Moral Hazard in Remote Teams*

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Abstract

We re-examine the ability of teams to credibly self-impose group punishments and prevent free-riding when individual inputs are unobservable. Holmström (1982) shows that group punishments are not credible in static games. We formulate self-imposed group punishments as performance under-reporting by the team, and we ask whether the team can credibly under-report in a repeated game. We develop simple strategies that sustain under-reporting, and show that the threat of under-reporting improves welfare only if team members' preferences between shirking and team output consumption are non-separable. Our results suggest that self-assessments can replace increased managerial monitoring and mitigate free-riding in remote work environments.

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1 Introduction

Teams exist in many economic settings, ranging from teams of individuals working together in clubs or firms, to teams of companies in the form of cartels and lobby groups, to teams of nations in the form of political alliances and economic unions. In each of these settings, teams aim to improve outcomes by coordinating efforts across members and are often successful in doing so. Organizing as a team, however, may also introduce moral hazard problems, especially when team outcomes are shared and individual effort is not perfectly observed (Alchian and Demsetz, 1972; Holmström, 1982).

In static environments of team production, Holmström (1982) shows that an effective way to alleviate moral hazard problems is to rely on an outsider who, following individual deviations, punishes the entire team by taking away some share of the team's output. Holmström argues that the intervention of this outsider is also necessary to implement such punishments in a repeated environment, as the team might not want to enforce group punishments once team production outcomes are realized: *"There is a problem [...] in enforcing such group penalties if they are are self-imposed by the worker team. [...] Ex post it is not in the interest of any of the team members to waste some of the outcome. But if it is expected that penalties will not be enforced, we are back in the situation with budget-balancing, and the free-rider problem reappears."*

In this paper, we ask if and under what conditions outsiders are truly needed to enforce group punishments in a repeated context. Specifically, we ask whether the ability of individual team members to punish other team members in the future enables the team to enforce group punishments which occur *after* aggregate outcomes are realized but *before* the realization of individual payoffs in the current period. We call such within-the-period punishments *static group punishments*.

We fix ideas with a concrete and relevant example. We consider a team where individuals "work-from-home" (WFH), so that individual efforts are unobservable by the team's manager and by other team members.[1] In this setting, management may ask the team to

[1] See Jensen, Lyons, Chebelyon, Le Bras, and Gomes (2020) for a discussion of the additional frictions present

submit a self-reported assessment of the team's joint effort, and condition individual wages on observable team outcomes as well as on this report.[2] Our paper asks if and under what conditions the team will credibly commit to produce a low self-reported assessment to punish free-riding deviations by its members. Our main result shows that the team can enforce static group punishments (i.e., under-report) and the threat of under-reporting is welfare enhancing only if team output and private effort are not separable in producers' utility.

We start our analysis in a repeated team production model with imperfect input observability closely resembling Holmström (1982). In each period, different team members, or agents, exert an unobservable amount of effort for the production of team output. Following our WFH example, the team may have more information about the true team output than the manager, and can send a self-reported assessment of the team's own output after observing its true realization. When management offers a wage that is (weakly) increasing in the team's self-reported assessment, we interpret a self-assessment below the true output as a static group punishment. In practice, a low self-reported assessment effectively corresponds to the team asking for a lower wage than originally agreed-upon with the manager.

Just as individual team members cannot commit to a given effort profile, the team cannot commit to a long-term reporting strategy. Instead, the team's reporting strategy maximizes the team's static income (given the wage scheme put in place by the manager), plus the sum of future discounted stage-game payoffs of all agents. Absent commitment, a threat of under-reporting that would reduce static income if implemented must be sustained through dynamic incentives. The central question of our paper is when such sustainable threats strictly improve the outcomes attainable by the team.

In our environment, since self-reporting occurs sequentially after individuals choose their private actions, our game fits the definition of a repeated *extensive form* game. Mailath, Nocke, and White (2017) show that conventional recursive methods to characterize the set of

in monitoring workers who work from home.

[2]Similarly, in academic settings, faculty frequently request self-reported assessments when students work in teams, typically on project work that happens outside the classroom.

equilibrium payoffs (Abreu, 1986) generally do not apply in repeated extensive form games, suggesting the need for computationally tractable methods (e.g., Abreu, Pearce, and Stacchetti, 1990) to solve our equilibrium payoffs' set. However, we show that our natural repeated extension of Holmström (1982) does admit a simple recursive characterization of the equilibrium set. In our model, individual team members' contributions to total output are not observable, and hence group punishments may not be tailored to the identity of the deviator. This, and the fact that individuals can impose sufficiently large losses on each other, makes subsequent histories for both the deviator and the other agents independent of the past history of deviations, and allows us to characterize the equilibrium set using simple penal codes.[3]

When team members are sufficiently patient, the threat of wasting continuation values in the future is enough to sustain high levels of effort (as in Abreu, 1986); that is, group punishments are not needed to sustain high levels of effort. Consider instead the case when team members are impatient. Intuitively, a threat of a group punishment improves incentives by reducing the static payoff associated with a deviation by any individual team member. However, because the group punishment must be sustainable, the continuation value of the team following a group punishment is higher than the worst continuation value sustainable by the team. This increased continuation value worsens incentives by raising the continuation value the deviating team member receives following such a deviation.

We show that that if team members' enjoyment of their wage (a function of team output) is separable from the effort they contribute, then the countervailing effects of group punishments on individual team members' incentives exactly offset. As a result, the threat of under-reporting in our dynamic setting is not a necessary feature of the best equilibrium as conjectured by Holmström. If, however, team members' enjoyment of their wage is not separable from the effort they contribute (as, for example, when leisure and utility from one's wage are complements), then on net, static group punishments strictly reduce team mem-

[3] In a similar spirit, Horner, Klein, and Rady (2018) uses simple penal codes to characterize strongly-symmetric equilibria in bandit games.

bers' incentives to deviate. Therefore, static group punishments are necessarily a feature of the best equilibrium.

We then argue that, for a range of discount factors, self-reporting is more effective when producers' inputs are highly substitutable. When producers' inputs are more substitutable, individual team members have stronger incentives to deviate and the scope for under-reporting to improve team welfare rises. These results uncover cases where task assignment is an important determinant of a team's ability to prevent free-riding. We find that self-reporting is more effective to prevent moral hazard in teams with homogeneous tasks as opposed to teams with differentiated tasks.[4]

Our findings uncover a novel economic benefit of self-reporting. While the existing literature focuses on self-reporting and self-policing *by individual agents* vis-à-vis public detection and enforcement (Malik, 1993; Kaplow and Shavell, 1994; Innes, 1999, 2000; Short and Toffel, 2008), our paper shows that self-reported assessments create opportunities *for teams* to resolve free-riding incentives beyond any incentives provided by their manager. To the best of our knowledge, we are the first to study self-reporting by teams and to argue that team self-reporting can *mitigate* free-riding.[5]

Our findings also shed light on a growing debate about how managers may best resolve the free-riding problem that arises when teams of workers work remotely.[6] Since WFH features a strong interaction between private leisure and wage consumption due to easier access to leisurely activities (see, e.g., Bloom, Liang, Roberts, and Ying, 2015), our results suggest that allowing the team to self-report may be an effective solution to prevent free-

[4]One can also interpret input substitutability as a measure of work specialization. In this sense, our results are consistent with Dutcher (2012), who shows that free-riding incentives are stronger when WFH teams perform monotonous tasks.

[5]In this sense, our paper speaks to two strands of literature on self-managed teamwork and remote work. See, e.g., Cohen and Ledford Jr (1994), Stewart and Barrick (2000), Sundstrom, McIntyre, Halfhill, and Richards (2000), Sparrowe, Liden, Wayne, and Kraimer (2001), and Chan (2016) on self-managed teamwork and Dockery and Bawa (2014), Dutcher and Saral (2014) Bloom, Liang, Roberts, and Ying (2015) Raffaele and Connell (2016), Groen, van Triest, Coers, and Wtenweerde (2018), Jensen, Lyons, Chebelyon, Le Bras, and Gomes (2020) on remote work.

[6]Bloom, Liang, Roberts, and Ying (2015) and Groen, van Triest, Coers, and Wtenweerde (2018) suggest that managerial fear of moral hazard ("shirking from home") is the main reason why many firms refrained from adopting WFH before the pandemic.

riding in WFH contexts.

Our paper speaks to a large literature on moral hazard in team production settings. In two seminal papers, Alchian and Demsetz (1972) and Holmström (1982) formalize the concept of moral hazard in teams and emphasize the role of an external monitor or budget breaker to prevent shirking by team members. Subsequent studies show that team members' shirking can be prevented by the team, provided that team members' actions and payoffs satisfy specific assumptions and team members can bear arbitrarily large losses (see Rasmusen, 1987 and Legros and Matthews, 1993 for leading examples).[7] In our paper, we use a general linear formulation for team members' payoffs to ask if and under what conditions group punishments that are not credible in static settings are sustainable and welfare-improving when team members interact infinitely many times. Our main contribution is to show that non-separability between individual actions and aggregate outcomes is a necessary condition for static group punishments to improve welfare in a repeated setting.[8]

More generally, our analysis is concerned with repeated team production. In these settings, the agents have an opportunity to retaliate against the team in future periods if shirking is detected (Fudenberg and Maskin, 1986; Ostrom, Walker, and Gardner, 1992). Peer evaluations and relative performance rankings can become strategic problems in their own right, as exemplified by Che and Yoo (2001), Fuchs (2007), and Cheng (2016). Our repeated team production game bears resemblance to a renegotiation game where agents are allowed to renegotiate over aggregate outcomes in any given period (i.e. the team can decide whether to shirk from the prescribed self-punishment), but are not permitted to update their continu-

[7] Other studies solve the team moral hazard problem using dynamic strategies (see Radner, 1986 and Radner, Myerson, and Maskin, 1986), or by injecting a degree of competition among team members via tournaments, rankings, or other relative performance measures (see Hart and Holmström, 1986 for a survey). More recently, Huddart and Liang (2005), Liang, Rajan, and Ray (2008), and Fu, Subramanian, and Venkateswaran (2016) study the relationship between effort provision, monitoring incentives, and equilibrium team characteristics such as size and output in static settings.

[8] An alternative to group punishments is to allow the agents to make side payments to each other (Goldlücke and Kranz, 2012, 2013). Harrington and Skrzypacz (2007, 2011) describe how the lysine and citric acid cartels successfully used these types of contracts, employing monitors to audit the money-transfer process. This class of models offers a recursive characterization of the equilibrium set using simple penal codes, but is limited to teams of two agents or to settings in which individual actions are observable.

ation strategies as in Farrell and Maskin (1989). Our game also bears resemblance to Hurwicz (2008), Rahman (2012), Acemoglu and Wolitzky (2015) , and Aldashev and Zanarone (2017), which study how to provide incentives to the monitor of the team (i.e., how to "guard the guardian"). In our setup the "guardian" is the reporter-team itself, and we provide new conditions under which it is incentive-compatible (and welfare-improving) for individual team members to retaliate against the team when group punishments are not enforced.

2 Team Production and Self-Assessments

We begin by describing a model of repeated team production subject to managerial monitoring. We formalize the difference between workplace and remote teams as a difference in the quality of the manager's information about team members' individual actions. To fix ideas, we consider a WFH environment where contributions are unobservable by managers and by other team members, and we equip the team with the ability of sending a message about its own output to the manager by means of a benevolent reporter. We derive conditions on team members' payoffs such that the reporter under-reports team performance following a free-riding deviation by any agent. We argue that the assumptions under which the presence of a reporter is useful to improve team welfare are more likely to be relevant for remote teams.

2.1 Stage Game

A team consists of n *agents* indexed by $i = 1, \dots, n$, and one *reporter*. Each agent chooses a nonnegative action $a_i \in [0, 1]$, representing a level of effort, with associated cost $c(a_i)$. Moreover, we write

$$a_{-i} = (a_1, \dots, a_{i-1}, a_{i+1}, \dots, a_n), a = (a_i, a_{-i}),$$

where the vector a constitutes an *effort profile*. An effort profile determines the aggregate outcome of team production according to an outcome function $Y : \mathbb{R}^n_+ \to \mathbb{R}_+$.

After observing the agents' outcome, Y, the reporter costlessly provides a report of the outcome, $m \in \mathbb{R}_+$ to the manager. The reporter chooses m to maximize the post-outcome utility of the team, treating individual contributions to Y as sunk.We interpret the reporter as a construct for the collective incentives of the team to break its own budget *after* production costs are paid and team output is realized, but *before* team members can consume the output. An alternative interpretation sees the reporter as separate player, a "budget breaker" who moves after observing aggregate team outcomes (similar to Holmström, 1982). The key difference between the budget breaker in Holmström (1982) and the reporter in our model is that Holmström's budget breaker has her own incentives to seize the team's output after observing an aggregate deviation. In our model, the benevolent reporter has no static incentives to break the team's budget by under-reporting, and the team must provide dynamic incentives for her to do so.

The manager observes the report m and a signal $\hat{Y}$ of the team's output and offers a wage $x(m, \hat{Y}) = \kappa + \theta \min\{m, \hat{Y}\}$ to the agents, with $\kappa \in \mathbb{R}$ and $\theta \in \mathbb{R}_+$.[9] To simplify the exposition, we assume that $Y = \hat{Y}$, but our results extend to an environment where the reporter has an informational advantage over the manager and the manager cannot perfectly observe team output—i.e., $\hat{Y} \neq Y$.[10]

Consistent with Holmström (1982) and Holmström and Milgrom (1991), we posit an output-based incentive pay scheme where the manager rewards the team when she observes a high signal of output. As argued by Jensen, Lyons, Chebelyon, Le Bras, and Gomes (2020), output-based compensation is particularly suitable in remote work environments due to managers' heightened inability to observe workers' inputs. Novel to our setting is the introduction of the team's report, m, into the pay structure which allows us to examine the

[9]Note that this linear wage implements optimal team incentives under certain team utility functions when there is no noise as in Holmström (1982).

[10]Details on this exercise are available upon request.

usefulness of potential self-reporting. Self evaluations and peer evaluations that directly or indirectly affect the team's overall payoffs are prevalent in classroom settings (Gueldenzoph and May, 2002; Ohland, Loughry, Woehr, Bullard, Felder, Finelli, Layton, Pomeranz, and Schmucker, 2012) and even in some corporations such as Netflix.[11]

Given the restriction that the manager always ignores over-reporting, it is without loss of generality to restrict the message space $m \leqslant Y$. We find it convenient to define the report in terms of the extent of under-reporting by the reporter, i.e., in terms of a "group punishment" $\tau \equiv Y - m \geqslant 0$ that negatively affects the entire team. Under-reporting in our model can therefore be interpreted as the team asking for *lower wages* than those initially agreed-upon with the manager. Under these restrictions, we express the wage as a function of the aggregate outcome and and the group punishment, $x(Y, \tau)$. Agents have identical preferences, which depend both on the wage x and on their individual effort. Utility, net of effort costs, is given by $\pi : (0, \infty) \times [0, 1] \to \mathbb{R}$.

Assumption 1. *We make the following functional form assumptions on* $c(\cdot)$, $Y(\cdot)$, *and* $\pi(\cdot, \cdot)$*:*

1. $c(\cdot)$ *is continuous and twice continuously differentiable, with* $c'(a_i) > 0$, $c''(a_i) \geqslant 0$. *Moreover,* $c(0) = 0$, *and* $c(1)$ *is finite.*

2. $Y(\cdot)$ *is continuous and twice continuously differentiable. For all* i, i', *and for all* $a \in \mathbb{R}^n_+$, $Y_{a_i}(a) = Y_{a_{i'}}(a) > 0$ *and* $Y_{a_i a_{i'}}(a) \leqslant 0$, *where the subscripts denote partial derivatives.* $Y(\cdot)$ *is bounded above by* $\bar{Y}$.

3. $\pi(\cdot, \cdot)$ *is continuous and twice continuously differentiable in both arguments. Moreover,* $\pi_x > 0$, $\pi_{xx} \leqslant 0$, $\pi_{a_i} \neq 0$, $\pi_{a_i a_i} \leqslant 0$, *and* $\pi_{x a_i} \leqslant 0$.

Assumption 1.2 provides assumptions on agents' contributions to total output, $Y(\cdot)$, and brings our model in line with the static team production model of Holmström (1982). The assumptions $\pi_x > 0$ and $\pi_{xx} \leqslant 0$ imply that agents' utility is increasing and concave in their

[11] See https://hbr.org/2014/01/how-netflix-reinvented-hr.

wage. The assumption $\pi_{a_i a_i} \leqslant 0$ is sufficient but not necessary to prove Lemma A.1 in the Appendix. The assumption $\pi_{x a_i} \leqslant 0$ is necessary to obtain an interior solution to the model, and it implies that each agent's marginal utility from consumption is weakly decreasing in her private production effort.[12]

As we show below, the condition $\pi_{x a_i} \leqslant 0$ plays a key role in determining the effectiveness of self-reporting in our repeated team production model (see Section 2.2.3). When $\pi_{x a_i} < 0$, one additional unit of wage consumption is less valuable the higher the private production effort. For example, when $\pi = \log(x)(1 - a_i)$, an additional unit of wage consumption is more valuable to the agent who is contributing less effort than to the agents who are contributing more effort, and under-reporting has a relatively larger impact on the deviating agent's utility. A natural interpretation of the function $\pi = \log(x)(1 - a_i)$ is that agent i spends a_i units of time contributing effort to the team at cost $c(a_i)$, and $1 - a_i$ units of time enjoying consumption of her wage x. When $\pi_{x a_i} = 0$, utility from wage consumption and disutility from private effort are additively separable as, for example, when $\pi = \log(x) + (1 - a_i)$.

We view the reporter as a construct to represent team-wide reporting incentives, so she does not have private utility. In the stage game, the reporter observes only aggregate output, and treats team members' effort costs $c(\cdot)$ as sunk.[13] Stage game payoffs to the agents and to the reporter are, respectively,

$$u(a_i, a_{-i}, \tau) = \pi\left(x(Y(a), \tau), a_i\right) - c(a_i), \tag{1}$$

$$w(a, \tau) = \sum_{i=1}^{n} \pi\left(x(Y(a), \tau), a_i\right). \tag{2}$$

Assumptions 1.2 and 1.3 jointly ensure that for any $\kappa > 0$, stage game payoffs are bounded. To see this, note that wages, $x(Y, \tau)$, lie in the compact set $[\kappa, \kappa + \theta\bar{Y}]$ where we impose the upper bound of output, $\bar{Y}$. Since the payoff function $\pi(x, a_i)$ is continuous on $[\kappa, \kappa + \theta\bar{Y}] \times$

[12] If $\pi_{x a_i} > 0$, agents derive higher marginal utility from effort when team output is high, which gives them incentives to work more.

[13] In the repeated game, the reporter still treats current period effort costs as sunk, but internalizes team members' *future* effort costs. See the discussion around Equation (6).

$[0,1]$ and increasing in x, given $\kappa > 0$,

$$-\infty < \min_{a_i} \pi(\kappa, a_i) - c(a_i) \leqslant \max_{a_i} \pi(\kappa + \theta\bar{Y}, a_i) - c(a_i) < \infty. \tag{3}$$

In other words, the minimum and maximum of the stage game payoffs exist so that for a given $\kappa > 0$, stage game payoffs are bounded.

2.1.1 Stage Game Equilibrium

A symmetric *perfect-public equilibrium* of the stage game consists of effort choices a_i by agents and a punishment choice $\tau(Y)$ by the reporter such that for every Y, $\tau(Y)$ maximizes (2), and such that given τ and a_{-i}, a_i maximizes (1).

Since in a static setting it is clearly optimal for the reporter not to under-report (i.e. to set $\tau(Y) = 0$), the optimal effort of the static equilibrium, which we denote by a_i^N, is given by

$$a_i^N = \text{argmax}_{a_i} \left[\pi(x(Y(a_i, a_{-i}^N), 0), a_i) - c(a_i) \right]. \tag{4}$$

Facing the reporter's optimal decision not to under-report, the socially-optimal level of effort a^* that maximizes the sum of individual agents' utilities is given by

$$a^* = \text{argmax}_a \sum_{i=1}^{n} u(a_i, a_{-i}, 0). \tag{5}$$

In the Appendix, we establish two intermediate results for this static game. First, we show that the Nash equilibrium level of effort of the static game is lower than the socially-optimal level of effort, that is $0 < a_i^N < a_i^*$. Second, we show that when all other agents are contributing less effort than the static Nash level (that is, when $a_{-i} < a_i^N$), individual agents' optimal response is to *increase* their effort in the interior of $[a_i^N, a_i^*]$. These two intermediate results allow us to characterize optimal punishments in the infinitely-repeated game.

Note that if the reporter were able to commit to under-reporting when the aggregate outcome is smaller than $Y(a^*)$, then each agent contributing a_i^* would be an equilibrium. For example, for a given effort profile a, if the reporter's strategy was to implement some $\tau(Y(a)) > 0$ such that $x = 0$ if $Y(a) < Y(a^*)$, and conversely, to implement $\tau = 0$ if $Y(a) = Y(a^*)$, then each agent's best response to a_{-i}^* would be to choose $a_i = a_i^*$.[14] In this sense, the threat of under-reporting would be useful if the reporter (i.e., the team) could commit to such strategy. In the next section, we investigate whether under-reporting may be sustainable and welfare-improving when the agents and the reporter interact repeatedly.

2.2 Infinitely-Repeated Game

In this section, we illustrate how agents may incentivize the reporter to under-report output to the manager when called upon, even if the reporter is benevolent and lacks commitment to under-reporting. We show that along the best equilibrium path, under-reporting never occurs. However, under certain conditions, the threat of under-reporting allows agents to attain strictly higher welfare than they would in an economy where the team's actions are restricted to exclude the possibility of self-reporting.

2.2.1 Histories, Perfect Equilibria, and One-Shot Deviations

We start by describing the infinitely-repeated game, define perfect-public equilibria in our game, and simplify our equilibrium characterization by appealing to the one-shot deviation principle. Proposition A.3 in the Appendix shows that the entire set of perfect-public equilibria in our game can be attained by preventing single-period (one-shot) deviations in the infinitely-repeated game.

Let $h_t^w \in H^w$, where $H^w = \mathbb{R}_+^2$ denote the public outcomes (Y_t, τ_t) observed at the end of period t. Then, let $\mathcal{H}^w$ denote the set of public histories, with $\mathcal{H}^w = \bigcup_{t=0}^{\infty} (H^w)^t$. Similarly, define the set of histories for agent i as $\mathcal{H}^i = \bigcup_{t=0}^{\infty} (\mathbb{R}_+ \times H^w)^t$. A pure strategy for agent i is

[14]In this example, we assume that for each a, there always exists some $\tau(Y(a)) > 0$ such that $x(Y(a), \tau(Y(a))) = 0$. In words, we assume that there exists a message low enough that agents can receive no payment from the manager.

a mapping from the set of all possible agent i histories to the set of pure actions,

$$\sigma_i : \mathcal{H}^i \to \mathbb{R}_+.$$

A pure strategy for the reporter is a mapping from the set of public histories and an observation of the aggregate outcome into the set of pure actions for the reporter,

$$\sigma_w : \mathcal{H}^w \times \mathbb{R}_+ \to \mathbb{R}_+.$$

Since the reporter is a construct for team-wide preferences after output is realized, we make the natural assumption that agents and the reporter have a common discount factor $\delta \in (0, 1)$, and we restrict attention to public strategies that are functions only of the public history. Given a strategy profile $\sigma = (\{\sigma_i\}_{i=1}^n, \sigma_w)$, if $h^{wt} \in H^{wt}$ denotes a generic period-t history, we let $U_i^t(h^{wt}, \sigma)$ denote the discounted continuation payoff that agent i obtains from period t onwards. The reporter's discounted continuation payoffs satisfy

$$U_t^w(h^{wt}, \sigma) = \sum_i U_t^i(h^{wt}, \sigma) + (1 - \delta) \sum_i c(\sigma_i(h^{wt})), \tag{6}$$

where the second term represents the present value of the team's *future* effort costs, treating period-t effort decisions as sunk.

In Appendix A.2 we define continuation games and strategies, perfect-public equilibria, and one-shot deviations. We prove that perfect-public equilibria can be constructed recursively by ensuring that, for any history, neither the reporter nor the agents have a profitable one-shot deviation.

Finally, we denote by $B(\kappa)$ the min-max payoff of this game, which depends on the exogenous parameter κ. Note that the worst perfect-public equilibrium, $\underline{v}$ satisfies $\underline{v} \geqslant B(\kappa)$.

Assumption 2. *We assume that for all* $\delta \in (0,1)$*, there exists a* $\kappa(\delta)$ *such that*

$$(1-\delta)\pi(\kappa(\delta),0) + \delta V(1) < B(\kappa(\delta)). \tag{7}$$

In (7), $V(1)$ is the continuation payoff when all other agents contribute $a_{-i} = 1$, the reporter does not impose group punishments, and agent i chooses their optimal level of effort forever. The value $V(1)$ is clearly an upper bound on the best perfect-public equilibrium value, and choosing κ to satisfy (7) ensures that for any continuation of the game, if all other agents provide no effort in the current period, then it can never be optimal for a single agent to also exert zero effort in the current period. Assumption 2 imposes that for any δ with $0 < \delta < 1$, there exists $\kappa(\delta) > 0$. Hence, for each δ and for a given $\kappa(\delta) > 0$ satisfying (7), stage game payoffs are bounded as discussed in the conclusion to Section 2.1. Assumption 2 is analogous to Assumption A4 in Abreu (1986), and it allows us to attain the worst equilibrium using a one-period punishment phase (see Appendix A.3). Note that one set of assumptions on primitives that ensures that such κ always exists is that $\pi(x,1) = 0$—which guarantees for all κ that $B(\kappa)$ is bounded below—and that $\lim_{x\to 0} \pi(x,\cdot) = -\infty$.

Characterizing equilibria by preventing one-shot deviations as described in this section, while standard practice in normal-form repeated games, is not obvious in our context. Since agents and the reporter move in sequence within each time period, our game fits the definition of a repeated *extensive form* game. Mailath, Nocke, and White (2017) shows that static (current-period) punishments affect the subsequent histories of all players (both the deviator and the punishing agents). As such, repeated extensive-form games cannot in general be characterized using simple penal codes calling for a one-period punishment of the deviator and subsequent reversal to the best equilibrium.

The appendix shows that the two elements that allow us to solve our model by preventing one-shot deviations are the imperfect observability of individual agents' actions, and the ability of individual agents to impose arbitrarily large punishments on the reporter when the reporter fails to under-report. Together, these elements make continuation payoffs independent of the identity of the deviator, and allow us to construct the equilibria of our model recursively.

2.2.2 Problem Setup

We focus on characterizing *strongly symmetric* perfect-public equilibria—hereafter, perfect equilibria or equilibria—whereby different agents play the same strategies even after asymmetric histories, and the reporter's strategy is independent of the identity of the deviator.[15]

In this section, we describe a procedure to characterize the set of equilibrium payoffs using modified "carrot-and-stick" strategies similar to those found in Abreu (1986). While individual deviations by agents may be subject to under-reporting by the reporter, the agents need to impose discipline on the reporter in the event that the reporter attempts *not to under-report*. Nonetheless, our results show that the extremal equilibrium payoffs (both the best and the worst equilibrium payoff) need not feature under-reporting (on path).

Since we focus on strongly symmetric equilibria, we simplify our notation by dropping i subscripts and by using a in place of $(a, a, \ldots, a)$ for the agents' strategies, $u(a, 0)$ in place of $u_i(a, a, \ldots, a, \tau = 0)$ for the agents' payoffs, and so on.

Under the one-shot deviation principle, given the worst perfect equilibrium payoff $\underline{v}$, the best perfect equilibrium payoff $\bar{v}$ can be constructed as the solution to the following program:

$$\bar{v} \quad = \max_{a, \tau(\cdot), v(\cdot, a, \tau(\cdot))} u(a, 0), \tag{8}$$

subject to, for all a',

$$u(a, 0) \geqslant (1 - \delta) u\left(a', a, \tau\left(Y\left(a', a\right)\right)\right) + \delta v\left(a', a, \tau\left(Y\left(a', a\right)\right)\right), \tag{9}$$

$$v\left(a', a, \tau\left(Y\left(a', a\right)\right)\right) \in [\underline{v}, \bar{v}], \tag{10}$$

[15] The restriction to such equilibria is without loss of generality if, for a given discount rate, the worst such equilibrium coincides with the min-max value.

and

$$(1-\delta)\, w\left(a', a, \tau\left(Y\left(a', a\right)\right)\right) + n\delta v\left(a', a, \tau\left(Y\left(a', a\right)\right)\right) \geqslant (1-\delta)\, w\left(a', a, 0\right) + n\delta \underline{v}. \qquad (11)$$

Inequality (9) represents the incentive-compatibility constraint for each agent, which requires the symmetric payoff $u(a, 0)$ to be greater than or equal to the payoff associated with a deviation effort a' with static payoff $u(a', a, \tau(Y(a', a)))$ and continuation payoff $v(a', a, \tau(Y(a', a))$. Equation (10) represents the feasibility constraint for the continuation payoff $v(a', a, \tau(Y(a', a))$, which must lie between the worst equilibrium payoff $\underline{v}$ and the best equilibrium payoff $\bar{v}$. Equation (11) is the incentive-compatibility constraint for the reporter, requiring that the reporter has sufficient incentives to under-report once one of the n agents deviates to a'.[16] The left-hand side of (11) is the reporter's payoff when under-reporting as prescribed, while the right-hand side is the payoff from a deviation to $\tau = 0$, followed by the worst perfect equilibrium payoff $\underline{v}$. Finally, note that the best equilibrium in (8) features no group punishments on path, a result that we confirm formally in Lemma A.4.[17]

It is useful to reduce the dimensionality of the problem by eliminating the reporter's incentive-compatibility constraint. Without loss of generality, we may assume (11) binds in any solution to the above program.[18] The continuation payoff following a deviation by an agent must then satisfy

$$v\left(a', a, \tau\left(Y\left(a', a\right)\right)\right) \quad = \quad \underline{v} + \frac{1-\delta}{\delta}\frac{1}{n}\left[w\left(a', a, 0\right) - w\left(a', a, \tau\left(Y\left(a', a\right)\right)\right)\right]. \qquad (12)$$

[16] Note that the game is symmetric among producers, such that, for a prescribed action a, the reporter can infer a deviation to a' by one of the producers from the realized output Y'. The program (8)-(11) is defined only for one-shot deviations by the producers and hence while the identity of the deviator is unknown, the reporter may evaluate payoffs exactly.

[17] Intuitively, if the best equilibrium featured on-path group punishments, we could construct another equilibrium delivering the same continuation values but featuring no static group punishments (and therefore higher period utility for both the agents and the reporter) for some history. This contradicts the assumption that the original equilibrium was the best.

[18] If (11) were slack for some (a', a), since $w(a', a, 0) \geqslant w(a', a, \tau(Y(a', a)))$, necessarily $v(a', a, \tau(Y(a', a))) > \underline{v}$. As a result, one could just reduce $v(a', a, \tau(Y(a', a)))$ until (11) binds, relaxing (9) and therefore not changing the solution to the program.

Hence, for any deviation a', we may write (9) as

$$u(a,0) \geqslant (1-\delta)\left[u\left(a',a,\tau\left(Y\left(a',a\right)\right)\right)+\frac{1}{n}\left[w\left(a',a,0\right)-w\left(a',a,\tau\left(Y\left(a',a\right)\right)\right)\right]\right]+\delta\underline{v}. \tag{13}$$

Let $g\left(a',a,\tau\left(Y\left(a',a\right)\right)\right)$ denote the static payoff for an individual agent exerting effort a' when all the other agents contribute a—the term in the outer square brackets on the right-hand side of (13). We call this quantity the *total static deviation payoff*. Using this definition, we re-write the problem (8)-(11) as

$$\bar{v} \;=\; \max_{a,\tau(\cdot)} u(a,0), \tag{14}$$

subject to, for all a',

$$u(a,0) \;\geqslant\; (1-\delta)\, g\left(a',a,\tau\left(Y(a',a)\right)\right)+\delta\underline{v}, \tag{15}$$

$$\bar{v} \;\geqslant\; \frac{1-\delta}{\delta}\frac{1}{n}\left[w\left(a',a,0\right)-w\left(a',a,\tau\left(Y(a',a)\right)\right)\right]+\underline{v}, \tag{16}$$

and

$$g\left(a',a,\tau\left(Y(a',a)\right)\right)=u\left(a',a,\tau\left(Y(a',a)\right)\right)+\frac{1}{n}\left[w\left(a',a,0\right)-w\left(a',a,\tau\left(Y(a',a)\right)\right)\right]. \tag{17}$$

Note that the total static deviation payoff (17) features two components. The first component, $u\left(a',a,\tau\left(Y(a',a)\right)\right)$, represents the agent's static utility from a deviation to a' (under the expectation that the reporter will implement the prescribed punishment τ). The second component, $\left[w\left(a',a,0\right)-w\left(a',a,\tau\left(Y(a',a)\right)\right)\right]/n$, represents the deviating agent's share of the net benefit that the reporter generates by deviating and not submitting the prescribed report.

The program (14)-(17) allows us to draw a comparison between our economy and an economy without a reporter (i.e., an economy where reporting is not part of the team's action set). We refer to such economy as an economy where self-reporting "is not allowed," and we let $\bar{v}^A$ and $\underline{v}^A$ denote its best and worst perfect equilibrium values. In the economy where self-reporting is not allowed, the reporter's incentive-compatibility constraint disappears, and (14)-(17) become

$$\bar{v}^A = \max_a u(a, 0), \tag{18}$$

subject to, for all a',

$$u(a, 0) \geqslant (1-\delta) u\left(a', a, 0\right) + \delta \underline{v}^A. \tag{19}$$

Under Assumptions 1 and 2, the program (18)-(19) can be solved using a simple "carrot-and-stick" strategy as in Abreu (1986), whereby i) team members exert the best perfect equilibrium level of effort (the "carrot," denoted by $\bar{a}^A$) on path; ii) deviations from the "carrot" are followed by an arbitrarily low level of effort (the "stick," denoted by $\tilde{a}^A$) for one period and by reversion to the "carrot" thereafter; iii) deviations from the "stick" are also followed by the "stick" and subsequent reversion to the "carrot." Importantly, the threat of future retaliation by other agents can prevent deviations in the current period, and allows the team to sustain the most collusive level of effort provided that agents are sufficiently patient.

Lemma 1. *There exists a $\delta^{A*} \in (0, 1)$ such that for all $\delta \geqslant \delta^{A*}$, $u\left(\bar{a}^A\right) = u(a^*)$.*

Proof. See Abreu (1986). □

In the next section, we ask if and under what conditions *static* under-reporting by the reporter (as opposed to future retaliation by other team members) allows the team to achieve higher welfare when agents are not sufficiently patient (i.e., when $\delta < \delta^{A*}$).

2.2.3 Self-Assessments and Static Deviation Incentives

To proceed in our analysis of static self-assessments, we start by studying under what conditions under-reporting deters static deviations. To do so, we revert to the program (14)-(17), and we define the maximum deviation payoff that an agent can achieve by deviating to a' from profile a by $\hat{g}\,(a, \tau\,(\cdot))$. This payoff satisfies

$$\hat{g}\,(a, \tau\,(\cdot)) = \max_{a'} g\left(a', a, \tau(Y(a', a))\right). \tag{20}$$

In the next proposition, we derive two important intermediate results. First, we show that when $\pi_{xa_i} < 0$, group punishments *increase* static deviation incentives if the prescribed level of effort is smaller than the static Nash equilibrium level of effort. Second, we show that under-reporting reduces static deviation incentives *only in the presence of complementarities* between individual effort and wages (i.e., only if $\pi_{xa_i} < 0$). When agents' utility is separable between individual effort and wages (i.e., when $\pi_{xa_i} = 0$), under-reporting does not reduce static deviation payoffs and therefore does not alter agents' incentives to deviate from their prescribed actions.[19]

Proposition 2. *Suppose that $\pi_{xa_i} < 0$ and $a \leqslant a^N$. Then, for all continuous functions $\tau(\cdot)$ with $\tau(Y) \geqslant 0$, $\hat{g}\,(a, \tau\,(\cdot)) \geqslant \hat{g}\,(a, \tau = 0)$. Suppose next that $\pi_{xa_i} < 0$ and $a > a^N$. There exists $\epsilon > 0$ such that for all continuous functions $\tau(\cdot)$ with $\tau(Y(a, a)) = 0$ and for $a' \neq a$, $0 < \tau(Y(a', a)) \leqslant \epsilon$, $\hat{g}(a, \tau(\cdot)) < \hat{g}(a, \tau = 0)$. Finally, suppose that $\pi_{xa_i} = 0$. In this case, under-reporting has no impact on static deviation incentives.*

Proof. Consider first an agent's static deviation payoff when she chooses effort level a', the other $n-1$ producers choose effort a, and the group punishment is a tax $\tau(Y(a', a))$. Apply-

[19] As mentioned below Assumption 1, we need $\pi_{xa_i} \leqslant 0$ to guarantee an interior solution to the model.

ing (17), these incentives may be written as

$$g(a', a, \tau) = \pi(x(Y(a', a), \tau), a') - c(a') + \frac{1}{n}\left[(n-1)\pi(x(Y(a', a), 0), a) + \pi(x(Y(a', a), 0), a')\right], \tag{21}$$

where for convenience we omit the dependence of τ on Y. Contrasting these deviation incentives to those that arise when $\tau = 0$ yields

$$g(a', a, \tau) - g(a', a, 0) = \frac{n-1}{n}\left\{[\pi(x(Y,0), a) - \pi(x(Y,\tau), a)] - \left[\pi(x(Y,0), a') - \pi(x(Y,\tau), a')\right]\right\}. \tag{22}$$

Since π is submodular, or $\pi_{xa_i} \leqslant 0$, the increment to the agent's utility of increasing x from $x(Y,\tau)$ to $x(Y,0)$ is larger when her own effort is smaller. In other words, (22) is positive when $a' \geqslant a$ and is negative when $a' \leqslant a$.

Suppose that $\pi_{xa_i} < 0$ and $a \leqslant a^N$. Let a' represent the most profitable deviation from a when $\tau(Y) = 0$ for all Y. Note that the most profitable deviation satisfies $a' \geqslant a$ when $a \leqslant a^N$ (see Lemma A.2 in the Appendix). It then immediately follows from (22) that $g(a', a, \tau(Y(a', a))) \geqslant g(a', a, 0)$. We conclude that $\hat{g}(a, \tau(\cdot)) \geqslant \hat{g}(a, \tau = 0)$.

Suppose next that $\pi_{xa_i} < 0$ and $a > a^N$. Let a' represent the most profitable deviation from a when $\hat{\tau}(Y) = 0$ for all Y and a'' the most profitable deviation under the group punishment satisfying $0 < \tau(Y(a', a))$ when $a' \neq a$. When $a > a^N$ and $\pi_{xa_i} < 0$, the best deviation without group punishments satisfies $a' < a$ (see Lemma A.2). Since the best response function is upper hemicontinuous, there exists $\epsilon > 0$ such that if $\max_Y \tau(Y) \leqslant \epsilon$, then $a'' < a$; that is, a'' is sufficiently close to a' so that $a'' < a$. Applying (22) implies $g(a'', a, \tau(Y(a'', a))) < g(a'', a, 0)$. Since $g(a'', a, 0) \leqslant \hat{g}(a, \tau = 0)$, we conclude that for such punishment functions, $\tau(\cdot)$, $\hat{g}(a, \tau(\cdot)) < \hat{g}(a, \tau = 0)$.

The proof of the final result (if $\pi_{xa_i} = 0$, then under-reporting has no impact on static deviation incentives) immediately follows from (22). When $\pi_{xa_i} = 0$, the term in curly brackets is zero and static deviation payoffs are independent of τ. $\square$

The intuition behind Proposition 2 is that an increase in the group punishment comes with a cost and a benefit to the deviator. The cost comes in the form of lower static utility due to a reduction in the wage x paid by the manager. The benefit comes in the form of a higher continuation value necessary to incentivize the team to implement the punishment. From (12), the benefit is proportional to the per-capita share of the group losses associated with increasing the group punishment (i.e., to the term in square brackets in (17)).

When $\pi_{xa_i} < 0$ and $a \leqslant a^N$, the deviator's benefit from team under-reporting (i.e., the benefit when the team implements some positive τ, resulting in higher continuation values) is higher than the deviator's static cost. Under these conditions, the deviator contributes *more effort* than the other $n-1$ agents and so enjoys lower marginal utility of the common wage. The cost the deviator bears from a reduced wage is smaller than the increased continuation value to compensate the reduced wages borne by the whole team. As a result, sustainable under-reporting strengthens the deviator's incentives to deviate. On the other hand, when $a > a^N$, the deviator receives the same wage but she contributes *less effort* than the other agents, enjoying higher marginal utility. The cost from receiving a lower wage following a group punishment is larger than her continuation-value benefit, and under-reporting decreases her deviation incentives.

Proposition 2 also shows that a necessary condition for self-reporting to reduce deviation incentives is the non-separability of aggregate outcomes and private actions in individual agents' utility (i.e., the condition that $\pi_{xa_i} < 0$). When preferences are separable, each team member receives the same marginal utility from team output consumption. Reducing the reported output has no effect on agents' deviation incentives, because the deviator's cost when the team under-reports is exactly equal to the continuation-value benefit required to make under-reporting credible. As a corollary, in absence of complementarities the equilibrium set of our problem (14)-(17) coincides with that of the problem (18)-(19) where group punishments are not allowed.

Corollary 3. *If $\pi_{xa_i} = 0$, then $\bar{v} = \bar{v}^A$ for all $\delta \in (0,1)$.*

Proof. When $\pi_{xa_i} = 0$, from Proposition 2 we have that for any τ, $\hat{g}(a', a, \tau) = \hat{g}(a', a, 0) = u(a', a, 0)$. This reduces the agent's incentive-compatibility constraint (15) to (19). Moreover, since the best equilibrium never features on-path group punishments, and since group punishments have no impact on agent's deviation incentives, we can always pick τ that satisfies constraint (16), say by setting $\tau = 0$ for all histories. $\square$

2.2.4 Equilibrium Characterization and Welfare Gains

Using the results of Proposition 2, we propose a modified "carrot-and-stick" strategy to characterize simple equilibrium strategies that obtain the worst perfect equilibrium payoff $\underline{v}$. With a small abuse of notation, we denote this strategy by $\sigma((\tilde{a}, \bar{a}), (0,0))$. This strategy calls for the agents to play some "stick" level of effort $\tilde{a}$ and subsequently revert to the "carrot" level $\bar{a}$—the level of effort prescribed in the best perfect equilibrium. If either the "carrot" or the "stick" is played by all agents as prescribed by the strategy, the reporter chooses $\tau = 0$. If the reporter detects an aggregate deviation $Y(a', \bar{a}) \neq Y(\bar{a})$ from the "carrot" $\bar{a}$, the reporter implements a group punishment $\tau(Y(a', \bar{a})) > 0$, and the agents consequently revert to some strategy with value $v(a', \bar{a}, \tau(Y(a', \bar{a}))$. If the reporter observes an aggregate deviation $Y(a', \tilde{a}) \neq Y(\tilde{a})$ from the stick $\tilde{a}$, the reporter chooses $\tau(Y(a', \tilde{a})) = 0$, and the agents consequently revert to the carrot-and-stick strategy $\sigma((\tilde{a}, \bar{a}), (0,0))$ with value $\underline{v}$. Any deviation by the reporter causes the carrot-and-stick strategy to be repeated.

In the Appendix, we prove that $\sigma((\tilde{a}, \bar{a}), (0,0))$ is an optimal punishment, that any value $v \in [\underline{v}, \bar{v}]$ can be attained by a perfect equilibrium strategy σ, and that the "carrot" level of effort $\bar{a}$ and the "stick" level of effort $\tilde{a}$ can be jointly determined as solutions to the following system of equations in two unknowns (see the computational appendix for details on how we solve for $\bar{a}$ and $\tilde{a}$ recursively):

$$\hat{g}(\tilde{a},0) = (1-\delta)u(\tilde{a},0)+\delta u(\bar{a},0) = \underline{v}, \tag{23}$$

$$\hat{g}(\bar{a},\tau(\cdot)) = u(\bar{a},0)+\delta(u(\bar{a},0)-u(\tilde{a},0)) \text{ if } \bar{a} < a^*, \tag{24}$$

and

$$\hat{g}(\bar{a},\tau(\cdot)) \leqslant u(\bar{a},0)+\delta(u(\bar{a},0)-u(\tilde{a},0)) \text{ if } \bar{a} = a^*. \tag{25}$$

Then, the value $\underline{v}$ of the worst equilibrium is equal to the value of the stick for one period, followed by the carrot forever after (and no group punishments on path), as in (23).

In the following proposition, we formally establish our main result. We show that, under the maintained assumption that $\pi_{xa_i} < 0$, the threat of under-reporting *strictly* improves team welfare when agents are not sufficiently patient and the threat of future retaliation is not enough to sustain a^* (i.e., when $\delta < \delta^{A*}$).

Proposition 4. *Suppose that $\pi_{xa_i} < 0$ and $\delta < \delta^{A*}$ (so that that $u(a^N) < \bar{v}^A < u(a^*)$). The best equilibrium in the model with self-reporting features group punishments for some history and, therefore, $\bar{v}^A < \bar{v}$.*

Proof. Suppose $0 < \delta < \delta^{A*}$ so that $u(a^N) < \bar{v}^A < u(a^*)$. Let $\bar{\sigma}^A$ be the equilibrium strategy that yields the best perfect equilibrium payoff $\bar{v}^A$ in the game without self-reporting. Further denote by $\bar{a}^A$ the carrot output associated with strategy $\bar{\sigma}^A$. Note that $\bar{\sigma}^A_t$ is a function only of the history of aggregate output, Y^t since in that model there are no self-reports or group punishments.

Next, let σ^A be the strategy in the game *with* self-reporting that coincides with $\bar{\sigma}^A$. That is, let $\sigma^A_t(h^{wt}) = \bar{\sigma}^A_t(Y^t)$ for all public histories h^{wt} and let the strategy of the reporter be to impose zero group punishments for all public histories. It is straightforward to show that this strategy is an equilibrium in the game with self-reporting. The reporter expects her actions to have no impact on subsequent play by the agents and therefore any group punishment is only harmful from her perspective. In any (off-equilibrium-path) history where the reporter has imposed a group punishment, her action is sunk and therefore only the dynamic incentives provided by other agents matter; these incentives by assumption are sufficient to sustain the given level of effort.

We will construct an equilibrium strategy that delivers strictly higher utility than σ^A. Specifically, let $a^1 = \bar{a}^A + \epsilon$ for some $\epsilon > 0$. The strategy we propose to support a^1 has the following features. As long as $Y = Y(a^1)$, there are no static group punishments and agents continue to play a^1. We define $\tau(Y, \epsilon)$ as a continuous function with $\tau(Y(a^1), \epsilon) = 0$ and $\tau(Y(a', a^1), \epsilon)$ small but strictly positive for $Y(a', a^1) \neq Y(a^1)$.

If in some history, the aggregate outcome is $\hat{Y} \neq Y(a^1)$, then the reporter imposes a small group punishment, $\tau(\hat{Y}; \epsilon)$. We include ϵ in the punishment function, $\tau(\cdot)$ to indicate how our construction of this function depends on the size of the proposed deviation, a^1. Following $\hat{Y}$ and $\tau(\hat{Y}; \epsilon)$, agents play a continuation strategy that features no group punishments (on or off the continuation path) and delivers value v^1 as defined in equation (26) below. Following $\hat{Y}$ and a deviation from $\tau(\hat{Y}; \epsilon)$, agents play the worst perfect equilibrium that features no group punishments, $\underline{v}^A$.

To show this is an equilibrium, we first show that for any ϵ and $\hat{Y}$, we may choose $\tau(\hat{Y}; \epsilon)$ so that the implied continuation value, v^1 may be obtained using an equilibrium strategy that does not require group punishments; that is, the implied values of v^1 lie in the set $[\underline{v}^A, \bar{v}^A]$ so such an equilibrium continuation strategy exists. Next, we prove that when $\pi_{xa_i} < 0$, for ϵ sufficiently small, agents strictly prefer to play a^1 than to deviate to any a'.

Suppose $\hat{Y} = Y(a', a^1)$. Given $\tau(\cdot)$, we define v^1 as

$$v^1\left(a', a^1, \tau\left(\hat{Y}; \epsilon\right)\right) \equiv \underline{v}^A + \frac{1-\delta}{\delta}\frac{1}{n}\left[w\left(a', a^1, 0\right) - w\left(a', a^1, \tau\left(\hat{Y}; \epsilon\right)\right)\right]. \tag{26}$$

Since $v^1(a', a^1, 0) = \underline{v}^A$, and $w(a', a^1, 0) > w(a', a^1, \tau(\hat{Y}; \epsilon))$ for any $\tau(\hat{Y}; \epsilon) > 0$, we may always choose $\tau(\hat{Y};\epsilon) > 0$ but small enough so that $v^1(a', a^1, \tau(\hat{Y};\epsilon)) \in [\underline{v}^A, \bar{v}^A]$ for all $a' \neq a^1$. Hence, the group punishment $\tau(\hat{Y};\epsilon)$ may be sustained with continuation strategies that do not require group punishments in any future period.

Next, since δ is such that $\bar{a}^A < a^*$, results from Abreu (1986) imply that the incentive compatibility constraint (19) binds:

$$u(\bar{a}^A, 0) = (1-\delta)\hat{g}(\bar{a}^A, 0) + \delta \underline{v}^A. \tag{27}$$

Since $\bar{a}^A > a^N$ and $\pi_{xa_i} < 0$, it follows from Proposition 2 that

$$u(\bar{a}^A + \epsilon, 0) > u(\bar{a}^A, 0) > (1-\delta)\hat{g}(\bar{a}^A, \tau(\cdot, \epsilon)) + \delta \underline{v}^A. \tag{28}$$

Moreover, by continuity of $\hat{g}$, for ϵ sufficiently small, we must have

$$u(\bar{a}^A, 0) > (1-\delta)\hat{g}(\bar{a}^A + \epsilon, \tau(\cdot, \epsilon)) + \delta \underline{v}^A \tag{29}$$

so that indeed for such ϵ, $u(a^1, 0) \geqslant (1-\delta)\hat{g}(a^1, \tau(\cdot, \epsilon)) + \delta \underline{v}^A$ and agents' incentive constraints are satisfied as desired.

□

The key assumption in Proposition 4 is that individual actions and team outcomes are non-separable. Absent this non-separability, static deviation payoffs are not affected by the threat of under-reporting, and reporting has no scope for improving welfare.

While the remote nature of our model arises from differences in output observability between the manager and the team, we also argue that the non-separability assumption may be more likely to be verified in WFH contexts. Team members working from home are more exposed to leisurely activities which interact with the utility that they derive from their wage (Bloom, Liang, Roberts, and Ying, 2015). This section shows that this interaction is *necessary* for under-reporting to mitigate shirking incentives in these environments, because under-reporting disproportionately affects the shirking agent who is enjoying higher marginal utility from wage consumption by working less.

3 Self-Assessments and Team Welfare

We parameterize our repeated team production model to study under what conditions self-reporting is most effective in improving team welfare. We show that, if agents' utility satisfies the non-separability conditions of Proposition 2, self-reporting is most effective in improving welfare for intermediate levels of the discount factor and when team members' inputs are more substitutable in the production of the aggregate outcome. We discuss the implications of these findings for WFH team production.

3.1 Stage Game

We assume that team output is the sum of individual team members' inputs (i.e., $Y(a) = \sum_i a_i$). Hence, the team's net outcome function is

$$x(a, \tau) = \kappa + \theta \max\left\{\sum_{i=1}^{n} a_i - \tau;\ 0\right\}, \tag{30}$$

with $a_i \in (0, 1)$ and $\tau \geqslant 0$. Individual agents' utility is therefore

$$u(a_{-i}, a_i, \tau) = \log(x(a, \tau))(1 - a_i) - ca_i. \tag{31}$$

As described above, (31) can be interpreted as a case in which agent i is endowed with one unit of time, spends a fraction a_i of this unit working at a marginal cost c, and spends the remaining $1 - a_i$ fraction of her time enjoying her wage x. From (31), it is clear that agent i derives higher utility from her wage the less she contributes to production. In the presence of such interactions between total output and individual effort, self-reporting can increase team welfare by imposing relatively harsher punishments on team members that contributed less effort to production.

Since the reporter chooses τ after observing total output and production costs are sunk, the reporter's stage-game (static) payoff is

$$w(a,\tau) = \sum_{i=1}^{n} \log\left(x(a,\tau)\right)(1-a_i). \tag{32}$$

3.2 Infinitely-Repeated Game and Agents' Discount Factors

As in the previous sections, we focus on characterizing strongly symmetric equilibria. Following the same steps as in Section 2.2, the program (14)-(17) maps to

$$\bar{v} = \max_{a,\tau(\cdot)} u(a,0), \tag{33}$$

subject to, for all a',

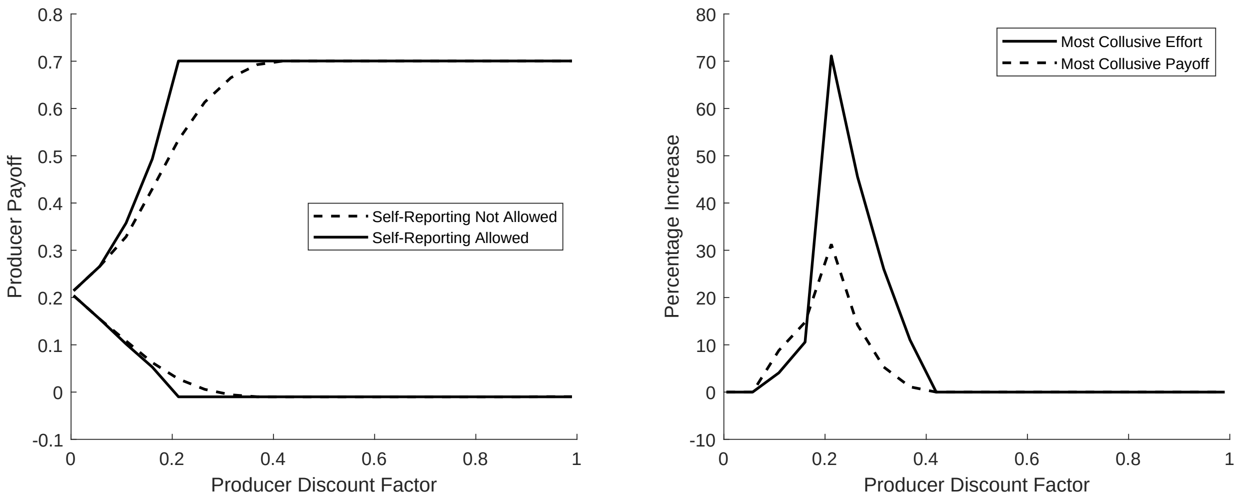
$$\begin{aligned} u(a,0) &\geqslant (1-\delta)\, g\left(a', a, \tau\left(a' + (n-1)\,a\right)\right) + \delta \underline{v}, & (34) \\ \bar{v} &\geqslant \frac{1-\delta}{\delta}\frac{1}{n}\left[w\left(a' + (n-1)\,a, 0\right) - w\left(a' + (n-1)\,a, \tau\left(a' + (n-1)\,a\right)\right)\right] + \underline{v}, & (35) \end{aligned}$$

where $\underline{v}$ and $\bar{v}$, respectively, denote the worst and the best perfect equilibrium payoffs of the repeated game, and where the total static deviation payoff is

$$\begin{aligned} g\left(a', a, \tau\left(a' + (n-1)\,a\right)\right) &= u\left(a', a, \tau\left(a' + (n-1)\,a\right)\right) \\ &\quad + \frac{1}{n}\left[w\left(a', a, 0\right) - w\left(a', a, \tau(a' + (n-1)\,a)\right)\right]. \end{aligned} \tag{36}$$

Given our assumptions, the results in Proposition 2 apply. As long as the prescribed effort is smaller than the static Nash equilibrium level of effort, the maximum deviation payoff $\hat{g}(a, \tau(\cdot))$ is minimized when $\tau = 0$.

Using Proposition 2, the results of the previous section extend to this environment. We



(a) Equilibrium Value Sets

(b) Impact of Group Punishments

Figure 1: Numerical illustration of the equilibrium value sets (Panel (a)) and impact of self-reporting on best equilibrium effort and best equilibrium payoff (Panel (b)).

find that the worst perfect equilibrium payoff can be attained by strategies that do not feature under-reporting on path, and the best and the worst equilibria can be jointly characterized as solutions to (33)-(36). Moreover, under-reporting strictly improves welfare relative to a model where self-reporting is not allowed.

In Figure 1, we provide a numerical illustration of how self-reporting can increase the welfare of the team when their discount factor changes.[20] In Figure 1a, we fix the number of agents n to 20, and we plot the value of the best and worst perfect equilibria for each level of the discount factor δ. In Figure 1a, for any δ, the lower branches, below the static Nash equilibrium value (roughly 0.2) represent worst equilibrium values, while the upper branches represent best equilibrium values.

The solid lines in Figure 1a represent the best and worst equilibrium values when self-reporting is allowed, while the dashed lines show these values when self-reporting is not allowed. Since the solid lines lie outside the dashed lines, for all levels of the discount factor, the model where self-reporting is allowed yields weakly larger best equilibrium payoffs than

[20]Details on the algorithm that we use to produce the figures are given in the appendix.

the model where self-reporting is not allowed. The repeated interaction between agents and the reporter leads to relatively large welfare gains for intermediate values of the discount factor, to relatively small gains when the discount factor is low, and to no welfare gains when the discount factor is high.

For low values of δ, the reporter has weak incentives to under-report. The continuation value that the producers have to promise to the reporter for under-reporting is high and, as a result, only very small under-reporting with small welfare gains can be sustained. On the other hand, for high values of δ the repeated interaction of producers is sufficient to guarantee the static most collusive level of effort even in the absence of the reporter.

For intermediate levels of δ, the reporter's ability to under-report increases welfare. To illustrate the gains associated with these sustainable group punishments, in Figure 1b we show the effect of the reporter's punishments on the level of effort in the best equilibrium. The solid line shows the percentage increase in effort in the best equilibrium that is obtained in our model relative to a model where under-reporting is not allowed, while the dashed line represents the associated percentage increase in welfare. Our model features a most collusive effort level as much as 70% higher than that of the model where self-reporting is not allowed, corresponding to a 30% higher level of welfare.

The results of Figure 1 suggest that under-reporting might not be effective in increasing team welfare if team members' production horizon (pinned down, for example, by the length of their employment contract) is too long or too short. If team members' horizon is too long, producers that heavily discount their future payoffs can be disciplined by the threat of future punishments (i.e., low effort) by other team members, even if these threats do not involve under-reporting in the current period. If the production horizon is short, then the threat of future punishments is not large enough to induce large welfare gains from under-reporting.

4 Substitutability, Externalities, and Self-Assessments

In this section, we provide additional results on how different degrees of interaction between team producers can change the effectiveness of self-reporting. We keep the same parameterization as in the previous section, but we rely on a new output function to allow for different degrees of substitutability between agents' efforts. We assume that the total output of the team is

$$Y(a) = \left(\sum_{i=1}^{n} a_i^{\rho}\right)^{\frac{1}{\rho}}, \tag{37}$$

where $\rho \in (0,1)$ is a parameter that governs the substitutability of individual agents' efforts in the production of total output. As in Dixit and Stiglitz (1977), a high level of ρ implies a higher degree of substitutability between producers' inputs. When $\rho = 1$, agents efforts' are perfect substitutes, team output is the sum of all agents' efforts, and a small change in one agent's effort has a small relative impact on team output. When ρ is small, agents' efforts are imperfect substitutes, team output is larger than the sum of all agents' efforts, and a small change in one agent's effort has a large relative impact on team output.

We extend the analysis of the previous sections to analyze the relationship between the usefulness of self-reporting and the substitutability parameter ρ. Specifically, we ask how the effectiveness of self-reporting in improving welfare (relative to a model where self-reporting is not allowed) changes as the substitutability of producers' effort changes. We show that, near the threshold of δ needed to sustain $\bar{a} = a^*$ in the model without self-reporting, the effectiveness of self-reporting in improving welfare increases as the substitutability parameter ρ increases.[21]

Proposition 5. *Fix* $\rho \in (0,1)$ *There exist a* $\delta \in (0,1)$ *and a* $\bar{\rho} > 0$ *such that for all* $\rho' \in (\rho, \bar{\rho})$, *the welfare gains from allowing self-reporting are increasing in* ρ'.

[21] Numerical simulations suggest that this result may hold for other levels of δ.

Proof. See Appendix A.4. □

We give here a sketch of our argument, and leave the formal proof to the appendix. For a fixed level of the substitutability parameter ρ, our model achieves the first-best level of effort a^* at a lower level of the discount factor than the model where self-reporting is not allowed. Let $\delta^A(\rho)$ be the threshold level of the discount factor at which the model where self-reporting is not allowed first achieves a^* as the most collusive level of effort, and let $\delta(\rho)$ be the level of the discount factor at which our model first achieves a^* as the most collusive level of effort. By Proposition 4, the threat of under-reporting always weakly enlarges the equilibrium set, and strictly enlarges the equilibrium set when agents are sufficiently impatient so that $\delta(\rho) < \delta^A(\rho)$.

For a small interval of substitutability parameters $[\rho, \bar{\rho}]$, we can easily construct an interval of discount rates $[\delta^0, \delta^1]$ that is a strict subset of $[\delta(\rho), 1]$ such that for all such δ and all $\hat{\rho} \in [\rho, \bar{\rho}]$ the model with self-reporting achieves the first-best level of effort. On this same interval of δ, however, the model without self-reporting does not achieve the first-best level of effort. Moreover, it is straightforward to show that as ρ increases, the best equilibrium level of effort and welfare in the model without self-reporting decrease. Consequently, for all $\rho \in [\rho, \bar{\rho}]$, since effort and welfare in the best equilibrium with self-reporting are constant and effort and welfare in the best equilibrium without self-reporting are declining, the gains from self-reporting must be increasing in ρ on this interval.

In Figure 2 we provide a numerical illustration of our result. The figure shows the value of the best equilibrium under a relatively low value of the substitutability parameter ($\rho = 0.60$) and under a high value of the substitutability parameter ($\rho = 0.81$). As in Figure 1a, the solid lines in Figure 2 represent the best equilibrium payoffs in the economies where self-reporting is allowed, and the dashed lines represent the equilibrium payoffs in the economies where self-reporting is not allowed. The difference between the solid lines and the dashed lines represent the welfare gains from allowing self-reporting.

The figure provides a clear illustration of our result that self-reporting yields larger in-

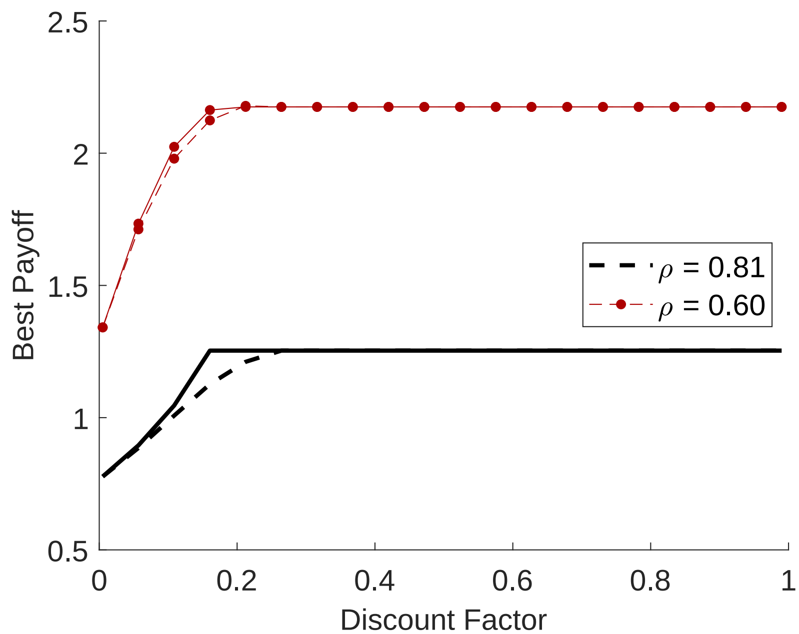


Figure 2: Best equilibrium values for $\rho = 0.60$ and $\rho = 0.81$ when self-reporting is not allowed (dashed lines) and when it is allowed (solid lines).

creases in best equilibrium values when producers' effort is more substitutable relative to when producers' effort is less substitutable. For example, for a discount factor of roughly 0.16, the model with self-reporting achieves the best equilibrium payoff both when $\rho = 0.60$ and when $\rho = 0.81$. However, at this discount rate the model without self-reporting achieves a value that is (relatively) much lower than the best equilibrium value when the substitutability of producers' output is high.

Collectively, this section shows that when producers' efforts are more substitutable, allowing the team to self-report can improve team welfare. Intuitively, when inputs are more substitutable, the incentives to shirk rise (both with and without group punishments). Proposition 5 shows that when the incentives to shirk are larger, the set of discount factors for which first best is unobtainable *without* group punishments is also larger. In this case, there is more scope for group punishments that occur before discounting is realized to improve outcomes.

The findings of this section have implications for the provision of incentives in worker-teams with unobservable inputs and decentralized structures. One interpretation sees the substitutability parameter ρ as a measure of job specialization: high ρ entails a lower marginal effect from the same worker input on total team output. Following this interpretation, our results suggest that when team members perform relatively more substitutable "dull" tasks, shirking incentives are higher (as documented by Dutcher, 2012), and self-reporting has a higher potential to deter such incentives. A second interpretation sees the parameter ρ as a measure of job similarity. Following this interpretation, our results suggest teams where team members have similar tasks are more prone to free-riding. An example of such structures is a (remote) team of developers jointly collaborating on the same section of a code. Conversely, when team members perform heterogeneous tasks, then shirking incentives are lower and self-reporting is less effective in deterring deviations. One example of such structure is a team of individual developers separately working on different sections of the code.

5 Conclusion

In this paper, we ask whether allowing a team to report its output to the manager can mitigate free-riding when team members' actions are unobservable. We uncover new conditions under which our repeated extensive form game can be characterized using simple strategies, and we show that a necessary condition for reporting to improve team welfare is the non-separability between individual actions and team output in producers' payoffs. Provided that this non-separability condition is satisfied, group punishments are most effective in reducing free riding for intermediate levels of producers' discount factors, and when producers' inputs are highly substitutable.

Our theoretical results have implications for incentive provision in WFH team production settings where only team output is observable. First, we argue that the interaction between team members' private leisure and team output provides credible incentives for the team to under-report because it imposes larger penalties on shirking agents. Second, task assignment affects shirking incentives and teams' ability to discipline these incentives through self-reporting. When team members perform homogeneous tasks, their shirking incentives are higher, and the welfare improvements from allowing the team to under-report their output to the manager are larger.

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Appendix: For Online Publication

A Definitions, Lemmas, and Propositions

A.1 Static Game Results

Lemma A.1. $0 < a_i^N < a_i^*$.

Proof. An individual agent's first-order conditions yield (recalling that in the static game $\tau = 0$, and suppressing the dependency of x on its arguments in the interest of notation)

$$\pi_x(x, a_i) x_Y Y_{a_i}(a_i, a_{-i}) + \pi_{a_i}(x, a_i) = c'(a_i). \tag{A.1}$$

The profile a^N necessarily satisfies (A.1) for all agents $i = 1, \ldots, n$. That is,

$$\pi_x(x, a_i^N) x_Y Y_{a_i}(a^N) + \pi_{a_i}(x, a_i^N) \quad = \quad c'(a_i^N). \tag{A.2}$$

The first-order condition for the socially-optimal level of effort, on the other hand, implies that for all i

$$\pi_x(x, a_i^*) x_Y Y_{a_i}(a^*) + \pi_{a_i}(x, a_i^*) + \sum_{i' \neq i} \pi_x(x, a_{i'}^*) x_Y Y_{a_i}(a^*) \quad = \quad c'(a_i^*). \tag{A.3}$$

Conditions (A.1) and (A.3) differ by an additional term in (A.3), which represents the positive externality of one agent's additional effort on the remaining $(n-1)$ agents. Since $\pi_x > 0$, $x_Y > 0$, and $Y_{a_i} > 0$, this additional term is positive. This implies that

$$\begin{aligned} \pi_x(x, a_i^N) x_Y Y_{a_i}(a^N) + \pi_{a_i}(x, a_i^N) - c'(a_i^N) \quad &> \\ \pi_x(x, a_i^*) x_Y Y_{a_i}(a^*) + \pi_{a_i}(x, a_i^*) - c'(a_i^*). \end{aligned} \tag{A.4}$$

The result follows from our assumptions on the functions π, x, and c. Given Assumption 2, we rule out the boundary solution $a_i^N = 0$, so $0 < a_i^N < a_i^*$. □

Lemma A.2. *If $a_{-i} < a_i^N$, then the most profitable deviation a_i' is such that $a_i' \geqslant a_i^N > a_{-i}$,* *and* $a_i' > a_i^N > a_{-i}$ *if* $\pi_{a_i x} < 0$*. If* $a_{-i} > a_i^N$*, then the most profitable deviation* a_i' *is such that* $a_i' \leqslant a_i^N < a_{-i}$*, and* $a_i' < a_i^N < a_{-i}$ *if* $\pi_{a_i x} < 0$*.*

Proof. Define by $\Omega(a_i, a_{-i})$ the marginal benefit of agent i's deviation, net of her leisure cost, when the other $n-1$ agents are contributing effort a_{-i}. That is,

$$\Omega(a_i, a_{-i}) \equiv \pi_x(x(a_i, a_{-i}), a_i)\,\theta Y_{a_i}(a_i, a_{-i}) + \pi_{a_i}(x(a_i, a_{-i}), a_i). \tag{A.5}$$

Since $\pi_{xx} \leqslant 0$, $Y_{a_i a_i'} \leqslant 0$, $\pi_{a_i a_i} \leqslant 0$, and $\pi_{a_i x} \leqslant 0$ from Assumption 1, it follows that $\Omega_{a_i} \leqslant 0$ and $\Omega_{a_{-i}} \leqslant 0$ (both strict when $\pi_{a_i x} < 0$). Next, consider the condition that is satisfied when $a_i = a_i^N$ for $i = 1, \ldots, n$:

$$\Omega\left(a_i^N\right) = c'(a_i^N), \tag{A.6}$$

and suppose that the effort by all the other agents but i decreases from a_{-i}^N to $a_{-i}' < a_{-i}^N$. Then,

$$\Omega\left(a_i^N, a_{-i}'\right) \geqslant c'(a_i^N). \tag{A.7}$$

The optimal response a_i' by agent i must satisfy the first-order condition

$$\Omega(a_i', a_{-i}') = c'(a_i'), \tag{A.8}$$

which means that the right-hand side of (A.7) must increase, its left-hand side must decrease, or both. Therefore, $a_i' \geqslant a_i^N$. Following the same reasoning, we get that if $\pi_{a_i x} < 0$, then (A.7) holds as a strict inequality and therefore $a_i' > a_i^N$. Following the same reasoning, we also get that if $a_{-i} > a_i^N$, then the most profitable deviation a_i' is such that $a_i' \leqslant a_i^N < a_{-i}$ (and $a_i' < a_i^N < a_{-i}$ if $\pi_{a_i x} < 0$). □

A.2 Histories, Perfect Equilibria, and One-Shot Deviations

Definition A.1. *For any history* $h^{wt} \in \mathcal{H}^w$, *the continuation game is the infinitely-repeated game that begins in period* t, *following history* h^{wt}. *For any strategy profile* $\sigma = (\{\sigma_i\}_{i=1}^n, \sigma_w)$, *agent* i*'s continuation strategy induced by* h^{wt} *is given by* $\sigma_i\left(h^{wt}h^{ws}\right)$ *for all* $h^{ws} \in \mathcal{H}^w$, *where* $h^{wt}h^{ws}$ *is the concatenation of history* h^{wt} *followed by history* h^{ws}. *Similarly, the reporter's continuation strategy induced by* h^{wt} *is given by* $\sigma_w\left(\left(h^{wt}h^{ws}\right), Y\left(\sigma_1\left(h^{wt}h^{ws}\right), \sigma_2\left(h^{wt}h^{ws}\right), \ldots, \sigma_n\left(h^{wt}h^{ws}\right)\right)\right)$ *for all* $h^{ws} \in \mathcal{H}^w$.

Definition A.2. *A perfect-public equilibrium is* $\sigma = (\{\sigma_i\}_{i=1}^n, \sigma_w)$ *such that, for all histories* $h^{wt} \in \mathcal{H}^w$,

$$U_i^t\left(h^{wt}, \sigma\right) \geqslant U_i^t\left(h^{wt}, (\tilde{\sigma}_i, \sigma_{-i}, \sigma_w)\right), \tag{A.9}$$

for all i, $\tilde{\sigma}_i$, *and*

$$U_w^t\left(h^{wt}, \sigma\right) \geqslant U_w^t\left(h^{wt}, (\{\sigma_i\}_{i=1}^n, \tilde{\sigma}_w)\right), \tag{A.10}$$

for all $\tilde{\sigma}_w$.

Definition A.3. *A one-shot deviation for agent* i *from strategy* σ_i *is a strategy* $\tilde{\sigma}_i \neq \sigma_i$ *such that there exists a unique history* $\tilde{h}^{wt} \in \mathcal{H}^w$ *such that for all* $h^{ws} \neq \tilde{h}^{wt}$,

$$\sigma_i\left(h^{ws}\right) = \tilde{\sigma}_i\left(h^{ws}\right). \tag{A.11}$$

Similarly, a one-shot deviation for the reporter from strategy σ_w *is a strategy* $\tilde{\sigma}_w \neq \sigma_w$ *such that for* *all* $h^{wt} \in \mathcal{H}^w$ *there exists a level of the total outcome* $\tilde{Y}_t$ *such that for all* $Y_t \neq \tilde{Y}_t$*,*

$$\sigma_w\left(h^{wt}, Y_t\right) \;=\; \tilde{\sigma}_w\left(h^{wt}, Y_t\right). \tag{A.12}$$

Definition A.4. *A one-shot deviation* $\tilde{\sigma}_i$ *from the agent's strategy* σ_i *is profitable if at history* $\tilde{h}^{wt}$ *for which* $\tilde{\sigma}_i\left(\tilde{h}^{wt}\right) \neq \sigma_i\left(\tilde{h}^{wt}\right)$*,*

$$U_i^t\left(\tilde{h}^{wt}, (\tilde{\sigma}_i, \sigma_{-i}, \sigma_w)\right) \;>\; U_i^t\left(\tilde{h}^{wt}, \sigma\right). \tag{A.13}$$

A one-shot deviation $\tilde{\sigma}_w$ *from the reporter's strategy* σ_w *is profitable if for all* $h^{wt} \in \mathcal{H}^w$*, at the outcome level for which* $\tilde{\sigma}_w\left(\tilde{h}^{wt}, Y_t\right) \neq \sigma_w\left(\tilde{h}^{wt}, Y_t\right)$*,*

$$U_w^t\left(\tilde{h}^{wt}, \left(\{\sigma_i\}_{i=1}^n, \tilde{\sigma}_w\right)\right) \;>\; U_w^t\left(\tilde{h}^{wt}, \sigma\right). \tag{A.14}$$

Proposition A.3. *A strategy profile* $\sigma = \left(\{\sigma_i\}_{i=1}^n, \sigma_w\right)$ *is a perfect-public equilibrium if and only if there are no profitable one-shot deviations either for the agents or for the reporter.*

Proof. If a profile is a perfect-public equilibrium, clearly there are no profitable one-shot deviations. Now suppose that the profile σ is not a perfect-public equilibrium. We want to show that there must be a profitable one-shot deviation. Since σ is not a perfect-public equilibrium, there exists a history $\tilde{h}^{wt}$, an agent i and a strategy $\tilde{\sigma}_i$ (the proof for the reporter follows the same steps) such that

$$U_i^t\left(\tilde{h}^{wt}, \sigma\right) \;<\; U_i^t\left(\tilde{h}^{wt}, (\tilde{\sigma}_i, \sigma_{-i}, \sigma_w)\right). \tag{A.15}$$

Let $\varepsilon = U_t^i\left(\tilde{h}^{wt}, (\tilde{\sigma}_i, \sigma_{-i}, \sigma_w)\right) - U_t^i\left(\tilde{h}^{wt}, \sigma\right)$. Let $m = \min_{i,a,\tau} u_i(a, \tau)$ and $M = \max_{i,a,\tau} u_i(a, \tau)$. Recall that for a given $\delta \in (0, 1)$ and a given $\kappa(\delta) > 0$ satisfying Assumption 2, Assumption 1 ensures that M and m exist. Thus, there exists T sufficiently large so that $\delta^T(M - m) < \varepsilon/2$.

Finally, for any agent i and history $h^{ws} \in \mathcal{H}^{w}$, let

$$u_i^s\left(\left(\tilde{h}^{wt}h^{ws}\right),\sigma\right) \;=\; u_i\left(\left\{\sigma_i\left(\tilde{h}^{wt}h^{ws}\right)\right\}_{i=1}^{n},\sigma_w\left(\left(\tilde{h}^{wt}h^{ws}\right),Y\left(\tilde{h}^{wt}h^{ws}\right)\right)\right), \tag{A.16}$$

where $Y\left(\tilde{h}^{wt}h^{ws}\right)$ is short-hand notation for $Y\left(\sigma_1\left(\tilde{h}^{wt}h^{ws}\right),\sigma_2\left(\tilde{h}^{wt}h^{ws}\right),\ldots,\sigma_n\left(\tilde{h}^{wt}h^{ws}\right)\right)$, and denote by $\tilde{h}^{ws}$ the period-s history induced by $(\tilde{\sigma}_i,\sigma_{-i},\sigma_w)$. Then,

$$\begin{aligned}
&(1-\delta)\left[\sum_{s=t}^{T-1}\delta^s u_i^s\left(\left(\tilde{h}^{wt}h^{ws}\right),\sigma\right)+\sum_{s=T}^{\infty}\delta^s u_i^s\left(\left(\tilde{h}^{wt}h^{ws}\right),\sigma\right)\right]\\
&=(1-\delta)\left[\sum_{s=0}^{T-1}\delta^s u_i^s\left(\left(\tilde{h}^{wt}\tilde{h}^{ws}\right),(\tilde{\sigma}_i,\sigma_{-i},\sigma_w)\right)+\sum_{s=T}^{\infty}\delta^s u_i^s\left(\left(\tilde{h}^{wt}\tilde{h}^{ws}\right),(\tilde{\sigma}_i,\sigma_{-i},\sigma_w)\right)\right]-\varepsilon,
\end{aligned} \tag{A.17}$$

so that

$$(1-\delta)\sum_{s=t}^{T-1}\delta^s u_i^s\left(\left(\tilde{h}^{wt}h^{ws}\right),\sigma\right) < \quad (1-\delta)\sum_{s=0}^{T-1}\delta^s u_i^s\left(\left(\tilde{h}^{wt}\tilde{h}^{ws}\right),(\tilde{\sigma}_i,\sigma_{-i},\sigma_w)\right)-\frac{\varepsilon}{2}. \tag{A.18}$$

Then the strategy $\hat{\sigma}_i$ such that

$$\hat{\sigma}_i\left(h^{ws}\right)=\begin{cases}\tilde{\sigma}_i\left(h^{ws}\right) & \text{if } s<T,\\ \sigma_i\left(h^{ws}\right) & \text{if } s\geqslant T,\end{cases} \tag{A.19}$$

is a profitable deviation from $\sigma_i\left(\tilde{h}^{wt}\right)$. Now let $\hat{h}^{w(T-1)}$ denote the period $T-1$ history induced by $(\hat{\sigma}_i,\sigma_{-i},\sigma_w)$. There are two possibilities. First, suppose

$$U_i^{T-1}\left(\left(\tilde{h}^{wt}\hat{h}^{w(T-1)}\right),\sigma\right) \;<\; U_i^{T-1}\left(\left(\tilde{h}^{wt}\hat{h}^{w(T-1)}\right),(\hat{\sigma}_i,\sigma_{-i},\sigma_w)\right). \tag{A.20}$$

Then, since $\hat{\sigma}_i$ agrees with σ_i in period T and after T, we have a profitable one-shot deviation after history $\tilde{h}^{wt}\hat{h}^{w(T-1)}$. Alternatively, suppose

$$U_i^{T-1}\left(\left(\tilde{h}^{wt}\hat{h}^{w(T-1)}\right),\sigma\right) \geqslant U_i^{T-1}\left(\left(\tilde{h}^{wt}\hat{h}^{w(T-1)}\right),(\hat{\sigma}_i,\sigma_{-i},\sigma_w)\right), \tag{A.21}$$

and construct the strategy

$$\bar{\sigma}_i\left(h^{ws}\right) = \begin{cases} \hat{\sigma}_i\left(h^{ws}\right) & \text{if } s < T-1, \\ \sigma_i\left(h^{ws}\right) & \text{if } s \geqslant T-1. \end{cases} \tag{A.22}$$

Since

$$\begin{aligned} U_i^{T-2}\left(\left(\tilde{h}^{wt}\hat{h}^{w(T-2)}\right),(\hat{\sigma}_i,\sigma_{-i},\sigma_w)\right) &= (1-\delta)\,u_i^{T-2}\left(\left(\tilde{h}^{wt}\hat{h}^{w(T-2)}\right),(\hat{\sigma}_i,\sigma_{-i},\sigma_w)\right) \\ &+ \delta U_i^{T-1}\left(\left(\tilde{h}^{wt}\hat{h}^{w(T-1)}\right),(\hat{\sigma}_i,\sigma_{-i},\sigma_w)\right) && \text{(A.23)} \\ &\leqslant (1-\delta)\,u_i^{T-2}\left(\left(\tilde{h}^{wt}\hat{h}^{w(T-2)}\right),(\hat{\sigma}_i,\sigma_{-i},\sigma_w)\right) \\ &+ \delta U_i^{T-1}\left(\left(\tilde{h}^{wt}\hat{h}^{w(T-1)}\right),\sigma\right) && \text{(A.24)} \\ &= U_i^{T-2}\left(\left(\tilde{h}^{wt}\hat{h}^{w(T-2)}\right),(\bar{\sigma}_i,\sigma_{-i},\sigma_w)\right), && \text{(A.25)} \end{aligned}$$

then

$$U_i^t\left(\tilde{h}^{wt},(\hat{\sigma}_i,\sigma_{-i},\sigma_w)\right) \leqslant U_i^t\left(\tilde{h}^{wt},(\bar{\sigma}_i,\sigma_{-i},\sigma_w)\right), \tag{A.26}$$

and $\bar{\sigma}_i$ is a profitable deviation at $\tilde{h}^{wt}$ that only differs from σ_i in the first $T-1$ periods. Proceeding in this way, we find a profitable one-shot deviation. □

A.3 Equilibrium Set Characterization

Lemma A.4. *The best equilibrium in the program (8)-(11) features no group punishments on-path.*

Proof. The best equilibrium in the program (8)-(11) is defined by agents' strategies a, group punishments $\tau(Y(a))$, and continuation utilities $\nu(a,\tau(Y(a)))$. Suppose that the best equilibrium features punishments on path (i.e., $\tau(Y(\bar{a})) > 0$). Construct a new strategy that keeps agents' strategies a fixed, features zero group punishments *only* at $Y(\bar{a})$ (i.e., $\hat{\tau}(Y(\bar{a})) = 0$ and $\hat{\tau} = \tau$ for all $a \neq \bar{a}$), and keeps continuation utilities fixed (i.e., $\hat{v}(a, \hat{\tau}(Y)) = v(a, \tau(Y))$ for all a). This strategy increases the on-path utility of the agents and the reporter, while leaving continuation utilities unchanged by construction. In other words, the new strategy increases the left-hand side of (9) while leaving (10)-(11) unchanged. As a result, the new strategy is incentive-compatible but delivers strictly higher welfare to agents, which contradicts the that the original strategy was the best. □

Proposition A.5. *There exists a level of effort $\tilde{a}$ such that the carrot-and-stick strategy $\sigma\left((\tilde{a}, \bar{a}), (0,0)\right)$ attains the value $\underline{v}$—that is, $\sigma\left((\tilde{a}, \bar{a}), (0,0)\right)$ is an optimal punishment.*

Proof. Let $\underline{v}$ be the infimum of equilibrium payoffs and $\bar{a}$ the value that attains the maximum $\bar{v}$ in the program (14)-(17). Again let $B(\kappa)$ be the min-max payoff of this game, where $\underline{v} \geqslant B(\kappa)$. The assumption that

$$(1-\delta)\pi(\kappa, 0) + \delta V(1) \quad < \quad B(\kappa), \tag{A.27}$$

ensures that we may obtain $\tilde{a}$ such that

$$\underline{v} = (1-\delta)\, u\left(\tilde{a}, 0\right) + \delta u\left(\bar{a}, 0\right). \tag{A.28}$$

To see this, note that $V(1) \geqslant \bar{v} = u(\bar{a}, 0)$ and consider $u(\tilde{a}, 0)$ as $\tilde{a} \to 0$. We have

$$\lim_{\tilde{a} \to 0} u(\tilde{a}, 0) = \lim_{\tilde{a} \to 0} \pi(x(\tilde{a}, 0), \tilde{a}) - c(\tilde{a}) = \pi(\kappa, 0) - c(0) = \pi(\kappa, 0). \tag{A.29}$$

Hence,

$$\lim_{\tilde{a} \to 0}(1-\delta)u(\tilde{a}, 0) + \delta u(\bar{a}, 0) = (1-\delta)\pi(\kappa, 0) + \delta u(\bar{a}, 0) \leqslant (1-\delta)\pi(\kappa, 0) + \delta V(1) < \underline{v}. \tag{A.30}$$

As a result, we can find $\tilde{a}$ so that

$$(1-\delta)u(\tilde{a},0)+\delta u(\bar{a},0)=\underline{v}. \tag{A.31}$$

We next argue that the carrot-and-stick strategy $\sigma((\tilde{a},\bar{a}),(0,0))$ is an equilibrium. By construction, the punishment has value $\underline{v}$. Since deviations from $\bar{a}$ are unprofitable when punished by $\underline{v}$, they are by construction unprofitable when punished by $\sigma((\tilde{a},\bar{a}),(0,0))$.

To show that no agent wishes to deviate when prescribed to contribute effort $\tilde{a}$, we must show that for all a',

$$\underline{v}=(1-\delta)\,u\,(\tilde{a},0)+\delta u\,(\bar{a},0) \;\;\geqslant\;\; (1-\delta)\,g\left(a',\tilde{a},0\right)+\delta\underline{v}, \tag{A.32}$$

and in particular

$$\underline{v}=(1-\delta)\,u\,(\tilde{a},0)+\delta u\,(\bar{a},0) \;\;\geqslant\;\; (1-\delta)\,\hat{g}\,(\tilde{a},0)+\delta\underline{v}. \tag{A.33}$$

We proceed by contradiction. Suppose (A.33) does not hold. Then there must exist another (strongly symmetric) equilibrium σ^y with first-period output $a^y\leqslant a^N$ such that

$$(1-\delta)\,\hat{g}\,(\tilde{a},0)+\delta\underline{v}>(1-\delta)\,u\,(a^y,0)+\delta U\,(\sigma^y|_{a^y})\geqslant\underline{v}, \tag{A.34}$$

where $U\,(\sigma^y|_{a^y})$ is the continuation payoff to a single agent from σ^y after contributing a^y in the first period.[A.1]

[A.1] Since repeated play of the static Nash equilibrium output a^N with no punishments must be an equilibrium, it is straightforward to show that the prescribed effort under the "stick" must satisfy $\tilde{a}\leqslant a^N$. If $a^y>a^N$, however, (A.34) implies that

$$\hat{g}(\tilde{a},0)>\hat{g}(a^N,0).$$

Since the best deviation payoff in the absence of punishments is increasing in a, this would imply $a^N<\tilde{a}$, a contradiction.

Replacing the definition of $\underline{v}$ in (A.34) implies

$$(1-\delta)\,u\left(a^{y},0\right)+\delta U\left(\sigma^{y}|_{a^{y}}\right) \;\;\geqslant\;\; (1-\delta)\,u\left(\tilde{a},0\right)+\delta u\left(\bar{a},0\right). \tag{A.35}$$

Since $U\left(\sigma^{y}|_{a^{y}}\right) \leqslant u\left(\bar{a},0\right)$, it must be that $u\left(a^{y},0\right) \geqslant u\left(\tilde{a},0\right)$ and therefore $a^{y} \geqslant \tilde{a}$. However, we show that if σ^{y} is a perfect equilibrium, $\tilde{a} > a^{y}$, yielding the necessary contradiction. Since σ^{y} is an equilibrium,

$$(1-\delta)\,u\left(a^{y},0\right)+\delta U\left(\sigma^{y}|_{a^{y}}\right) \;\;\geqslant\;\; (1-\delta)\,\hat{g}\left(a^{y},\tau\left(Y(a^{y})\right)\right)+\delta\underline{v}, \tag{A.36}$$

so that from (A.34)

$$(1-\delta)\,\hat{g}\left(\tilde{a},0\right)+\delta\underline{v} \;\;>\;\; (1-\delta)\,\hat{g}\left(a^{y},\tau\left(Y(a^{y})\right)\right)+\delta\underline{v}. \tag{A.37}$$

Since $a^{y} \leqslant a^{N}$, Proposition 2 implies that

$$\hat{g}\left(a^{y},\tau\left(Y(a^{y})\right)\right) \geqslant \hat{g}\left(a^{y},0\right), \tag{A.38}$$

so that

$$\hat{g}\left(\tilde{a},0\right) > \hat{g}\left(a^{y},0\right). \tag{A.39}$$

Since $\hat{g}\left(a,0\right)$ is increasing in a, (A.39) implies $\tilde{a} > a^{y}$ providing the needed contradiction. □

Proposition A.6. *If the strategy σ is an equilibrium, then $u\left(\sigma\right) \in [\underline{v},\bar{v}]$. If $v \in [\underline{v},\bar{v}]$, then there exists an equilibrium strategy σ such that $u\left(\sigma\right) = v$.*

Proof. We only need to prove that for each $v \in [\underline{v},\bar{v}]$, there exists an equilibrium strategy that attains the value v. To construct such strategy, we start from the set of equilibrium strategies of the game where the reporter is not allowed to impose group punishments,

$\left[\underline{v}^A, \bar{v}^A\right]$. We know from Abreu (1986) that any equilibrium value v^A such that $v^A \in \left[\underline{v}^A, \bar{v}^A\right]$ can be achieved with an equilibrium strategy σ^A. Under σ^A, the reporter never imposes group punishments and the agents exert effort a^A such that $u\left(a^A\right) = v^A$ on path, and deviations by both team and agents are punished by reversion to the worst (carrot-and-stick) equilibrium with value $\underline{v}^A$. Therefore, we focus on characterizing the equilibrium strategies for the cases in which $\left[\underline{v}^A, \bar{v}^A\right] \subset [\underline{v}, \bar{v}]$.

Consider a new strategy σ^1. Define by $\bar{a}^A$ the carrot output in the model where group punishments are not allowed. Under σ^1, for some $\varepsilon^1 > 0$, the agents choose $a^1 = \bar{a}^A + \varepsilon^1$ as long as the aggregate outcome Y^1 is such that $Y^1 = Y\left(a^1\right)$, and the reporter never imposes punishments. Suppose that an agent deviates to some a', such that the observed aggregate outcome is $Y^1 = Y\left(a', a^1\right)$. In this case, the reporter imposes an arbitrarily small punishment $\tau^1\left(Y^1\right) > 0$ and agents follow a continuation strategy $\sigma^A\left(v^1\left(a', a^1, \tau^1\left(Y^1\right)\right)\right)$, featuring *no group punishments* after the current period, to deliver a value $v^1\left(a', a^1, \tau^1\left(Y^1\right)\right) \in \left[\underline{v}^A, \bar{v}^A\right]$. Specifically, we let the value of this strategy be

$$v^1\left(a', a^1, \tau^1\left(Y^1\right)\right) \equiv \underline{v}^A + \frac{1-\delta}{\delta}\frac{1}{n}\left[w\left(a', a^1, 0\right) - w\left(a', a^1, \tau^1\left(Y^1\right)\right)\right], \tag{A.40}$$

where the term in square brackets is positive. For any $\delta > 0$, we can pick any τ^1 positive and small enough such that $v^1\left(a', a^1, \tau^1\left(Y^1\right)\right) < \bar{v}^A$, thus ensuring that $v^1\left(a', a^1, \tau^1\left(Y^1\right)\right) \in \left[\underline{v}^A, \bar{v}^A\right]$. From Abreu (1986), there exists a continuation strategy that never calls for equilibrium self-reporting and achieves $v^1\left(a', a^1, \tau^1\left(Y^1\right)\right)$ as an equilibrium value.

If an agent deviates and the reporter implements the prescribed punishment, then the agents follow the strategy $\sigma^A\left(v^1\left(a', a^1, \tau^1\left(Y^1\right)\right)\right)$, and the reporter never imposes group punishments either on or off-path. As a result, the continuation value promised to the agents when one of the agents deviates and the reporter imposes $\tau^1\left(Y^1\right)$ can be achieved with an equilibrium strategy σ^A. Conversely, deviations by the agents followed by deviations by the reporter are punished by the worst equilibrium strategy $\sigma^A\left(\underline{v}^A\right)$. Clearly, this strategy is an equilibrium. Moreover, it achieves a value $u\left(a^1\right) \equiv \bar{v}^1 > \bar{v}^A$.

Next, note that reversion to the equilibrium $\bar{v}^1 > \bar{v}^A$ allows to construct a new carrot-and-stick strategy in which the agents contribute an effort level $\tilde{a}^1 < \tilde{a}^A$ for one period and then revert to $\bar{v}^1$, with deviations from the prescription causing the prescription to be repeated. Moreover, note that this new carrot-and-stick strategy has value $\underline{v}^1 < \underline{v}^A$. Hence, for any value $v^1 \in \left[\underline{v}^1, \bar{v}^1\right]$, we can find an equilibrium strategy σ^1 such that $u\left(\sigma^1\right) = v^1$.

Now take some $k \geqslant 2$ and set $\left[\underline{v}^k, \bar{v}^k\right]$ such that $\left[\underline{v}^1, \bar{v}^1\right] \subset \left[\underline{v}^k, \bar{v}^k\right] \subset [\underline{v}, \bar{v}]$, and assume that for any $v^k \in \left[\underline{v}^k, \bar{v}^k\right]$ we can construct an equilibrium strategy σ^k such that $u\left(\sigma^k\right) = v^k$. Denote by $\bar{a}^k$ the effort level with value $\bar{v}^k$, and construct a new strategy σ^{k+1}. Under σ^{k+1}, for some $\varepsilon^{k+1} > 0$ the agents contribute $a^{k+1} = \bar{a}^k + \varepsilon^{k+1}$ as long as the observed aggregate outcome Y^{k+1} is such that $Y^{k+1} = Y\left(a^{k+1}\right)$, and the reporter never imposes punishments. Suppose that an agent deviates to some a', such that the observed aggregate outcome is $Y^{k+1} = Y\left(a', a^{k+1}\right)$. In this case, the reporter imposes a punishment $\tau^{k+1}\left(Y^{k+1}\right) > 0$ such that the punishment is feasible. That is, such that $v^{k+1}\left(a', a^{k+1}, \tau^{k+1}\left(Y^{k+1}\right)\right) \in \left[\underline{v}^k, \bar{v}^k\right]$, where

$$v^{k+1}\left(a', a^{k+1}, \tau^{k+1}\left(Y^{k+1}\right)\right) \equiv \underline{v}^k + \frac{1-\delta}{\delta}\frac{1}{n}\left[w\left(a', a^{k+1}, 0\right) - w\left(a', a^{k+1}, \tau^{k+1}\left(Y^{k+1}\right)\right)\right]. \tag{A.41}$$

Note that since $\bar{v}^k > \bar{v}^1$, the range of punishments that can be sustained is larger than $\left[0, \sup_{Y^1} \tau^1\left(Y^1\right)\right]$. If an agent deviates and the reporter implements the prescribed punishment, then the agents follow the strategy $\sigma^k\left(v^{k+1}\left(a', a^{k+1}, \tau^{k+1}\left(Y^{k+1}\right)\right)\right)$. Therefore, the continuation value promised to agents when one of them deviates and the reporter imposes $\tau^{k+1}\left(Y^{k+1}\right)$ can be achieved with an equilibrium strategy. Conversely, deviations by the agents followed by deviations by the reporter are punished by the worst equilibrium strategy $\sigma^k\left(\underline{v}^k\right)$. Clearly, this strategy is an equilibrium. Moreover, it achieves a value $u\left(a^{k+1}\right) \equiv \bar{v}^{k+1} > \bar{v}^k$. Next, note that reversion to the equilibrium $\bar{v}^{k+1} > \bar{v}^k$ allows us to construct a new carrot-and-stick strategy in which the agents exert an effort level $\tilde{a}^{k+1} > \tilde{a}^k$

for one period and then revert to $\bar{v}^{k+1}$, with deviations from the prescription causing the prescription to be repeated. Moreover, note that this new carrot-and-stick strategy has value $\underline{v}^{k+1} < \underline{v}^{k}$. Hence, for any value $v^{k+1} \in \left[\underline{v}^{k+1}, \bar{v}^{k+1}\right]$, we can find an equilibrium strategy σ^{k+1} such that $u\left(\sigma^{k+1}\right) = v^{k+1}$. The proof is completed by induction. □

Proposition A.7. *The optimal carrot-and-stick punishment satisfies*

$$\hat{g}(\tilde{a}, 0) = (1-\delta) u(\tilde{a}, 0) + \delta u(\bar{a}, 0) = \underline{v}, \tag{A.42}$$

$$\hat{g}(\bar{a}, \tau(\cdot)) = u(\bar{a}, 0) + \delta(u(\bar{a}, 0) - u(\tilde{a}, 0)) \text{ if } \bar{a} < a^*, \tag{A.43}$$

and

$$\hat{g}(\bar{a}, \tau(\cdot)) \leqslant u(\bar{a}, 0) + \delta(u(\bar{a}, 0) - u(\tilde{a}, 0)) \text{ if } \bar{a} = a^*. \tag{A.44}$$

Proof. Suppose $\sigma((\tilde{a}, \bar{a}), (0, 0))$ is an optimal carrot-and-stick punishment. Recalling from Proposition A.5 that $\tilde{a} \leqslant a^N$, the requirements that the agents do not deviate from the stick and carrot outputs $\tilde{a}$ and $\bar{a}$ are, respectively:

$$(1-\delta) u(\tilde{a}, 0) + \delta u(\bar{a}, 0) \geqslant (1-\delta) \hat{g}(\tilde{a}, 0) + \delta(1-\delta) u(\tilde{a}, 0) + \delta^2 u(\bar{a}, 0), \tag{A.45}$$

$$u(\bar{a}, 0) \geqslant (1-\delta) \hat{g}(\bar{a}, \tau(\cdot)) + \delta(1-\delta) u(\tilde{a}, 0) + \delta^2 u(\bar{a}, 0). \tag{A.46}$$

Rearranging these inequalities, we get

$$\hat{g}(\tilde{a}, 0) \leqslant (1-\delta) u(\tilde{a}, 0) + \delta u(\bar{a}, 0) = \underline{v}, \tag{A.47}$$

$$\hat{g}(\bar{a}, \tau(\cdot)) \leqslant u(\bar{a}, 0) + \delta(u(\bar{a}, 0) - u(\tilde{a}, 0)). \tag{A.48}$$

If (A.47) holds strictly, we can decrease $\tilde{a}$ and hence reduce $u(\tilde{a}, 0)$ while preserving (A.48). But this yields a lower punishment value than the infimum $\underline{v}$, a contradiction. Hence (A.47) holds with equality. Now suppose that if $\bar{a} < a^*$, (A.48) holds as a strict inequality. Then we can simultaneously decrease $\tilde{a}$ by a small amount (therefore not violating (A.48)) and increase $\bar{a}$ to preserve (A.47). But then since $\hat{g}(\tilde{a}, 0)$ is increasing in $\tilde{a}$ and (A.47), we also found a lower punishment value than the infimum, again a contradiction. $\square$

A.4 Proof of Proposition 5

Fix $\rho \in (0, 1)$. We know that there exists a unique $\delta^{A*}(\rho)$ in the model where under-reporting is not allowed such that $\bar{a}^A(\rho) = a^*$ for all $\delta \geqslant \delta^{A*}(\rho)$. This $\delta^{A*}(\rho)$ simultaneously solves

$$\hat{g}(a^*, 0) = \left(1 + \delta^{A*}(\rho)\right) u(a^*) - \delta^{A*}(\rho) u\left(\tilde{a}^A(\rho)\right), \tag{A.49}$$

$$\hat{g}\left(\tilde{a}^A(\rho), 0\right) = \left(1 - \delta^{A*}(\rho)\right) u\left(\tilde{a}^A(\rho)\right) + \delta^{A*}(\rho) u(a^*), \tag{A.50}$$

and represents the threshold level of the discount factor for which the model where under-reporting is not allowed achieves the first-best level of effort a^*. Similarly, for the same ρ we know that there exists a unique $\delta^*(\rho)$ in the model where under-reporting is allowed such that $\bar{a}(\rho) = a^*$, which simultaneously solves

$$\hat{g}(a^*, \tau(\cdot)) = (1 + \delta^*(\rho)) u(a^*) - \delta^*(\rho) u(\tilde{a}(\rho)), \tag{A.51}$$

$$\hat{g}(\tilde{a}(\rho), 0) = (1 - \delta^*(\rho)) u(\tilde{a}(\rho)) + \delta^*(\rho) u(a^*). \tag{A.52}$$

Next, note that for $\bar{a}^A < a^*$ if $\bar{a}$ is sustained by a positive punishment threat (for some $a' \neq \bar{a}$, $\tau(a' + (n-1)\bar{a}) > 0$), then $\bar{a}^A < \bar{a} \leqslant a^*$. Then, for all $\delta < \delta^{A*}(\rho)$, $\left[\underline{v}^A; \bar{v}^A\right] \subset [\underline{v}; \bar{v}]$, and $\delta^*(\rho) < \delta^{A*}(\rho)$ (i.e., the model where group punishments are allowed achieves the first-best level of effort a^* at a lower value of the discount factor than the model where group punishments are not allowed). Next, let δ^0 be such that $\delta^*(\rho) < \delta^0 < \delta^{A*}(\rho)$. Note that at δ^0, $\bar{a}(\rho) = a^*$ and $\bar{a}^A(\rho) < a^*$. Now let $\bar{\rho}' > \rho$, and let $\delta^*(\bar{\rho}')$ in the model where group punishments are allowed be such that $\bar{a}(\bar{\rho}') = a^*$, which solves

$$\hat{g}(a^*, \tau(\cdot)) = \left(1 + \delta^*(\bar{\rho}')\right) u(a^*) - \delta^*(\rho) u\left(\tilde{a}(\rho')\right) \tag{A.53}$$

$$\hat{g}\left(\tilde{a}\left(\bar{\rho}'\right), 0\right) = \left(1 - \delta^*(\bar{\rho}')\right) u\left(\tilde{a}(\bar{\rho}')\right) + \delta^*(\rho) u(a^*). \tag{A.54}$$

By continuity we can always choose $\bar{\rho}'$ small enough such that $\delta^*(\bar{\rho}') < \delta^0$. Therefore, in the model where group punishments are allowed $\bar{a}(\bar{\rho}') = \bar{a}(\rho) = a^*$ for all $\delta \in (\delta^0, 1)$. We also know that there exists a $\delta^1 \geqslant \delta^A(\rho)$ such that for all $\delta \in (\delta^{A*}(\rho), \delta^1)$, then $\bar{a}^A(\bar{\rho}') < \bar{a}^A(\rho) = a^*$.[A.2] Hence, at δ^A

$$\Delta U\left(\rho'\right) \equiv \frac{u\left(\bar{a}(\bar{\rho}')\right) - u\left(\bar{a}^A(\bar{\rho}')\right)}{u\left(\bar{a}^A(\bar{\rho}')\right)} > \frac{u\left(\bar{a}(\rho)\right) - u\left(\bar{a}^A(\rho)\right)}{u\left(\bar{a}^A(\rho)\right)} \equiv \Delta U(\rho). \tag{A.55}$$

In words, as ρ increases to $\bar{\rho}'$, the model with group punishments achieves higher welfare than the model without group punishments. Following the same argument, we have that for all $\rho' \in (\rho, \bar{\rho}')$, $d\Delta U(\rho')/d\rho' > 0$.

[A.2]The symmetric equilibrium payoff function is decreasing in ρ, such that if ρ increases for a fixed $\delta \geqslant \delta^{A*}(\rho)$, then the best equilibrium level of effort is lower than a^*.

B Computational Algorithm

In this appendix, we describe the computational algorithm for our numerical results in Section 3. Define $\hat{a} \equiv \arg\max_{a'} g\left(a', \bar{a}, \tau\left(a' + (n-1)\,\bar{a}\right)\right)$. For each level of the discount factor δ, we want to find $\tilde{a}$, $\bar{a}$, $\hat{a}$, and τ that solve the following system of equations:

$$
\begin{aligned}
\hat{g}\left(\tilde{a}, 0\right) &= (1-\delta)\, u\left(\tilde{a}, 0\right) + \delta \mu\left(\bar{a}, 0\right), && \text{(B.1)} \\
\hat{g}\left(\hat{a}, \tau(\cdot)\right) &\leqslant u\left(\bar{a}, 0\right) + \delta\left(u\left(\bar{a}, 0\right) - u\left(\tilde{a}, 0\right)\right), && \text{(B.2)} \\
u\left(\bar{a}, 0\right) &\geqslant \frac{1-\delta}{\delta}\frac{1}{n}\left[w\left(\hat{a} + (n-1)\,\bar{a}, 0\right) - w\left(\hat{a} + (n-1)\,\bar{a}, \tau\left(\hat{a} + (n-1)\,\bar{a}\right)\right)\right] \\
&\quad + \hat{g}\left(\tilde{a}, 0\right). && \text{(B.3)}
\end{aligned}
$$

From Proposition A.7, Equation (B.2) holds with equality only when $\bar{a} < a^*$ and is slack when $\bar{a} = a^*$. The algorithm works as follows:

1. For each level of the discount factor δ, we know that $\tau \in [0, (n-1)a^* + a'(a^*)]$, where $a'(a^*)$ is the most profitable deviation from a^*. Start with $\hat{\tau} = (n-1)a^* + a'(a^*)$.

   (a) Check if a^* can be supported:

      i. Set $\bar{a} = a^*$. Solve (B.1) for $\tilde{a}$.

      ii. Obtain $\hat{a} = \arg\max_{a' \in [a^N, a^*]} g\left(a', \bar{a}, \hat{\tau}\right)$. We do this by searching for $\hat{a}$ over a fine grid for a'. Evaluate $\hat{g}\left(\bar{a}, \hat{\tau}\right)$.

      iii. Check if the resulting values for $\tilde{a}$ and $\bar{a}$ satisfy (B.2) (with inequality) and (B.3). If so, the algorithm is finished.

   (b) If either (B.2) or (B.3) is not satisfied (a^* cannot be supported), jointly solve for $\bar{a}$ and $\tilde{a}$. We do this by using a nested bisection algorithm to solve (B.1) and (B.2) with equality (also solving for $\hat{a}$ as before).

      i. The nested bisection algorithm proceeds as follows. The outer bisection algorithm searches for $\tilde{a} \in [\tilde{a}_\ell, \tilde{a}_h]$. The inner bisection algorithm solves for the corresponding $\bar{a}$.

ii. At each iteration of the double bisection algorithm, check whether (B.1)-(B.3) are all satisfied.

2. If (B.1) and (B.3) are satisfied, the algorithm is finished. If not, decrease $\hat{\tau}$ by a small amount and return to step 1.