

Optimal Relay Placement for Cellular Coverage Extension

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Abstract—In this paper, we address the problem of optimal relay placement in cellular networks for maximum extension of coverage area. We present a novel definition of the coverage radius after the introduction of relays. Using this, we determine the optimal relay positions to maximize the coverage radius and estimate the number of relays required per cell. We also analyze relay placement in the multi-cell scenario, which takes into account inter-cell interference, a dominant factor in the next generation cellular Orthogonal Frequency Division Multiple Access (OFDMA) systems. Considering inter-cell interference in the multi-cell scenario, leads to an interesting iterative algorithm which is used to determine the optimal relay station (RS) positions.

Index Terms—Coverage Radius, Relay placement, Inter-cell interference, Cellular OFDMA

I. INTRODUCTION

With the rapid growth of the number of cellular subscribers, and the scarcity of frequency spectrum, cellular systems are facing difficulty in providing satisfactory signal to noise ratio (SNR) to users, especially at the cell edge. One solution to support the ever increasing number of subscribers per cell is to decrease the cell radius. This results in more base stations (BSs) required per area thus escalating the infrastructure costs. Also, smaller cell radius causes higher inter-cell interference, thereby calling for interference management techniques such as sectorization and adaptive interference cancellation.

An alternate solution being employed in next generation cellular systems [1] is to deploy low-cost relay stations (RSs) in each cell. The deployment of RSs has two key benefits - increase in cell capacity, and coverage extension. There is an increase in capacity since a mobile station (MS) has the diversity advantage of two possible links - the direct link to BS, and the link via RS. As a result incoming calls may experience lower blocking probability, and thus the cell can support a greater traffic density of users in the same cell area. Alternately, for the same traffic density, deployment of cellular RSs helps increase the cell radius. This is because RSs being closer to the cell edge MSs than the BS, improve received SNR to these MSs. Due to increase in cell radius, the infrastructure cost of deploying more BSs is also reduced.

The increase in cell radius depends upon the radial position of the RSs in the cell since the location of an RS affects the SNR of the received signal on the BS-RS and RS-MS links. An RS placed close to the cell edge, will result in low received SNR on the BS-RS link and will also cause higher interference to the neighboring cells. On the other hand, placing the RS away from the cell edge, results in a low SNR on the RS-MS link, causing cell edge users to be more prone to outage. Thus in order to achieve maximum extension in cell radius there is a need to determine optimal radial position of the RSs.

Only a few researchers so far have addressed the issue of optimal placement of cellular RSs. The authors in [2] and [3] analyze RS placement for wireless sensor networks, where the objective is to achieve maximum connectivity between pairs of ad-hoc relay nodes. [4] considers a dual-relay architecture with cooperative RS pairs and proposes an algorithm to select the two best RS locations from a predefined set of candidate positions. The analyzes presented in [2]–[4] involve inter-RS communication. However, the existing standards, including the ongoing work in IEEE 802.16m standard [1] do not currently support inter-RS communication because it involves a significant communication cost. In [5] and [6], the RS placement problem is analyzed from the perspective of increasing system capacity rather than coverage extension. In [7], an iterative RS placement algorithm is proposed which divides all points in the cell into good and bad coverage points and places RSs at the good points whose neighbors have bad coverage. Though the RS placement problem has been addressed by all the aforementioned works, they have not considered realistic channel conditions such as channel fading and inter-cell interference.

In this paper, we take into account shadow fading as well as inter-cell interference. We define the coverage radius of the cell in terms of the probability of correct decoding at a point. Using this notion, we determine the optimal RS position to achieve maximum coverage radius, both for single cell and multi-cell scenarios. The multi-cell scenario takes into account inter-cell interference, which is a dominant factor in the next generation cellular Orthogonal Frequency Division Multiple Access (OFDMA) systems. The extended coverage radius after RS placement, determined in this paper is useful for designing the inter-BS distance during system planning.

The rest of the paper is organized as follows. In Section II, we describe the system model. In Section III, we compute the optimal RS position to maximize the coverage radius and estimate the number of RSs required in a relay-assisted cell. In Section IV, we present an iterative algorithm to solve the problem in a multi-cell scenario. The results are presented in Section V. Finally, Section VI concludes the paper and provides directions for further investigation.

II. SYSTEM MODEL

We consider downlink data transmission in a relay-assisted cellular system. Cellular RSs can be classified into two broad types - transparent and non-transparent RSs. Transparent RSs do not transmit pilot signals to the MS and hence the MS is unaware of their existence. A transparent RS functions like a repeater which merely forwards the signal from the BS to the MS. On the other hand, a non-transparent RS transmits pilot signals to the MS and performs most of the functions of a full-fledged BS such as inter-RS and RS-BS handover. The IEEE 802.16m standard [1] currently supports non-transparent RSs with no direct communication between BS and MS, after the MS has been handed over to the RS.

We consider a topology with N_R non-transparent RSs placed symmetrically at radial distance R_1 around every BS as shown in Fig. 1. Although we focus our attention on the two-hop case with data transmission from the BS to MS via only one RS, our analysis can be extended to multi-hop relay architecture. Let the downlink transmit power of the BS be P_B dBm and that of the RSs be P_R dBm ($P_R < P_B$). The pathloss exponent is denoted by η . Let the thermal noise level be N dBm. We consider log-normal shadowing ξ on each link, where ξ is a Gaussian random variable with mean 0 and standard deviation σ , σ_1 and σ_2 for the BS-MS, BS-RS and RS-MS links respectively. Since our aim is to evaluate the optimal RS positions from a long term coverage perspective, we ignore the effect of fast fading on all wireless links.

Initially in Section III, we consider a single cell scenario, and assume that there is no inter-cell interference from neighboring cells. The assumption of zero inter-cell interference is relaxed in Section IV where we analyze RS placement for a multi-cell scenario by taking into account the interference from the first tier of neighboring cells only.

III. SINGLE CELL SCENARIO

In this section, we solve the optimal RS placement problem for a single cell scenario, assuming zero inter-cell interference

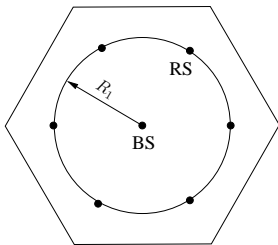


Fig. 1. Topology with $N_R = 6$ RSs placed symmetrically around the BS

from neighboring cells¹. We define the coverage radius as a metric of cellular coverage, and determine the optimal R_1 which maximizes it. We also estimate the number of RSs N_R required in each cell.

A. Definition: Coverage Radius

We first consider a direct transmission from the BS to an MS located at a distance d from it. The received SNR at the MS is, $SNR_{BS-MS} = P_B - 10\eta \log d - N + \xi$, where ξ is a Gaussian random variable with standard deviation σ on the BS-MS link. Let T (in dB) be the threshold of the minimum SNR required for correct decoding of the received signal. We define the probability of correct decoding p_c , as the probability that the received SNR is greater than threshold T . Thus, for the direct transmission from the BS to MS,

$$\begin{aligned} p_c &= Pr(SNR_{BS-MS} > T), \\ &= Pr(P_B + \xi - 10\eta \log d - N > T), \\ &= Pr(\xi > T + N - P_B + 10\eta \log d), \\ &= Q\left(\frac{T + N - P_B + 10\eta \log d}{\sigma}\right). \end{aligned} \quad (1)$$

where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{x^2}{2}} dx$. We define that a point is said to be covered if the probability of correct decoding p_c at that point is greater than or equal to a required value. We consider this minimum required value of p_c to be 0.5. We note that when $p_c \geq 0.5$, $T + N - P_B + 10\eta \log d < 0$ in (1). Or equivalently, the expected value of the received SNR, $E(SNR_{BS-MS}) = P_B - 10\eta \log d - N$ is greater than or equal to threshold T . Thus, for every point with $p_c \geq 0.5$, the expected value of the received SNR is greater than decoding threshold T . This justifies our choice of 0.5 as the required minimum value for the probability of correct decoding p_c . The coverage area of the BS is a circular disc of radius R_{cov} such that $p_c = 0.5$ at the circumference. We define coverage radius R_{cov} as the distance from the BS at which the MS experiences $p_c = 0.5$, such that all locations of the MS at a distance

¹Though it seems restrictive, the analysis of RS placement in the single cell scenario is applicable to a system which employs frequency planning to ensure that inter-cell interference is negligible, for example the Global System for Mobile communications (GSM) cellular systems.

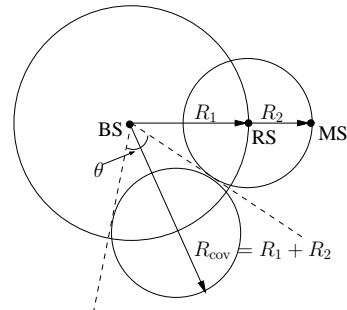


Fig. 2. Illustration of the definition of coverage radius R_{cov} for the relay-assisted cellular system. Also shown is the method to evaluate the angle θ subtended by each RS at the BS. $\theta = \sin^{-1}(R_2/R_1)$.

$d > R_{\text{cov}}$ from the BS experience $p_c < 0.5$. By substituting $p_c = 0.5 = Q(0)$ in (1) we obtain $R_{\text{cov}} = 10^{\frac{P_B - T - N}{10\eta}}$.

Now we consider the relay-assisted cellular system described in Section II. When an MS moves outside the coverage area of the BS, it is handed over to one of RSs in the cell. It stops direct communication with the BS and starts receiving all further data via the RS. Thus, the probability of correct decoding is,

$$\begin{aligned} p_c &= p_{c_1} \cdot p_{c_2}, \\ &= \Pr(SNR_{BS-RS} > T) \times \Pr(SNR_{RS-MS} > T), \\ &= Q\left(\frac{T + N - P_B + 10\eta \log R_1}{\sigma_1}\right) \\ &\quad \times Q\left(\frac{T + N - P_R + 10\eta \log R_2}{\sigma_2}\right), \end{aligned} \quad (2)$$

where p_{c_1} and p_{c_2} are the probabilities of correct decoding on the BS-RS and RS-MS links respectively. R_1 is the relay placement radius and R_2 is distance from the RS to the MS. For this two-hop cellular system, we define the coverage radius R_{cov} as the *maximum* distance from the BS at which transmission via an RS results in $p_c \geq 0.5$. Equivalently, for given relay placement radius R_1 , R_{cov} is the maximum distance from the BS at which both $E(SNR_{BS-RS})$ and $E(SNR_{RS-MS})$ are greater than threshold T . From Fig. 2 we see that distance R_{cov} at which the condition $p_c \geq 0.5$ is satisfied is maximum when the BS, RS and MS are collinear. Thus, $R_{\text{cov}} = R_1 + R_2$ where R_1 is the RS placement radius, and R_2 is the RS-MS distance such that $p_c = p_{c_1} \cdot p_{c_2} = 0.5$.

B. Optimal Relay Placement

Given an RS placement radius R_1 , R_2 can be evaluated as a function $f(R_1)$, by setting $p_c = 0.5$ in (2) as follows,

$$R_2 = f(R_1) = 10^{\left(\frac{P_R - N - T}{10\eta} + \frac{\sigma_2}{10\eta} \cdot Q^{-1}\left(\frac{0.5}{p_{c_1}}\right)\right)}. \quad (3)$$

It is clear from (1) that greater the distance between a pair of nodes, lower is the p_c on that link. We observe that in (3) R_2 is inversely proportional to the RS placement radius R_1 . This is because if R_1 is large, p_{c_1} , being inversely proportional to it will be small. Now in order to maintain $p_{c_1} p_{c_2} = 0.5$, p_{c_2} will have to be large. Hence, R_2 being inversely proportional to p_{c_2} will be small. Thus, there is a tradeoff between the values of R_1 and R_2 and we can determine the optimum value of R_1 which maximizes the coverage radius $R_{\text{cov}} = R_1 + R_2$ as follows,

$$\begin{aligned} R_1^* &= \arg \max_{R_1 \in (0, R_1^{\text{max}}]} R_1 + R_2 \quad \text{s.t. } p_{c_1} \cdot p_{c_2} = 0.5, \\ &= \arg \max_{R_1 \in (0, R_1^{\text{max}}]} R_1 + f(R_1), \end{aligned}$$

where, R_1^{max} is the maximum possible RS placement radius. It is the R_1 at which the probability of correct decoding of BS-RS transmission, p_{c_1} is equal to 0.5. If RS is placed at a greater distance, p_{c_1} will fall below 0.5 and it will not be possible to satisfy the condition $p_{c_1} \cdot p_{c_2} = 0.5$. Thus,

$$R_1^{\text{max}} = 10^{\left(\frac{P_B - N - T}{10\eta} + \frac{\sigma_1}{10\eta} \cdot Q^{-1}(0.5)\right)} = 10^{\left(\frac{P_B - N - T}{10\eta}\right)} \quad (4)$$

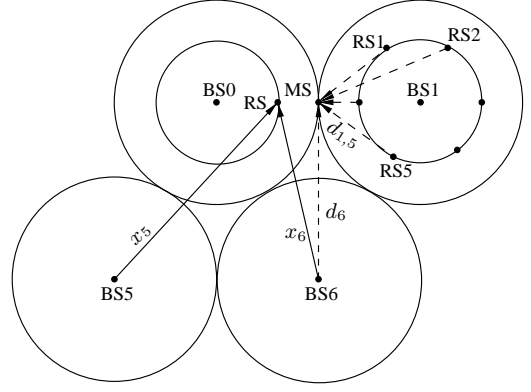


Fig. 3. Illustration of computation of inter-cell interference. Solid lines denote distances x_i from the reference RS to the BS if i^{th} neighboring cell. Dashed lines denote distances $d_{i,j}$ the reference MS to the j^{th} RS in the i^{th} neighboring cell

C. Number of Relays required

Now we determine the number of RSs required in the cell. We assume that the number of RSs is chosen such that the coverage discs of the RSs are tangent to each other, as shown in Fig. 2. We consider coverage areas of RS which are *just* non-overlapping because placing RSs in this fashion gives the minimum number of RSs required. There may be coverage holes between adjacent RS coverage discs where the probability of correct decoding falls below 0.5. By increasing the number of RSs, the coverage holes can be reduced.

The number of RSs required is inversely proportional to the coverage radius R_2 of each RS. Let θ be the angle subtended by each RS's coverage disc at the BS, as shown in Fig. 2. The number of RSs required is,

$$\begin{aligned} N_R &= \left\lceil \frac{2\pi}{\theta} \right\rceil \\ &= \left\lceil \frac{\pi}{\sin^{-1}\left(\frac{R_2^*}{R_1^*}\right)} \right\rceil. \end{aligned} \quad (5)$$

IV. MULTI-CELL SCENARIO

Cellular OFDMA systems usually employ 1:1 frequency reuse. Thus, inter-cell interference becomes significant and affects the optimal placement of RSs in the cell. In this section, we determine the optimal RS positions by taking into account the inter-cell interference. We describe the system model, followed by the computation of inter-cell interference and finally present an iterative algorithm for determining the optimal RS placement radius R_1 .

A. System Model

In cellular OFDMA, a set of subcarriers, called a subchannel is allocated for each data transmission. Thus, in the OFDMA context, P_B and P_R shall denote power transmitted per subchannel by the BS and RS respectively. We assume that the BS-RS and RS-MS links are assigned disjoint frequency bands for their signal transmissions. Thus, the MS receives inter-cell interference only from the RSs in the neighboring cells, and the RS receives interference only from the BSs of

the neighboring cells. We consider inter-cell interference from the first-tier of neighboring cells only. The inter-BS distance is equal to two times the coverage radius $2R_{\text{cov}}$. Let p_{act} be the probability that a subcarrier is being used for data transmission in the cell. It depends upon the traffic load in each cell. We assume uniform traffic load across all the cells in the system. Hence p_{act} is constant across all cells in a multi-cell system with 1:1 frequency reuse.

B. Inter-cell Interference

Let us evaluate the total inter-cell interference I^r at the reference RS, and I^m at a reference MS at the cell edge as shown in Fig. 3. For simplicity of analysis, we ignore shadowing on the interfering links and consider only the path loss while evaluating the inter-cell interference. We evaluate the total interference power I^r received at the RS shown in Fig. 3. It is the sum of the interference received from each neighboring BS, I_i^r . To determine I_i^r , we multiply the received power from an interfering BS by the probability of subcarrier activity p_{act} . Thus, the total interference at the RS is,

$$I^r = \sum_{i=1}^6 I_i^r = \sum_{i=1}^6 p_{\text{act}} P_B x_i^{-\eta}. \quad (6)$$

where $x_i = \sqrt{(2R_{\text{cov}})^2 + R_1^2 - 4R_1 R_{\text{cov}} \cos(i\frac{\pi}{3})}$, the distance from i^{th} neighboring BS to the reference RS. Similarly, the interference I^m received at an MS at the cell edge as shown in Fig. 3, is the sum of the interference I_i^m received from each neighboring BS, which in turn is the sum of the interference $I_{i,r}^m$ from each RS in the neighboring cells. Thus,

$$I^m = \sum_{i=1}^6 I_i^m = \sum_{i=1}^6 \frac{p_{\text{act}}}{N_R} \sum_{r=1}^{N_R} P_R d_{i,r}^{-\eta} \quad (7)$$

where $d_{i,r}$ is the distance from the reference MS to the r^{th} RS in the i^{th} neighboring cell. For example, $d_{1,r} = \sqrt{R_{\text{cov}}^2 + R_1^2 - 2R_1 R_{\text{cov}} \cos(\frac{2\pi r}{N_R})}$. We assume that the sub-carrier allocation algorithm is such that, each subcarrier has probability $1/N_R$ of being allotted to each RS in every neighboring cell. Hence we have the factor $1/N_R$ in (7).

In order to simplify the evaluation of the interference in (7), we determine the interference power from the first neighboring cell, $i = 1$, and scale it by the path loss $d_i^{-\eta}$ from each of the other neighboring BSs to the MS. For example, if I_1^m is the interference power from RSs in neighboring cell 1, the interference from cell i , I_i^m is approximated as $I_1^m d_i^{-\eta} / d_1^{-\eta}$. Here, $d_i = \sqrt{4R_{\text{cov}}^2 + R_{\text{cov}}^2 - 4R_{\text{cov}}^2 \cos(\frac{\pi i}{3})}$ is the distance from the reference MS to BS- i . Thus,

$$I^m \approx \frac{p_{\text{act}}}{N_R} \left(\sum_{r=1}^{N_R} P_R d_{1,r}^{-\eta} \right) \left(\sum_{i=1}^6 \frac{d_i^{-\eta}}{d_1^{-\eta}} \right) \quad (8)$$

C. Iterative Algorithm for Relay Placement

In the single cell scenario, the SNR_{BS-RS} and SNR_{RS-MS} depend only the distances R_1 and R_2 and the respective transmit powers respectively as given in (2).

Algorithm 1 Iterative evaluation of R_1 , N_R , R_{cov}

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 $R_{\text{cov}}^{(1)} \leftarrow R_{\text{cov}}^{(init)}$ 
 $R_{\text{cov}}^{(0)} \leftarrow 0$ 
 $N_R^{(1)} \leftarrow N_R^{init}$ 
 $i \leftarrow 1$ 
while  $|R_{\text{cov}}^{(i)} - R_{\text{cov}}^{(i-1)}| > \epsilon$  do
  for each  $R_1 \in (0, R_{\text{cov}})$  do
     $\mathbf{R}_{\text{cov}} \leftarrow \{\emptyset\}$ 
    Compute  $I^r$  and  $I^m$ 
     $p_{c_1} = Q\left(\frac{T + 10 \log(10 \frac{N}{10} + I^r) + 10\eta \log R_1 - P_B}{\sigma_1}\right)$ 
    if  $p_{c_1} < 0.5$  then
      break from for loop
    end if
     $p_{c_2} = Q\left(\frac{T + 10 \log(10 \frac{N}{10} + I^m) + 10\eta \log R_2 - P_R}{\sigma_2}\right)$ 
    Solve  $p_{c_1} \cdot p_{c_2} = 0.5$  for  $R_2$ 
    Append  $(R_1 + R_2)$  to  $\mathbf{R}_{\text{cov}}$ 
  end for
   $i \leftarrow i + 1$ 
   $R_{\text{cov}}^{(i)} = \max \mathbf{R}_{\text{cov}}$ 
   $R_1^{*(i)} = \arg \max \mathbf{R}_{\text{cov}}$ 
   $N_R^{(i)} = \lceil \frac{\pi}{\sin^{-1}(\frac{R_{\text{cov}}^{(i)} - R_1^{*(i)}}{R_1^{*(i)}})} \rceil$ 
end while

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However in the multi-cell scenario, since we consider inter-cell interference, the received signal to interference plus noise ratio (SINR) at the RS and MS is also a function of the inter-BS distance $2R_{\text{cov}}$, and the number of RSs N in every cell. As a result the knowledge of R_{cov} is required to determine R_2 , given a relay placement radius R_1 . We use an iterative Algorithm 1 to determine the RS placement radius R_1 that maximizes the cell coverage radius R_{cov} . The algorithm uses the value R_{cov} from the previous iteration to evaluate R_2 as a function of R_1 . Then we determine the R_1 which maximizes $R_1 + R_2$, and set this as the new value R_{cov} .

V. NUMERICAL RESULTS

In this section, we present the numerical results of the optimal RS placement problem formulated in Section III and Section IV. The system parameters are chosen according to Table I. In Fig. 4 we plot the coverage radius $R_{\text{cov}} = R_1 + R_2$ versus the RS placement radius R_1 . Given a value of R_1 , R_2 is evaluated as shown in (3). We also plot the number

| SYSTEM PARAMETERS | |
|--|------------------------|
| BS transmit power | $P_B = 36$ dBm |
| RS transmit power | $P_R = 28$ dBm |
| Path loss exponent | $\eta = 3.5$ |
| Shadowing standard deviation BS-RS | $\sigma_1 = 3$ dB |
| Shadowing standard deviation RS-MS | $\sigma_2 = 6$ dB |
| Noise level | $N = -100$ dBm |
| Decoding Threshold SINR | $T = 10$ dB |
| Probability of subcarrier being active | $p_{\text{act}} = 0.2$ |

of RSs required $N_R = \frac{\pi}{\sin^{-1}(R_2/R_1)}$. The maximum R_{COV} is attained approximately at $R_1 = 3550$ m. At this radial location of the RSs, $R_{COV} = 5475$ m. Thus the RSs are placed at $R_1^*/R_{COV} = 0.65$ fraction of the coverage radius. Also, we evaluate from (5) that at the optimal value, 6 RSs are required to cover the cell area with minimum coverage gaps and without the coverage discs of RSs overlapping each other.

For the multi-cell scenario, we use Algorithm 1 to evaluate the optimal RS placement radius R_1 and the corresponding R_{COV} . We set the initial values of $R_1^{(init)}$ and $R_{COV}^{(init)}$ to the R_1^* and R_{COV} determined in the single cell case. For the system parameters in Table I, and $\epsilon = 0.01$, the algorithm converges to the values $R_{COV} = 3900$ m and $R_1^* = 2338$ m. Fig. 6 shows the convergence of R_{COV} for $P_R = 26, 27$ and 28 dBm. R_{COV} reduces as compared to the single cell case due to inter-cell interference from the neighboring cells.

In Fig. 5, we plot the ratio R_1^*/R_{COV}^* versus the RS transmit power P_R , for the single cell and multi-cell scenarios. The BS transmit power is constant at $P_B = 36$ dBm. As the P_R increases, the RS can serve MS further away from it, and hence the ratio decreases. We also observe that the ratio is greater for the single cell case. In the multi-cell scenario, the optimal RS radius moves away from the cell edge in order to reduce the interference to neighboring cells.

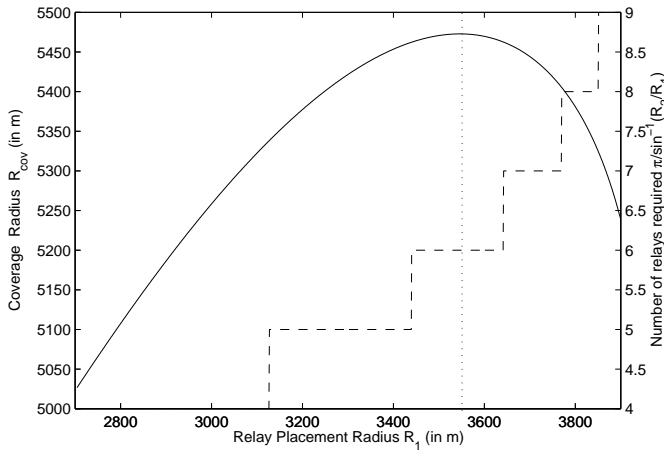


Fig. 4. Plots of the coverage radius R_{COV} and number of RSs N_R versus the RS placement radius R_1 .

VI. CONCLUSIONS

In this paper we have analyzed RS placement in cellular networks for maximum coverage improvement. We present a novel approach to determine the optimal RS positions by defining the coverage radius in terms of the probability of correct decoding at a point. The optimal RS positions have been determined both for the single cell and multi-cell scenarios. For the multi-cell scenario, we take into account inter-cell interference and propose an iterative algorithm to determine the optimal RS positions. The results presented in this paper can be used to determine the RS positions for maximum extension of coverage radius.

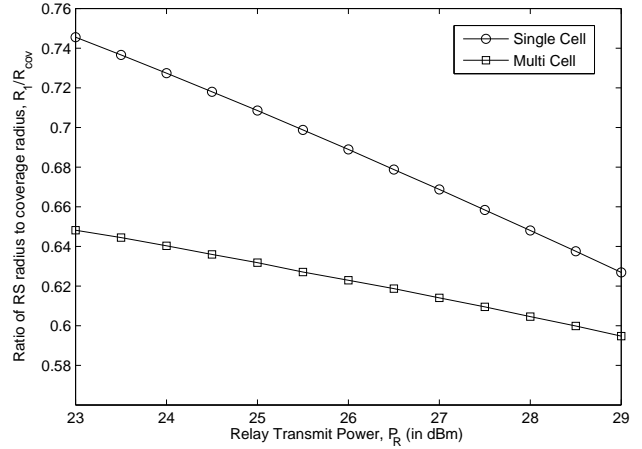


Fig. 5. Plots of the ratio R_1^*/R_{COV}^* versus RS transmit power for the single cell and multi-cell scenarios.

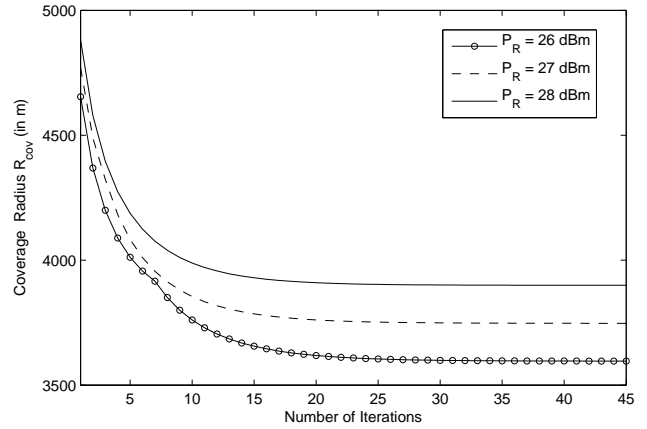


Fig. 6. Plots demonstrating the convergence of R_{COV} for the iterative algorithm proposed to determine the optimal R_1 in the multi-cell scenario.

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