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Selling goods of unknown quality: forward versus spot auctions

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Abstract We consider an environment where the sale can take place so early that both the seller and potential buyers have the same uncertainty about the quality of the good. We present a simple model that allows the seller to offer the good for sale before or after this uncertainty is resolved, namely via forward auction or spot auction, respectively. We solve for the equilibrium of these two auctions and then compare the resulting expected revenues. We also consider the revenue implications of insurance in forward auctions.

Keywords Forward trading · Forward auctions · Spot auctions · Insurance

JEL Classification D44

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1 Introduction

It is quite often the case that sellers and buyers are uncertain about the quality of the good that is up for sale. Examples can be drawn from a wide range of industries. In Australia, real estate is occasionally sold "off the plan" at auctions. In many countries, agricultural produce is often sold long before it is harvested. Moreover, in the Japanese racehorse industry, foals are often sold *before* they are born. Similarly, in many countries, including Australia, Canada, and the United States, livestock breeders sell embryos at auctions. In the UK gas industry, National Grid, which is in charge of balancing the supply and demand of gas in the network, auctions off the transmission capacity rights - the right to insert gas into the network - well in advance of the realization of demand, that is, before observing the availability of capacity (see McDaniel and Neuhoff 2004).[1]

These examples share three salient features: One, the selling mechanism is an auction. Two, the sale takes place so early that both the seller and potential buyers share the same uncertainty (*symmetric uncertainty*) about the quality of the object. And three, this uncertainty is resolved simultaneously for both seller and buyers. While there is a line of literature on transactions under asymmetric uncertainty [see, e.g., Lewis (2011) and Kogan and Morgan (2010)], to the best of our knowledge, no theoretical work has been done on auctions with symmetric uncertainty.[2] This paper is a first attempt toward filling in this gap.

The uncertainty present in the aforementioned examples can be captured by considering two possible outcomes: a good state (or high quality) and a bad state (or low quality). Both sides of the transaction anticipate that the object confers full value to its owner in the good state but only a fraction of value in the bad state.[3] In extreme cases, the object yields no value in the bad state (e.g., in gas transmission and embryo auctions).

Along this line, we present a simple model in which the seller can sell the object either before the uncertainty is resolved, via *forward auction* , or afterward, via *spot auction*. In the former case, the terms of sale (i.e. regarding the winning bidder and winning price) are pre-committed before the ex-post values are observed by the bidders; in the latter case, the terms are fixed after the ex-post values are observed, as is usually assumed in auction theory literature.

We consider a symmetric environment with independent and private values. In this setup, symmetric and strictly increasing equilibrium bidding strategies yield the same efficient allocation in both forward auctions and spot auctions. If buyers are risk neutral, one can confirm that forward and spot auctions generate the same expected

[1] Note that, in almost all of these industries, the goods are also sold after the uncertainty is resolved. For example, sale of transmission capacities also occurs when there are extra capacities left after the demand is realized; livestock and race horses are also sold after they are born.

[2] In an experimental paper, Phillips et al. (2001) test behavior in forward and spot *double auctions*. Their focus, however, is on the implications of inventory costs due to advance production and not on the uncertainty of the quality.

[3] Note that bidders' private information does not change over time. Uncertainty refers to the quality parameter that determines the ex-post valuation together with private information.

revenue. The coexistence of both types of auctions in the real world, however, suggests that in fact they could yield different revenues.

Moreover, in forward auctions, the price the winner commits to pay could exceed his ex-post value for the object, yielding a negative payoff. The seller, therefore, could naturally consider insuring the winner against such a risk. Interestingly, sellers tend to completely remove this risk in some industries but not in others. For instance, National Grid guarantees they will buy back capacity rights if the capacity ends up being unavailable. Similarly, in embryo auctions many sellers guarantee pregnancy. Both of these examples suggest that fully insuring the bidders is profitable to the seller only if the good has not much economic value in the bad state. To shed some light on these observations, we also look at the revenue implications of insurance (in the form of a buy-back guarantee) in forward auctions. In this paper, we provide a rationale (i) why spot and forward auctions can each be optimal when we relax the standard model by assuming that buyers are risk averse over gains and risk seeking over losses,[4] and (ii) why buy-back guarantees tend to be offered in some scenarios and not in others.

In the majority of the examples mentioned above, the auctions typically use a second price rule. Therefore, in our model we consider “second-price sealed bid auctions.”[5] We solve for the equilibrium of three auction formats: spot auctions, forward auctions, and forward auctions with insurance. We then compare the resulting expected revenues. First, we show that the forward auction results in higher revenues than the spot auction if and only if the quality of the object is more likely to be high. Interestingly, the revenue ranking is independent of the curvature parameter.[6] Next, we show that–under a mild convexity assumption that is satisfied by all convex distributions and power distributions–the spot auction generates higher expected revenue than the forward auction with insurance. This obtains because while the insurance induces buyers to bid more aggressively (relative to the expected bids in spot auction), it also results in inefficiencies, as the seller sometimes retains the object. It turns out that the loss of revenue due to inefficiency dominates the gain from aggressive bidding. Lastly, we compare the expected revenues of forward auctions with and without insurance. We find that there is no general revenue ranking between the two. Without insurance, bidders might be very cautious about their bids in a forward auction. If that is the case, then a forward auction with insurance can be better for the seller than a forward auction without.

We observe that among the three auction formats, there is only one general revenue ranking, and it is between the forward auction with insurance and the spot auction. It is interesting to note that neither spot auctions nor forward auctions with insurance

4 Seminal works on this type of preferences (prospect theory) are due to Kahneman and Tversky (1979) and Tversky and Kahneman (1991). Primary contributions on risk aversion in auctions are due to Maskin and Riley (1984) and Matthews (1987).

5 In some of these examples English auctions are used as selling mechanisms. Yet, given our assumption of private values they can be conveniently modeled as a second-price sealed bid auction.

6 We assume that the bidders are less sensitive to changes in wealth the further away they are from their wealth levels prior to the auction. The curvature parameter represents the level of bidders’ diminishing sensitivity to changes in wealth and it measures relative risk aversion over gains.

dominate forward auctions revenue-wise. The usual intuition from principal-agent models with risk-neutral principal and risk-averse agents suggests that the principal could extract more revenue by reducing the risk the agent is facing. This intuition, however, does not carry over to our setup. This is because the auction formats we study here are suboptimal revenue-wise, and because the fact that there is less risk to the bidders (value-wise[7]) in one environment does not necessarily result in more revenue extraction.[8]

2 Model

Consider a seller who would like to sell an object (a foal), the quality of which is unknown in period 1 (unborn foal), but becomes common knowledge in period 2 (after birth). The quality can take one of two forms: high with probability p and low with probability $(1 - p)$.

There are n bidders with the utility function,

$$u(x) = \begin{cases} x^{\alpha} & \text{if } x \geq 0 \\ -(-x)^{\alpha} & \text{if } x < 0, \end{cases} \tag{1}$$

where $\alpha \in (0, 1]$ is the curvature parameter.[9] Note that this utility specification is equivalent to the standard risk-neutral utility when $\alpha = 1$.

The seller's value for the object is normalized to zero. The bidders independently draw their private valuations from an atomless cumulative distribution F on $[0, 1]$ with $f(\cdot) = F'(\cdot) > 0$. Distribution F is common knowledge. If the object turns out to be of high quality, then it will confer the full value, x, to its buyer. On the other hand, if it is of low quality, then it will confer a fraction of the full value, kx, where $k \in [0, 1)$. For the unborn foal example, x could represent the value associated with the ability of the buyer to train the horse, whereas quality (1 vs. $k < 1$) could represent the inherited quality of the foal.

The seller can sell the object either before the quality is realized (forward auction) or afterward (spot auction). In either case, he adopts a second-price sealed bid auction. The highest bidder gets to keep the object irrespective of its quality and pays the second-highest bid.

[7] Bidders face two kinds of uncertainty, one surrounding the value of the object to the winner and the other surrounding the payment that the winner will make. While the latter is common to both forward and spot auctions, the former is specific to forward auctions.

[8] We should also note here that in contrast to standard principle-agent models, we consider a utility specification that is in line with prospect theory.

[9] This utility function is in line with Kahneman and Tversky (1979)'s prospect theory with a probability weighting function equal to the identity function, and reference points equal to current levels of wealth. It exhibits a constant relative risk aversion (risk-seeking) over gains (losses) with the same coefficient $0 \leq 1 - \alpha < 1$. To model loss aversion, this utility function could be modeled with a slope over losses by a constant factor $\gamma \geq 1 : u(x) = \begin{cases} x^{\alpha} & \text{if } x \geq 0 \\ -\gamma(-x)^{\alpha} & \text{if } x < 0. \end{cases}$ The results of our model would also hold in this more general setting. However, the loss aversion parameter γ seems to add little to our main results. Most of our interesting results are generated by the curvature paramenter α.

When using a forward auction, the seller could also offer insurance in the form of a "buy-back guarantee," as the price the winner pays could exceed his ex-post value for the object (if the quality turns out to be low), a risk that would hinder aggressive bidding. The buy-back guarantee would allow the winner to default on buying the object. Note that in spot auction no such insurance is needed, as the price paid in equilibrium would always be less than the ex-post value.

In Sects. 3, 4, and 5, we analyze spot auctions, forward auctions, and forward auctions with insurance, respectively. We then compare the respective revenues in Sect. 6 and conclude in Sect. 7.

3 Spot auction

In a spot auction, in both contingencies (high quality or low quality), bidders bid their values in (weakly dominant) equilibrium; therefore the seller's expected revenue is given by

$$(p + (1 - p)\, k)\, E\left[Y_2^{(n)}\right] \tag{2}$$

where $Y_2^{(n)}$ is the random variable representing the second-highest of n independent and identical draws from distribution F.

4 Forward auction

Here we consider a seller who uses a forward auction but offers no insurance. Then, the expected utility of a bidder with value x who bids as if his value is z is given by

$$\begin{aligned} &G(z)\, E\left[pu\left(x - \beta^F(y)\right) + (1-p)\, u\left(kx - \beta^F(y)\right) \mid y < z\right] \\ &= \int_0^z \left(pu\left(x - \beta^F(y)\right) + (1-p)\, u\left(kx - \beta^F(y)\right)\right) g(y)\, dy \end{aligned}$$

where $G(z) = F(z)^{n-1}$ represents the distribution of the highest value of the remaining $n - 1$ bidders, i.e. $Y_1^{(n-1)}$, $g(z) = G'(z)$ represents the corresponding density, and $\beta^F(\cdot)$ is the symmetric strategy they follow. Differentiating with respect to z and evaluating at $z = x$ gives the following necessary first-order condition:

$$\left(pu\left(x - \beta^F(x)\right) + (1-p)\, u\left(kx - \beta^F(x)\right)\right) g(x) = 0$$

Since $g(x) > 0$, it follows that

$$pu\left(x - \beta^F(x)\right) + (1-p)\, u\left(kx - \beta^F(x)\right) = 0. \tag{3}$$

For this to be true, we should have

$$x \geq \beta^F(x) \geq kx.$$

Since u is assumed to be of the form in (1), we obtain

$$p\left(x - \beta^F(x)\right)^{\alpha} = (1-p)\left(\beta^F(x) - kx\right)^{\alpha},$$

or

$$\frac{x - \beta^F(x)}{\beta^F(x) - kx} = \left(\frac{1-p}{p}\right)^{\frac{1}{\alpha}} \equiv t.$$

Rearranging this expression gives us the following equilibrium bidding strategy,

$$\beta^F(x) = \frac{1+tk}{1+t}x$$

or[10]

$$\beta^F(x) = \left(\frac{1}{1+t} + \frac{t}{1+t}k\right)x.$$

We see that, in equilibrium, bidders calculate a weighted average of the qualities and bid proportionaly to this weighted average. For example, when bidders are risk neutral, i.e. when $\alpha = 1$, the weights attached to respective qualities correspond to bidders' priors p and $(1-p)$, implying that risk-neutral bidders would bid their expected valuations [i.e. $(p + (1-p)k)x$] in a forward auction. Yet, in general, depending on the parameter values, bidders might bid more or less than their expected valuations. We elaborate on this below when comparing the forward and spot auction revenues.

For the above equilibrium, the seller's expected revenue is given by

$$\frac{1+tk}{1+t}E\left[Y_2^{(n)}\right]. \tag{4}$$

5 Forward auction with insurance

We showed in the previous section that, in equilibrium, the winner in a forward auction could end up with a negative payoff, which hinders aggressive bidding. To induce more aggressive bidding, the seller can offer insurance. Here, we consider insurance in the form of a buy-back guarantee: the seller guarantees to buy back the object at the selling price if the winner demands so. In this way, a forward auction with insurance provides the successful buyer with an option to default (if the outcome is bad) without penalty.

10 Note that we obtain the equilibrium bidding strategy by using only the necessary condition. It can be confirmed that for this strategy, global deviations are not profitable and $\beta^F(\cdot)$ constitutes a symmetric Bayesian Nash equilibrium.

Consider a bidder with value x who pretends his value is z. Then his expected payoff is given by

$$G(z)\,E\left[pu\left(x-\beta^{FI}(y)\right)+(1-p)\max\left\{u\left(kx-\beta^{FI}(y)\right),0\right\}\mid y<z\right]$$
$$=\int_0^z\left(pu\left(x-\beta^{FI}(y)\right)+(1-p)\max\left\{u\left(kx-\beta^{FI}(y)\right),0\right\}\right)g(y)\,dy,$$

where $G(z)=F(z)^{n-1}$, $g(z)=G'(z)$, and $\beta^{FI}(\cdot)$ is the symmetric strategy that the remaining bidders follow. Differentiating with respect to z and evaluating at $z=x$ yields the following first-order condition:

$$\left(pu\left(x-\beta^{FI}(x)\right)+(1-p)\max\left\{u\left(kx-\beta^{FI}(x)\right),0\right\}\right)g(x)=0.$$

Since $g(\cdot)>0$, it follows that

$$pu\left(x-\beta^{FI}(x)\right)+(1-p)\max\left\{u\left(kx-\beta^{FI}(x)\right),0\right\}=0.$$

Hence, we should have

$$\beta^{FI}(x)=x.$$

That is, the buyers bid truthfully. Note that this equilibrium holds for any utility specification (with $u(0)=0$). Note also that, in equilibrium, the seller buys the object back with positive probability. Hence, with a buy-back guarantee, the outcome of the forward auction is not fully efficient, whereas both the standard forward auction and the spot auction are efficient.

The seller's expected revenue from the forward auction with insurance is then given by

$$p\int_0^1\int_0^{x_1}x_2 f_{1,2}(x_1,x_2)\,dx_2dx_1+(1-p)\int_0^1\int_0^{kx_1}x_2 f_{1,2}(x_1,x_2)\,dx_2dx_1 \qquad (5)$$

where x_1 represents the highest order statistic and x_2 represents the second-highest order statistic of n random draws, and $f_{1,2}(x_1,x_2)$ is the corresponding joint density. Note that, in equilibrium, the winner pays the second-highest value whenever he keeps the object, and he does so *unless* the quality is low and the price exceeds his ex-post value for the low quality object, kx_1; hence the upper bound of the second integral in the second term.

6 Comparison of revenues in different auction formats

6.1 Spot auction versus forward auction

Proposition 1 *Consider* $\alpha\in(0,1)$; *then the expected revenue of the forward auction is greater than that of the spot auction if and only if* $p>\frac{1}{2}$.

Proof From (2) and (4), the expected revenue in forward auctions is greater than that of spot auctions when

$$\frac{1}{1+t} + \frac{t}{1+t}k > p + (1-p)\,k$$

where

$$t = \left(\frac{1-p}{p}\right)^{\frac{1}{\alpha}}.$$

Rearranging, we obtain

$$\frac{1-p}{p} > \left(\frac{1-p}{p}\right)^{\frac{1}{\alpha}}$$

which is true when $p > \frac{1}{2}$. □

Note that this condition is independent of k and α. This follows from the fact that in forward auctions, buyers bid according to some weighted average of the two qualities (1 and k), and in spot auctions, the ex-ante average bid is also a weighted average of these qualities. While the weight attached to the high quality is $\frac{1}{1+t}$ in the former, it is p in the latter. Thus, which average is greater depends only on the attached weights, not the qualities.

The intuition of why $p = \frac{1}{2}$ emerges as the threshold probability can be seen from Eq. (3). In essence, buyers are weighing two risks: the risk of not winning when the good is high quality versus the risk of winning when the good is low quality. When $p = \frac{1}{2}$, these risks exactly counterbalance each other and the bidder will choose a bid halfway between the high- and low-quality values, exactly as if risk neutral. On the other hand, if high quality is more likely, the bidder will skew his bid upward to better avoid this higher-probability risk, skewing more if more sensitive to the risk. However, the fact that he skews upward given any curvature parameter yields the unambiguous ranking above or below $p = \frac{1}{2}$.[11]

Remark 1 If $\alpha = 1$, that is, if bidders are risk neutral, then spot and forward auctions are revenue equivalent. This can also be seen from Milgrom and Weber (1982). The forward auction with no buy-back can be interpreted as a second-price auction where a signal k—the common factor—affecting the values is not revealed to bidders. The spot auction is the second-price auction where the signal k has been revealed. Milgrom and Weber (1982) have shown a "linkage principle," according to which a policy of revealing such a signal k weakly increases the expected revenue. From their proof, one sees that the expected revenues stay the same after revelation when the signal k and bidders' signals x_i are independently distributed, which is the case here.[12]

[11] We are grateful to an associate editor for providing this nice intiution.

[12] We thank an anonymous referee for pointing this out.

6.2 Spot auction versus forward auction with insurance

Remember that in both the spot auction and the forward auction with insurance, bidders bid their true valuations and the corresponding expected revenues are given by (2) and (5). We obtain the following revenue ranking.

Proposition 2 *Under a mild convexity assumption—the assumption that $xf(x)$ is increasing—the expected revenue from the spot auction is greater than that from the forward auction with insurance.*

Proof The difference in the two revenues is given by

$$(p+k(1-p))\,E\left[Y_2^{(n)}\right]-\left(pE\left[Y_2^{(n)}\right]+(1-p)\int_0^1\int_0^{kx_1} x_2 f_{1,2}(x_1,x_2)\,dx_2dx_1\right)$$

$$=(1-p)\left(kE\left[Y_2^{(n)}\right]-\int_0^1\int_0^{kx_1} x_2 f_{1,2}(x_1,x_2)\,dx_2dx_1\right)$$

$$=(1-p)\left(k\int_0^1\int_0^{x_1} x_2 f_{1,2}(x_1,x_2)\,dx_2dx_1-\int_0^1\int_0^{kx_1} x_2 f_{1,2}(x_1,x_2)\,dx_2dx_1\right)$$

Define $A(k)=k\int_0^1\int_0^{x_1} x_2 f_{1,2}(x_1,x_2)\,dx_2dx_1-\int_0^1\int_0^{kx_1} x_2 f_{1,2}(x_1,x_2)\,dx_2dx_1$ and note that $A(0)=A(1)=0$. We differentiate $A(k)$ once to obtain

$$A'(k)=\int_0^1\int_0^{x_1} x_2 f_{1,2}(x_1,x_2)\,dx_2dx_1-\int_0^1 x_1kx_1 f_{1,2}(x_1,kx_1)\,dx_1,$$

and by differentiating twice we get

$$A''(k)=-\int_0^1 (x_1)^2 f_{1,2}(x_1,kx_1)+k(x_1)^3\frac{\partial}{\partial x_2}f_{1,2}(x_1,kx_1)\,dx_1.$$

Since the valuations are independent, we have

$$f_{1,2}(x_1,x_2)=n(n-1)\,f(x_1)\,f(x_2)\,F(x_2)^{n-2}$$

and

$$\frac{\partial}{\partial x_2}f_{1,2}(x_1,x_2)=n(n-1)\,f(x_1)\left[f'(x_2)\,F(x_2)^{n-2}+(f(x_2))^2(n-2)\,F(x_2)^{n-3}\right]$$

$$=n(n-1)\,f(x_1)\,f(x_2)\,F(x_2)^{n-2}\left[\frac{f'(x_2)}{f(x_2)}+\frac{f(x_2)}{F(x_2)}(n-2)\right].$$

Therefore, we obtain

$$A''(k) = -\int_0^1 (x_1)^2 \left(f_{1,2}(x_1, kx_1) + kx_1 \frac{\partial}{\partial x_2} f_{1,2}(x_1, kx_1) \right) dx_1$$

$$= -\int_0^1 (x_1)^2 f_{1,2}(x_1, kx_1) \left(1 + kx_1 \left(\frac{f'(kx_1)}{f(kx_1)} + \frac{f(kx_1)}{F(kx_1)} (n-2) \right) \right) dx_1.$$

Note that $(x_1)^2 f_{1,2}(x_1, kx_1)$ is positive for all k and x_1. Moreover,

$$1 + kx_1 \left(\frac{f'(kx_1)}{f(kx_1)} + \frac{f(kx_1)}{F(kx_1)} (n-2) \right) \geq 1 + kx_1 \frac{f'(kx_1)}{f(kx_1)}$$

where the right-hand side of the inequality is greater than zero for the distributions that satisfy $(f(x)x)' = f'(x)x + f(x) \geq 0$.

Since $A(k)$ is zero at $k = 0$ and $k = 1$ and concave in between, we conclude that $A(k) \geq 0$ for all k. □

Note that all convex distributions and all power distributions (whether convex or concave) satisfy the above-mentioned mild-convexity assumption.[13] The intuition behind the result is the following. When the quality of the object is high, both the spot auction and the forward auction with insurance obtain the same expected revenue. When the quality is low, however, the spot auction always obtains an ex-post revenue of $kY_2^{(n)}$, whereas the forward auction with insurance obtains ex-post revenue of either $Y_2^{(n)}$ or 0. When the value distribution becomes sufficiently convex, the revenue becomes more likely to be 0. Therefore, the spot auction obtains a higher expected revenue.

The above proposition establishes that under a mild convexity assumption, the spot auction is revenue-superior to the forward auction with insurance. This finding then begs the question, "Why do we see in practice forward auctions with insurance?" To the extent that this model captures the main aspects of the motivating examples, provision of full insurance in forward auctions is optimal only if the seller does not choose to use a spot auction, for reasons not included in our model. This result conforms with practice, as full insurance is provided by the seller only in auctions for transmission capacity rights and in embryo auctions. In these two industries, forward auctions are preferred over spot auctions because of the need for balancing the network in a timely fashion and the need for fast reproduction of livestock, respectively. In the other three industries mentioned, insurance is typically provided not by the seller, but by third parties.

[13] The assumption of increasing $xf(x)$ is only a sufficient condition for the inequality between revenues of spot auctions and forward auctions with insurance to hold. For instance, consider the distribution family F defined over [0, 1] with $F(x) = \frac{1}{a} - \frac{1}{x^{\alpha}+a}$ for $\alpha \in (0, 1)$ and $a = \frac{1}{2}\sqrt{5} - \frac{1}{2}$. It can be confirmed that $xf(x)$ is not increasing. Yet, for all values of α, the inequality holds. On the other hand, we can find distributions that violate increasing $xf(x)$ where the inequality is reversed.

6.3 Forward auction versus forward auction with insurance

Finally, we compare the expected revenues of forward auctions with and without insurance.

Proposition 3 *There is no general revenue ranking between forward auctions and forward auctions with insurance.*

Proof The expected revenue from forward auctions with insurance is given by (5). For uniform value distributions, $F(x) = x$, $f_{1,2}(x_1, x_2) = n(n-1)(x_2)^{n-2}$ and (5) simplifies to $(p + (1-p)k^n) E\left[Y_2^{(n)}\right]$. Therefore, for uniform distributions, the expected revenue from the forward auction is greater than that with full insurance if

$$\frac{1+tk}{1+t} > p + (1-p)k^n$$

$$\frac{1+\left(\frac{1-p}{p}\right)^{\frac{1}{\alpha}} k}{1+\left(\frac{1-p}{p}\right)^{\frac{1}{\alpha}}} > p + (1-p)k^n.$$

We conclude that there is no unambiguous ranking. We can find two sets of parameter values that show either auction format can be better than the other: $\{n=2, p=\frac{1}{3}, \alpha = \frac{1}{2}, k = 0.1\}$ gives 0.28 versus 0.34 and $\{n = 2, p = \frac{1}{3}, \alpha = \frac{1}{2}, k = 0.4\}$ gives 0.52 versus 0.44. □

Remark 2 When buyers are risk neutral, i.e. $\alpha = 1$, (with the mild convexity assumption), both forward and the spot auctions are revenue superior to forward auctions with insurance.

With insurance (in the form of a buy-back guarantee), it might seem that the seller would be worse off, as he is offering insurance for free. But this is not quite the case, as without this insurance bidders would be very cautious in their bids. That is why, for some parameter values, providing insurance in a forward auction is preferable for the seller.

7 Conclusion

We observe a wide use of forward transactions (auctions-markets-contracts) in the real world, yet a theoretical analysis of forward auctions is virtually absent. The emergence of forward auctions might stem from various reasons, such as credit constraints, risk hedging, or scheduling purposes. We provide an alternative explanation, namely, the revenue implications of buyers' attitudes toward risk. We present a simple model with risk-averse buyers and compare the equilibria of forward auctions with spot auctions on the basis of expected revenue. We also consider the revenue effects of insurance provided by the seller. We observe that either the spot or the forward auction can be revenue superior to the other. Interestingly, this superiority is independent of the quality of the object being sold. We also observed that, under a mild assumption, the spot auction is always revenue-superior to the forward auction with insurance, whereas forward auctions with insurance can be revenue superior to forward auctions without insurance.

The extension of our work to the multi-unit setting is the next natural step. Forward auctions for transmission capacity rights, for instance, would more naturally be modeled as multi-unit auctions where buyers have multi-unit demand. In a multi-unit setting, one would also be interested in seeing how the seller endogenously determines the number of units she sells in a forward auction. Insights from multi-unit forward auctions would also help us better understand forward markets in general.

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