (1 - \phi)p^2m_S + (1 - (1 - \phi)p)b \) and \( b_I^* = pm_I \).

Similar to Scenario III, depending on the values of the parameters, either firm may end up at the top position. And once again, as \( \phi \) increases, the inferior firm will bid higher, consistent with the differential cost intuition. Furthermore, if the inferior firm wins the auction, it will obtain more clicks than the superior firm only if \( \phi < p/(1 + p) \), otherwise the superior firm will obtain more clicks even though it is at the bottom position.

**Scenario V:** \( s > p(V + Q) \)

In Scenario V, if the superior firm is placed on top, the probability that it will be clicked is 1. If it is placed at the bottom, this probability is \( \phi \). If the inferior firm is placed on top, the probability that it will be clicked is \( 1 - \phi \), while if it is at the bottom, this probability
is zero.

If the superior firm pays $b_S$ per click and is placed on top, then the expected firm profits are $E[\Pi_{S,1}] = pm_S - b_S$ and $E[\Pi_{I,2}] = 0$. If the inferior firm pays $b_I$ per click and is placed on top, then the expected firm profits are $E[\Pi_{S,2}] = \phi(pm_S - b)$ and $E[\Pi_{I,1}] = (1 - \phi)(pm_I - b_I)$.

The equilibrium bids can be derived as $b^*_S = (1 - \phi)pm_S + \phi b$ and $b^*_I = pm_I$.

Once again, either firm may end up at the top position and as $\phi$ increases, the inferior firm bids higher and will win the auction. Furthermore, if the inferior firm wins, then it will obtain more clicks than the superior firm only if $\phi < 1/2$, otherwise the superior firm will obtain more clicks even though it is at the bottom position.

Note that an implicit assumption made in the above analysis is that in all equilibria, both firms’ bids as specified in the equations are larger than or equal to the minimum bid ($b$). This technical assumption is made so that we can focus on the insights of the model without being inconvenienced by technical details. The assumption will hold as long as both firms’ margins are considerably larger than the minimum bid, and when the match probability is not too close to 0 or 1. (If any of these is true, then it is likely one of the firms will not participate in the auction, while our model is not concerned with entry decisions.)

A2 Weighted Bids

Search engine firms such as Google and Yahoo! often weight bids by quality scores for the firms to decide rankings. The exact formulae used by different search engines are not known. However, it is well accepted that the weights are, in general, positively correlated with firm qualities. Hence, in this scheme, a high-quality firm can be ranked higher even with a slightly lower bid than a low-quality firm. The intuition is that a higher-quality firm can generate more clicks and should be favored for positions that generate higher revenue per click.

In this extension, we model such an auction. Our intuitions such as residual demand, incremental value and differential cost still hold, so we again observe the position paradox. We illustrate this in the pay-per-click mechanism in Scenario IV. The same analysis leads to
similar conclusions for the other scenarios.

Our setting remains the same except the ranking rule: instead of ranking the higher bidder on top, in a scheme of weighted bids, the superior firm will be ranked on top as long as \( b^*_S > w b^*_I \), where \( w < 1 \) is a “discount” placed on the inferior firm due to its low quality. (For instance, within the scope of our model, one way to derive this score could be from product qualities as \( w = V/(V + Q) \).

In Scenario IV, the expected profits with the superior firm on top are given by \( E[\Pi_{S,1}] = p m_S - b_S \) and \( E[\Pi_{I,2}] = 0 \). If the inferior firm is on top, the profits are given by \( E[\Pi_{S,2}] = (\phi + (1 - \phi)(1 - p))(p m_S - b) \) and \( E[\Pi_{I,1}] = (1 - \phi)(p m_I - b_I) \). The equilibrium bids can thus be calculated as \( b^*_S = (1 - \phi)p^2 m_S + (1 - (1 - \phi)p)b \) and \( b^*_I = p m_I \).

As we can see, the equilibrium bids, which are determined by the incremental values of the top position to both firms, remain the same as in the standard pay-per-click case. What is different is the winner determination — the inferior firm will be on top as long as \( b^*_S < w b^*_I \). Therefore, if \( w p m_I > (1 - \phi)p^2 m_S + (1 - (1 - \phi)p)b \), the inferior firm will still be placed on top, and the position paradox still remains. However, since \( w < 1 \), it occurs for a smaller range of parameter values as compared to the standard pay-per-click case (in which, effectively, \( w = 1 \)).

A further variation is to not only change the ranking mechanism by weighting the bids, but also change the paying mechanism — instead of paying the bid of the firm ranked beneath it, a firm would have to pay at least a minimum amount required to stay in the position. In this case, the equilibrium bids will still remain the same, and the position paradox will still remain, although the inferior firm needs to pay more to stay in the top position.
A3 Technical Details for Model with Heterogeneous Search Costs

A3.1 Pay-Per-Impression Auction with Uniformly Differentiated Consumers

With pay-per-impression and uniformly differentiated consumers, both firms will bid for the top position based on the expected incremental values it provides over the bottom position. Consider first the case where \( V \geq Q \). With probability \( pQ \), the customer will have a search cost between 0 and \( pQ \). For these customers (Scenario I), the incremental value of the top position is 0 to both the superior and the inferior firm. With probability \( p(V - Q) \), the customer will have a search cost between \( pQ \) and \( pV \) (Scenario III). For these customers, the incremental value of the top position is \((1 - \phi)p^2m_S\) for the superior firm and \((1 - \phi)p^2m_I\) for the inferior firm. With probability \( pQ \), the customer will have a search cost between \( pV \) and \( p(V + Q) \) (Scenario IV). For these customers, the incremental value of the top position is \((1 - \phi)p^2m_S\) for the superior firm and \((1 - \phi)p^2m_I\) for the inferior firm. Finally, with probability \( 1 - p(V + Q) \) the customer has a search cost between \( p(V + Q) \) and 1 (Scenario V). For these customers, the incremental value of the top position is \((1 - \phi)pm_S\) for the superior firm and \((1 - \phi)pm_I\) for the inferior firm. The expected incremental value of the top position for the superior firm is given by

\[
0 \times pQ + (1 - \phi)p^2m_S \times p(V - Q) + (1 - \phi)p^2m_S \times pQ + (1 - \phi)pm_S \times (1 - p(V + Q))
= (1 - \phi)pm_S \times (1 - p(V + Q)).
\]

Similarly, the expected incremental value of the top position for the inferior firm is given by

\[
0 \times pQ + (1 - \phi)p^2m_I \times p(V - Q) + (1 - \phi)pm_I \times pQ + (1 - \phi)pm_I \times (1 - p(V + Q))
= (1 - \phi)pm_I \times (1 - p(V + Q)).
\]
In equilibrium, both firms will bid the respective incremental value plus the minimum bid. The case where $V < Q$ is analyzed similarly and the result is the same as above. This completes the proof of the proposition.

A3.2 Pay-Per-Click Auction with Uniformly Differentiated Consumers

The analysis for pay-per-click is similar to that for pay-per-impression. Consider first the case where $V \geq Q$. With probability $pQ$, the customer will have a search cost between 0 and $pQ$. For these customers (Scenario I), the incremental revenue of the top position is 0 to both the superior and the inferior firm. The incremental cost of the top position over the bottom position for the superior firm is $b_1 - b$, where $b_1$ is the amount the superior firm pays per click at top position, and that for the inferior firm is $(\phi(1-p) + (1-\phi)b_2 - (1-p)b$, where $b_2$ is the amount the inferior firm pays per click at top position. With probability $p(V - Q)$, the customer will have a search cost between $pQ$ and $pV$ (Scenario III). For these customers, the incremental revenue of the top position is $(1-\phi)p^2m_S$ for the superior firm and $(1-\phi)p^2m_I$ for the inferior firm. The corresponding incremental cost of the top position is $b_1 - (\phi + (1-p)(1-\phi)b$ for the superior firm and $(\phi(1-p) + (1-\phi)b_2 - (1-p)b$ for the inferior firm. With probability $p(V + Q)$, the customer will have a search cost between $pV$ and $p(V + Q)$ (Scenario IV). For these customers, the incremental revenue of the top position is $(1-\phi)p^2m_S$ for the superior firm and $(1-\phi)p^2m_I$ for the inferior firm. The corresponding incremental cost of the top position is $b_1 - (\phi + (1-p)(1-\phi)b$ for the superior firm and $(1-\phi)b_2$ for the inferior firm. Finally, with probability $1 - p(V + Q)$ the customer has a search cost between $p(V + Q)$ and 1 (Scenario V). For these customers, the incremental revenue of the top position is $(1-\phi)p m_S$ for the superior firm and $(1-\phi)p m_I$ for the inferior firm. The incremental cost is $b_1 - \phi b$ for the superior firm and $(1-\phi)b_2$ for the inferior firm.

Combining the above scenarios, $b_1 = p(1 - p(1-p)V - pQ)(1-\phi)m_S + (\phi + p(1-p)(1-\phi)Q)\phi$ will make the superior firm indifferent between the top and bottom
position, while \( b_2 = \frac{p(1-p)(1-p)V-p^2Q(1-\phi)m_I+p(1-p)Vb}{1-\phi+p(1-p)V} \) will make the inferior firm indifferent. These bids are thus the equilibrium bids.

The case where \( V < Q \) is analyzed similarly. The equilibrium bid for the superior firm is the same as in the case where \( V \geq Q \), while that for the inferior firm is \( b_2 = \frac{p(1-p)(1-p)V-p^2Q(1-\phi)m_I+p(1-p)Vb}{(1-\phi)(1+p^2V-p^2Q)+\phi p(1-p)V} \). This completes the proof of the proposition.

### A3.3 Pay-Per-Impression Auction with Arbitrarily Differentiated Consumers

In this section, we model the distribution of search costs across consumers using a four-point discrete distribution with arbitrary mass on each point. In contrast to the uniform distribution discussed in the main text, the distribution used here is very flexible. We look at pay-per-impression only, as the case for pay-per-click is similar.

From the analysis in the main text, it is clear that, for any distribution, what really matters is the proportion of consumers that fall into each scenario. That is, if two different distributions result in the same proportion of consumers in each of the scenarios, then they will result in the same bidding strategy and position placement from the firms. Consequently, we characterize the distribution by the proportions of consumers. Let \( \alpha \) represent the portion of consumers who fall into Scenario I, \( \beta \) represent the portion who fall into Scenario II (if \( Q > V \)) or Scenario III (if \( Q \leq V \)) and \( \gamma \) represent the portion who fall into Scenario IV. Then the portion of consumers who fall into Scenario V is \( 1 - \alpha - \beta - \gamma \). We make the natural assumptions that \( \alpha, \beta, \gamma \geq 0 \) and \( \alpha + \beta + \gamma \leq 1 \).

With the proportions described by the three parameters \( \alpha, \beta \) and \( \gamma \), if \( Q \leq V \), firm S will get the following additional profit if it is placed at the top position and pays \( b_S \) than when placed at the bottom (in which case it pays \( \bar{b} \)):

\[
E[\Delta \Pi_S] = E[\Pi_{S,1}] - E[\Pi_{S,2}] = (1 - \phi)pm_S(1 - \alpha - \beta(1 - p) - \gamma(1 - p)) - b_S + \bar{b}.
\]
Similarly, the additional profit for firm I by being on top is:

\[
E[\Delta \Pi_I] = E[\Pi_{I,1}] - E[\Pi_{I,2}] = (1 - \phi)pm_I(1 - \alpha - \beta(1 - p)) - b_I + \underline{b}.
\]

The equilibrium bids can be derived as:

\[
b_S^* = (1 - \phi)pm_S(1 - \alpha - (\beta + \gamma)(1 - p)) + \underline{b}
\]

and

\[
b_I^* = (1 - \phi)pm_I(1 - \alpha - \beta(1 - p)) + \underline{b}.
\]

Therefore, firm S will outbid firm I and be placed on top if:

\[
m_S > \frac{1 - \alpha - \beta(1 - p)}{1 - \alpha - (\beta + \gamma)(1 - p)} m_I.
\]

Otherwise, firm I will win the auction and be placed on top.

In the other case, if \( Q > V \), the additional revenue for firm S placed on top is:

\[
E[\Delta \Pi_S] = E[\Pi_{S,1}] - E[\Pi_{S,2}] = (1 - \phi)pm_S(1 - \alpha - \beta - \gamma(1 - p)) - b_S + \underline{b}
\]

and the additional revenue for firm I placed on top is:

\[
E[\Delta \Pi_I] = E[\Pi_{I,1}] - E[\Pi_{I,2}] = (1 - \phi)pm_I(1 - \alpha - p) - b_I + \underline{b}.
\]

The equilibrium bids can be derived as:

\[
b_S^* = (1 - \phi)pm_S(1 - \alpha - \beta - \gamma(1 - p)) + \underline{b}
\]

and

\[
b_I^* = (1 - \phi)pm_I(1 - \alpha - \beta p) + \underline{b}.
\]
Therefore, firm S will outbid firm I and be placed on top if:

\[ m_S > \frac{1 - \alpha - \beta p}{1 - \alpha - \beta - \gamma (1 - p)} m_I. \]

Otherwise, firm I will win the auction and be placed on top.

In both cases, we find that the size of the informed consumer group has no influence on which firm will win the auction. This result, which is discussed in the main text, thus holds with generality.

Assessing how a change of product value or quality premium influences the auction outcome is difficult in this general setting, as it is not as easy to characterize how it affects the size of each group as in the uniform distribution case. Nonetheless, we can view the proportion parameters as functions of product values and quality premiums: \( \alpha(V, Q) \), \( \beta(V, Q) \), and \( \gamma(V, Q) \), and assess the comparative statics. We illustrate this with the case where \( Q \leq V \).

Define \( m_{SV}^* = \frac{1 - \alpha - \beta (1 - p)}{1 - \alpha - (\beta + \gamma)(1 - p)} m_I \) as the threshold value of the superior firm’s margin, as a function of \( \alpha, \beta \) and \( \gamma \), below which the inferior firm wins the auction. Based on the corresponding scenarios, if \( Q \leq V \), we can see that \( \frac{\partial \alpha}{\partial Q} > 0, \frac{\partial \beta}{\partial Q} < 0, \frac{\partial \gamma}{\partial Q} > 0 \), and \( \frac{\partial (\alpha + \beta)}{\partial Q} = 0 \). Putting this back into the threshold value \( m_{SV}^* \), we can see that \( \frac{\partial m_{SV}^*}{\partial Q} > 0 \), i.e., the threshold value increases. Therefore, the higher the product premium offered by the superior firm, the more likely it is that the inferior firm will win the auction. This again confirms the results in the main text when a uniform distribution is assumed. Other analyses can be carried out in a similar manner.