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Reputation and Persistence of Adverse Selection in Secondary Loan Markets†

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The volume of new issuances in secondary loan markets fluctuates over time and falls when collateral values fall. We develop a model with adverse selection and reputation that is consistent with such fluctuations. Adverse selection ensures that the volume of trade falls when collateral values fall. Without reputation, the equilibrium has separation, adverse selection is quickly resolved, and trade volume is independent of collateral value. With reputation, the equilibrium has pooling and adverse selection persists over time. The equilibrium is efficient unless collateral values are low and originators' reputational levels are low. We describe policies that can implement efficient outcomes. (*JEL* D82, G11, G21, G28)

Secondary loan markets allow loan originators to sell all or part of their loan portfolios to other financial institutions. These markets include the market for syndicated loans as well as the market for publicly traded securitized loans. This market is economically important. In 2007, for example, approximately $1.3 trillion of new loans were syndicated, and from 1986 to 2012, approximately $500 billion of new loans were syndicated each year.[1] The volume of new issuances in secondary loan markets arising from originators' sales decisions fluctuates a great deal and sometimes collapses, typically when the value of the assets underlying the original loans falls. Ivashina and Scharfstein (2010), for example, present data that shows that the volume of loans to large corporations, almost all of which are syndicated, declined by 37 percent in August and September 2008 relative to the previous year. In Chari,

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[1] These statistics were computed using data from Thomson Reuters LPC. This dataset includes loans made to a subset of nonfinancial corporations.

Shourideh, and Zetlin-Jones (2010), we document that in the late 1920s, the volume of new issuances in the securitized loan market fell sharply.

In the wake of the recent financial crisis, policymakers were concerned about the effect of the collapse on broader economic activity and initiated a variety of asset purchase policies intended to restore the volume of new issuances in secondary loan markets. Evaluating the effects of these policies requires models in which trade volume falls when the underlying asset values fall.

In this paper, we develop a dynamic model of the volume of new issuances of secondary loans, show that reductions in underlying asset values can generate a fall in this volume, and use the model to evaluate policies. Adverse selection and reputation play key roles in our model. We assume that sellers are better informed than buyers about the quality of their loan portfolios, so our model features adverse selection. Adverse selection ensures that the volume of trade falls when the value of the underlying asset falls. Incorporating reputational concerns is necessary in the sense that in the absence of these concerns, adverse selection is quickly resolved. When such concerns are sufficiently strong, our model has pooling outcomes in the sense that banks of different quality levels make the same decisions. This pooling implies that buyers cannot use past actions to perfectly infer the quality levels of individual banks, so adverse selection persists.

The persistence of adverse selection implies that our model can generate fluctuations in trade volume associated with fluctuations in underlying asset values even in the long run. In terms of policy, we show that our equilibrium outcomes are efficient unless underlying asset values are low and secondary loan issuers have poor reputations. Thus, intervention is needed only when underlying asset values are low and is best directed at issuers with poor reputations.

In our model, financial institutions that originate primary loans, called *banks* for convenience, sell all or part of these loans to other institutions. Motivated by an empirical literature discussed below, we assume that banks are better informed than buyers about the likelihood of default of these loans. This asymmetry of information creates an adverse selection problem because buyers understand that banks have an incentive to sell loans with a high likelihood of default. In the static version of our model, buyers use the quantity sold as a screening device to induce high-quality banks to separate themselves from low-quality banks. In particular, buyers offer nonlinear pricing schedules in which the price per unit of loans falls with the fraction of a bank's loan portfolio that is purchased. Banks with high-quality loans are attracted to contracts that offer a high per unit price with a small fraction sold because holding loans is not very costly. Banks with low-quality loans are attracted to contracts that offer a low per unit price and allow a large fraction of their loans to be sold. The equilibrium is necessarily separating. Pooling outcomes in which banks with different qualities of loan portfolios choose the same amount of loans to be sold cannot be equilibria because buyers have strong incentives to offer cream-skimming contracts that are relatively attractive only to high-quality banks. Because the equilibrium is separating, the quality level of individual banks becomes known to future buyers.[2]

[2] A familiar issue with adverse selection models is that a separating equilibrium in pure strategies sometimes does not exist. We follow Dasgupta and Maskin (1986) in studying equilibria in mixed strategies by uninformed

Adverse selection is particularly acute when the value of the loans in default, called *collateral value*, is low. In the static version of our model, we show that when the collateral value is low, the fraction of loans sold by high-quality banks is low. Thus, declines in collateral values lead to declines in volume. This feature of our model also appears in other models in economics and finance that study the endogenous determination of trade volume with adverse selection. See, for example, Glosten and Milgrom (1985); Garleanu and Pedersen (2004); Guerrieri and Shimer (2012); Fishman and Parker (2012); and Kurlat (2013), among many others.

While adverse selection is promising in accounting for volume fluctuations, the feature that the equilibrium is separating creates a challenge in accounting for fluctuations in volume in periods other than the first period in our dynamic model. The challenge arises because with separating equilibria, the quality levels of banks are revealed to buyers in future periods. There is then no adverse selection in future periods if the same banks participate regularly in secondary markets and if quality levels are persistent. The empirical literature on secondary loan markets discussed below documents that issuers of secondary loans are typically well-established institutions that regularly participate in these markets and are therefore not anonymous traders. This literature also suggests that quality and reputations are persistent. These considerations lead us to develop a dynamic model in which reputational considerations play a central role.

Our main contribution is to show that a concern for reputation is necessary and sufficient for fluctuations in collateral values to lead to fluctuations in trade volume.[3] In our dynamic model, we show that when reputational concerns are absent, the equilibrium has no asymmetry of information in any period other than the first one. We go on to show that when reputational concerns are sufficiently strong, asymmetry of information must persist in the sense that there is no equilibrium with complete revelation of information about quality levels of banks.

We focus on equilibria that maximize trade. When reputational concerns are sufficiently strong, low-quality banks have strong incentives to mimic the behavior of high-quality banks. We show that these mimicking incentives are so strong that they overcome cream-skimming attempts by buyers. Specifically, when the reputational level of the bank is relatively high, the mimicking incentives are so strong that the equilibrium outcome has complete pooling with full trade of loan portfolios by banks of all qualities. Since the equilibrium has complete pooling, no information is revealed about the quality of bank portfolios. When a bank's reputational level is relatively low, the mimicking incentives are still very strong. The equilibrium has partial pooling in the sense that high-quality banks sell a relatively small portion of their loan portfolios and low-quality banks randomize between selling all their loans and pooling with high-quality banks. This randomization implies that the contract choice reveals some but not complete information about the quality levels of the bank's portfolio. The equilibrium has partial pooling because banks with

buyers. See Guerrieri, Shimer, and Wright (2010) for an alternative approach to resolving the nonexistence of equilibria in pure strategies.

[3] An important reason why reputation is necessary for adverse selection to persist in our model is that quality levels of banks are persistent over time. Obviously, adverse selection could persist without reputation if the quality levels of banks are independent over time. The empirical evidence lead us to develop a model in which quality levels are persistent over time.

high-quality loans have strong incentives to retain their entire loan portfolios rather than accepting a low complete pooling price.[4]

The potential for our model to be consistent with substantial fluctuations in the volume of trade is best exemplified in the infinite horizon version of our model with aggregate shocks to collateral values. We show that the aggregate volume of trade is relatively low when collateral values are low and high when collateral values are high. We show that the model can generate discontinuous fluctuations in the volume of trade at critical levels of collateral values. We present an illustrative simulation in which the volume of trade rises gradually over time and then abruptly falls.

We address policymakers' concerns about fluctuations in the volume of trade in secondary loan markets by analyzing the efficiency properties of our model. We show that contracts that induce greater separation in allocations between banks of different qualities reduce welfare by distorting current allocations but raise future welfare by revealing more information about the quality level of banks. We show that this trade-off is always resolved by reducing the extent of separation in allocations and the amount of information revelation to its lowest feasible value. Indeed, when a bank's reputation level is high, efficient allocations have no separation and full trade. These allocations therefore coincide with maximal trade equilibrium outcomes. Efficient allocations and equilibrium allocations also coincide when reputation is low as long as adverse selection is not too severe. The two differ from each other only when a bank's reputation level is low and adverse selection is severe. In this case, competitive forces induce more separation and more information revelation than do efficient allocations. We show that efficient outcomes can be implemented by policies that limit private trade.

Our model also generates inefficient fluctuations in the volume of trade through another channel. Like many signaling models, our model has multiple equilibria,[5] and fluctuations can occur if market participants switch from coordinating on one equilibrium to coordinating on another equilibrium. We analyze policies that are intended to eliminate inefficient equilibria and ensure a unique equilibrium. In some models, such as those analyzing deposit insurance policies (see Diamond and Dybvig 1983, for example), inefficient equilibria can be eliminated by a commitment from the government to pay off depositors using the proceeds from bank assets. This policy does not need to be activated in equilibrium and therefore does not use external resources. In our model, we argue that conventional asset purchase policies in which the government offers to purchase loans at an actuarially fair price are ineffective in the sense that these policies do not eliminate inefficient equilibria. We show that more unconventional policies that limit private trade can be effective in ensuring uniqueness of equilibrium.

Evidence and Related Literature.—Here, we present evidence that adverse selection and reputation play important roles in secondary loan markets as well as evidence supporting implications of our model. We also discuss related literature.

[4] Pooling equilibria also emerge in the literature on the so-called Ratchet effect. See, for example, Freixas, Guesnerie, and Tirole (1985) and Laffont and Tirole (1988).

[5] See Spence (1973) for the classic example of this phenomenon and Vickers (1986) for an application in the Barro and Gordon (1983) model.

Ivashina (2009), in a study of the syndicated loan market, finds that when the share of the loan retained by the originator rises, other participants accept a lower per unit return on the loan. This feature of the data is consistent with adverse selection models and is hard to reconcile with other models. Downing, Jaffee, and Wallace (2009) find that loans that banks held on their balance sheets yielded higher returns on average than did similar loans that they securitized and sold. Drucker and Mayer (2008) argue that underwriters of prime mortgage-backed securities are better informed than buyers. Specifically, the tranches on which such underwriters bid perform better than the tranches on which they do not bid. See also Dewatripont and Tirole (1994), Ashcraft and Schuermann (2008), and Arora et al. (2009) for arguments and evidence of adverse selection in secondary loan markets.

Ivashina (2009) shows that the syndicated loan market is dominated by the same banks over time so that originators are not anonymous traders. Ross (2010) and Fang (2005) present evidence on the importance of reputation in secondary loan markets. Ross (2010) finds that when borrowers obtain loans from high reputation banks, the stock price response of borrowers is favorable relative to borrowers who obtain primary loans from lower reputation banks. He also finds that borrowers who obtain loans from high reputation banks receive lower interest rates than borrowers who obtain loans from lower reputation banks. He interprets these findings as suggesting that banks are heterogeneous in their ability to screen and monitor primary loans. Since this heterogeneity persists in the data, Ross's evidence provides strong support for our assumption that the quality of loans originated by banks is persistent over time.

Fang (2005) studies the role of bond underwriting in the investment banking industry. She finds that reputable investment banks charge higher fees but obtain larger issues for the borrowers whose loans they underwrite. She argues that these findings are evidence both of heterogeneity in the quality of underwriting services and of the importance of reputational considerations that induce reputable investment banks to provide underwriting services conscientiously.

Our model implies that when adverse selection problems become more severe, the average quality of loans that are sold falls compared with the average quality of loans that are retained. Elul (2011) presents evidence on the quality of loans that were sold relative to those that were held and finds that in 2006 there was not much difference in these quality levels. Starting in 2006, the quality of loans that were sold worsened relative to the quality of loans that were held. Our model is consistent with the finding in Elul (2011) given that underlying asset values seem to show signs of weakening in 2006. Mian and Sufi (2009) present evidence that securitized loans were more likely to default than nonsecuritized loans. This evidence is also consistent with our model.

We build on an extensive literature on adverse selection in asset markets, including the work of Myers and Majluf (1984); Glosten and Milgrom (1985); Kyle (1985); and Garleanu and Pedersen (2004); as well as to the related securitization literature, specifically the work of DeMarzo and Duffie (1999) and DeMarzo (2005). Our work is particularly related to that of Fishman and Parker (2012) and Guerrieri and Shimer (2012), who study trade volume fluctuations in models with adverse selection. As in Fishman and Parker (2012) and in Dang, Gorton, and Holmström (2012), the equilibrium in our model sometimes features inefficiently high levels of information revelation. Our work is also related to Eisfeldt (2004) and Kurlat (2013), who study dynamic environments with persistent adverse selection. In these papers, either the type of an agent is not persistent over time or agents are anonymous. In our paper, in contrast, we have argued that the natural assumptions are that types are persistent and agents are not anonymous. With these assumptions, reputational concerns necessarily play a central role.[6]

We also build on an extensive literature on reputation. See, for example, Kreps and Wilson (1982); Milgrom and Roberts (1982); Diamond (1989); Mailath and Samuelson (2001); Ely and Välimäki (2003); Ely, Fudenberg, and Levine (2008); and Ordoñez (2013). Much of that literature develops reputational models in which certain types of players play fixed strategies rather than maximizing payoffs. In those models, players develop reputations by deciding whether or not to mimic their nonstrategic counterparts. We have no nonstrategic players. In our model, banks of all quality types must decide whether or not to mimic each other.

Our analysis of policy is closely related to recent work by Philippon and Skreta (2012) as well as Tirole (2012), who analyze policies in models with adverse selection. The main difference between our work and theirs is that we focus on the incentives induced by reputation, whereas these authors analyze static models.

I. Static Model of Adverse Selection in Secondary Loan Markets

In this section, we introduce and analyze a static model of a secondary loan market that features adverse selection. The static model sets the stage for the dynamic models with reputation. In the model, the primary economic role of the secondary loan market is to allow loans to be reallocated from originators to buyers, who have a comparative advantage in holding and managing loans. Adverse selection arises because originators are better informed about the quality of their loans. We show that such a model has a unique separating equilibrium, and we show that fluctuations in collateral values induce fluctuations in aggregate trading volume.

A. *Model*

Consider an economy with a large number of loan originators referred to as banks and a large number of buyers. Both banks and buyers are risk neutral. Each bank is endowed with a loan portfolio. We normalize the size of the loan portfolio to be 1. The loans in a portfolio can also be thought of more generally as investment opportunities such as projects, mortgages, or asset-backed securities.

Loans are risky because of the possibility of a default. If no default occurs, they yield a return of $\overline{v}$ and if a default occurs, they yield a return of $\underline{v}$. We refer to $\underline{v}$ as the *collateral value* of the loan and to $v = \overline{v} - \underline{v}$ as the *spread* in returns. Note for future reference that a fall in the collateral value, $\underline{v}$, induces an increase in the spread, v. The probability of no default is denoted by π. For simplicity, we assume

[6] Camargo and Lester (2011) and Daley and Green (2012) also study adverse selection in dynamic models with persistent types. In their models, sellers are endowed with a single asset or a commodity at the beginning of the game and must decide when to sell. In our model, in contrast, in each period sellers are endowed with a loan, and current decisions about whether to sell or hold the loan affect the prices at which banks can sell their loans in the future.

that the probability of default is the same for all loans in a given bank's portfolio. Banks are heterogeneous in the sense that the probability of default differs across different banks. We assume that there are two types of banks so that the probability of no default can take on one of two values $\pi \in \{\underline{\pi}, \overline{\pi}\}$ with $\underline{\pi} < \overline{\pi}$. We refer to a bank that has a loan portfolio of type $\overline{\pi}$ as a *high-quality bank* and one with a loan portfolio of type $\underline{\pi}$ as a *low-quality bank*.

We assume that the buyers have a comparative advantage in managing loans. We model this comparative advantage by assuming that the cost to the bank of holding and managing a loan is $c > 0$, and we normalize the holding and management cost for buyers to be 0. The holding and management costs represent funding liquidity costs, servicing costs, renegotiation costs in the event of a loan default, and costs associated with holding a loan that may be correlated in a particular way with the rest of the bank's portfolio, among other potential factors.

The bank chooses how much of its loan portfolio to sell in the secondary market. Let x denote the fraction of the loan portfolio that the bank sells. Let t denote the payment the bank receives from buyers for selling x loans so that the ratio t/x is the price per loan. The payoff for a bank of type π that sells a fraction x of its loan portfolio at payment t can be written, subject to a normalization, as

$$t + (1 - x)(\pi v - c), \tag{1}$$

and the profit of a buyer who purchases a fraction of loans x for a payment t from a bank of type π is given by $x\pi v - t$. (In the online Appendix, we provide details of this normalization.) If buyers are perfectly informed about the types of banks, it is clearly efficient to have banks sell their entire loan portfolios.

We introduce adverse selection by assuming that the bank knows the type of loans in its portfolio and that potential buyers do not. Buyers believe that the bank is high quality with probability μ and low quality with probability $1 - \mu$. We refer to μ as the *reputation* of the bank. Banks differ in their reputation levels, and the distribution of banks by reputation levels is given exogenously. In the dynamic models that follow, this reputation evolves endogenously. We model buyers as engaging in Bertrand-style price competition, so it suffices to restrict the number of potential buyers to two.

We begin by analyzing the secondary loan market for a given bank with reputation μ and then extend the analysis to aggregate outcomes. In the secondary loan market, buyers simultaneously offer contracts to the bank. A contract specifies the fraction of loans that the buyer will purchase and the payment for such a purchase. Since we have two types of banks, a *contract*, z, consists of a four-tuple (x_h, t_h, x_l, t_l). Here, a pair (x_i, t_i) is an *offer* that is intended for a bank of type $i = l, h$. After buyers offer contracts, the bank chooses which buyer's contract and offer to accept. Since the bank can choose which offer to accept, we can restrict attention to contracts that are incentive-compatible in the sense that they satisfy

$$t_h + (1 - x_h)(\overline{\pi} v - c) \geq t_l + (1 - x_l)(\overline{\pi} v - c) \tag{2}$$

$$t_l + (1 - x_l)(\underline{\pi} v - c) \geq t_h + (1 - x_h)(\underline{\pi} v - c). \tag{3}$$

The incentive constraints (2) and (3) ensure that facing a contract z, a bank of quality type $i, i = l, h$ prefers the offer (x_i, t_i) intended for it to the offer intended for the bank of the other type. Let $\mathcal{Z}$ denote the set of incentive-compatible contracts.

A well-known problem with adverse selection models is that equilibria in pure strategies sometimes do not exist (see, for example, Rothschild and Stiglitz 1976). Dasgupta and Maskin (1986) show that in many such environments, mixed strategy equilibria do exist. We follow Rosenthal and Weiss (1984) in allowing for mixed strategies on the part of buyers and pure strategies by banks.

A strategy for buyer $j = 1, 2$ is a distribution function $F_j(z)$ over the set $\mathcal{Z}$. A strategy for the bank consists of an action $\delta_j(z_1, z_2; \pi) \in [0, 1]$ for $j = 1, 2$, where δ_j denotes the probability that contract j is accepted. Obviously, the bank may not accept both contracts. But we do allow the bank to reject both contracts. Given the mixed strategy by the other buyer, F_{-j}, and the strategy of the bank, δ, the profits earned by a buyer offering contract z are given by

$$\int \left[\mu \delta_j\left(z, z_{-j}; \overline{\pi}\right)(x_h \overline{\pi} v - t_h) + (1 - \mu)\delta_j\left(z, z_{-j}; \underline{\pi}\right)(x_l \underline{\pi} v - t_l)\right] dF_{-j}\left(z_{-j}\right). \tag{4}$$

An *equilibrium* consists of strategies for buyers and a strategy for the bank such that (i) for all z_j in the support of F_j no alternative contract $\hat{z}_j$ earns strictly higher profits, and (ii) the bank's strategy specifies that its choice maximizes its payoff. For some of our analyses, we require that our equilibria be *monotone* in the sense that a low-quality bank prefers a contract $\hat{z}$ to a contract z if and only if a high-quality bank also prefers the contract $\hat{z}$ to z. In any monotone equilibrium, $\delta_j\left(z, z_{-j}; \overline{\pi}\right) = 1$ if and only if $\delta_j\left(z, z_{-j}; \underline{\pi}\right) = 1$. We restrict ourselves to tie-breaking rules in which if contracts offered by both buyers give the same payoff to a bank of given quality, the bank accepts either offer with probability $1/2$. This assumption is purely for convenience. Since buyers can offer a contract $(0, 0, 0, 0)$, we can restrict ourselves to equilibria in which the bank chooses some offer. An equilibrium is *symmetric* if $F_1 = F_2 = F$.

We say that an equilibrium is *separating* if the offers accepted by low- and high-quality banks are different. In such a situation, after trades have occurred, the type of the bank is known. Separating equilibria are of interest in dynamic versions of our model because future buyers could exploit knowledge of choices by the bank in previous periods and thus affect the behavior of the bank in the current period.

We now show that our model has a unique separating equilibrium. We begin by using arguments similar to those in Dasgupta and Maskin (1986) to show that any monotone equilibrium outcome must have four key properties. (See the online Appendix for the proof.) At each contract in the support of F, the low-quality bank sells its entire loan portfolio $(x_l = 1)$, the incentive constraint for the low type, (3), holds with equality, buyers make zero profits in the sense that for each point in the support of F, the expression in brackets in (4) is zero, and the intended offers to the low-quality bank do not yield positive profits so that $t_l \geq \underline{\pi} v$. Since $x_l = 1$, the zero profit condition at each point in the support of F can be written as

$$\mu\left(x_h \overline{\pi} v - t_h\right) + (1 - \mu)\left(\underline{\pi} v - t_l\right) = 0. \tag{5}$$

Since any contract must also satisfy the incentive constraint of the low-quality bank with equality and $x_l = 1$, it follows that given any payment to the low-quality bank t_l, the equations (3) with equality and (5) can be solved to uniquely obtain the other elements of the contract, x_h, t_h. Thus, any distribution F can be represented by an associated distribution over payments to the low-quality bank t_l. With some abuse of notation, we use F to denote the distribution over t_l.

Next we characterize the pure strategy equilibrium and the range of reputation levels for which such an equilibrium exists. Following Rothschild and Stiglitz (1976), it is straightforward to show that any pure strategy equilibrium must have buyers breaking even on each type of bank so that $t_l = \underline{\pi} v$ and $t_h = x_h \overline{\pi} v$. Since the incentive constraint for the low-quality bank (3) must hold with equality, we can substitute for t_l and t_h into (3) to obtain that the fraction of loans sold by the high-quality bank is given by

$$x_h = \frac{1}{1 + \frac{(\overline{\pi} - \underline{\pi}) v}{c}} = \frac{1}{1 + d}, \tag{6}$$

where $d = (\overline{\pi} - \underline{\pi}) v/c$. We refer to d as the *adverse selection discount.* This discount captures the extent to which adverse selection reduces trade compared with the full information volume of trade. Note that as the dispersion in quality $\overline{\pi} - \underline{\pi}$ or the loan spread v increases, the adverse selection discount increases. Recall that $v = \overline{v} - \underline{v}$. Thus, the adverse selection discount increases as the collateral value $\underline{v}$ falls.

We say that the contract $(x_h, x_h \overline{\pi} v, 1, \underline{\pi} v)$, where x_h is given by (6), is the *least-cost separating outcome*. This outcome is that associated with the pure strategy equilibrium. In the proof of the proposition below, we show that when the bank's reputation is below a threshold, $\tilde{\mu}$, the least-cost separating outcome is an equilibrium outcome. To determine the threshold $\tilde{\mu}$, compare the payoffs of the high-quality bank in the least-cost separating outcome given by $x_h \overline{\pi} v + (1 - x_h)(\overline{\pi} v - c)$ with the payoffs it would receive from accepting a zero profit full trade pooling contract. Such a contract specifies $x_h = x_l = 1$ with $t_h = t_l = \hat{p}(\mu)$, where $\hat{p}(\mu)$ is given from the zero profit condition for buyers as

$$\hat{p}(\mu) = \mu \overline{\pi} v + (1 - \mu) \underline{\pi} v. \tag{7}$$

Using (6) and (7), straightforward algebra shows that the high-quality bank is indifferent between the least-cost separating outcome and the zero profit pooling contract at a reputation level

$$\tilde{\mu} = \frac{d}{1 + d}. \tag{8}$$

Above $\tilde{\mu}$, the high-quality bank strictly prefers the zero profit pooling contract to the least-cost separating outcome. Thus, when the bank's reputation is relatively high, no pure strategy equilibrium exists because a deviation by one of the buyers to an allocation near the pooling outcome is profitable. With appropriately chosen mixed strategies, deviations are not profitable. The idea is to construct the mixed strategies so that any deviation attracts low-quality banks with disproportionate probability. In the online Appendix, we show that our model has a mixed strategy equilibrium in which the distribution F is given by

$$(9) \qquad F(t_l) = \left(\frac{t_l - \underline{\pi}v}{\mu(\overline{\pi} - \underline{\pi})v}\right)^{\frac{\mu}{d(1-\mu)}-1}$$

with support given by $[\underline{\pi}v, \hat{p}(\mu)]$, where the pooling price $\hat{p}(\mu)$ is given by (7). Notice that since the payments to the low-quality bank are higher than $\underline{\pi}v$ with positive probability, the equilibrium features cross-subsidization in the sense that profits from high-quality banks are used by buyers to subsidize losses from low-quality banks. We summarize this discussion in the following proposition which is proved in the online Appendix.

PROPOSITION 1: *The static model has a separating equilibrium. If* $\mu \leq \tilde{\mu}$, *the equilibrium outcome is the least-cost separating outcome. If* $\mu \geq \tilde{\mu}$, *then the equilibrium has mixed strategies by buyers and the distribution over contracts is given by* (9). *Furthermore*, *the separating equilibrium is unique in the class of monotone equilibria.*

This static equilibrium induces payoffs for the high- and low-quality banks as a function of their initial reputation levels, μ, given by

$$(10) \qquad V(\mu;\pi) = \begin{cases} \underline{\pi}v + \dfrac{\max\{\mu - \tilde{\mu}, 0\}}{1 - \tilde{\mu}}(\overline{\pi} - \underline{\pi})\,v & \pi = \underline{\pi}, \\ \hat{p}(\tilde{\mu}) + \dfrac{(\max\{\mu - \tilde{\mu}, 0\})^2}{\mu(1 - \tilde{\mu})}(\overline{\pi} - \underline{\pi})\,v & \pi = \overline{\pi}. \end{cases}$$

Note that in the least-cost separating allocation, the payoff of the high-quality bank is equal to its payoff in a pooling outcome with $\mu = \tilde{\mu}$ given by $\hat{p}(\tilde{\mu})$. This value function plays a central role in our dynamic model with reputational concerns.

Note that we have described an equilibrium in which buyers possibly mix and banks do not. Using standard cream-skimming arguments, it is straightforward to show that there is no equilibrium in which banks use mixed strategies. When we turn to the dynamic model, we will show that standard cream-skimming arguments fail so that there are equilibria in which banks do mix.

Trade Volume and the Spread in the Static Model.—Next, we turn to the promise of adverse selection models in generating fluctuations in the volume of trade associated with changes in collateral values. Specifically, we will show that an increase in the adverse selection discount results in a fall in trade volume for all levels of reputation. To show this result, consider the expected volume of trade for a given bank with reputation level μ. For low levels of reputation, $\mu \leq \tilde{\mu}$, from Proposition 1, the equilibrium outcome is the least-cost separating outcome so that the expected volume of trade, $T_L(\mu)$, is given by

$$(11) \qquad T_L(\mu) = \mu\frac{1}{d+1} + (1 - \mu).$$

For high levels of reputation, $\mu > \tilde{\mu}$, from Proposition 1, the equilibrium outcome has mixed strategies. Straightforward computation using the form of F in (9) yields that the expected volume of trade in our model $T_H(\mu)$ is given by

$$T_H(\mu) = \mu\left[1 - \frac{1-\mu}{\mu}\left[\frac{1}{d} + \frac{1}{d^2}\right]^{-1}\right] + (1-\mu). \tag{12}$$

Consider now an increase in the adverse selection discount, d. From (8) we see that the threshold $\tilde{\mu}$ increases as d increases. Therefore, a bank with reputation level μ in the least-cost separating equilibrium stays in that equilibrium, and from (11) we see that the expected volume of trade falls for such a bank. A bank with reputation level μ sufficiently greater than $\tilde{\mu}$ stays in the mixed strategy equilibrium, and from (12) it is easy to show that the expected volume of trade falls for such a bank. A bank with reputation level near $\tilde{\mu}$ switches from the mixed strategy equilibrium to the least-cost separating equilibrium. Straightforward computation shows that $T_L(\mu) \leq T_H(\mu)$. Thus, for such banks, the volume of trade falls as well.

We have shown that if the adverse selection discount increases, due either to a decrease in the collateral value or to an increase in the dispersion in bank quality, the expected volume of trade for a bank of a given reputation μ falls. Since the expected volume of trade for banks of each reputation level falls, it follows that for any distribution of banks by reputation levels, the aggregate volume of trade falls as well. We summarize this discussion in the following proposition.

PROPOSITION 2 (The Promise of Adverse Selection Models in Generating Trade Volume Fluctuations): *Decreases in collateral values and increases in the dispersion of bank quality reduce expected trade volumes for banks of all reputational levels and decrease aggregate trade volume.*

Although the static adverse selection model is promising in generating a fall in the expected volume of trade when collateral values fall, note that all of the decline is due to a decline in volume of trade of high-quality banks. Low-quality banks always sell their entire loan portfolios. Declines in the volume of trade in secondary markets seem very widespread, suggesting that the volume of trade across banks of varying qualities is highly correlated. Since the static adverse selection model cannot generate a decline in volume of trade by all quality types of banks, it faces a correlation challenge when confronted with the data. Next we turn to dynamic models, primarily to show that adverse selection can persist and secondarily to show that such models can address the correlation challenge.

II. Reputational Concerns in a Two-Period Model

We begin our analysis of reputation in financial markets by analyzing a two-period model. We show that when reputational concerns are weak, adverse selection cannot persist because the quality of the bank can be perfectly inferred from its past actions. When reputational concerns are strong, we show that adverse selection persists in the sense that the quality of the bank cannot be perfectly inferred from its past actions. We show that any equilibrium necessarily features partial pooling for low values of bank reputation in the sense that future buyers are able to obtain some but not complete information about the bank's quality from its past actions. For high values of reputation, equilibria with the highest volume of trade have complete pooling in the sense that future buyers are able to obtain no information about the bank's quality from its past actions. The incompleteness of information revelation implies that fluctuations in collateral values can induce fluctuations in the aggregate volume of trade in both periods as well as in the volume of trade by banks of different qualities in the first period.

The two-period model is a straightforward extension of the static model. As in the static model, in each period of the dynamic model, banks originate a loan portfolio of size normalized to be 1, and then buyers offer contracts intended for high- and low-quality banks. Banks then choose the contract that gives them the highest payoffs. Banks discount future payoffs at rate β. In order for reputation to play a role, we assume that buyers observe the contract chosen by an individual bank in the previous period. Buyers use this observation to update their beliefs about the quality level of banks.

We make a variety of simplifications that are intended to allow us to focus on the role of reputation and to suppress other links between buyers and banks over time. We assume that the quality type of banks is completely persistent in the sense that it is the same in both periods. (In our infinite horizon model we analyze equilibria with partial persistence.) We assume that loans originated in any period can only be sold in that period and that each bank interacts with a new set of buyers in each period. We also assume that buyers do not observe the returns on loans in previous periods. Our simplifications imply that the only variable that links behavior over time is the reputation level of the bank, which is endogenously determined by the bank's past choices.

A Markov equilibrium for this economy consists of (possibly mixed) strategies by both banks and buyers, updating rules for buyers that satisfy Bayes' rule and is defined in the usual fashion.

A. *The Model without Reputational Concerns*

We begin by analyzing a dynamic adverse selection model without reputational concerns. Formally, we suppress reputational concerns by setting banks' discount factor β equal to 0. Note that buyers still update their beliefs about the quality level of an individual bank from past contract choices.

Since the discount factor β is zero and since each buyer interacts with an individual bank only in one period, all decisions in the first period are unaffected by future payoffs. It follows that the equilibrium in the first period coincides with the equilibrium in the static model. Since this equilibrium features complete separation of banks by quality type, it also features complete learning by buyers. In the second period, buyers believe that the bank is either of high or low quality depending on its first-period contract choice. Hence, the equilibrium in the second period has the bank selling its entire loan portfolio at a price of $\overline{\pi}v$ if buyers believe the bank is of high quality and at a price of $\underline{\pi}v$ if buyers believe the bank is of low quality. The volume of trade in the second period is independent of the collateral value and the dispersion in bank quality. We summarize this discussion in a proposition.

PROPOSITION 3: *Suppose that the discount factor of banks, β, equals 0. The equilibrium of the dynamic model features full separation and complete learning in the* *first period. The volume of trade in the second period is independent of the collateral value and the dispersion in bank quality.*

By continuity, it follows that for sufficiently small values of the discount factor β, the equilibrium has full separation. This proposition can be extended trivially to a model with many periods. It illustrates starkly that when reputational concerns are weak, adverse selection cannot persist and raises challenges for theories based on adverse selection in accounting for fluctuations in the volume of trade.

B. *The Model with Reputational Concerns*

Here we show that if banks are sufficiently patient, complete separation in the first period cannot be an equilibrium outcome. The proof of this result is by contradiction. Suppose that an equilibrium exists with complete separation. With such separation, buyers in the last period know the quality level of the bank, so their beliefs are either 1 or 0. The equilibrium in the last period coincides with the static equilibrium with beliefs of 1 or 0, and the associated payoffs of banks can be calculated from (10).

Incentive compatibility of buyers' contracts in the first period implies

$$t_h + (1 - x_h)(\overline{\pi} v - c) + \beta V(1; \overline{\pi}) \geq t_l + (1 - x_l)(\overline{\pi} v - c) + \beta V(0; \overline{\pi}), \tag{13}$$

$$t_l + (1 - x_l)(\underline{\pi} v - c) + \beta V(0; \underline{\pi}) \geq t_h + (1 - x_h)(\underline{\pi} v - c) + \beta V(1; \underline{\pi}), \tag{14}$$

where the continuation value $V(\mu, \pi)$ is given in (10). Adding the constraints (13) and (14) and substituting for the continuation values from (10) gives

$$x_l - x_h \geq \beta \frac{d}{1 + d}. \tag{15}$$

Clearly, if the right side of (15) is greater than 1, since $x_l \leq 1$ and $x_h \geq 0$, no incentive-compatible contract can achieve full separation in the first period. When the right side of (15) is bigger than 1, we say that banks are *sufficiently patient*. We have proved the following proposition.

PROPOSITION 4 (Patience and Persistence of Adverse Selection): *Suppose that banks are sufficiently patient. Then no equilibrium in the two-period economy has complete separation of high- and low-quality banks in the first period.*

This proposition shows that if banks are sufficiently patient, any equilibrium must have partial or complete pooling by banks. Since any equilibrium must have partial or complete pooling, banks of both quality types must either use mixed strategies or choose the same contract. In what follows, we focus on equilibria in which in the first period, buyers use pure strategies. As in the static model, buyers simultaneously offer contracts $z = (x_h, t_h, x_l, t_l)$ and banks choose which offer, if any, to accept. If they choose to accept one of the offers, high-quality banks accept the offer (x_h, t_h) with probability α_h and the offer (x_l, t_l) with complementary probability, and low-quality banks choose (x_l, t_l) with probability α_l and (x_h, t_h) with the complementary probability. We will say that z has complete pooling if $(x_h, t_h) = (x_l, t_l)$ and has partial pooling if $(x_h, t_h) \neq (x_l, t_l)$. Incentive compatibility implies that

$$(16)\quad t_h + (1 - x_h)(\overline{\pi}v - c) + \beta V(\mu_h'; \overline{\pi}) \geq t_l + (1 - x_l)(\overline{\pi}v - c) + \beta V(\mu_l'; \overline{\pi}),$$

$$(17)\quad t_l + (1 - x_l)(\underline{\pi}v - c) + \beta V(\mu_l'; \underline{\pi}) \geq t_h + (1 - x_h)(\underline{\pi}v - c) + \beta V(\mu_h'; \underline{\pi}),$$

where $\mu_h'(\mu_l')$ is the belief of future buyers that the bank is of high quality upon observing the accepted offer (x_h, t_h) $((x_l, t_l))$. In addition, when $\alpha_i < 1, i = h, l$, the relevant incentive constraint holds with equality. If the contract has complete pooling, Bayes' rule implies that beliefs of future buyers are the same as initial beliefs so that $\mu_h' = \mu_l' = \mu$. If the contract has partial pooling, Bayes' rule implies that

$$(18)\qquad \mu_h' = \frac{\mu\alpha_h}{\mu\alpha_h + (1 - \mu)(1 - \alpha_l)}, \qquad \mu_l' = \frac{\mu(1 - \alpha_h)}{\mu(1 - \alpha_h) + (1 - \mu)\alpha_l}.$$

An *equilibrium* consists of an incentive-compatible contract z, mixing probabilities α_h and α_l, and a belief function $\mu'(x, t)$ such that (i) banks' choices about which offer to accept, if any, maximize their payoffs, (ii) buyers maximize expected profits given by

$$(19)\qquad \mu\,\alpha_h(x_h\overline{\pi}v - t_h) + \mu(1 - \alpha_h)(x_l\overline{\pi}v - t_l) + (1 - \mu)\alpha_l(x_l\underline{\pi}v - t_l) + (1 - \mu)(1 - \alpha_l)(x_h\underline{\pi}v - t_h),$$

and (iii) the belief function satisfies Bayes' rule when applicable. We focus on equilibria in which buyers make zero profits and hence all the surplus is captured by banks.

If the bank chooses to reject both offers, its payoff is given by $\pi v - c + \beta V(\mu'; \pi)$. Since the continuation value V is non-decreasing in μ, it follows that a necessary condition for an equilibrium is that it satisfy a participation constraint for the high-quality bank given by

$$(20)\qquad t_h + (1 - x_h)(\overline{\pi}v - c) + \beta V(\mu_h'; \overline{\pi}) \geq \overline{\pi}v - c + \beta V(0; \overline{\pi}),$$

and an analogous participation constraint for the low-quality bank. Note that any equilibrium in which banks hold their loans can be represented as an equilibrium in which buyers offer to buy none of the loans at a zero payment. Therefore, we can restrict attention to equilibria in which the bank accepts one of the offers and require that any equilibrium satisfy (20) and its analogue for the low-quality bank.

In much of what follows, we focus on equilibria with the highest volume of trade referred to as *maximal trade equilibria*. These equilibria are attractive for three reasons. First, in our analysis of optimal policy below, we show that such equilibria Pareto dominate any other equilibrium. Second, the maximal trade outcomes are closest to the maximal trade outcomes under full information. Third, as we show below, such equilibria also maximize first-period ex ante expected payoffs of banks.

Given our focus on maximal trade outcomes, we ask when complete pooling with full trade, $x_h = x_l = 1$, and associated payments $t_h = t_l = \hat{p}(\mu)$ where $\hat{p}(\mu)$ is given by (7) is an equilibrium outcome. We show that full trade is an equilibrium outcome if $\mu \geq \mu^*$ where

$$\mu^* = 1 - \frac{c}{(\overline{\pi} - \underline{\pi})v} = 1 - \frac{1}{d}. \tag{21}$$

The threshold μ^* is set so that the high-quality bank is just indifferent between accepting the full trade contract and holding its loans, so that (20) is satisfied with equality. To determine this threshold, note first that with complete pooling, $\mu' = \mu$. Since $\mu^* < \tilde{\mu}$, the continuation values are equal so that $V(\mu'; \overline{\pi}) = V(0; \overline{\pi})$. Evaluating (20) with equality at the full trade contract using (7) gives us (21). If $\mu \geq \mu^*$, the high-quality bank strictly prefers to sell all of its loans at the pooling price to holding, and if $\mu < \mu^*$, it prefers to hold rather than to sell. Thus, full trade is potentially an equilibrium outcome for $\mu \geq \mu^*$ and cannot be an equilibrium for $\mu < \mu^*$. In what follows, we assume that $\mu^* \geq 0$ so that $d \geq 1$.

Consider supporting full trade as an equilibrium by the following out of equilibrium belief function

$$\mu'(x', t') = \begin{cases} 0 \text{ if } t' + (1 - x')(\overline{\pi}v - c) \leq \hat{p}(\mu) \\ 1 \text{ otherwise} \end{cases}.$$

Note that this belief function says that future buyers believe that the bank is of low quality if and only if it accepts an offer that is statically less favorable for the high-quality bank than the pooling contract. In the Appendix, we show that these strategies and beliefs are indeed equilibria. For some intuition for why full trade is an equilibrium, note that as in the static model, buyers would like to offer cream-skimming contracts that are attractive to high-quality banks and unattractive to low-quality banks. With our beliefs, attracting high-quality banks requires offering them a price higher than $\hat{p}(\mu)$ in return for some retention, $x_h < 1$. The key step in our proof is to show that if the right side of (15) is greater than 1, low-quality banks are also attracted to such a deviation. Therefore, buyers cannot make positive profits with this deviation. We summarize this result in the following lemma.

LEMMA 1: *Suppose that $\mu \geq \mu^*$ and that banks are sufficiently patient. Then full trade by banks of both quality types is an equilibrium outcome.*

In the Appendix, we prove a more general version of Lemma 1 in which we characterize the entire set of complete pooling equilibria with $x_l = x_h$, possibly less than 1.

Recall that under full information, it is efficient to have banks of all qualities sell their entire portfolio, so that, for high values of reputation, these outcomes coincide in the first period with full information outcomes.

Consider next the case in which banks are sufficiently patient and $\mu < \mu^*$. We have already shown that full trade cannot be an equilibrium outcome. Since banks are sufficiently patient, complete separation cannot be an equilibrium outcome. Next, we argue that banks of both quality types cannot mix in any equilibrium. If they both did, then any contract together with mixing probabilities must satisfy (16)–(17) with equality as well as the zero profit condition. Straightforward but extensive algebraic calculations available on request show that any such contract violates (20).[7] Thus, the equilibrium must have partial pooling with $0 < \alpha_l < 1$.

Consider then the problem of maximizing trade volume given by

$$T = \left[\mu + (1-\mu)(1-\alpha_l)\right] x_h + (1-\mu)\alpha_l x_l \tag{22}$$

subject to the requirement that the contract be an equilibrium. We have shown that incentive compatibility (16)–(17), zero profits (namely, that (19) is zero), and (20) are necessary conditions that any equilibrium must satisfy. It turns out that Bertrand competition between the buyers requires us to impose an additional necessary condition that

$$\frac{1}{2}\mu(x_h\overline{\pi}v - t_h) + (1-\mu)\left(\underline{\pi}v - t_l - (1-x_l)(\underline{\pi}v - c)\right) \leq 0. \tag{23}$$

To see why this condition is necessary, consider a deviation by one of the buyers to a contract that has the same offer for high-quality banks and an offer to the low-quality bank that makes that bank slightly better off and delivers the highest profits to the buyer. Specifically, let the deviation offer be given by $z' = \left(x_h, t_h, 1, t_l'\right)$, where $t_l' = t_l + (1-x_l)(\underline{\pi}v - c) + \varepsilon$ with a small enough value of ε. In such a deviation, the low-quality bank chooses $\left(1, t_l'\right)$ with probability 1, independent of the beliefs assigned to $\left(1, t_l'\right)$. To see why this choice is optimal for the low-quality bank, note first that the low-quality bank is statically better off. Since the value function is weakly increasing and $\mu_l' = 0$, the continuation values following $\left(1, t_l'\right)$ cannot be lower than $V(0; \underline{\pi})$, so the low-quality bank is weakly better off dynamically. The high-quality bank is indifferent between the buyers' contracts and chooses the two buyers with equal probability. Hence, the profits for the buyer from such deviation converge to the left side of (23) as ε converges to 0. The inequality in (23) guarantees that such a deviation is not profitable.

A candidate contract for the maximal trade equilibrium then is a contract that maximizes (22) subject to (16)–(17), the expression in (19) is nonnegative, (20), and (23). In the Appendix, we show that this candidate contract is indeed an equilibrium by showing that (16), (17), zero profits, (20), and (23) are not just necessary but also sufficient conditions that any equilibrium must satisfy. Hence, we have the following lemma.

LEMMA 2: *Suppose that* $\mu < \mu^*$ *and that banks are sufficiently patient. Then the maximal trade outcome maximizes* (22) *subject to incentive compatibility* (16)–(17), *zero profits*, (20), *and* (23).

[7] It is immediate that if banks are sufficiently patient, no equilibrium can have only the high-quality bank mixing. The argument is identical to that in Proposition 4.

Notice that this characterization of equilibrium allocations does not depend on any specification of beliefs off the equilibrium path.

Next we show that the maximal trade equilibrium maximizes first-period ex ante payoffs given by

$$\mu[t_h + (1 - x_h)(\overline{\pi}v - c)] \\ + (1 - \mu)[\alpha_l(t_l + (1 - x_l)(\underline{\pi}v - c) + (1 - \alpha_l)(t_l + (1 - x_h)(\underline{\pi}v - c))]. \tag{24}$$

Substituting for t_h and t_l from the zero profits constraint, (24) reduces to $\hat{p}(\mu) - c(1 - T)$. Clearly, maximizing T is equivalent to maximizing (24). By the same type of reasoning, if $\mu \geq \mu^*$, the maximal trade equilibrium maximizes first-period ex ante expected payoffs.

Let $\overline{z} = (\overline{x}_h, \overline{t}_h, \overline{x}_l, \overline{t}_l)$ and $\overline{\alpha}_l$ denote the maximal trade allocation. Here we provide a characterization of this allocation. Clearly, it must have $\overline{x}_l = 1$ because it is feasible to increase volume by increasing x_l by ε and t_l by $(\underline{\pi}v - c)\varepsilon$. Next, we show that the constraints to the problem of maximizing trade volume can be reduced to two inequalities. To obtain these inequalities, substitute for t_h and t_l from the incentive constraint (17) and the zero profit condition into (20) and (23) to obtain

$$x_h \leq \overline{x}(\mu_h', v) = \frac{\beta\left[\Delta(\mu_h';\overline{\pi}) - \Delta(\mu_h';\underline{\pi})\left(1 - \frac{\mu}{\mu_h'}\right)\right] + \left(1 - \frac{\mu}{\mu_h'}\right)c}{\overline{\pi}v - c - \hat{p}(\mu) + \left(1 - \frac{\mu}{\mu_h'}\right)c} \tag{25}$$

$$\frac{\frac{2 - \mu - \mu_h'}{\mu_h'(1 - \mu)}}{(\overline{\pi} - \underline{\pi})v + c\frac{2 - \mu - \mu_h'}{\mu_h'(1 - \mu)}}\left(c - \beta\Delta(\mu_h';\underline{\pi})\right) = \underline{x}(\mu_h', v) \leq x_h, \tag{26}$$

where $\Delta(\mu;\pi) = V(\mu;\pi) - V(0;\pi)$. (The algebra underlying these substitutions can be found in the online Appendix.) The maximal trade outcome maximizes (22) subject to (25) and (26). Since the volume of trade is increasing in x_h and since (25) imposes an upper bound on x_h, it follows that in the maximal trade equilibrium, (25) must be binding. Furthermore, in the proof of the following lemma in the Appendix, we show that if (26) is not binding, then $\mu_h' = \tilde{\mu}$. We summarize this characterization in the following lemma.

LEMMA 3: *Suppose that* $\mu \leq \mu^*$ *and that banks are sufficiently patient. Then, in the maximal trade equilibrium,* $x_l = 1$, (25) *is binding. In addition,* $\mu_h' = \tilde{\mu}$ *if* (26) *is not binding and* $\mu_h' \geq \tilde{\mu}$ *otherwise.*

In summary, Lemmas 1, 2, and 3 show that the maximal trade equilibrium features pooling in the sense that buyers in the second period remain uncertain about the quality levels of individual banks. Thus, adverse selection persists in our model.

Volume of Trade and the Spread.—Here, we show that in the maximal trade equilibrium, a temporary unexpected aggregate increase in the spread results in a reduction in the aggregate trading volume. It is straightforward to adapt the proposition for the two-period model to allow for the adverse selection discount to be different in the two periods.

Consider the effect on volume associated with such an increase for a bank with a high reputation level so that $\mu \geq \mu^*$. From (21), it follows that μ^* increases. If the increase in v leaves the bank in the full trade equilibrium, the volume of trade is unaffected. If the increase raises the threshold μ^* above the bank's reputation level, the equilibrium switches from full trade to partial pooling so that the volume of trade falls. Thus, all banks with reputation levels in a neighborhood above μ^* experience a decline in the volume of trade.

Consider next the effect on volume associated with an increase in the spread for a bank with reputation level $\mu \leq \mu^*$. Since μ^* increases, such a bank continues to remain in the partial pooling equilibrium. Recall that in the maximal trade equilibrium, $\bar{x}_l = 1$ and (25) is binding. Moreover, because the shock to the spread is temporary, it has no effect on continuation values. Therefore, an increase in v reduces the right side of (25) for given μ_h'. We have shown that when (26) is not binding, $\mu_h' = \tilde{\mu}$ so that in this case, the volume of trade decreases. In the Appendix, we show that when (26) is also binding, the volume of trade decreases. We have established the following proposition for the maximal trade equilibrium.

PROPOSITION 5 (Fluctuations in Trade Volume with Reputational Concerns): *If banks are sufficiently patient, a temporary decrease in collateral values weakly reduces expected trade volumes for banks of all reputational levels. Furthermore, if the distribution of reputation has mass below μ^* or in a neighborhood above μ^*, aggregate trade volume strictly falls in the first period.*

Recall from Proposition 2 that an unanticipated shock to collateral values in the last period reduces volume in that period. These propositions taken together illustrate that if reputational concerns are sufficiently strong, adverse selection can persist and temporary fluctuations in collateral values can induce fluctuations in the volume of trade in both periods. Note also that the volume of trade declines for both high- and low-quality banks so that reputation can help address the correlation challenge.

Notice that the changes in aggregate volume associated with changes in the spread depend on the nature of the initial distribution of reputation. In our infinite horizon model, we endogenize this distribution and show that aggregate volume falls in response to increases in the spread. In that model, we also analyze the role of anticipations of future shocks to loan spreads on the current volume of trade.

Multiplicity of Equilibria.—Although the main focus of our analysis is maximal trade equilibria, for the sake of completeness we describe other equilibria. Since our model has multiple equilibria, the volume of trade may fluctuate even when fundamentals are unchanged.

Consider the set of equilibria for values of $\mu \leq \mu^*$. Recall that incentive compatibility (16)–(17), zero profits, (20), and (23) are not just necessary but also sufficient conditions for an equilibrium. Thus, any contract that satisfies these conditions can be supported as an equilibrium. Clearly, a range of values of x_l and x_h can be supported as equilibria. These other equilibria have lower volumes of trade.

Consider next equilibria for $\mu \geq \mu^*$. Recall that our model has a range of complete pooling equilibria with lower volumes of trade than the maximal trade equilibrium. In our proof of Lemma 1, we show that the range of complete pooling equilibria is characterized by a lower bound $\underline{x}(\mu)$ and that any value of $x \in [\underline{x}(\mu), 1]$ can be sustained as a complete pooling equilibrium. In that proof, we also show that $\underline{x}(\mu)$ equals 0 for sufficiently high values of μ. This result illustrates that the model is consistent with complete collapses in the volume of trade that are independent of fundamentals, at least for high values of reputation.

Clearly, sunspot-like fluctuations can induce a switch from one equilibrium to another and thereby induce fluctuations in the volume of trade that are unrelated to fundamentals. In Chari, Shourideh, and Zetlin-Jones (2010), we use global game techniques to refine the multiplicity of equilibria in a similar environment. In the resulting unique equilibrium, there is a critical threshold in the collateral value above which aggregate trading volume is high and below which aggregate trading volume is low.

Next we ask whether policy interventions can ensure uniqueness of equilibrium and also whether policy interventions are needed to address possible inefficiency of equilibria.

III. Implications for Policy

In the wake of the 2007 collapse of secondary loan markets, policymakers proposed a variety of programs intended to restore the volume of trade and to remedy perceived inefficiencies in the market for secondary loans. Under some of these policies, labeled *conventional asset purchase policies*, the government planned to purchase asset-backed securities at prices roughly equal to its perception of the value.

One possible motivation for intervention is that policymakers perceived equilibrium outcomes in the secondary loan market as inefficient. We address this motivation by analyzing the efficiency of equilibria in our model. We show that when the adverse selection discount is low, the maximal trade equilibrium is efficient for all reputation levels of banks and that the equilibrium outcomes are inefficient only when the adverse selection discount is high and reputation levels are low. We show that when equilibrium outcomes are inefficient, unconventional policies that limit private trade can support efficient outcomes.

A second possible motivation for such policies is that policymakers desired to restore the volume of trade because they perceived the collapse as arising from a switch by private agents from one equilibrium to another. Given that our model has multiplicity of equilibria, we ask whether conventional asset purchase policies are effective. We show that policies in which the government attempts to purchase securities at prices that prevailed before the market collapsed do not by themselves eliminate multiplicity and therefore do not induce increased volume of trade. In this sense, conventional asset purchase policies are ineffective when reputational concerns are strong. We show that unconventional policies can eliminate multiplicity.

A. *Optimal Policy*

Here, we analyze the efficiency properties of our equilibria. Consider the problem of a planner who is able to control only allocations in the first period and takes as given the continuation payoff functions as well as the way future buyers infer reputation levels. We assume that the planner maximizes ex ante expected payoffs of banks (including the discounted continuation values). To derive the objective of the planner, let $\bar{W}(z,\alpha,\overline{\pi})$ denote the left side of (16), $\underline{W}(z,\alpha,\overline{\pi})$ denote the right side of (16), $\bar{W}(z,\alpha,\underline{\pi})$ the left side of (17), and $\underline{W}(z,\alpha,\underline{\pi})$ the right side of (17), where future beliefs are given from the mixing probabilities $\alpha = (\alpha_h,\alpha_l)$ from (18). The planner's objective is to maximize

$$\mu[\alpha_h\bar{W}(z,\alpha,\overline{\pi}) + (1-\alpha_h)\underline{W}(z,\alpha,\overline{\pi})] + (1-\mu)[\alpha_l\bar{W}(z,\alpha,\pi) + (1-\alpha_l)\underline{W}(z,\alpha,\pi)] \tag{27}$$

by choosing the first-period contract, z, and mixing probabilities (α_h,α_l).[8] The idea that the planner can only control first-period allocations respects the restriction in our model that buyers can only offer one-period contracts. A more general analysis, which we leave for future work, would allow for limited commitment by the planner.

Formally, a first-period allocation $z = (t_h, x_h, t_l, x_l)$ and mixing probabilities (α_h,α_l) is *ex ante efficient* if it maximizes (27) subject to (i) incentive compatibility, (16) and (17), (ii) the Bayes' rule requirement (18), (iii) the participation constraint for the high-quality bank in (20) and its analogue for low-quality banks, and (iv) the break-even constraint for buyers, or feasibility, which implies that the expression in (19) be nonnegative.

We now characterize the set of ex ante efficient allocations.[9] As we have shown above, the maximal trade equilibrium maximizes ex ante first-period payoffs. Ex ante efficiency includes continuation payoffs as well. Thus, ex ante efficiency yields different outcomes only if the static loss from deviating from the maximal trade equilibrium is outweighed by a dynamic gain.

Consider $\mu \geq \mu^*$. Here, we prove that the maximal trade equilibrium allocation is ex ante efficient and provide some intuition for the result. We show that the static loss of a small deviation from this allocation is of an order of magnitude greater than the dynamic gain from doing so. Consider a small perturbation of the maximal trade outcome in which the mixing probabilities are given by $\alpha_h = 1/2 + \varepsilon$

[8] The restriction to allocations that consist of offers to two types of agents in an environment with limited commitment is without loss of generality. See Bester and Strausz (2001).

[9] Analogously to the analysis of equilibrium, it is possible to prove that in any ex ante efficient allocation, it is optimal to set $\alpha_h = 1$ so that high-quality banks do not mix.

and $\alpha_l = 1/2$ together with a perturbed contract $\left(x_h', t_h', 1, t_l'\right)$, which is incentive compatible and yields nonnegative profits. From Bayes' rule (18), this perturbation raises μ_h' and lowers μ_l' by approximately $2\mu(1-\mu)\,\varepsilon$ for small values of $\varepsilon > 0$. Consider the increase in the continuation value for the high-quality bank given by

$$\alpha_h V\left(\mu_h'; \overline{\pi}\right) + \left(1-\alpha_h\right) V\left(\mu_l'; \underline{\pi}\right) - V\left(\mu; \overline{\pi}\right) = \Delta_\varepsilon.$$

Taking a Taylor series expansion of this expression shows that Δ_ε is of an order of ε^2. A similar argument applies for the low-quality banks. Notice that this perturbation induces a spread in continuation values, $V\left(\mu_h'; \pi\right) - V\left(\mu_l'; \pi\right)$. To ensure that the incentive constraints (17) and (16) are satisfied, this increase in spread in continuation values implies that x_h must fall by an amount proportional to ε. Therefore, first period ex ante expected payoffs also fall by an amount proportional to ε. Since continuation payoffs rise by a term in the order of ε^2 and first-period payoffs fall by a term in the order of ε, it follows that such a perturbation reduces ex ante payoffs. In the Appendix, we show that this argument generalizes to any perturbation of the maximal trade equilibrium. We then have the following proposition.

PROPOSITION 6: *Suppose that $\mu \geq \mu^*$. The maximal trade equilibrium is ex ante efficient.*

Next, consider ex ante efficient allocations when $\mu \leq \mu^*$. Consider a partial pooling allocation in which $\alpha_h = 1$ and α_l is such that $\mu_h' = \tilde{\mu}$. This allocation has the feature that the spread in continuation values $\Delta\left(\mu_h'; \pi\right) = V\left(\mu_h'; \pi\right) - V(0; \pi)$ is zero. As the discussion of the preceding proposition shows, increasing the spread in continuation values reduces the volume of trade in the first period. In the Appendix, we show that by using the strong submodularity property of the value function $V(\mu; \pi)$, the static losses from reducing volume in the first period outweigh the dynamic gains. Hence, any ex ante efficient partial pooling allocation must have no spread in continuation values, so that $\mu_h' = \tilde{\mu}$.

Hence, maximal trade equilibria are efficient if they have the feature that $\mu_h' = \tilde{\mu}$ and are inefficient otherwise. Recall from Lemma 3 that if (26) is not binding, $\mu_h' = \tilde{\mu}$. In the Appendix, we show that if $d \leq 2$, the efficient allocation satisfies (23) and violates (23) only if $d > 2$ and μ is sufficiently below μ^*.

In order to get some intuition for this result, notice that holding the contract fixed, an increase in the spread v raises the right side of (20) by more than the left side. Thus, an increase in the spread creates a force that makes a reduction in x_h necessary. A fall in x_h reduces profits generated from the high-quality bank and therefore reduces the subsidy available to low-quality banks at the offer $\left(1, t_l\right)$. This reduction in the subsidy is particularly large when μ is low. These considerations suggest that when v is large and μ is low, the payment t_l tends to be low. This low payment creates strong incentives for one of the buyers to deviate to a contract that induces the low-quality bank to accept an offer close to $\left(1, t_l\right)$ with probability 1 and thereby not to accept the offer (x_h, t_h). When these incentives are sufficiently strong (that is, when (23) is violated), the ex ante efficient allocation cannot be supported as an equilibrium.

Next, we compare volume of trade in the efficient allocations and the equilibrium outcomes. It is possible to show that an increase in μ_h' reduces the right side of (25). Since this constraint is binding in the maximal trade equilibrium, such a reduction reduces x_h and therefore trade volume. Thus, when the equilibrium allocation is inefficient so that $\mu_h' > \tilde{\mu}$, the volume of trade is lower than in the efficient allocation. Note also that when $\mu_h' > \tilde{\mu}$, the equilibrium inefficiently reveals too much information. We summarize these findings in the following proposition.

PROPOSITION 7: *Suppose that $\mu \leq \mu^*$ and that banks are sufficiently patient. If $d \leq 2$, the maximal trade equilibrium is ex ante efficient. If $d > 2$, there exists some $\hat{\mu} < \mu^*$ such that for $\mu \geq \hat{\mu}$, the maximal trade equilibrium is efficient and for $\mu < \hat{\mu}$, the maximal trade equilibrium has inefficiently low levels of volume.*

This proposition shows that intervention is desirable only when the adverse selection discount is high and that this intervention should be targeted to banks with low levels of reputation.

Efficient Interventions.—Here, we discuss policies that can implement ex ante efficient allocations. We show how ex ante efficient allocations can be weakly implemented in the sense that one equilibrium of the resulting game is ex ante efficient.

Propositions 6 and 7 show that we need only focus on cases in which $\mu \leq \hat{\mu}$ and $d \geq 2$. We have argued that in this case, the efficient allocation violates the Bertrand competition constraint. In particular, if one of the buyers (say, buyer 2) offers the efficient contract, buyer 1 would find it optimal to deviate it to a contract $(x_h, t_h, x_l, t_l + \varepsilon)$. Such a contract attracts the low-quality bank to the offer $(x_l, t_l + \varepsilon)$ with probability 1 and attracts high-quality banks with probability $1/2$. The resulting payoffs to buyer 1 are given by the left side of (23), and, since that constraint is violated in the efficient allocation, this deviation is profitable.

Implementing the efficient allocations requires designing a policy (say, a tax policy) that makes such deviations unprofitable. To design such a policy, consider the efficient allocation and consider the following set of potentially profitable deviation offers by buyers:

$$\mathcal{A} = \Big\{(x,t) : t + (1-x)(\underline{\pi}v - c) > t_l + (1-x_l)(\underline{\pi}v - c),$$
$$(1-\mu)(x\underline{\pi}v - t) + \frac{1}{2}\mu(x_h\overline{\pi}v - t_h) \geq 0\Big\}.$$

The set $\mathcal{A}$ is a set of offers that attract the low-quality bank with probability 1 and make positive profits against the efficient allocation. Since the efficient allocation violates (23), $\mathcal{A}$ is non-empty. Consider a tax policy with a tax on buyers τ_l chosen so that

$$(1-\mu)(x_l\underline{\pi}v - t_l - \tau_l) + \frac{1}{2}\mu(x_h\overline{\pi}v - t_h) = 0. \tag{28}$$

Apply the tax defined by (28) to any offer in the set $\mathcal{A}$. Clearly, if one of the buyers offers the efficient allocation, the other buyer does not have an incentive to make an offer that satisfies the Bertrand competition constraint. We have proved the following proposition.

PROPOSITION 8: *Suppose that* $\mu \leq \hat{\mu}$ *and* $d \geq 2$. *Then the ex ante efficient allocation is an equilibrium outcome given the tax policy defined by* (28).

B. *Equilibrium Multiplicity and Asset Purchase Policies*

Here, we analyze asset purchase policies that can eliminate multiplicity of equilibria discussed earlier. We show that conventional asset purchase policies do not eliminate multiplicity of equilibria in our model, but unconventional asset purchase policies can do so.

To set the stage for the discussion of conventional asset purchase policies in our model, we begin by describing an environment in which such policies do eliminate multiplicity of equilibria. Consider a nonstrategic version of our static model. In this version, banks and buyers have the same payoffs but are price takers. In this environment, when μ is sufficiently high, it is straightforward to show that the model has two equilibria. In one equilibrium, the price is $\hat{p}(\mu)$ and both types of banks sell their entire loan portfolios, and in the other the price is $\underline{\pi}v$ and only the low-quality bank sells its portfolio. This model can clearly generate sudden collapses in trade volume associated with a switch from the high to the low trade equilibrium. Also note that the low trade equilibrium is inefficient in the sense that the high trade equilibrium Pareto dominates the low trade equilibrium. In this environment, if the government commits to a policy of purchasing assets at $\hat{p}(\mu)$, it can eliminate the low price/volume equilibrium and the government does not actually have to buy any loans. This policy resembles that of deposit insurance in the Diamond and Dybvig (1983) model and is desirable since it is Pareto improving.

We find this way of modeling sudden collapses in trade unattractive for two reasons. First, buyers have strong incentives to offer nonlinear contracts intended to separate high-quality banks from low-quality banks. This nonlinearity is a pervasive feature of models with adverse selection. For example, in this model, a buyer who makes an offer to purchase a small quantity of loans for a price per loan close to $\overline{\pi}v$ will attract only high-quality banks. Second, even if we restricted ourselves to linear contracts, we find the low volume equilibrium unappealing. The reason is that in such an environment, each buyer has a strong incentive to offer to buy the entire loan portfolio at a price per loan slightly less than $\hat{p}(\mu)$ and attract both types of banks.[10] Such strategic interaction would eliminate the low volume equilibrium.

Notwithstanding these critiques, we are sympathetic to the idea that asset purchase policies intended to eliminate multiplicity of equilibria could possibly be effective. As we have shown in Section IIB, if $\mu \geq \mu^*$, our model has a range of complete pooling equilibria with varying values of volume. We ask whether asset purchase policies can eliminate multiplicity of equilibria in our two-period model. Consider

[10] It has been suggested to us that models with linear contracts and capacity constraints could possibly generate multiple equilibria. We think that this conjecture is unlikely to be true based on the results of Guerrieri, Shimer, and Wright (2010), who develop an adverse selection model with capacity constraints and matching frictions and obtain a unique equilibrium.

an asset purchase policy in which the government offers a contract $\left(1, \hat{p}(\mu)\right)$ for any bank type in the first period and does not intervene in the market in the second period. We will show that such a policy does not eliminate pooling equilibria with $x < 1$.

To show this result, consider a low trade pooling equilibrium with $x < 1$. Recall that in the equilibrium without the government, it was feasible for one of the buyers to make the offer $\left(1, \hat{p}(\mu)\right)$, but such a deviation was not profitable. The reason is that under our equilibrium beliefs, a bank that accepts such an offer ends up with a future reputation of 0. With such a future reputation level, high-quality banks are unwilling to accept the offer, because the static gains are outweighed by the dynamic losses. In the proof of the following proposition in the Appendix, we show that the offer $\left(1, \hat{p}(\mu)\right)$ does not attract low-quality banks either. We then have the following proposition.

PROPOSITION 9: *Suppose that* $\mu \geq \mu^*$ *and the government offers to purchase loans at price* $\hat{p}(\mu)$ *in the first period. Then the two-period game has an equilibrium where neither bank type sells to the government and both types sell a fraction of their portfolio*, x, *at price* $\hat{p}(\mu)$.

This proposition shows that when reputational concerns are sufficiently strong, conventional asset purchase policies do not restore the volume of trade if equilibrium selection is unaffected by the policy. Clearly, a policy of purchasing all assets at a sufficiently high price will restore the volume of trade. But such a policy necessarily requires using tax revenues from other sources.

Multiplicity of equilibria can be eliminated by more interventionist policies. One example of such a policy prohibits private trade at any contract other than the maximal trade equilibrium. This policy clearly ensures that the maximal trade equilibrium is implemented. Another example is a tax policy similar to that discussed above. In this sense, unconventional policies that limit private trade can eliminate multiplicity of equilibria.

IV. The Infinite Horizon Model with Reputational Concerns

We now turn to a stochastic infinite horizon model with reputational concerns. In this model, we show that the main results from our two-period model are robust in the infinite horizon. In addition, the infinite horizon model captures the effect of anticipated stochastic fluctuations in collateral values on aggregate trading volume and allows us to endogenize the initial distribution of reputations. We show that the model is capable of producing a slow buildup of volume in secondary loan markets followed by abrupt collapses.

Our stochastic infinite horizon model is a straightforward extension of the two-period model studied above. We introduce stochastic fluctuations in collateral values that are perfectly correlated across banks. We assume that the spread v_t is independently distributed over time and that it is drawn from an identical distribution $G(v_t)$ with support $[v_{\min}, v_{\max}]$, where the lower bound v_l is given by $c/(\overline{\pi} - \underline{\pi})$. At this lower bound, there is no adverse selection problem in the sense that for all reputation levels, the high-quality bank is willing to sell its entire loan portfolio at the pooling price $\hat{p}(\mu)$. Let Ev denote the expected value of the spread v. For simplicity, we assume that the cost of holding loans, c, and the dispersion in bank quality $\overline{\pi} - \underline{\pi}$ are constant, so that fluctuations in collateral values are the only shocks that induce fluctuations in the adverse selection discount.

We allow the quality of the banks to change according to a Markov process. In particular, at the end of each period, with probability λ, each bank draws a new quality level. These draws are independent across banks, and buyers can observe whether a bank has drawn a new quality level. If the bank draws a new quality level, the bank is of high quality with probability μ, where μ is distributed according to the distribution function $\Psi(\mu)$ and $\Psi(\mu)$ is continuous with support on $[0, 1]$. Allowing the quality levels of banks to change ensures that the model has a unique invariant distribution of banks' reputations.

To simplify the analysis, we restrict banks to either sell or hold their entire loan portfolios so that $x = 0$ or $x = 1$. In the online Appendix, we show that the equilibrium features that we characterize are robust to relaxing this restriction.

Next, we turn to characterizing a stationary Markov equilibrium in which equilibrium outcomes in any period depend only on the beliefs of buyers about the bank's quality and the spread v_t. In particular, these outcomes do not depend on calendar time. As in the two-period model, if banks are sufficiently patient, a separating equilibrium does not exist in the infinite horizon model. We will say that banks are sufficiently patient in the infinite horizon model if

$$\frac{\beta(1 - \lambda)}{1 - \beta(1 - \lambda)} \geq \frac{(\overline{\pi} - \underline{\pi})v_{\max} + c}{(\overline{\pi} - \underline{\pi})Ev}. \tag{29}$$

The following proposition is the analogue of Proposition 4. (The proof is identical to the two-period model and is available upon request.)

PROPOSITION 10 (Patience and Persistence of Adverse Selection): *If banks are sufficiently patient in the infinite horizon economy, then no equilibrium has complete separation of high- and low-quality banks in any period.*

Consider now an equilibrium for the infinite horizon model when banks are sufficiently patient. We show that the infinite horizon model has an equilibrium that is similar to the first-period equilibrium of the two-period model. Figure 1 displays the nature of the equilibrium in the infinite horizon model. As in the two-period model, when reputation levels are low, the equilibrium has partial pooling in the sense that high-quality banks use pure strategies and low-quality banks use mixed strategies. In the infinite horizon model, partial pooling is an equilibrium if current reputation μ_t is below a threshold function $\mu^*(v)$ and below a threshold level μ_h. The threshold function $\mu^*(v)$ is chosen so that when $\mu_t = \mu^*(v)$, high-quality banks are statically indifferent between holding their loans and receiving a payoff of $\overline{\pi}v - c$ and selling them at the pooling price and receiving a payoff of $\hat{p}(\mu)$. Equating these payoffs yields

$$\mu^*(v) = 1 - \frac{c}{(\overline{\pi} - \underline{\pi})v}.$$

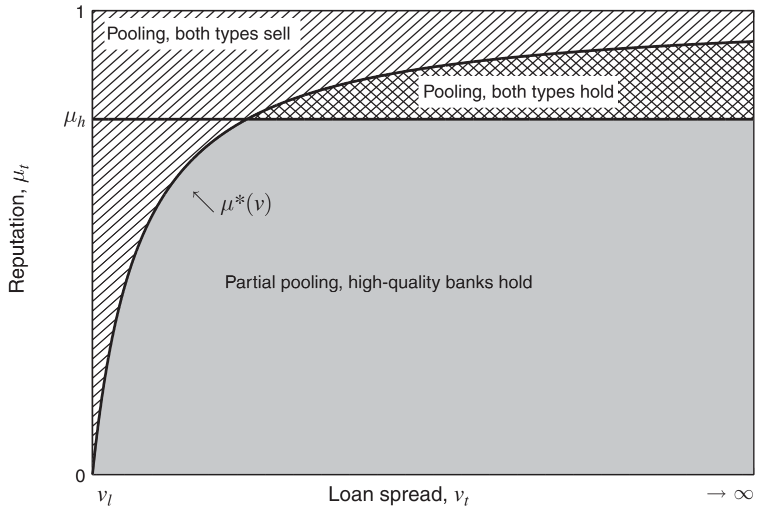

Figure 1. Equilibrium Regions in the Infinite Horizon Model with Stochastic Spreads

If $\mu_t > \mu^*(v)$, banks of both quality types statically strictly prefer to sell their loans rather than hold them. If $\mu_t < \mu^*(v)$, banks of both quality types statically strictly prefer to hold their loans rather than sell them. Note that the indifference condition used to define $\mu^*(v)$ is the same as the indifference condition that defines the threshold μ^* in the two-period model.

The threshold level μ_h is chosen so that the low-quality bank is indifferent between selling their loans for a payment of $\underline{\pi} v_t$ and receiving a future reputation of 0 and holding their loans and receiving a future reputation of μ_h. This indifference condition requires that μ_h satisfies

$$(30)\quad \underline{\pi} v_t + \beta(1-\lambda)V(0;\underline{\pi}) + \beta\lambda W = \underline{\pi} v_t - c + \beta(1-\lambda)V(\mu_h;\underline{\pi}) + \beta\lambda W,$$

where W, the continuation value in the event of replacement, is given by

$$W = \int_0^1 \left[\mu V(\mu;\overline{\pi}) + (1-\mu)\, V(\mu;\underline{\pi})\right] \, d\Psi(\mu).$$

Notice that μ_h is independent of the spread v_t. The analogue of μ_h in the two-period model is an equilibrium in which μ_h' is sufficiently greater than $\tilde{\mu}$ and in which $x_h = 0$. One difference between the infinite horizon and the two-period model is that, in the infinite horizon model, the value function $V(\mu;\pi)$ is increasing in reputation levels μ even for relatively low values of μ, whereas in the two-period model, the value function is constant for low values of reputation. The value function is increasing in the infinite horizon model because of the possibility that low future values of the spread v might make it possible for low- and high-quality banks to sell at a favorable pooling price.

When reputation levels are high, the equilibrium features complete pooling. Depending on the reputation levels, this complete pooling equilibrium has both high- and low-quality banks both selling or both holding. Note that the feature that the equilibrium can have both types of banks holding their loans is unlike that in the two-period model. This feature arises because in our stochastic model, low-quality banks have incentives to hold on to their loans in the hope of being able to sell their loans at high prices in the future.

The equilibrium strategy of the high-quality banks is to sell their loans if $\mu_t > \mu^*(v)$ and to hold their loans if $\mu_t \leq \mu^*(v)$. The equilibrium strategy of the low-quality banks is, in a sense, to mimic the behavior of high-quality banks. These banks sell their loans if $\mu_t > \mu^*(v)$, hold their loans if $\mu_t \leq \mu^*(v)$ and $\mu_t \geq \mu_h$, and mix between selling their loans and holding them if $\mu_t \leq \mu^*(v)$ and $\mu_t > \mu_h$. The mixing probabilities $\alpha_l(\mu_t, v_t)$ are chosen so that the reputation level in the next period is μ_h, so that

$$\mu_h = \frac{\mu_t}{\mu_t + (1 - \mu_t)(1 - \alpha_l(\mu_t, v_t))}. \tag{31}$$

These strategies induce continuation payoffs $V(\mu, \pi)$ in the obvious fashion. Clearly, these continuation payoffs are increasing in reputation levels μ.

Next, consider the beliefs that make these strategies optimal for banks. When $\mu_t \geq \mu^*(v_t)$, the equilibrium strategies are for banks of both quality types to sell, so that $\mu_{t+1} = \mu_t$. If a bank deviates to holding its loan, we assume that future buyers assign the same reputation level as under the equilibrium, so that $\mu_{t+1} = \mu_t$. These beliefs ensure that banks of both quality types care only about static incentives and therefore prefer to sell their loan portfolios. When $\mu_t \leq \mu_h$ and $\mu_t < \mu^*(v_t)$, that is, when the equilibrium falls in the partial pooling region, in equilibrium future buyers assign a reputation of $\mu_{t+1} = \mu_h$ to a bank that holds its loan portfolio and a reputation of 0 to a bank that sells its loan portfolio. Consider finally reputation levels such that $\mu_h < \mu_t < \mu^*(v_t)$. For such reputation levels, the equilibrium prescribes that both types of banks hold their entire loan portfolios and future beliefs are given by $\mu_{t+1} = \mu_t$. We assume that if a bank deviates and sells its loan portfolio, future buyers assign the bank a reputation $\mu_{t+1} = 0$. With these out-of-equilibrium beliefs, high-quality banks statically and dynamically prefer to hold their loans rather than sell them. Therefore low-quality banks can only receive a price of $\underline{\pi} v_t$ for selling their loans. Thus, holding loans is preferred to selling them for low-quality banks if

$$\underline{\pi} v_t - c + \beta(1 - \lambda)V(\mu; \underline{\pi}) + \beta\lambda W \geq \underline{\pi} v_t + \beta(1 - \lambda)V(0; \underline{\pi}) + \beta\lambda W. \tag{32}$$

Since continuation payoffs are increasing in μ, (30) implies (32) for $\mu \leq \mu_h$.

We have shown that bank strategies are optimal given the equilibrium offers by buyers. In the Appendix, we construct out-of-equilibrium beliefs in essentially the same manner as in our two-period model to show that there are no profitable deviations by buyers. We summarize this discussion in the following proposition.[11]

[11] In the online Appendix, we show that this proposition also holds when banks can sell a fraction of their loans. The proof is similar to that of the proof of Lemma 2 and uses submodularity of the value function.

PROPOSITION 11: *Suppose that banks are sufficiently patient in the infinite horizon model. The model has a stationary Markov equilibrium with partial separation at best. For a given value of the loan spread* v_t, *there is a threshold such that when reputation is below this threshold*, *there is partial learning as the low-quality bank reveals its type with probability* $\alpha_l(\mu_t, v_t)$ *satisfying* (31). *When reputation is above the threshold*, *the equilibrium has no learning with both quality types selling either all or none of their loan portfolios.*

This proposition shows that when reputational concerns are sufficiently strong, adverse selection persists.

A. *Volume of Trade and the Spread in the Infinite Horizon*

Next we use Proposition 11 to show that the aggregate volume of trade falls in response to a decline in collateral values, and that this decline is discontinuous at a critical value of the spread. To show this result, we begin by characterizing properties of the equilibrium distribution over buyers' beliefs about bank quality. Let H_t denote the equilibrium distribution over reputation levels μ in period t for a particular history of realizations of collateral shocks, and suppose that t is large enough so that the initial distribution H_0 has an insignificant effect on H_t. Proposition 11 implies that since Ψ has full support over $[0, 1]$, H_t has full support with mass points at 0 and μ_h.

To understand how shocks to collateral value v_t affect volume of trade, consider an increase in the spread v_t from v^1 to v^2. Consider the effect on the expected volume of trade for a bank with a given reputation μ_t. If the increase in spread does not induce the bank to switch regions, clearly its volume of trade does not change. Consider therefore a bank that switches from selling all its loans to the complete pooling with holding region or to the partial pooling region. If such a bank has reputation $\mu_t > \mu_h$, it switches from selling to complete pooling with holding and the expected volume falls from 1 to 0. If such a bank has reputation $\mu_t \leq \mu_h$, then it switches to the partial pooling region so that the high-quality bank switches from selling to holding and the low-quality bank switches from selling with probability 1 to selling with probability $(1 - \alpha_l(\mu_t))$. Since H_t has full support, an increase in the spread leads to a decline in the aggregate volume of trade. Since the distribution H_t has a mass point at μ_h, if the increase in spread induces banks with reputation levels $\mu_t = \mu_h$ to switch regions, the decline in the aggregate volume of trade is discontinuous. A similar argument implies that increases in volume of spread are discontinuous at a critical value of the spread.

We have established the following proposition.

PROPOSITION 12: *If banks are sufficiently patient and if shocks to collateral values are independent over time*, *then the aggregate volume of trade is declining in the spread* v. *Changes in the volume of trade are discontinuous at critical values of the collateral shock.*

As the proof of the proposition makes clear, an increase in the spread leads to a decline in the volume of new issuances by banks of both quality types. In this sense, reputational forces can help generate a decline in trading volume by banks

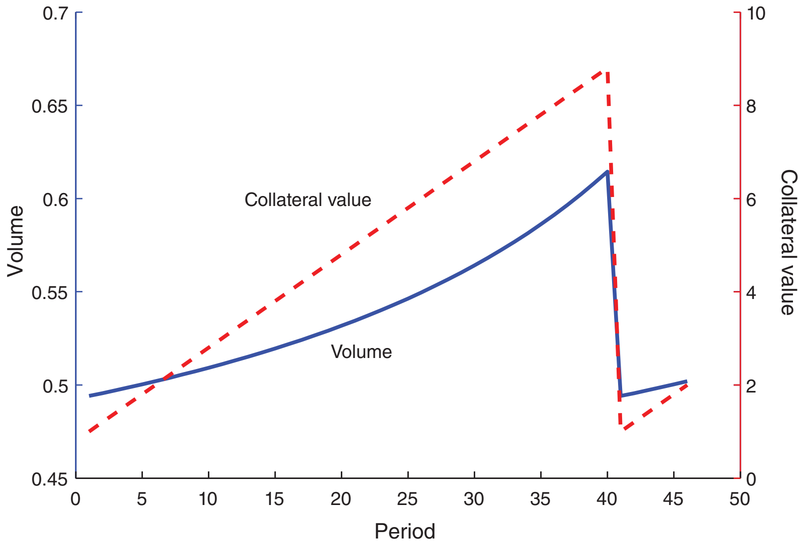

FIGURE 2. NUMERICAL SIMULATION OF COLLATERAL VALUES AND VOLUME OF TRADE

of all quality levels. Recall that in the static model, an increase in the spread leads to a reduction in the volume of trade only by high-quality banks. Thus, reputational forces can help overcome the correlation challenge present in static adverse selection models.

Figure 2 reports results from a numerical example intended to illustrate fluctuations in the trade volume that our model can generate.[12] We have plotted trade volume for a particular simulation of our model. In this simulation, collateral values gradually rise at a linear rate from period 1 to period 40. In period 41, collateral values fall abruptly and then rise at a linear rate from that period onward. The figure shows a sizable increase in volume from period 0 to period 40 and a sharp fall in period 41, after which volume continues to grow.

This illustrative simulation captures the idea that the model can generate slow buildups in the volume of trade followed by abrupt collapses. Although exploring whether this model can generate fluctuations of the kind seen in the data is beyond the scope of this paper, the simplicity of our infinite horizon model makes it amenable to future quantitative analyses.

V. Conclusion

We have argued that adverse selection is a promising candidate in accounting for fluctuations in the volume of trade in secondary loan markets. Since issuers of secondary loans are not anonymous and since buyers can observe past transactions, one would expect the market to reveal information about sellers, and one would expect that eventually adverse selection would cease to be a factor in generating fluctuations in

[12]The parameter values for this simulation are as follows: $\overline{\pi} = 0.8, \underline{\pi} = 0.2, c = 1, \beta = 0.9$. $\lambda = 0.1, v_{\min} = (5/3)$. $v_{\max} = 15, v_t \sim U[v_{\min}, v_{\max}]$ (or $G(v_t) = (v_t - v_{\min})/(v_{\max} - v_{\min})$), $\Psi(\mu) = \mu$. The value of the loan if no default occurs is set to 16 so that the collateral value is given by $\underline{v}_t = 16 - v_t$.

the volume of trade. We have shown that reputational concerns can impede the revelation of information and indeed, in some cases, can lead buyers to never learn about the quality of sellers. Thus, reputational concerns imply that adverse selection can play a role in generating fluctuations in the volume of trade even in the long run.

The techniques developed here should be widely applicable to the study of other markets that feature both private information and lack of anonymity. Applying these techniques to these other markets can help us to understand price formation, volume of trade, information revelation, and the welfare properties of outcomes in other markets.

Appendix

A. *Proof of Lemma 1*

Our construction of out-of-equilibrium beliefs is similar for both $\mu \geq \mu^*$ and $\mu \leq \mu^*$. Letting $z = \left(x_h, t_h, x_l, t_l\right)$ be the equilibrium contract, let the out-of-equilibrium belief function following acceptances of offers that are not part of the equilibrium contract be

(A1)

$$\mu'(\hat{x},\hat{t}) = \begin{cases} 0 & \text{if} \quad \hat{t} + (1-\hat{x})(\overline{\pi}v - c) \\ & \qquad \leq \max\left\{\hat{x}\hat{p}(\mu) + (1-\hat{x})(\overline{\pi}v - c), t_h + (1-x_h)(\overline{\pi}v - c)\right\} \\ 1 & \text{otherwise} \end{cases}.$$

Note that these beliefs say that future buyers believe that the bank is high quality if the bank accepts an offer that is both statically more favorable to the high-quality bank than the equilibrium and above the market-odds line.

We prove the following extension of Lemma 1. This extension characterizes all complete pooling equilibria for $\mu \geq \mu^*$.

LEMMA 4: *Suppose* $\mu \in \left[\mu^*, 1\right]$ *and banks are sufficiently patient. Then the two-period model has an equilibrium for any value of* $x \in \left[\underline{x}(\mu), 1\right]$, *where* $\underline{x}(\mu) = 1$ *when* $\mu \leq \tilde{\mu}$ *and otherwise it satisfies*

$$\text{(A2)} \quad \hat{p}(\mu) + \beta V(0;\overline{\pi}) = \underline{x}(\mu)\hat{p}(\mu) + (1-\underline{x}(\mu))(\overline{\pi}v - c) + \beta V(\mu;\overline{\pi}).$$

Note that $\underline{x}(\mu)$ is a lower bound on the set of complete pooling equilibria. Values of $x < \underline{x}(\mu)$ cannot be supported as equilibria because one of the buyers can deviate to a pooling outcome in which both types sell their entire loan portfolios at a price slightly less than $\hat{p}(\mu)$, attract banks of both quality levels, and make strictly positive profits.

PROOF:

The proof is by contradiction and has three steps. Let $(x, x\hat{p}(\mu))$ denote a pooling contract that satisfies $x \geq \underline{x}(\mu)$, and let $z = \left(\hat{x}_h, \hat{t}_h, \hat{x}_l, \hat{t}_l\right)$ denote a deviating contract that makes positive profits. Let $\hat{\mu}_h$, $\hat{\mu}_l$ denote the reputation levels associated with acceptance of the deviating offers. Clearly, any contract that makes positive profits must attract the high-quality bank and make strictly positive profits from such banks. We prove the claim in three steps. The first step is to show that $\hat{t}_h \geq \hat{x}_h \hat{p}(\mu)$. The second step is to show that $\hat{t}_l \geq \hat{x}_l \hat{p}(\mu)$. The key part of this step is to show that if $(\hat{x}_h, \hat{t}_h)$ attracts the high-quality bank, it also attracts the low-quality bank. The third step shows that $\hat{z}$ makes nonpositive profits.

Step 1: Given our belief function in (A1), any offer to the high-quality bank that makes it statically worse off also makes it dynamically worse off and cannot attract it. Thus, $(\hat{x}_h, \hat{t}_h)$ must satisfy

$$\hat{t}_h + (1 - \hat{x}_h)(\overline{\pi} v - c) \geq t_h + (1 - x)(\overline{\pi} v - c), \tag{A3}$$

where $t_h = x\hat{p}(\mu)$. In addition, if $\hat{t}_h < \hat{x}_h \hat{p}(\mu)$, then (A1) implies that $\hat{\mu}_h = 0$, so that the payoffs associated with an acceptance of $\hat{x}_h, \hat{t}_h$ for the high-quality bank are less than the left side of (A2). The payoffs associated with the equilibrium contract are at least as large as the right side of (A2). If the deviation is to attract high-quality banks, it must offer them a payoff higher than the equilibrium payoff, so that $\hat{t}_h \geq \hat{x}_h \hat{p}(\mu)$. Note for later that from (A1), the beliefs following an acceptance of $(\hat{x}_h, \hat{t}_h)$ are given by $\hat{\mu}_h = 1$.

Step 2: Since $(\hat{x}_h, \hat{t}_h)$ makes positive profits from high-quality banks, we have $\hat{t}_h < \hat{x}_h \overline{\pi} v$. Using this inequality in (A3), and $t_h = x\hat{p}(\mu)$, we obtain

$$\hat{x}_h c > x\hat{p}(\mu) - x(\overline{\pi} v - c). \tag{A4}$$

Using the results that the beliefs following an acceptance of $(\hat{x}_h, \hat{t}_h)$ are given by $\hat{\mu}_h = 1$, we claim that the low-quality bank gets a higher utility by accepting $(\hat{x}_h, \hat{t}_h)$ than (x, t_h), or that

$$\hat{t}_h + (1 - \hat{x}_h)(\underline{\pi} v - c) + \beta V(1; \underline{\pi}) > x\hat{p}(\mu) + (1 - x)(\underline{\pi} v - c) + \beta V(\mu; \underline{\pi}). \tag{A5}$$

We show (A5) by substituting for $\hat{t}_h$ from (A3) into (A5) and simplifying to obtain

$$\beta V(1; \underline{\pi}) - \beta V(\mu; \underline{\pi}) > (x - \hat{x}_h)(\overline{\pi} v - \underline{\pi} v). \tag{A6}$$

Using (A4) to substitute for $\hat{x}_h$ in (A6), and $d = (\overline{\pi} v - \underline{\pi} v)/c$, we have that (A6) is satisfied if

$$\beta V(1; \underline{\pi}) - \beta V(\mu; \underline{\pi}) > x(1 - \mu) d(\overline{\pi} v - \underline{\pi} v). \tag{A7}$$

Suppose that $\mu \geq \tilde{\mu}$. Substituting for the continuation payoffs from (10), (A7) is satisfied if

$$\frac{\beta}{1 - \tilde{\mu}} > xd. \tag{A8}$$

Since $\tilde{\mu} = d/(1 + d)$ and $\beta \geq (1 + d)/d$, the left side of (A8) is greater than $(1 + d)^2/d$. Since the right side of (A8) is less than d, the needed inequality is satisfied. If $\mu < \tilde{\mu}$, substituting from (10), (A7) becomes $\beta > x(1 - \mu)d$. This inequality is satisfied because $\mu \geq \mu^* = (1 - 1/d)$ and $\beta \geq (1 + d)/d$. Thus, the low-quality bank is attracted to the deviating offer to the high-quality bank.

Suppose now that $\hat{x}_l \hat{p}(\mu) > \hat{t}_l$. We show that the low-quality bank weakly prefers the equilibrium allocation to $(\hat{x}_l, \hat{t}_l)$. Together with the result that the low-quality bank is attracted to the high-quality offer, we have a contradiction (namely, these results imply that the deviation contract is not incentive compatible). Since from (A1), beliefs are zero for all accepted offers with $\hat{x}\hat{p}(\mu) > \hat{t}$, the highest possible payoff for the low-quality bank from such a contract is given by $\hat{p}(\mu) + \beta V(0; \underline{\pi})$. We show that

$$\text{(A9)} \qquad \hat{p}(\mu) + \beta V(0; \underline{\pi}) \leq x\hat{p}(\mu) + (1 - x)(\underline{\pi} v - c) + \beta V(\mu; \underline{\pi}).$$

If $\tilde{\mu} \geq \mu \geq \mu^*$, $V(0; \underline{\pi}) = V(\mu; \underline{\pi})$, and (A9) clearly holds (recall for such μ, $x = 1$). Suppose $\mu \geq \tilde{\mu}$. Using (10), we can rewrite (A9) as

$$(1 - x)\big(\hat{p}(\mu) - \underline{\pi} v + c\big) \leq \beta \frac{(\mu - \tilde{\mu})}{(1 - \tilde{\mu})}(\overline{\pi} - \underline{\pi})\, v.$$

Since $x \geq \underline{x}(\mu)$ and $\mu \geq \tilde{\mu}$, (A2) implies that

$$\hat{p}(\mu) + \beta V(0; \overline{\pi}) \leq x\hat{p}(\mu) + (1 - x)(\overline{\pi} v - c) + \beta V(\mu; \overline{\pi}),$$

which using (10) can be rewritten as

$$\text{(A10)} \qquad (1 - x)\big(\hat{p}(\mu) - \overline{\pi} v + c\big) \leq \beta \frac{(\mu - \tilde{\mu})^2}{\mu(1 - \tilde{\mu})}(\overline{\pi} - \underline{\pi})\, v.$$

Thus, (A9) holds if

$$\text{(A11)} \qquad (1 - x)\big(\hat{p}(\mu) - \underline{\pi} v + c\big) \leq \frac{\mu}{\mu - \tilde{\mu}}(1 - x)\big(\hat{p}(\mu) - \overline{\pi} v + c\big).$$

Substituting for $\hat{p}(\mu)$ from (7), using $d = (\overline{\pi} - \underline{\pi})\, v/c$, and simplifying, (A11) holds if

$$\text{(A12)} \qquad \mu d + 1 \leq \frac{\mu}{\mu - \tilde{\mu}}\big(-(1 - \mu)d + 1\big).$$

Using $\tilde{\mu} = d/(1 + d)$ and simplifying, it is straightforward to check that (A12) holds, so (A9) does as well.

Step 3: Adding the incentive compatibility constraints (16) and (17) and simplifying, we have that

$$\big(\hat{x}_l - \hat{x}_h\big)(\overline{\pi} - \underline{\pi})v \geq \beta\Big[\big(V(1; \underline{\pi}) - V(\hat{\mu}; \underline{\pi})\big) - \big(V(1; \overline{\pi}) - V(\hat{\mu}; \overline{\pi})\big)\Big] \geq 0,$$

where $\hat{\mu}$ is the belief associated with $(\hat{x}_l, \hat{t}_l)$, so that $\hat{x}_l \geq \hat{x}_h$. The profits from $\hat{z}$ are given by

$$\begin{aligned}\Pi &= \mu(\hat{x}_h \overline{\pi} v - \hat{t}_h) + (1 - \mu)(\hat{x}_l \underline{\pi} v - \hat{t}_l) \\ &\leq \mu(\hat{x}_h \overline{\pi} v - \hat{x}_h \hat{p}(\mu)) + (1 - \mu)(\hat{x}_l \underline{\pi} v - \hat{x}_l \hat{p}(\mu)) \\ &= \mu(1 - \mu)(\overline{\pi} - \underline{\pi}) v(\hat{x}_h - \hat{x}_l).\end{aligned}$$

Here, the first inequality follows from step 2. Since $\hat{x}_l \geq \hat{x}_h$, profit Π is nonpositive.

B. *Proof of Lemma 2*

Here, we prove the following extension of Lemma 2.

LEMMA 5: *Suppose that banks are sufficiently patient and that $\mu \leq \mu^*$. Suppose an allocation satisfies (i) $\alpha_h = 1$ and $\mu'_h \geq \mu$, (ii) the incentive constraint for the low-quality bank holds with equality, and (iii) (19) equals 0, and (20) and (23) are satisfied. Then the allocation can be supported as an equilibrium. In addition, any equilibrium of the game has this form.*

PROOF:

The proof is by contradiction. Suppose first that a deviation contract attracts high-quality banks and makes strictly positive profits on such banks so that $\hat{x}_h > 0$. As in the proof of the preceding lemma, attracting such banks requires that the analogue of (A3) hold. In the online Appendix, we show that (20) implies that

$$\text{(A13)} \qquad x_h \leq \frac{\beta}{(\overline{\pi} - \underline{\pi})v}\left[V(1;\underline{\pi}) - V(\mu'_h;\underline{\pi})\right],$$

so that

$$\text{(A14)} \qquad \beta\left[V(1;\underline{\pi}) - V(\mu'_h;\underline{\pi})\right] > (x_h - \hat{x}_h)(\overline{\pi} - \underline{\pi})v,$$

which holds since $\hat{x}_h > 0$. Adding (A3) and (A14), we obtain

$$\text{(A15)} \qquad \begin{aligned}&\hat{t}_h + (1 - \hat{x}_h)(\overline{\pi} v - c) + \beta V(1;\underline{\pi}) - \beta V(\mu'_h;\underline{\pi}) \\ &> t_h + (1 - x_h)(\overline{\pi} v - c) + (x_h - \hat{x}_h)(\overline{\pi} - \underline{\pi})v.\end{aligned}$$

Simplifying (A15), we get

$$\text{(A16)} \quad \hat{t}_h + (1 - \hat{x}_h)(\underline{\pi} v - c) + \beta V(1;\underline{\pi}) > t_h + (1 - x)(\underline{\pi} v - c) + \beta V(\mu;\underline{\pi}),$$

which implies that low-quality banks are attracted to the offer intended for the high-quality bank. The argument that the profits to the deviating buyer are then negative, taking account of the offer intended for the low-quality banks, is the same as in the preceding lemma.

Next we show that a deviation offer that attracts low-quality banks is not profitable. Clearly, it is impossible to attract only low-quality banks and make positive profits. What is left to be shown is that a deviation to a contract of the form $z' = (x_h, t_h, x_l', t_l')$ cannot make positive profits. Note that the contract that maximizes buyers' payoffs and attracts the low-quality bank is given by

$$\left(x_h, t_h, 1, t_l + (1 - x_l)(\underline{\pi}v - c)\right).$$

This contract attracts the high-quality bank with probability $1/2$ and attracts the low-quality bank with probability 1. Hence, the profits from this deviation are given by the left side of (23) and are, therefore, nonpositive.

To show that any equilibrium must have the specified form, it suffices to show that there is no equilibrium in which the high-quality bank uses mixed strategies. Suppose, to the contrary, that an equilibrium exists in which the high-quality bank mixes. Then, it must be that the low-quality bank is also mixing. To see this, suppose that only the high-quality bank uses mixed strategies. Then on-path continuation beliefs are given by 1 and $\mu' < \mu$. Since $V(0; \underline{\pi}) = V(\mu'; \underline{\pi})$ and banks are sufficiently patient, a proof similar to Proposition 4 implies that no incentive-compatible contract can exist. Hence, we focus on contracts in which both banks mix. Such an equilibrium can be described by $(x_h, t_h, x_l, t_l, \alpha_h, \alpha_l)$ with $\alpha_h, \alpha_l \in (0, 1)$ and $\alpha_h > 1 - \alpha_l$. From (18), we have that $\mu_l' < \mu$. Since $\mu < \tilde{\mu}$, $V(\mu_l'; \pi) = V(0; \pi)$. Since both types are mixing, it must be that both types are indifferent between the two contracts. Subtracting (16) from (17), we obtain

$$x_h(\overline{\pi} - \underline{\pi})v + \beta(\underline{\Delta} - \overline{\Delta}) = x_l(\overline{\pi} - \underline{\pi})v,$$

where $\overline{\Delta} = V(\mu_h'; \overline{\pi}) - V(0; \overline{\pi})$ and $\underline{\Delta}$ is defined similarly for the low type. Since $V(\cdot\,;\,\cdot)$ is submodular, $\underline{\Delta} > \overline{\Delta}$ and therefore $x_l > x_h$. Furthermore, it follows that $t_l > t_h$. We can rewrite the above as

$$x_l = x_h + \beta\left[\frac{\mu_h' - \tilde{\mu}}{1 - \tilde{\mu}} - \frac{(\mu_h' - \tilde{\mu})^2}{\mu_h'(1 - \tilde{\mu})}\right]$$

$$= x_h + \beta\frac{\mu_h' - \tilde{\mu}}{1 - \tilde{\mu}}\left[1 - \frac{\mu_h' - \tilde{\mu}}{\mu_h'}\right]$$

$$= x_h + \beta\frac{(\mu_h' - \tilde{\mu})\tilde{\mu}}{(1 - \tilde{\mu})\mu_h'}.$$

Hence,

$$\begin{aligned} t_h &= t_l + (x_h - x_l)(\overline{\pi}v - c) - \beta\overline{\Delta} \\ &= t_l - \beta\frac{(\mu_h' - \tilde{\mu})\tilde{\mu}}{(1 - \tilde{\mu})\mu_h'}(\overline{\pi}v - c) - \beta\frac{(\mu_h' - \tilde{\mu})^2}{\mu_h'(1 - \tilde{\mu})}(\overline{\pi} - \underline{\pi})v \\ &= t_l - \beta\frac{\mu_h' - \tilde{\mu}}{\mu_h'(1 - \tilde{\mu})}\left[\tilde{\mu}(\overline{\pi}v - c) + (\overline{\pi} - \underline{\pi})v(\mu_h' - \tilde{\mu})\right] \\ &= t_l - \beta\frac{\mu_h' - \tilde{\mu}}{\mu_h'(1 - \tilde{\mu})}\left[(\overline{\pi} - \underline{\pi})v\mu_h' + \tilde{\mu}(\underline{\pi}v - c)\right] = t_l - a. \end{aligned}$$

Extensive algebra shows that $t_l - x_l\hat{p}(\mu) \leq 0$ (this derivation is available upon request). Therefore, $t_l - (\overline{\pi}v - c)x_l < 0$. That is, the high type strictly prefers holding to trading at (x_l, t_l). This holds true statically and since $V(\mu_l'; \overline{\pi}) = V(0; \overline{\pi})$, it must also hold dynamically. This implies that an offer of the form $(\varepsilon, \hat{p}(\mu)\varepsilon - \delta)$ for small enough ε and δ would attract the high-quality bank and make positive profits. Hence, the original allocation cannot be an equilibrium.

C. *Basic Properties of Partial Pooling Allocations*

LEMMA 6: *Consider the partial pooling allocation described in Section IIB with* $x_l = 1$. *Then we have the following:*

(i) If the constraints (16)–(17) are satisfied and the expression in (19) is nonnegative, then (20) and (23) can be simplified to $x_h \leq \overline{x}_h(\mu_h, v)$, and $x_h \geq \underline{x}_h(\mu_h, v)$, where $\overline{x}_h(\mu_h, v)$ is given in (25) and $\underline{x}_h(\mu_h, v)$ is given in (26).

(ii) The upper bound implied by (20) is decreasing in μ_h' when $\mu_h' \geq \tilde{\mu}$ and increasing otherwise.

(iii) The lower bound $\underline{x}_h$, when positive, is decreasing in μ_h'.

(iv) Aggregate volume implied by the upper bound in (20) is maximized at $\mu_h' = \tilde{\mu}$.

(v) Ex ante welfare implied by the upper bound in (20) is maximized at $\mu_h' = \tilde{\mu}$.

(vi) When μ_h' is large enough, $\underline{x}_h \leq \overline{x}_h$.

The proof is in the online Appendix.

D. *Proof of Lemma 3*

Lemma 6 establishes that if (26) is not binding, volume is maximized when $\mu_h' = \tilde{\mu}$ and $x_h = \overline{x}_h$. Thus, if (26) is binding, then $\overline{x}(\tilde{\mu}, v) < \underline{x}(\tilde{\mu}, v)$. Also, in Lemma 6 above, we have shown that $\underline{x}(\mu_h', v)$ is a decreasing function of μ_h' and $\overline{x}(\mu_h', v)$ is increasing when $\mu_h' \leq \tilde{\mu}$. Thus, for all values $\mu_h' \leq \tilde{\mu}$, $\overline{x}(\mu_h', v) < \underline{x}(\mu_h', v)$. Thus, if (26) is binding, $\mu_h' > \tilde{\mu}$.

E. *Proof of Proposition 5*

Note that the upper bound in (25) is decreasing in v. If (26) is not binding, $\mu_h' = \tilde{\mu}$ and does not change with v, so that an increase in v lowers total volume. Suppose (26) is binding. As is clear from the formula in (26), the lower bound on x_h is a decreasing function of v. Furthermore, as we have established in Lemma 6 above, an increase in v decreases the upper bound in (25). Since the upper bound is always binding in the maximal trade equilibrium, μ_h' should change in order to satisfy both the upper and the lower bound. In particular, μ_h' should increase so as to satisfy both (25) and (26). As we show in Lemma 6, aggregate volume is decreasing in μ_h'. This implies that the change in μ_h' decreases volume even further.

F. *Basic Properties of Mixed Strategy Allocations*

LEMMA 7: *Consider an allocation* (x_h, t_h, x_l, t_l) *with mixed strategies* (α_l, α_h) *in which both types mix. Then*:

(i) If $\tilde{\mu} \le \mu_l' \le \mu \le \mu_h$, then ex ante welfare is given by

$$\kappa - (1 - x_l)\, c - \frac{\beta d^2}{1 + d} c \left(\frac{1}{\mu_l'} - \frac{1}{\mu_h'}\right) \frac{\mu}{\mu_h'},$$

where κ is a constant and independent of x_l, μ_l', and μ_h'.

(ii) If $\mu_l' \le \tilde{\mu} \le \mu_h'$, then ex ante welfare is given by

$$\kappa' - c(1 - x_l) - \beta d c \frac{\mu_h' - \tilde{\mu}}{\mu_h'} (1 - \mu)(1 - \alpha_l),$$

where κ' is a constant and independent of x_l, μ_l', and μ_h'.

PROOF:

Substituting for t_h and t_l from the incentive constraints with equality and the zero profit constraint, we obtain that ex ante welfare is given by

$$\begin{aligned} \text{(A17)} \quad & \hat{p}(\mu) - \mu\big(\alpha_h(1 - x_h)\, c + (1 - \alpha_h)(1 - x_l) c\big) \\ & \times (1 - \mu)\big(\alpha_l(1 - x_l)\, c + (1 - \alpha_l)(1 - x_h)\, c\big) \\ & + \beta\mu\big[\alpha_h V(\mu_h'; \overline{\pi}) + (1 - \alpha_h)\, V(\mu_l'; \overline{\pi})\big] \\ & + \beta(1 - \mu)\big[\alpha_l V(\mu_l'; \underline{\pi}) + (1 - \alpha_l)\, V(\mu_h'; \underline{\pi})\big]. \end{aligned}$$

Adding the incentive constraints for the banks, we obtain

$$(x_l - x_h)(\overline{\pi} - \underline{\pi}) v = \beta\big[V(\mu_h'; \underline{\pi}) - V(\mu_l'; \underline{\pi})\big] - \beta\big[V(\mu_h'; \overline{\pi}) - V(\mu_l'; \overline{\pi})\big].$$

Now consider case 1 where $\mu_l' \geq \tilde{\mu}$. Using the form of the value function (10) when $\mu_l' \geq \tilde{\mu}$, one can show

$$x_h - x_l = \frac{-\beta d^2}{1+d}\left(\frac{1}{\mu_l'} - \frac{1}{\mu_h'}\right).$$

Substituting for x_h and the value function (10) into (A17) yields the form of ex ante welfare asserted in the lemma. Following a similar procedure yields the stated result in the second case when $\mu_l' \leq \tilde{\mu} \leq \mu_h'$.

G. *Proof of Proposition 6*

We first show that any feasible allocation with both types mixing yields lower ex ante utility than the utility in the full trade allocation. With both types mixing, without loss of generality $\mu_h' \geq \mu \geq \mu_l'$. Suppose that $\mu_l' = \mu_h'$. Then continuation values must satisfy $V(\mu_l', \pi) = V(\mu', \pi)$, so that (16) and (17) imply that $x_h = x_l$ as well as $t_h = t_l$. Hence, ex ante welfare is highest at the full trade allocation.

Suppose, next, that $\mu_l' \geq \tilde{\mu}$. From Lemma 7 we have that welfare is given by

$$\kappa - c(1 - x_l) + \beta\frac{d^2}{1+d}c\frac{\mu}{\mu_h'}\left(\frac{1}{\mu_l'} - \frac{1}{\mu}\right).$$

Since $\mu_l' \leq \mu$, welfare is maximized at $\mu_l' = \mu$ and $x_l = 1$, which from (18) implies that $\mu_h' = \mu$. This allocation coincides with the full trade allocation.

Now suppose that $\mu_l' \leq \tilde{\mu}$. Here, welfare is given from Lemma 7 by

$$\kappa' - c(1 - x_l) - \beta dc\frac{\mu_h' - \tilde{\mu}}{\mu_h'}(1-\mu)(1-\alpha_l).$$

This expression is maximized at $\mu_h' = \max\{\tilde{\mu}, \mu\}$. When $\mu_h' = \mu$, we must have $\mu_l' = \mu$, and hence the allocation coincides with full trade allocation. If $\mu_h' = \tilde{\mu}$, then since $\mu_l' \leq \mu_h' = \tilde{\mu}$, the continuation values are equal, and therefore welfare cannot be higher than that of the full trade allocation.

Next, we consider partial pooling allocations where only the low type mixes and show that they cannot deliver a higher ex ante utility than a pooling allocation with full trade. Substituting for t_h and t_l using zero profits and for α_l from (18), we can write welfare, W, as

$$\text{(A18)} \qquad \hat{p}(\mu) - c(1 - x_h)\frac{\mu}{\mu_h'} + \beta\mu V(\mu_h'; \overline{\pi}) + \beta\left(\frac{\mu}{\mu_h'} - \mu\right)V(\mu_h'; \underline{\pi}) + \beta\left(1 - \frac{\mu}{\mu_h'}\right)V(0; \underline{\pi}).$$

Adding the two incentive compatibility constraints, we have the following upper bound for x_h:

$$\text{(A19)} \qquad (\overline{\pi} - \underline{\pi})\,v + \beta\big(\Delta(\mu_h'; \overline{\pi}) - \Delta(\mu_h'; \underline{\pi})\big) \geq x_h(\overline{\pi} - \underline{\pi})\,v.$$

Using the upper bound on x_h implied by (A19) and grouping terms, we have that

$$W \leq \kappa + \beta \frac{(\mu_h' - \tilde{\mu})\,\mu}{(1 - \tilde{\mu})\,\mu_h'}\left[-c\frac{\tilde{\mu}}{\mu_h'} - (\overline{\pi} - \underline{\pi})\,v\frac{\tilde{\mu}}{\mu_h'} + (\overline{\pi} - \underline{\pi})\,v\right],$$

where $\kappa = \hat{p}(\mu) + \beta\mu V(0;\overline{\pi}) + \beta(1-\mu)\,V(0;\underline{\pi})$. Since $\tilde{\mu} = (\overline{\pi} - \underline{\pi})v/(c + (\overline{\pi} - \underline{\pi})v)$ and $\mu_h' \leq 1$, it follows that

$$\begin{aligned} W &\leq \hat{p}(\mu) + \beta\mu V(0;\overline{\pi}) + \beta(1-\mu)\,V(0;\underline{\pi}) \\ &\leq \hat{p}(\mu) + \beta\mu V(\mu;\overline{\pi}) + \beta(1-\mu)\,V(\mu;\underline{\pi}), \end{aligned}$$

where the last inequality follows because $V(\mu;\pi)$ is an increasing function of μ.

H. *Proof of Proposition 7*

In Section I, we show that the high-quality bank does not mix in the efficient allocation. Hence, we only need to consider allocations in which the low-quality bank mixes. As shown in Lemma 6, when (20) is binding, aggregate welfare as a function of μ_h' is maximized at $\mu_h' = \tilde{\mu}$. Since in any ex ante efficient allocation with $\mu \leq \mu^*$, (20) must be binding, we have established that any efficient allocation must have $\mu_h' = \tilde{\mu}$. Since the continuation value functions are constant for $\mu \leq \tilde{\mu}$, it follows that (26) is satisfied in the efficient allocation if

$$\frac{c\dfrac{2 - \mu - \tilde{\mu}}{\tilde{\mu}(1 - \mu)}}{(\overline{\pi} - \underline{\pi})\,v + c\dfrac{2 - \mu - \tilde{\mu}}{\tilde{\mu}(1 - \mu)}} \leq \frac{c\dfrac{\tilde{\mu} - \mu}{\tilde{\mu}(\mu^* - \mu)}}{(\overline{\pi} - \underline{\pi})\,v + c\dfrac{\tilde{\mu} - \mu}{\tilde{\mu}(\mu^* - \mu)}}.$$

Straightforward algebra shows that the above inequality is equivalent to

$$\frac{2}{1 - \mu^*} - \frac{1}{1 - \tilde{\mu}} \leq \frac{1}{1 - \mu}$$

or, substituting for μ^* and $\tilde{\mu}$ in terms of d, we obtain

$$d - 1 \leq \frac{1}{1 - \mu}. \tag{A20}$$

The inequality in (A20) holds for all values of μ when $d \leq 2$. If $d \geq 2$, then there is a threshold $\hat{\mu}(d)$ defined by equality in (A20) such that the maximal trade equilibrium is efficient for $\mu \geq \hat{\mu}(d)$ and inefficient for $\mu \leq \hat{\mu}(d)$.

I. *No Mixed Strategy by the High-Quality Bank*

Here we show that, when $\mu \leq \mu^*$, the high-quality bank does not mix in the efficient allocation. Consider an allocation where $\alpha_h \in (0,1)$, so that the high-quality bank uses a mixed strategy. We show that this allocation can be perturbed so that ex ante welfare is higher and hence cannot be ex ante efficient. First notice that an argument similar to that of Proposition 4 implies that the low-quality bank should be mixing as well. Furthermore, note that the allocation cannot be a pooling allocation, since a pooling allocation does not satisfy (20). Given that the allocation is not pooling, we must have $\alpha_h > 1 - \alpha_l$. Furthermore, we must have $\mu'_h > \tilde{\mu} > \mu > \mu'_l$. If not, $V(\mu'_h; \pi) = V(\mu'_l; \pi)$ for $\pi = \overline{\pi}, \underline{\pi}$ and the incentive compatibility constraints imply that $x_h = x_l$ and $t_h = t_l$, a pooling allocation.

Before showing how to perturb the allocation, it is useful to write the payoff for the buyers as

$$\text{(A21)} \qquad 0 = \mu\alpha_h(\overline{\pi}vx_h - t_h) + (1-\mu)(1-\alpha_l)(\underline{\pi}vx_h - t_h) + (1-\mu)\alpha_l(\underline{\pi}vx_l - t_l) + \mu(1-\alpha_h)(\overline{\pi}vx_l - t_l)$$
$$= \big(\mu\alpha_h + (1-\mu)(1-\alpha_l)\big)\big(\hat{p}(\mu'_h)x_h - t_h\big) + \big(\mu(1-\alpha_h) + (1-\mu)\alpha_l\big)\big(\hat{p}(\mu'_l)x_l - t_l\big).$$

Now we consider three possible cases:

(i) Suppose that $\hat{p}(\mu'_h)x_h - t_h > \hat{p}(\mu'_l)x_l - t_l$. Consider the following allocation: (x_h, t_h, x_l, t_l) with mixing probabilities given by $\alpha_h(1+\varepsilon)$ and $\alpha_l - \varepsilon(1-\alpha_l)$ for high- and low-quality banks respectively and for a small value of $\varepsilon > 0$. Given this allocation, future beliefs are given by

$$\tilde{\mu}_h = \frac{\mu\alpha_h(1+\varepsilon)}{\mu\alpha_h(1+\varepsilon) + (1-\mu)(1-\alpha_l)(1+\varepsilon)} = \mu'_h$$

$$\tilde{\mu}_l = \frac{\mu\big(1-\alpha_h(1+\varepsilon)\big)}{\mu\big(1-\alpha_h(1+\varepsilon)\big) + (1-\mu)\big(\alpha_l - \varepsilon(1-\alpha_l)\big)}$$

$$= \frac{\mu(1-\alpha_h) - \varepsilon\mu\alpha_h}{\mu(1-\alpha_h) + (1-\mu)\alpha_l - \varepsilon\mu\alpha_h - \varepsilon(1-\mu)(1-\alpha_l)} < \mu'_l \leq \mu \leq \mu^* < \tilde{\mu},$$

where the last inequality follows from the fact that $1 - \alpha_l < \alpha_h$. Therefore, we have $V(\mu'_l; \pi) = V(\tilde{\mu}_l; \pi)$, so that the payoffs to the banks are the same as in the original allocation. The buyer's payoff given the new allocation is given by

$$(1+\varepsilon)\big(\mu\alpha_h + (1-\mu)(1-\alpha_l)\big)\big(\hat{p}(\mu'_h)x_h - t_h\big) + \big(\mu(1-\alpha_h) + (1-\mu)\alpha_l - \varepsilon\mu\alpha_h - \varepsilon(1-\mu)(1-\alpha_l)\big)\big(\hat{p}(\mu'_l)x_l - t_l\big).$$

Since $\hat{p}(\mu_h')\,x_h - t_h > \hat{p}(\mu_l')\,x_l - t_l$, the above expression must be higher than that in (A21) since it puts a higher weight on $\hat{p}(\mu_h')\,x_h - t_h$. This implies that this is a Pareto-improving allocation: we can use the extra revenues by buyers and increase transfers.

(ii) Suppose that $\hat{p}(\mu_h')\,x_h - t_h < \hat{p}(\mu_l')\,x_l - t_l$. In this case, a perturbation of the form $\big(x_h, t_h, x_l, t_l, \alpha_h(1-\varepsilon), \alpha_l + \varepsilon(1-\alpha_l)\big)$ Pareto improves the original allocation. The proof is similar to the first case.

(iii) Finally, suppose that $\hat{p}(\mu_h')\,x_h - t_h = \hat{p}(\mu_l')\,x_l - t_l = a \geq 0$. Then, we have the following incentive constraints:

$$t_h + (1 - x_h)\,(\overline{\pi} v - c) + \beta V(\mu_h'; \overline{\pi}) \;=\; t_l + (1 - x_l)\,(\overline{\pi} v - c) + \beta V(\mu_l'; \overline{\pi}).$$

Using the assumption above, the constraint becomes

$$\hat{p}(\mu_h')\,x_h - a + (1 - x_h)\,(\overline{\pi} v - c) + \beta V(\mu_h'; \overline{\pi})$$

$$= \hat{p}(\mu_l')\,x_l - a + (1 - x_l)\,(\overline{\pi} v - c) + \beta V(\mu_l'; \overline{\pi}).$$

After some algebra, we have

$$\big(\hat{p}(\mu_h') - \overline{\pi} v + c\big)\,x_h + \beta\big(V(\mu_h'; \overline{\pi}) - V(\mu_l'; \overline{\pi})\big) \;=\; x_l\big(\hat{p}(\mu_l') - \overline{\pi} v + c\big).$$

Note that $\mu_l' < \mu \leq \mu^* < \tilde{\mu} < \mu_h'$. Therefore, $\hat{p}(\mu_h') > \overline{\pi} v - c > \hat{p}(\mu_l')$. Therefore, the left side of the above equality is positive, whereas the right side is negative which is a contradiction.

J. *Proof of Proposition 9*

Consider the pooling equilibrium $\big(x, x\hat{p}(\mu)\big)$ where $x \in (\underline{x}(\mu), 1)$. Furthermore, suppose that beliefs following a sale of loans to the government are given by $\mu' = 0$. As we have shown in the proof of Lemma 1, (A2) implies that neither type chooses $(1, \hat{p}(\mu))$ when its associated belief is given by $\mu' = 0$.

K. *Proof of Proposition 11*

This equilibrium is supported by beliefs that specify that if any bank accepts a contract (x, xp) where $x \in \{0,1\}$ and

$$\mu'(\hat{x}, \hat{x}\hat{p}) = \begin{cases} 1 & \text{if } \hat{x}\hat{p} + (1-\hat{x})(\overline{\pi} v - c) \geq x^*(\mu, v)\hat{p}\,(\mu, v) + (1 - x^*(\mu, v))(\overline{\pi} v - c) \\ 0 & \text{otherwise} \end{cases},$$

where $x^*(\mu, v)$ specifies the equilibrium action of the high-quality bank, which is either 1 if $\mu \geq \mu^*(v)$ or 0 if $\mu \leq \mu^*(v)$.

To see that this is an equilibrium, we start by showing that μ_h exists and that the value functions for each bank quality are increasing in reputation. Clearly, if $\mu = 0$, then $V(0;\underline{\pi}) = (\underline{\pi}Ev)/(1-\beta(1-\lambda)) + \beta\lambda W$. Suppose $\mu \leq \mu_h$ and define $v^*(\mu) = \frac{c}{(1-\mu)(\overline{\pi}-\underline{\pi})}$ (note that $(\mu^*)^{-1}(\mu) = v^*(\mu)$), so that if $v \leq v(\mu)$, then $\mu \geq \mu^*(v)$. Recall that when v is below $v(\mu)$ so that $\mu \geq \mu^*(v)$, the bank sells its loan portfolio at the pooling price, and when v is above $v(\mu)$ so that $\mu \leq \mu^*(v)$ (and $\mu \leq \mu_h$), the bank is indifferent between selling at price $\underline{\pi}v$ and receiving a reputation of 0 and holding its loan and receiving a reputation of μ_h. The value function for the low-quality bank when $\mu \leq \mu_h$ is then given by

$$\text{(A22)} \qquad V(\mu;\underline{\pi}) = \int_{v_l}^{v^*(\mu)} \left[\hat{p}(\mu,v) + \beta(1-\lambda)\,V(\mu;\underline{\pi})\right] dG(v) + \int_{v^*(\mu)}^{v_{\max}} \left[\underline{\pi}v + \beta(1-\lambda)\,V(0;\underline{\pi})\right] dG(v) + \beta\lambda W.$$

Straightforward algebra then implies that for $\mu \leq \mu_h$,

$$\text{(A23)} \qquad V(\mu;\underline{\pi}) = \frac{G(v^*(\mu))}{1-\beta(1-\lambda)\,G(v^*(\mu))}\mu(\overline{\pi}-\underline{\pi})\,E[v|v \leq v^*(\mu)] + \frac{\underline{\pi}Ev}{1-\beta(1-\lambda)} + \beta\lambda W.$$

A consequence of (A23) is that in this equilibrium, the value μ_h always exists. To see this, substitute for $V(\mu;\underline{\pi})$ into (A22), which implies that μ_h must be chosen so that

$$\text{(A24)} \qquad c = \frac{\beta(1-\lambda)\,G(v^*(\mu_h))}{1-\beta(1-\lambda)\,G(v^*(\mu_h))}\mu_h(\overline{\pi}-\underline{\pi})\,E[v|v \leq v^*(\mu_h)].$$

Note that $v^*(1) = \infty$, $v^*(0) = v_l$, and the right side of (A24) is strictly increasing in μ_h. At $\mu_h = 0$, the right side is equal to $0 < c$, whereas at $\mu_h = 1$, the right side is given by $\frac{\beta(1-\lambda)}{1-\beta(1-\lambda)}(\overline{\pi}-\underline{\pi})\,Ev > c$, where the inequality follows from the assumption that $\frac{\beta(1-\lambda)}{1-\beta(1-\lambda)} > \frac{c+(\overline{\pi}-\underline{\pi})\,v_{\max}}{(\overline{\pi}-\underline{\pi})\,Ev}$. Hence, μ_h exists and is unique.

To finish the expression of the low-quality bank's value function, recall that when $\mu > \mu_h$, when v is below $v^*(\mu)$, the bank sells its loan portfolio at the pooling price, and when v is above $v^*(\mu)$, the bank holds its loan portfolio. The value function of the low-quality bank for $\mu > \mu_h$ then satisfies

$$V(\mu;\underline{\pi}) = \int_{v_l}^{v^*(\mu)} [\hat{p}(\mu,v) + \beta(1-\lambda)\,V(\mu;\underline{\pi})]\,dG(v) + \int_{v^*(\mu)}^{v_{\max}} [\underline{\pi}v - c + \beta(1-\lambda)\,V(\mu;\underline{\pi})]\,dG(v) + \beta\lambda W.$$

Straightforward algebra implies that for such μ,

$$V(\mu;\underline{\pi}) = \frac{1}{1-\beta(1-\lambda)}\Big[G(v^*(\mu))\big[\mu(\overline{\pi}-\underline{\pi})E[v\,|\,v\le v^*(\mu)] + c\big]+\underline{\pi}Ev-c+\beta\lambda W\Big],$$

which is clearly increasing in μ. Moreover, since for $\mu < \mu_h$ we can express the value function as

$$\begin{aligned} V(\mu;\underline{\pi}) \ = \ & \int_{v_l}^{v(\mu)}\big[\hat{p}(\mu,v) + \beta(1-\lambda)\,V(\mu;\underline{\pi})\big]\,dG(v) \\ & + \int_{v(\mu)}^{v_{\max}}\big[\underline{\pi}v - c + \beta(1-\lambda)\,V(\mu_h;\underline{\pi})\big]\,dG(v) + \beta\lambda W. \end{aligned}$$

It is immediate that $V(\mu;\underline{\pi})$ is increasing in μ.

Next consider the value for the high-quality bank and show that it is also increasing in μ. When $\mu \le \mu_h$,

$$\begin{aligned} V(\mu;\overline{\pi}) \ = \ & \int_{v_l}^{v(\mu)}\big[\hat{p}(\mu,v) + \beta(1-\lambda)\,V(\mu;\overline{\pi})\big]\,dG(v) \\ & + \int_{v(\mu)}^{v_{\max}}\big[\overline{\pi}v - c + \beta(1-\lambda)\,V(\mu_h;\overline{\pi})\big]\,dG(v) + \beta\lambda W, \end{aligned}$$

and when $\mu > \mu_h$,

$$\begin{aligned} V(\mu;\overline{\pi}) \ = \ & \int_{v_l}^{v(\mu)}\big[\hat{p}(\mu,v) + \beta(1-\lambda)\,V(\mu;\overline{\pi})\big]\,dG(v) \\ & + \int_{v(\mu)}^{v_{\max}}\big[\overline{\pi}v - c + \beta(1-\lambda)\,V(\mu;\overline{\pi})\big]\,dG(v) + \beta\lambda W. \end{aligned}$$

It is immediate from these expressions that $V(\mu;\overline{\pi})$ is also increasing in μ.

We have established that μ_h exists and the value functions for both quality types are increasing in reputation μ. Consider now possible deviations by buyers when facing a bank with reputation μ and current collateral value v, and show for all μ and v that there are no profitable deviation offers by buyers. Clearly, if a profitable deviation exists, it must satisfy $p' \le \hat{p}(\mu,v)$, since for all $p' > \hat{p}(\mu,v)$, such deviations at best attract both high- and low-quality banks and earn negative profits. We partition the (μ,v) space into cases. First suppose that $\mu \ge \mu^*(v)$. In this equilibrium outcome, both banks sell at price $\hat{p}(\mu,v)$. At any price p' higher than $\hat{p}(\mu,v)$, since $\mu'(1,p') = 1$, both banks would accept the offer, but such an offer would then earn negative profits. At any price p' below $\hat{p}(\mu,v)$, both banks would obtain a less static payoff and a reputation of 0, so such offers attract no banks. Thus, when $\mu \ge \mu^*(v)$, there are no profitable deviations by buyers.

Next suppose that $\mu < \mu^*(v)$. For any price $p' \le \hat{p}(\mu,v)$, the high type prefers the equilibrium since

$$\begin{aligned} \overline{\pi}v - c + \beta(1-\lambda)\,V(\mu;\overline{\pi}) \ & > \ \hat{p}(\mu,v) + \beta(1-\lambda)\,V(\mu;\overline{\pi}) \\ & \ge \ p' + \beta(1-\lambda)\,V(0;\overline{\pi}), \end{aligned}$$

which holds since $\overline{\pi} v - c > \hat{p}(\mu, v)$ and $V(\mu; \overline{\pi})$ is increasing in μ. Thus, when $\mu < \mu^*(v)$, if the deviation satisfies $\underline{\pi} v < p' \leq \hat{p}(\mu)$, it must earn nonpositive profits since at best it attracts no banks and earns zero profits.

Suppose further that $\mu_h < \mu < \mu^*(v)$. We now show that no deviation with $p' \leq \underline{\pi} v$ attracts low-quality banks since if they accept such a deviation offer, they would obtain a reputation of 0. To see this, note that at any price $p' \leq \underline{\pi} v$, the low-quality bank prefers the equilibrium since

$$\underline{\pi} v - c + \beta(1 - \lambda) V(\mu; \underline{\pi}) > \underline{\pi} v - c + \beta(1 - \lambda) V(\mu_h; \underline{\pi})$$

$$= \underline{\pi} v + \beta(1 - \lambda) V(0; \underline{\pi}).$$

Finally, suppose instead that $\mu \leq \min\{\mu_h, \mu^*(v)\}$. We again show that no deviation with $p' \leq \underline{\pi} v$ attracts the low-quality bank. In this case, the low-quality bank obtains utility equal to $\underline{\pi} v + \beta(1 - \lambda) V(0; \underline{\pi}) + \beta \lambda W$. Hence, at price any $p' \leq \underline{\pi} v$, the deviating buyer earns at most zero profits.

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