# Efficient Financial Crises 

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## Introduction

- Banks and Financial Institutions rely heavily on short-term debt to finance their assets
- Implies exposure to bank runs or rollover risk
- Bank runs play important role in understanding Great Depression, perhaps most recent financial crisis
- Why do banks find fragile capital structure optimal?


## Fragility of Bank Capital Structure in Data

- Largest $0.1 \%$ of banks finance between 40 and $60 \%$ of assets with uninsured short-term liabilities
- Largest $0.1 \%$ of banks now hold $50 \%$ of total bank assets (up from $20 \%$ in 1992)
- For comparison, largest $0.1 \%$ of non-financial firms finance up to $20 \%$ of assets with short-term debt
- Only account for $15 \%$ of total non-financial firm assets


## This Paper

- Develop theory of optimal capital structure of banks
- Show optimal capital structure of banks is fragile
- there are states in which bank is inefficiently liquidated (bank runs)
- Show short-term debt is critical for fragility
- Analyze implications of theory for portfolio choices of banks


## Key Contributions

- Short-term debt with many small lenders introduces a coordination problem which makes debt-roll over difficult
- Coordination problem resembles problem of public good provision
- In moral hazard framework with fixed asset portfolio, depositors and banker will optimally choose to use short-term debt
- Short-term debt allows depositors to commit to bank runs
- Commitment to bank runs beneficial for resolving moral hazard
- Optimal capital structure features bank runs in equilibrium


## Other Findings

- Endogenize asset portfolio decisions in model with multiple banks
- With independent banks and bank returns, short-term debt may not commit depositors to bank runs
- Short-term debt not sufficient to resolve commitment problem
- Commitment problem can be resolved with correlated bank returns
- Optimal financial system features crises
- Planner subject to same constraints cannot improve outcomes $\Rightarrow$ Efficiency of crises


## Related Literature

- Bank runs: Diamond and Dybvig (1983)
- Bank Runs as Disciplining Device: Calomiris and Kahn (1991), Diamond and Rajan (2001)
- Lender Coordination Problems: Bolton and Scharfstein (1990), Brunnermeier and Oehmke (2013)
- Many others on optimal capital structure, crises


## Outline

- Example: When debt roll-over resembles a public good problem
- Benchmark Model: Single Bank, Many Depositors, and limited commitment
- Optimal contracts resemble short-term debt
- Optimal contracts feature ex-post debt-rollover problems
- Extension: Model with Multiple Banks
- With limited commitment, correlated and risky returns across banks is optimal
- Policy Implications


# Simple Example: <br> When Debt Rollover Resembles a Public Good Problem 

## Environment of Simple Example

- Time: $\mathrm{t}=1,2$
- $N$ Depositors' each owed $I / N$ in period 1
- Preferences:
- Depositors: $c_{1}+v_{i} c_{2}$ with $c_{t} \geq 0$
- $v_{i}$ is an i.i.d. with $G_{i}\left(v_{i}\right)$ and support $[\underline{v}, \bar{v}]$
- $v_{i}$ is private information
- $v=\left(v_{1}, \ldots, v_{N}\right)$
- Debt-Rollover:
- Requires I resources in period 1
- Delivers $Y$ units of output in period 2


## The Game Between Depositors

- Each depositor has a right to claim resources $I / N$ in period 1
- A mechanism specifying payments to depositors in period 1 and 2 is proposed
- If each depositor (knowing $v_{i}$ ) agrees to waive their right, project is continued
- If any depositor refuses, project is discontinued


## Rollover and Depositors' Discount Factors

- Consider designing general (direct) mechanisms $\left(p_{1}^{i}(v), p_{2}^{i}(v), x(v)\right)$ which respect:
- Private information of Depositors
- Participation constraints of depositors
- Raise I resources
- Will compare full information and private information outcomes


## Full Information Outcomes

- When depositors' discount factors are observable, rollover dominates no-rollover if and only if there exist payments $p_{2}^{i}(v)$ such that

$$
v_{i} p_{2}^{i}\left(v_{i}, v_{-i}\right) \geq I / N
$$

where $\sum_{i} p_{2}^{i}\left(v_{i}, v_{-i}\right) \leq Y$

- Implies rollover is efficient if

$$
I \frac{1}{N} \sum_{i} \frac{1}{v_{i}} \leq Y
$$

Lemma
If IE $\left[1 / v_{i}\right]<Y$ then as $N \rightarrow \infty$, the probability rollover is ex-post efficient tends to 1 .

## Efficient Rollover with Private Information

- When depositors' discount factors are unobservable, incentive compatibility requires

$$
\begin{aligned}
\int_{v_{-i}} & {\left[x\left(v_{i}, v_{-i}\right) v_{i} p_{2}^{i}\left(v_{i}, v_{-i}\right)+\left(1-x\left(v_{i}, v_{-i}\right)\right) \frac{I}{N}\right] d G_{-i}\left(v_{-i}\right) } \\
& \geq \int_{v_{-i}}\left[x\left(\hat{v}_{i}, v_{-i}\right) v_{i} p_{2}^{i}\left(\hat{v}_{i}, v_{-i}\right)+\left(1-x\left(\hat{v}_{i}, v_{-i}\right)\right) \frac{I}{N}\right] d G_{-i}\left(v_{-i}\right)
\end{aligned}
$$

- Participation requires

$$
\int_{v_{-i}}\left[x\left(v_{i}, v_{-i}\right) v_{i} p_{2}^{i}\left(v_{i}, v_{-i}\right)+\left(1-x\left(v_{i}, v_{-i}\right)\right) \frac{I}{N}\right] d G_{-i}\left(v_{-i}\right) \geq \frac{I}{N}
$$

- Resources (in ex-ante terms)

$$
\int_{v} x(v)\left[Y-\sum_{i} p_{2}^{i}(v)\right] d G(v) \geq 0
$$

## Efficient Rollover with Private Information

- Can show: a rollover rule, $x(v)$ is implementable if and only if $x(v)$ is increasing and

$$
\int_{v} x(v)\left[Y-\frac{I}{N} \sum_{i}\left[\frac{1-G_{i}\left(v_{i}\right)}{v_{i}^{2} g_{i}\left(v_{i}\right)}+\frac{1}{v_{i}}\right]\right] d G(v) \geq 0
$$

Lemma
If discount factors are such that $\underline{v} Y<I$ and $\left(1-G_{i}\left(v_{i}\right)\right) /\left(v_{i}^{2} g_{i}\left(v_{i}\right)\right)$ is decreasing, then $x(v) \rightarrow 0$ as $N \rightarrow \infty$

- For large $N$, difficult to construct mechanisms which get all depositors to agree to waive rights
- Similar to standard results from public goods literature (Rob (1989) and Mailath and Postlewaite (1990))


## Efficient Rollover with Private Information

- Reason difficult to construct rollover contracts
- Most impatient type requires more than pro-rata share to participate
- Implies rollover contract must subsidize impatient types in favor of patient types
- Implies patient types have incentives to under-report discount factor:
- Benefit: receive larger share of future returns
- Cost: lower probability of roll-over
- Costs tend to 0 as $N \rightarrow \infty$, Benefits do not
- For large $N$, not rolling over debt is ex-post inefficient and resembles runs or panics
- Next, show depositors endogenously choose capital structure with these outcomes


## Benchmark Model with Single Bank and Many Depositors

## Model Ingredients

- Standard repeated moral hazard environment (Holmstrom (1979))
- Banker must be provided incentives to exert effort
- Effort affects distribution of future returns
- Depositors experience private discount factor shocks (Diamond and Dybvig (1983))
- Depositors must be provided incentives to report discount factor truthfully
- Limited enforcement of contracts


## Environment

- Agents: N depositors, 1 banker
- Time: $\mathrm{t}=0,1,2$
- Depositors' Endowments: identical, $\left(\frac{I}{N}, 0,0\right)$
- Preferences:
- Banker: $c_{0}+c_{1}+\beta c_{2}$
- Depositors: $c_{0}+c_{1}+v_{i} c_{2}$
- $v_{i}$ is i.i.d., distribution $G_{i}\left(v_{i}\right)$, support $[\underline{v}, \bar{v}]$ and $\beta<\underline{v}$
- $v_{i}$ is private information, $v=\left(v_{1}, \ldots, v_{N}\right)$
- $c_{t} \geq 0$


## Investment Technology

- Investment in period $t=0,1$ requires $I$ goods and banker's effort, $e \in\left\{\pi_{l}, \pi_{h}\right\}$ with $\operatorname{cost} \bar{q}=q\left(\pi_{h}\right), 0=q\left(\pi_{l}\right)$
- Output:
- Period 1:
- Output: $I+y_{1}$

$$
y_{1}= \begin{cases}y_{h} & \text { w/ prob } e_{0} \\ 0 & \text { w/ prob } 1-e_{0}\end{cases}
$$

- Continuation requires $I$ re-invested and effort $e_{1}$
- Period 2 (if continued)
- Output: $I+\rho y_{1}+z_{2}$

$$
z_{2}= \begin{cases}y_{h} & \text { w/ prob } e_{1} \\ 0 & \text { w/ prob } 1-e_{1}\end{cases}
$$

- $\rho>0$


## Investment Contracts

- Focus on direct mechanisms
- Investment contract specifies: banker's effort, transfers, continuation rule
- Payments to depositors, $p_{t}^{i}$ :

$$
P^{d}=\left\{\left(p_{1}^{i}\left(y_{1}\right), p_{1 c}^{i}\left(y_{1}, v\right), p_{1 n}^{i}\left(y_{1}, v\right), p_{2}^{i}\left(y_{1}, z_{2}, v\right)\right)_{i \in\{1, \ldots, N\}}\right\}
$$

- Payments to the banker, $p_{t}^{b}: P^{b}=\left\{p_{1}^{b}\left(y_{1}\right), p_{2}^{b}\left(y_{1}, z_{2}, v\right)\right\}$
- Continuation rule: $x\left(y_{1}, v\right)$
- Recommended effort: $e_{0}, e_{1}\left(y_{1}, v\right)$


## Timing of Events



## Constraints on Investment Contracts

- Resource Constraints
- Non-negativity constraints
- Banker's incentive constraints (to exert high effort)
- Depositors' incentive constraints (to report $v_{i}$ truthfully)
- Depositors' participation constraints
- Enforcement constraints (to not re-negotiate the contract)

Constraints on Investment Contracts

- Resource Constraints

$$
\begin{aligned}
& p_{1}^{b}\left(y_{1}\right)+\sum_{i=1}^{N}\left[p_{1}^{i}\left(y_{1}\right)+x\left(y_{1}, v\right) p_{1 c}^{i}\left(y_{1}, v\right)+\left(1-x\left(y_{1}, v\right)\right) p_{1 n}^{i}\left(y_{1}, v\right)\right] \\
& \quad \leq I+y_{1}-I x\left(y_{1}, v\right) \\
& E_{e_{1}\left(y_{1}, v\right)} \sum_{i=1}^{N} p_{2}^{i}\left(y_{1}, z_{2}, v\right) \leq I+\rho y_{1}+E_{e_{1}\left(y_{1}, v\right)}\left(z_{2}-p_{2}^{b}\left(y_{1}, z_{2}, v\right)\right)
\end{aligned}
$$

## Constraints on Investment Contracts

- Banker's Incentives in period 1

$$
\begin{gathered}
\beta\left[\pi_{h} p_{2}^{b}\left(y_{1}, z_{h}, v\right)+\left(1-\pi_{h}\right) p_{2}^{b}\left(y_{1}, z_{l}, v\right)\right]-\bar{q} \\
\quad \geq \beta\left[\pi_{l} p_{2}^{b}\left(y_{1}, z_{h}, v\right)+\left(1-\pi_{l}\right) p_{2}^{b}\left(y_{1}, z_{l}, v\right)\right] \\
p_{2}^{b}\left(y_{1}, z_{h}, v\right) \geq \frac{\bar{q}}{\beta\left(\pi_{h}-\pi_{l}\right)}+p_{2}^{b}\left(y_{1}, z_{l}, v\right)
\end{gathered}
$$

- Let $U_{1}\left(y_{1}, v\right)=x\left(y_{1}, v\right)\left[\beta E_{\pi_{h}} p_{2}^{b}\left(y_{1}, z_{2}, v\right)-\bar{q}\right]$
- Banker's incentives in period 0

$$
\begin{equation*}
p_{1}^{b}\left(y_{h}\right)+\int_{v} U_{1}\left(y_{h}, v\right) d G(v) \geq \frac{\bar{q}}{\pi_{h}-\pi_{l}}+p_{1}^{b}\left(y_{l}\right)+\int_{v} U_{1}\left(y_{l}, v\right) d G(v) \tag{1}
\end{equation*}
$$

## Constraints on Investment Contracts

- Define $w\left(y_{1}, \hat{v}_{i}, v_{i}\right)$ as value of reporting $\hat{v}_{i}$ when true discount factor is $v_{i}$ :

$$
\begin{aligned}
w_{i}\left(y_{1}, \hat{v}_{i}, v_{i}\right) & =\int_{v_{-i}} x\left(y_{1}, \hat{v}_{i}, v_{-i}\right)\left(p_{1 c}^{i}\left(y_{1}, \hat{v}_{i}, v_{-i}\right)+v_{i} p_{2}^{i}\left(y_{1}, \hat{v}_{i}, v_{-i}\right)\right) d G_{-i}\left(v_{-i}\right) \\
& +\int_{v_{-i}}\left(1-x\left(y_{1}, \hat{v}_{i}, v_{-i}\right) p_{1 n}^{i}\left(y_{1}, \hat{v}_{i}, v_{-i}\right) d G_{-i}\left(v_{-i}\right) .\right.
\end{aligned}
$$

- Incentive and Participation Constraints:

$$
\begin{aligned}
& w_{i}\left(y_{1}, v_{i}, v_{i}\right) \geq \max _{\hat{v}_{i}} w_{i}\left(y_{1}, \hat{v}_{i}, v_{i}\right) \\
& \pi_{h} \int_{v_{i}} w_{i}\left(y_{h}, v_{i}, v_{i}\right) d G_{i}\left(v_{i}\right)+\left(1-\pi_{h}\right) \int_{v_{i}} w_{i}\left(y_{l}, v_{i}, v_{i}\right) d G_{i}\left(v_{i}\right) \geq I / N
\end{aligned}
$$

## Nature of Limited Commitment Problem

- Allow depositors to construct new continuation contracts after $p_{1}^{i}\left(y_{1}\right)$ paid and $v$ realized
- New continuation contracts must be incentive feasible
- non-negativity of depositor's and banker's consumption
- Depositors' incentive and participation constraints
- Banker's incentive constraint
- Resource constraints


## Enforceable Contracts

- Contract is enforceable if no other continuation contract improves ex-ante welfare and is incentive feasible:


## Improve Ex-ante welfare

$$
\begin{aligned}
& \sum_{i} \int_{v}\left[\hat{x}(v)\left(\hat{p}_{1 c}^{i}(v)+v_{i} \hat{p}_{2}^{i}(v)\right)+(1-\hat{x}(v)) \hat{p}_{1 n}^{i}(v)\right] d G(v) \\
& >\sum_{i} \int_{v}\left[x\left(y_{1}, v\right)\left(p_{1 c}^{i}\left(y_{1}, v\right)+v_{i} p_{2}^{i}\left(y_{1}, v\right)\right)+\left(1-x\left(y_{1}, v\right)\right) p_{1 n}^{i}\left(y_{1}, v\right)\right] d G(v)
\end{aligned}
$$

$$
p_{1}^{i}\left(y_{1}\right)+\hat{x}(v) \hat{p}_{1 c}^{i}(v)+(1-\hat{x}(v)) \hat{p}_{1 n}^{i}(v) \geq 0
$$

- Do not require pareto improvements


## Benchmark Model: <br> Characterizing Optimal Contracts and Bank Runs

## Characterizing Optimal Contracts

- Outcomes under Full Commitment if moral hazard is severe
- Liquidate project after low period 1 output
- Continue project after high period 1 output
- Many state-contingent plans implement optimum
- Liquidation Outcomes resemble bank runs
- Outcomes under limited commitment mimic commitment outcomes
- With full info of discount factors, cannot commit to liquidate
- Short-term debt-like claims with private info needed
- Long-term debt-like claims with private info do not work


## Characterizing Optimal Contracts

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## Incentive Benefits of Liquidation

- Recall banker's effort constraints

$$
\begin{aligned}
& p_{2}^{b}\left(y_{1}, z_{h}, v\right) \geq \frac{\bar{q}}{\beta\left(\pi_{h}-\pi_{l}\right)}+p_{2}^{b}\left(y_{1}, z_{l}, v\right) \\
& p_{1}^{b}\left(y_{h}\right)+\int_{v} U_{1}\left(y_{h}, v\right) d G(v) \geq \frac{\bar{q}}{\pi_{h}-\pi_{l}}+p_{1}^{b}\left(y_{l}\right)+\int_{v} U_{1}\left(y_{l}, v\right) d G(v)
\end{aligned}
$$

- Moral hazard plus limited liability imply

$$
U_{1}\left(y_{l}, v\right)=x\left(y_{l}, v\right) \frac{\pi_{l} \bar{q}}{\pi_{h}-\pi_{l}}
$$

or $U_{1}\left(y_{l}, v\right)>0$ if $x\left(y_{l}, v\right)>0$

- Implies banker earns rents if project is continued
- Liquidating after low output reduces $U_{1}\left(y_{l}, v\right)$, relaxes banker's period 0 incentive constraint
- Liquidating after low output potentially costly for depositors (forgone surplus)


## Liquidation After Low Output

- Tradeoff involving reductions in $x\left(y_{l}, v\right)$ :
- Ex-ante benefit from reducing payment to banker, $p_{1}^{b}\left(y_{h}\right)$

$$
\pi_{h} \underbrace{\frac{\pi_{l} \bar{q}}{\pi_{h}-\pi_{l}}}_{\text {banker's rent }}
$$

- Ex-ante maximal cost from forgone surplus

$$
\left(1-\pi_{h}\right) \underbrace{\left[-I+\bar{v}\left(I+\pi_{h} z_{h}-\frac{\pi_{h} \bar{q}}{\beta\left(\pi_{h}-\pi_{l}\right)}\right)\right]}_{\text {maximum }(\bar{v}) \text { potential surplus }}
$$

## Lemma (Liquidate after Low Output)

The optimal contract satisfies $x\left(y_{l}, v\right)=0$ for all $v$ if

$$
\frac{\pi_{h} \pi_{l} \bar{q}}{\pi_{h}-\pi_{l}}-\left(1-\pi_{h}\right)\left[-I+\bar{v}\left(I+\pi_{h} z_{h}-\frac{\pi_{h} \bar{q}}{\beta\left(\pi_{h}-\pi_{l}\right)}\right)\right]>0
$$

## Continuation After High Output

- Increasing $x\left(y_{h}, v\right)$ reduces payment to banker and (potentially) increases surplus
- Incentive benefit: $\beta \pi_{h} p_{2}^{b}\left(y_{h}, z_{h}, v\right)-\bar{q}$
- Surplus benefit: $-I+\sum_{i} v_{i} p_{2}^{i}(v)$
- Surplus maximizing rule $x\left(y_{h}, v\right)=1$ if and only if $\sum_{i} v_{i} p_{2}^{i}\left(y_{h}, v\right)+\beta \pi_{h} p_{2}^{b}\left(y_{h}, z_{h}, v\right)-I-\bar{q} \geq 0$


## Lemma (Continute after High Output)

The optimal contract satisfies $x\left(y_{h}, v\right)=1$ for all $v$ if

$$
\beta\left(I+\rho y_{h}+\pi_{h} z\right) \geq I+\bar{q}
$$

- Assumption requires project to yield higher total surplus following high output under banker's discount factor than resource and effort cost


## Optimal Contracts

- Have found optimal continuation rule
- Can solve for optimal payments
- Focusing on period 1 payments
- Following low output, set $p_{1}^{i}\left(y_{l}\right)=I / N$ or $p_{1 n}^{i}\left(y_{l}, v\right)=I / N$ (or any combination)
- Following high output, depositors willing to pay $I / N$ for pro-rata share if

$$
I<\underline{v}\left[I+\rho y_{h}+\pi_{h} z_{h}-\frac{\pi_{h} \bar{q}}{\beta\left(\pi_{h}-\pi_{l}\right)}\right]
$$

(optimum more complicated typically)

- Optimum resembles short-term debt with liquidations, or long-term debt with bankruptcy, etc


## Inefficient Liquidations

- Will say liquidations resemble bank runs if they are ex-post inefficient
- Ex-post inefficient if under full info, depositor welfare can be improved (ex-post) by continuing


## Lemma (Ex-Post Inefficient Liquidations, Bank Runs)

 If$$
I E\left[\frac{1}{v_{i}}\right]<\underbrace{I+\pi_{h} z_{h}-\frac{\pi_{h} \bar{q}}{\beta\left(\pi_{h}-\pi_{l}\right)}}_{\begin{array}{c}
\text { Total Returns after low output } \\
\text { net of banker's rents }
\end{array}}
$$

then the probability that liquidation resembles a bank run tends to 1 as $N \rightarrow \infty$.

## Characterizing Optimal Contracts

- Outcomes under Full Commitment if moral hazard is severe
- Liquidate project after low period 1 output
- Continue project after high period 1 output
- Many state-contingent plans implement optimum
- Liquidation Outcomes resemble bank runs
- Outcomes under limited commitment mimic commitment outcomes
- With full info of discount factors, cannot commit to liquidate
- Short-term debt-like claims with private info needed
- Long-term debt-like claims with private info do not work


## Efficient Liquidations and Bank Runs

- If liquidations ex-post inefficient, for any long-term contract, depositors will re-negotiate (with high probability)


## Proposition (Time Inconsistency)

If liquidations resemble banks runs, or,

$$
I E\left[\frac{1}{v_{i}}\right]<I+\pi_{h} z_{h}-\frac{\pi_{h} \bar{q}}{\beta\left(\pi_{h}-\pi_{l}\right)}
$$

then under full information of discount factors as $N \rightarrow \infty$, no contract implements optimum with commitment. Equilibrium outcomes feature no liquidation.

- Proposition implies that if $v_{i}$ is observable, optimal continuation rule is not enforceable for large $N$


## Optimal Contracts with Limited Commitment

## Proposition (Sufficiency of Short-Term Debt)

Suppose $\left(1-G_{i}\left(v_{i}\right)\right) /\left(v_{i}^{2} g_{i}\left(v_{i}\right)\right)$ is decreasing in $v_{i}$ and

$$
\underline{v}\left[I+\pi_{h} z_{h}-\frac{\pi_{h} \bar{q}}{\beta\left(\pi_{h}-\pi_{l}\right)}\right]<I .
$$

As $N \rightarrow \infty$, the optimal continuation rule is enforceable if $p_{1}^{i}\left(y_{1}\right)=I / N$.

- Main result: choosing high first period transfers when depositors' discount factors are unobservable introduces a "public goods" problem that resolves the time-inconsistency problem
- Enforcement constraint slack (in terms of welfare) but determines timing of payments


## How Short-Term Debt Replicates Commitment

- Suppose $p_{1}^{i}\left(y_{l}\right)=I / N$
- Look for re-negotiation contracts that feature continuation with positive probability
- Aggregate Resources:

$$
\begin{gathered}
p_{1}^{b}\left(y_{l}\right)+\sum_{i} p_{1}^{i}\left(y_{l}\right)+\sum_{i}\left[\hat{x}(v) \hat{p}_{1 c}^{i}(v)+(1-\hat{x}(v)) \hat{p}_{1 n}^{i}(v)\right] \leq I-\hat{x}(v) I \\
\sum_{i}\left[\hat{x}(v) \hat{p}_{1 c}^{i}(v)+(1-\hat{x}(v)) \hat{p}_{1 n}^{i}(v)\right] \leq-\hat{x}(v) I
\end{gathered}
$$

- Limited Liability:

$$
\underbrace{\frac{I}{N}}_{p_{1}^{i}\left(y_{l}\right)}+\hat{x}(v) \hat{p}_{1 c}^{i}(v)+(1-\hat{x}(v)) \hat{p}_{1 n}^{i}(v) \geq 0
$$

- Implies $\hat{p}_{1 c}^{i}(v)=-I / N$


## How Short-Term Debt Replicates Commitment

- Then, the participation constraint (to waive right to $I / N$ ) is

$$
\frac{I}{N}+\int_{v_{-i}} \hat{x}\left(v_{i}, v_{-i}\right)\left[-\frac{I}{N}+v_{i} \hat{p}_{2}^{i}\left(v_{i}, v_{-i}\right)\right] d G_{-i}\left(v_{-i}\right) \geq \frac{I}{N}
$$

- Re-negotiation faces exact public good problem as above
- Choosing $p_{1}^{i}\left(y_{l}\right)=I / N$ makes it difficult to get depositors to waive right
- Implies depositors can commit to liquidate after low output


## Why Long-Term Debt Does Not Work

- $\operatorname{Suppose} p_{1}^{i}\left(y_{l}\right)=0$ but $p_{1 n}^{i}\left(y_{l}, v\right)=I / N$
- Look for re-negotiation contracts that feature continuation with positive probability
- Aggregate Resources:

$$
\sum_{i}\left[\hat{x}(v) \hat{p}_{1 c}^{i}(v)+(1-\hat{x}(v)) \hat{p}_{1 n}^{i}(v)\right] \leq I-\hat{x}(v) I
$$

Note: $I$ still "in the bank"

- Limited Liability: $\hat{x}(v) \hat{p}_{1 c}^{i}(v)+(1-\hat{x}(v)) \hat{p}_{1 n}^{i}(v) \geq 0$
- Participation:
$\int_{v_{-i}}\left[\hat{x}\left(v_{i}, v_{-i}\right)\left(\hat{p}_{1 c}^{i}\left(v_{i}, v_{-i}\right)+v_{i} \hat{p}_{2}^{i}\left(v_{i}, v_{-i}\right)\right)+\left(1-\hat{x}\left(v_{i}, v_{-i}\right)\right) \hat{p}_{1 n}^{i}\left(v_{i}, v_{-i}\right)\right] d G_{-i}\left(v_{-i}\right) \geq 0$


## Why Long-Term Debt Does Not Work

- Can choose $\hat{x}(v)=1, \hat{p}_{1 c}^{i}(v)=\hat{p}_{1 n}^{i}(v)=0$ and $\hat{p}_{2}^{i}(v)=Y / N$ where

$$
Y=I+\pi_{h} z_{h}-\frac{\pi_{h} \bar{q}}{\beta\left(\pi_{h}-\pi_{l}\right)}
$$

- Clearly, this alternative contract is IC, feasible, and satisfies participation
- Status quo welfare $=I$
- Re-negotiated welfare $=\frac{Y}{N} \sum_{i} E\left[v_{i}\right]$
- Since $I<E\left[v_{i}\right] Y$, as $N \rightarrow \infty, \hat{x}(v) \rightarrow 1$ (such a re-negotiation is successful)
- Long-term debt (or equity) with bankruptcy does not work


## Optimal Bank Maturity

- Constrained efficiency requires promising to re-pay entire principal $\left(\sum_{i} p_{1}^{i}\left(y_{1}\right) \geq I\right)$
- Contracts which do not promise to re-pay entire principal are worse
- Such contracts do not commit depositors to liquidate the bank ex-post
- Contracts which do not promise to re-pay entire principal resemble long-term debt or equity
- In this sense, optimal for banks to use short-term debt over longer-term contracts
- In paper, show this in decentralized economy with explicit short, long-term debt contracts


## Extended Model with Multiple Banks \& Policy Implications

## Crises vs. Individual Bank Failures

- Commitment to liquidate individual bank requires limited availability of external resources
- Show in environment with multiple banks, depositors and bankers also have incentives to choose investments that ensure limited availability of external resources
- Will consider two extreme examples:
- Replica economy of above with 2 bankers, $2 N$ depositors, fully independent
- Economy with perfectly correlated, riskier returns
- Will show strict preference for correlated, risky return economy
- Implies optimality of crises


## Independent Replica Economies

- 2 bankers, $2 N$ depositors
- Project returns and depositor discount factors drawn independently
- Immediate that optimal continuation rule under commitment is identical to one bank outcome $x\left(y_{h}, v\right)=1$ and $x\left(y_{l}, v\right)=0$ for both banks
- Ask, under limited commitment, can depositors enforce $x\left(y_{l}, v\right)=0$ ?


## Independent Replica Economies

- Answer:
- If $y^{1}, y^{2}=y_{h}, y_{l}$, then enforcement is possible
- If $y^{1}, y^{2}=y_{l}, y_{l}$, then enforcement is not possible
- Focus on case where both bank earn low returns
- Aggregate resources $2 I$, aggregate welfare from status quo $=2 I$
- Construct re-negotiation contract with pro-rata shares: $\hat{p}_{1 c}^{i}(v)=-I / N$ and $\left.\hat{p}_{2}^{i}(v)=\frac{1}{N}\left(I+\pi_{h} z_{h}-\pi_{h} \bar{q} /\left(\beta\left(\pi_{h}-\pi_{l}\right)\right)\right)\right)$
- Do $N$ most patient depositors want to undertake such a deviation?


## Independent Replica Economies

- If depositor with median patience under $G_{i}$ accepts, then for $N$ large, $N$ depositors will accept
- Implies exist incentive compatible continuation contracts which strictly improve depositor's welfare
- Consider incentives of a single banker
- From ex-ante perspective, under low effort, with probability $\left(1-\pi_{l}\right)\left(1-\pi_{h}\right)$, both banks will realize $y_{1}=y_{l}$
- For $N$ large, with probability $1 / 2, x(v)=1$
- Implies incentive constraint of banker given by

$$
p_{1}^{b}\left(y_{h}\right)+\int_{v} U_{1}\left(y_{h}, v\right) d G(v) \geq \frac{\bar{q}}{\pi_{h}-\pi_{l}}+\frac{1}{2}\left(1-\pi_{h}\right) \frac{\pi_{l} \bar{q}}{\pi_{h}-\pi_{l}}
$$

which is strictly tighter than the commitment outcome

## Correlated Return Economy

- Assume project returns are perfectly correlated and effort is leontief:

$$
\operatorname{Pr}\left[\left(y^{1}, y^{2}\right) \in\left\{\left(y_{l}, y_{h}\right),\left(y_{h}, y_{l}\right)\right\}\right]=0
$$

and

$$
\operatorname{Pr}\left[\left(y^{1}, y^{2}\right)=\left(y_{h}, y_{h}\right)\right]=\min \left\{e_{0}^{1}, e_{0}^{2}\right\}
$$

and similarly in period 2

- Leontief implies no added advantage in terms of incentive provision in commitment outcome
- Also assume $y_{1}=-I / 2$ so that if $y^{1}, y^{2}=y_{l}, y_{l}$, aggregate resources are I
- Increase $y_{h}$ so that planner under commitment with $x\left(y_{h}, v\right)=1$ and $x\left(y_{l}, v\right)=0$ indifferent between independent projects and correlated, risky projects


## Correlated Return Economy

- After high outcomes, continuation is feasible, optimal as before
- After low outcomes, each of $2 N$ depositors need to finance a single bank operation
- If financed with short-term debt, exact same public goods problem implies no incentive feasible continuation contract has $x\left(y_{l}, v\right)>0$ for either bank
- Implies commitment outcome enforceable


## Efficiency of Financial Crises

Proposition (Efficient Crises)
If returns are perfectly correlated and sufficiently risky, then commitment outcomes are enforceable.

- Strict preference for aggregate crises (all banks earn low returns, all banks are liquidated)
- Suggests fragile banks should undertake riskier returns more correlated with aggregate outcomes than non-fragile banks
- Besides forgone profits, no additional external cost to crises


## Policy Implications

- In absence of external costs, crises are efficient
- Optimal bank maturity responds to policies that distort moral hazard problem or income process of banks
- Implications for securitization and mortgage modification programs:
- Securitization creates a disperse group of debtors
- Inability to re-negotiate ex-post may be a feature of the system


## Conclusion

- Developed model and conditions under which banks prefer fragile capital structure
- Along equilibrium path, bank runs occur
- Short-term debt allows small depositors to commit to ex-post inefficient runs
- Long-term debt/equity may not attain same level of commitment
- Limited commitment problems imply preference for correlated, risky outcomes in financial sector

