Currency Stability Using Blockchain Technology

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Motivation

- Tokens used as a means of payment central component of most blockchain technologies
  - Bitcoin: means of payment primary purpose
  - Many others: "utility token" used to perform transactions on the blockchain

- To date, existing crypto-currencies too volatile to be effective medium of exchange or store of value
Price Instability with Current Crypto-Currencies

- Crypto-currencies have been too volatile to be used as money

- BTC order of magnitude more volatile than EUR, Gold, SP500

Source: Coinbase, FRBoF
This Paper

- Currency stability challenging, even with exchange rate peg
  - If not 100% backed, peg vulnerable to self-fulfilling attacks
  - Exchange rate pegs admit multiple equilibria as in Obstfeld (1996)

- We develop new theory of pegs with less than 100% backing
  - Show optimal exchange rate policy necessarily dynamic
  - Exchange rate adjusts (optimally) to trade requests
  - Optimal, gradual exchange capture much of stability of traditional peg, but immune to speculative attacks
    - Ex post exchange rate depreciations (under pressure) unwind ex ante incentives to speculate (as reduced convertibility in Green and Lin (2003) eliminates ex ante incentives to run)
Why Blockchain Matters

• Theory is agnostic to the currency involved
  ◦ Applies equally well to government issued fiat as to blockchain crypto-currency
  ◦ Theory shows how to resolve Obstfeld (1996) multiplicity “problem”

• Blockchain important primarily for implementation
  ◦ Optimal policy depends on real-time currency demand
  ◦ Specifying/communicating such a policy difficult (moral hazard)
  ◦ We use “smart contracts”—rich, state-contingent contracts verified and credibly enforced by an irreversible distributed ledger blockchain—to implement the optimal exchange rate policy (in progress)
Related Literature

Exchange Rate Stability


Suspension of Convertibility

○ Diamond and Dybvig (1983), Green and Lin (2003), many others

Existing Crypto-currency Stable-Coins (and white papers)

○ Tether, TrueUSD, Bridgecoin, Dai, NuBits, Nomin, Basecoin, Carbon, USD Fragments, AAA Reserve Currency

More on Stablecoins
Example: Using State Contingent Policy to Eliminate Speculative Attacks
Model Environment

• Two periods, \( t = 0, 1 \)

• Continuum (measure 1) of traders each own 1 Crypto-Peso

• Each trader is of type \( \theta \in \{C, F\} \) (Crypto, Foreign)
  
  ○ Crypto traders care about period 1 consumption in (mostly) Crypto-Peso currency goods
    
      - Crypto traders are potential speculators
  
  ○ Foreign traders care about period 0 consumption in foreign currency goods
  
  ○ \( \text{Prob}(\theta = C) = \mu_C \), i.i.d. across traders

• Let \( e_t \) denote the period \( t \) USD price of Crypto-Pesos
Actions and Payoffs

- **Foreign** traders’ action set in period 0: \{attack\}

- Attacking yields utility: \(u(e_0)\)

- Foreign traders, endowed with 1 crypto-peso, require immediate conversion into foreign reserves
Actions and Payoffs

- **Crypto** traders’ action set in period 0: \{*wait*, *attack*\}

- Waiting yields utility:  \( u((1 - \lambda)e_1 + \lambda) \)
  - \( \lambda \equiv \) exogenous fraction of period 1 pesos spent on foreign goods

- Attacking yields utility

\[
u \left( \left[ (1 - \lambda)e_1 + \lambda \right] \left[ \frac{e_0}{e_1} - t \right] \right)\]

- Convert Crypto-Peso to USD in period 0 at rate \( e_0 \)
- Convert USD back into Crypto-Pesos in period 1 at rate \( e_1 \)
- Fixed, round-trip transaction cost \( t > 0 \)
The Currency Board and Obstfeld Policies

- Currency board sets policy with initial USD reserves $R_0$

### Definition (Limited Contingency Policies)

A *limited contingency policy* converts crypto-pesos to USD at a fixed rate $e_0$ as long as feasible. When not feasible, converts fraction of demand uniformly at $e_0$.

- Restrict attention to *limited contingency* policies (Obstfeld (1996)):
  - If total conversion demand, $x$ satisfies $xe_0 < R_0$, convert at $e_0$
  - If $xe_0 > R_0$, convert as much as feasible uniformly, allow exchange rate to float at $e_f$ after
    - Implies each of $x$ demand convert $y = R_0 / (xe_0)$ at $e_0$
    - Think of $e_f$ as small implying Crypto-peso "overvalued"
    - Can think of $e_f$ as priced by arbitrageurs
Optimal Limited Contingency Policy

• Currency board using limited contingency policies solves

$$\max_{e^0,e^1} (1 - \mu_C)u(e^0) + \mu_C u((1 - \lambda)e^1 + \lambda)$$

subject to

$$(1 - \mu_C)e^0 \leq R_0$$

$$(1 - \mu_C)e^0 + \mu_C(1 - \lambda)e^1 \leq R_0$$

and the no-speculation incentive constraint,

$$u((1 - \lambda)e^1 + \lambda) \geq u\left(\begin{array}{c}
\frac{e^0}{e^1} - t \\
\text{Cons. per Crypto-Peso} \\
\text{Spec. Profit}
\end{array}\right)$$
Optimal Obstfeld Policy

- Since currency board exhausts reserves in period 1, if incentive constraint slack, then optimal $e_0$ satisfies

\[ u'(e_0) = u' \left( \frac{R_0 - (1 - \mu_C)e_0}{\mu_C} + \lambda \right) \]

or

\[ e_0^* = R_0 + \mu_C \lambda \]

- This policy is incentive-feasible if \((1 - \mu_C)e_0^* \leq R_0\) and

\[ (1 - \lambda)e_1^* + \lambda \geq [(1 - \lambda)e_1^* + \lambda] \left[ \frac{e_0^*}{e_1^*} - t \right] \]

or

\[ \frac{e_0^*}{e_1^*} \leq 1 + t. \]
Proposition (Optimal Obstfeld Policies and Multiplicity)

If \((1 - \mu_C)\lambda \leq R_0\) and \(\lambda\) sufficiently close to 1, then the optimal, incentive-feasible Obstfeld policy satisfies

\[
e_0^* = R_0 + \mu_C \lambda, \quad e_1^* = \frac{R_0 - (1 - \mu_C)\lambda}{1 - \lambda}.
\]

Moreover, if \(e_f\) sufficiently small, then this policy admits another equilibrium where all crypto traders speculate.

Proof of multiplicity:

- Conjecture equilibrium where all crypto traders demand conversion
- Since \(e_0^* > R_0\), currency board will run out of reserves and exchange rate will float
- Consumption from speculating:
  \[
  \left[(1 - \lambda)e_f + \lambda\right]\left[\frac{e_0^*}{e_f}y + 1 - y - t\right], \text{ where } y = \frac{R_0}{e_0^*}
  \]
- For \(e_f\) sufficiently small, speculation is worthwhile
Optimal, Contingent Policies

• Consider next fully contingent policies:
  ○ Let $x$ denote total demand for USD in period 0
  ○ A contingent policy is $e_t(x)$

• An obvious policy that is immune to speculative attacks:

\[
e_0(x) = \begin{cases} 
  e_0^* & \text{if } x = (1 - \mu_C) \\
  e_f & \text{if } x \neq (1 - \mu_C) 
\end{cases}, \quad e_1(x) = \begin{cases} 
  e_1^* & \text{if } x = (1 - \mu_C) \\
  e_f & \text{if } x \neq (1 - \mu_C) 
\end{cases}
\]
Contingent Policies

Proposition (Contingent Policies and Uniqueness)
There exist contingent policies that uniquely implement the efficient exchange rate policy.

Example very stylized

- Assumes total demand observed before setting exchange rates
- No sequential service constraints (not in Obstfeld (1996) either)
- Assumes no risk in foreign vs crypto currency demand; too stark for crypto-currencies
- Next, relax these assumptions
Efficient, History-Contingent Exchange Rate Protocols
A Finite Trader Economy

Two model modifications:

- Finitely many traders, $J$
  - $d^i_0 \equiv$ report of trader $j$ ($d^i_0 = 1$ implies foreign)
  - $D^j_0 = (d^1_0, \ldots, d^j_0)$

- Policies respect sequential service:
  - $e^j_0(D^j_0) \equiv$ history-contingent exchange rate offered to trader $j$
  - Sequential service: $e^j_0(D^j_0)$ measurable with respect to $D^j_0$

- These changes imply model is subject to aggregate risk
  - Interpret this risk as aggregate shock to demand for crypto-pesos
  - Risk indistinguishable (for currency board) from speculative attack
Policies and Objectives

Optimal policy solves

\[
\max \mathbb{E} \sum_{j=1}^{J} \left[ d^j_0 u(e^j_0(D^j_0)) + (1 - d^j_0)u \left( (1 - \lambda)e_1(D^j_0) + \lambda \right) \right]
\]

subject to the reserve transition equations

\[
R^j_0(D^j_0) = R^j_{0-1}(D^j_{0-1}) - d^j_0 e^j_0(D^j_0)
\]

the feasibility constraints

\[
\forall j \in \{1, \ldots, J\} \text{ and } D^j_{0-1}, \quad e^j_0(D^j_0) \leq R^j_{0-1}(D^j_{0-1})
\]

\[
\forall D^j_0, \quad (1 - \lambda)e_1(D^j_0) \sum_{j=1}^{J} (1 - d^j_0) \leq R_0 - \sum_{j=1}^{J} d^j_0 e^j_0(D^j_0)
\]

and the incentive constraints

\[
\forall D^j_0, \quad \mathbb{E} \left[ u \left( (1 - \lambda)e_1(D^j_0) + \lambda \right) \bigg| D^j_0 \right] \geq \mathbb{E} \left[ u \left( \left[ (1 - \lambda)e_1(\hat{D}^j_0) + \lambda \right] \left[ \frac{e^j_0(\hat{D}^j_0)}{e_1(\hat{D}^j_0)} - t \right] \right) \bigg| D^j_0 \right]
\]
Finding Optimal Policies

• Conjecture (and later verify) incentive constraints are slack

• Policy determined as solution to straightforward dynamic programming problem

• State variables:
  ○ $\Theta \equiv \text{sum of previous “crypto” reports}$
  ○ $R \equiv \text{remaining reserves}$

• Period 1:

$$W(\Theta; R) = \max_{e \leq R / [\Theta(1-\lambda)]} \Theta u \left( (1-\lambda)e + \lambda \right) = \Theta u \left( \frac{R}{\Theta} + \lambda \right)$$

• Period 0, trader $j$:

$$V_j^0(\Theta; R) = \max_{e \leq R} (1-\mu_C) \left[ u(e) + V_{j+1}^0(\Theta; R-e) \right] + \mu_C V_{j+1}^0(\Theta + 1; R)$$

where $V_{j+1}^0(\Theta; R) = W(\Theta; R)$
An Analytically Tractable Case
A Tractable Case

- Suppose $J = 3$ and $u(x) = -\exp(-\alpha x)$

- Straightforward to solve (by hand) dynamic program assuming incentive constraints slack

- Will show:
  - As $\lambda \to 1$ and $\mu_C \to 1$, not speculating a dominant strategy
  - Implies optimal policy admits a unique (no speculation) equilibrium

- Key feature of optimal policy for proof:
  - Government retains reserves if any traders report they are crypto
  - Period 1 exchange rate satisfies:

$$e_1(\Theta; R) = \begin{cases} \frac{R}{\Theta(1-\lambda)} & \text{if } \Theta \geq 1 \& \frac{R}{\Theta(1-\lambda)} \geq e_f \\ e_f & \text{otherwise} \end{cases}$$

- Implies as $\lambda \to 1$, traders expect large appreciation unless government out of reserves
Incentive Compatibility

• Incentives for Trader 3 require

\[
(1 - \lambda)e_1(\Theta + 1; R) + \lambda \geq \left[ (1 - \lambda)e_1(\Theta; R - e_3^3(\Theta; R)) + \lambda \right]\left[ \frac{e_0^3(\Theta; R)}{e_1(\Theta; R - e_0^3(\Theta; R))} - t \right].
\]

• When \( \Theta \geq 1 \), last trader knows government will retain reserves to period 1

• \( \lambda \rightarrow 1 \Rightarrow \) trader expects appreciation \( \Rightarrow \) speculation not profitable

• If \( e_f \) not too small, speculation also not profitable when \( \Theta = 0 \)

• Implies independent of previous players strategies, not speculating dominant strategy for Trader 3

• Also implies can use objective probability Trader 3 is crypto to evaluate incentives for Trader 2
Incentive Compatibility

- In case $\Theta = 0$ (interesting case), incentives for Trader 2 require

\[
(1 - \mu_C)u \left( (1 - \lambda)e_1(1;R_1 - e_0^3(1;R_1)) + \lambda \right) + \mu_C u \left( (1 - \lambda)e_1(2;R_1) + \lambda \right)
\]

\[
\geq (1 - \mu_C)u \left( (1 - \lambda)e_f + \lambda \left[ \frac{e_0^2(0;R_1)}{e_f} - t \right] \right)
\]

\[
+ \mu_C u \left[ (1 - \lambda)e_1(1;R_3) + \lambda \left[ \frac{e_0^2(0;R_1)}{e_1(1;R_3)} - t \right] \right]
\]

where $R_3 = R_1 - e_0^2(0;R_1)$

- As $\lambda \to 1$, if trader 3 is crypto, trader 2 expects appreciation

- But if trader 3 is foreign, trader 2 expects depreciation ($e_f$ small)

- As $\mu_C \to 1$, $Pr($exchange rate floats$) \to 0$

- As $\lambda \to 1$ and $\mu_C \to 1$, truth-telling dominant for trader 2 (similar idea for trader 1)
Optimal Policy Admits Unique Equilibrium

Proposition (Finite Complex Policies and Uniqueness)

For \( \lambda \) and \( \mu_C \) in a neighborhood of \( \lambda = \mu_C = 1 \) and \( 1 + t \geq R_0 \left[ \frac{1}{e_f} - 1 \right] \), the efficient exchange rate policy is incentive compatible. Moreover, truth-telling, or no speculation is the unique equilibrium.

- Have shown optimal exchange rate resembles a peg
- Optimal policy is immune to (purely) speculative attacks
- Optimal policy tolerates some currency appreciation or depreciation
Optimal Policy in Large Economies
Policy in Large Economies

- Model emphasizes speculative motives between $e^j_0$ and $e_1$

- Our aim is implementation via blockchain $\Rightarrow$ real-time dynamics (e.g. $e^j_0$ vs $e^{j+1}_0$) interesting

- Today:
  - Solve optimal policy in large economies
  - Explore key features of optimal policy:
    - When to appreciate/depreciate? how much?
    - Dynamic incentives? (in progress)
  - Parameterization:
    - $J = 100$, $\mu_C = 0.85$, $\lambda = 0.98$, $t = 0.01$, $R_0 = 2$
    - Generates mild depreciation in Period 1
• Mean policy resembles an exchange rate peg
Optimal Exchange Rate Policy

- Policy is incentive compatible (via same backwards induction argument)

- Policy eliminates speculative equilibria (additional volatility)

- Outcomes independent of floating rate, $e_f$ (within a window)

- Policy successfully eliminates additional sources of volatility
• Consider impact of “rare” event: Traders 90-92 all report $\theta = F$

• Use of reserves induces depreciation of the currency Policy reacts more aggressively to late “shocks”
Next Steps

- Check dynamic incentives *within* period 0

- Simulation on Ethereum’s test network (in progress)
  - Policy and traders’ strategies easy to implement as smart contracts
  - Smart contracts: software code (solidity) that implements state-contingent transfers of crypto-currency based on publicly observable (and defined) states

- Simulate transaction costs associated with complex, history-contingent policies
  - More complicated smart contracts require more “gas” / transaction costs to implement
  - Simulations useful to benchmark costs of averting speculative attacks

- Explore implementation without centralized control of reserves
Conclusions

• Developing protocol to issue to stable USD price crypto-currency with limited USD reserves

• Protocol requires history contingent USD reserve exchange policy

• Such policies implementable in transparent manner on blockchain ledger with smart contracts (as on Ethereum’s network)
Appendix Slides
Existing Stable Coins

Currently, three classes of stable (crypto)-coins

- 100% USD Reserve backed coins (Tether, TrueUSD)
  - Costly way to implement stability

- Protocol coins without redemption (Bridgecoin, Dai, NuBits, Nomin)
  - Users post collateral in exchange for stablecoin
  - If price of stable-coin were to fluctuate, users incentivized to redeem collateral or sell stable-coins
  - Requires over-collateralization to avoid margin risks

- Protocol coins with redemption in floating-rate crypto-currencies (Basecoin, Carbon, USD Fragments)
  - Stability dependent on stability of floating-rate coin