Currency Stability Using Blockchain Technology

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Abstract

To date, cryptocurrency prices are volatile and many cryptocurrency developers have adopted ad hoc approaches to stabilize their cryptocurrency price. When these currencies are not 100% backed by other valued assets, part of their price volatility may arise from self-fulfilling expectations of a speculative attack (as in Obstfeld (1996)). We show that an exchange rate policy, which is less than 100% backed and dynamically adjusts in response to traders' conversion demand eliminates speculative attacks while, under some conditions, preserving much of the desired exchange rate stability. This dynamic exchange rate policy admits a great deal of discretion to and requires commitment by the party implementing the policy. We demonstrate how to implement this policy using the Ethereum network—a smart contract blockchain environment—and how this implementation yields commitment to the policy.

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1 Introduction

A central component of most blockchain technologies is a token that can be used as a method of payment. For some blockchains like Bitcoin, the token is a crypto-currency and its use as a means of payment is its primary purpose. For other blockchains, the token serves as a "utility token" to perform transactions on the blockchain. An Ether token, for example, is a means of payment for computer processing on the Ethereum Virtual Machine, the runtime environment for smart contracts in Ethereum blockchain technology. In either case, the conversion between crypto-currencies and between crypto-currencies and government issued currencies is important. To date, existing crypto-currencies are simply too volatile to be an effective medium of exchange or a store of value. Figure 1 plots the standard deviation of daily USD price changes of Bitcoin from January 2015 to January 2021. Relative to the Euro-Dollar exchange rate, the price of gold, or even the US stock market, the volatility is an order of magnitude larger.

More generally, currency prices are volatile. Exchange rates fluctuate far more than country differences in economic activity or aggregate price levels (Rogoff (2001)). There is a long history of currency issuers—historically, governments—following policies to stabilize the price of their currency. Typically these policies involve using a "peg" to a more stable currency like the U.S. dollar or a commodity price like gold. To maintain the peg, the issuer maintains a stock of reserves (dollars, gold) to redeem their currency at a fixed rate. Currently, countries such as Qatar, Cuba, and Panama have pegged their exchange rate to the U.S. dollar. Previously, Mexico and Argentina had pegged to the dollar but abandoned that policy. For crypto-currencies, Tether has pegged their exchange rate to the U.S. dollar.

Currency stability, even with a policy of a peg is difficult. When a currency is not 100% backed, it is vulnerable to a speculative attack. If enough traders sell (short) the currency, these traders can force a full depletion of the issuer's dollar reserves leading to a devaluation of the currency. This means that if each trader believes that enough other traders will sell (short) the currency, then it is also optimal for each trader to short the currency in the expectation of a devaluation. As a result, a change in traders' beliefs alone about the likelihood that others traders will speculate against the currency is sufficient to induce a run on the currency. Obstfeld (1996) develops this mechanism to show how currency pegs may be subject to arbitrary speculative attacks.

The canonical "peg" policy uses a fixed amount of reserve currency (dollars), R, and redeems domestic currency (or cryptocurrency in our running example), into reserves at a fixed exchange rate e. Should reserves be exhausted, the currency floats at a depreciated market rate $e^f < e$. The policy "works" as long as demand for the reserve currency is not large (relative to R). The policy, however, permits equilibria with speculative attacks. A single trader, who does not have a fundamental need for the reserve currency, might choose to trade at e with the anticipation of unwinding the trade at the floating rate, e^f. The speculative profit, net of a transaction cost τ, is $e/e^f - \tau$. If this trader believes that all traders will demand the reserve currency, then the trader rationally anticipates that the domestic currency will depreciate and so also demands the reserve currency. This canonical "peg" policy is "unconditional" in the sense that the exchange rate e does not depend on demand (except of course, after reserves are exhausted).

In contrast, in this paper, we develop a new theory of optimal exchange rate pegs that are less than 100% backed by a reserve currency and are also immune to speculative attacks. Our first contribution is to show that the classical problem of speculative attacks arises from the ad hoc restriction that exchange rate policy be unconditional. We develop a simple, conventional model of speculative attacks with no aggregate uncertainty about the relative value of the currencies. In our model, some traders realize an immediate need for the reserve currency and some traders do not—the latter are those who may profit from speculation—and that these shocks are independently drawn across traders. Under certain conditions relating the stock of reserve currency, the floating rate, fundamental demand for the cryptocurrency, and the transaction costs of speculating, a canonical peg policy admits multiple equilibria—an equilibrium without an attack as well as one with an attack. In the attack equilibrium, all agents, even those without an immediate need for the reserve currency, attempt to convert their domestic currency into the reserve currency at the pegged exchange rate.

Under these same conditions, we show that a commitment to devalue the cryptocurrency if too many traders demand conversion (into reserve currency) eliminates the speculative attack equilibrium and therefore stabilizes the exchange rate. (Note, in this setting, "stability" is relative to the stability of the reserve currency.) Our argument builds on the observation that speculative attacks on an exchange rate resemble runs on a deposit-issuing bank. Just as a commitment to suspend convertibility in the most basic version of the model of Diamond and Dybvig (1983) eliminates runs, so too our devaluation policy eliminates speculative attacks.

Our second contribution is to generalize this result to settings with aggregate uncertainty about the relative value of the currencies. Using arguments analogous to those in Green and Lin (2003), we prove that the use of a dynamic exchange rate peg that adjusts in response to traders' conversion demand also eliminates speculative attack equilibria. We prove this result in a model where traders face idiosyncratic and independent shocks to their preferences over the domestic and the reserve currency, where there is aggregate risk to traders' preferences, and where the exchange rate policy must respect a sequential service constraint—a constraint requiring traders who demand conversion be paid in order of their demands and the exchange rate policy offered to each trader is measurable with respect to only the current history of traders' demands.

Formally, we show that if an exchange rate policy prevents speculation when all traders believe no other trader will speculate, then the only (perfect Bayesian) equilibrium involves no speculation. The logic of the argument follows a basic backward induction argument and relies on our assumption that traders preference shocks are independent.[1] Suppose a trader has an arbitrary place in line; that is, this trader has the ith opportunity to demand conversion into the reserve currency. For trader i to forecast her payoffs from speculating or not she must forecast the conversion demand of traders who follow her. Suppose she believes (inductively) that among traders who follow her, only those traders who realize a sudden need for the reserve currency will demand conversion. When traders' currency demand shocks are independent, the demands of traders in line *before* her do not influence her forecast of the conversion demand of traders *after* her. Moreover, any sequence of conversion demand of the traders before her in line is associated with some history of demands when no traders speculate. Since the exchange rate policy prevents speculation by trader i when no traders speculate, it must also prevent her from speculating in the history where some traders previous to her have speculated. In other words, trader i has a dominant strategy to not speculate, ensuring that the exchange rate policy admits a unique equilibrium among the traders without speculation.

We go on to characterize the optimal exchange rates that emerge from our model when the domestic currency is (on average) relatively valuable. Specifically, we show that when relatively few traders expect to realize an immediate need for the reserve currency and when those traders who do not realize this immediate need ultimately value owning mostly the domestic currency, then the ex post efficient exchange rate policy—the policy an issuer would implement if traders' realized needs for domestic and reserve currency were observable—is incentive compatible. Under these conditions, we show that the optimal exchange rate is stable: traders expect the exchange rate to not adjust very much. Nonetheless, in the unlikely event that many traders realize immediate needs for the reserve currency, the ex-

[1]Our logic is analogous to that used by Andolfatto, Nosal, and Wallace (2007) to prove a similar result on unique implementation in a banking context.

change rate does depreciate—the policy dynamically adjusts to realized traders needs for reserve currency.

Our theory is agnostic about the currencies involved. It applies equally well to a government issued fiat currency or a blockchain cryptocurrency. The particular importance of blockchain technology is two-fold. First, the new technology has spawned a large number of coins. CoinMarketCap tracks the market capitalization (price times outstanding currency) of over 4000 different cryptocurrencies with a total market cap of approximately $900 billion. The ten largest each have a market capitalization over five billion dollars.[2] Currency stability is directly relevant for these blockchain-driven businesses. For many of the coins like Bitcoin Cash, Ethereum's Ether, Litecoin, price volatility is as large as Bitcoin (Figure 1); the exchange rate with US dollars is extremely volatile. Several crypto-currencies have been designed with price stability as the objective. Tether, one of the largest coins (by market capitalization) with an exchange rate "peg" offers one-for-one conversion between its tokens and U.S. dollars. To defend its peg, Tether claims to hold $1 US in reserve for every Tether outstanding, but the exact mechanism Tether uses to maintain price stability is not transparent and has lead to claims of market manipulation[3]. It is often hard to verify issuers' reserve holdings. Tether, for example, severed its relationship with its auditor in January 2018 and then used $850 million (US dollar) of reserves to cover losses in a related company.[4] Other crypto-currencies that aim for price stability include Nubits, Dia, Basecoin. All of these protocols feature collateral or reserves at 100% (or more). Others, at least on the surface, appear to have no workable mechanism for stability other than the name.

Second, and perhaps more importantly, blockchain technology has the potential to credibly implement complicated peg policies. Specifying and communicating a policy that depends on real-time currency demand may not be easy. Moreover, conditional policies may appear less credible since they are more complicated to monitor and appear to feature more discretion (see Kydland and Prescott (1977) or, in a banking context Ennis and Keister (2009a)). "Smart Contracts," such as those on the Ethereum Network are rich state-contingent contracts that are credible since they are immutable and enforced by an irreversible distributed-ledger blockchain technology. To see how this can work, we code our dynamic exchange rate policy in Solidity, a computer language used for smart-contracts on Ethereum, and deploy the contract on a local instance of a blockchain (`https://github.com/azetlinjones/cryptopeg`). This exercise highlights policy choices

[2] These statistics reflect information as of January 12, 2021. `https://coinmarketcap.com/`.

[3] Griffin and Shams (2018). `https://www.bloomberg.com/graphics/2018-tether-kraken-trades/`

[4] See New York State Attorney General Letitia James press release of April 25, 2019. `https://on.ny.gov/356dsbf`.

like the information structure and demonstrates clearly how the blockchain setting facilitates commitment.

What follows is a model without aggregate risk to outline the basic structure of the speculative trading game. We show how enriching the policy space is useful for eliminating undesirable speculative attacks. Section 3 demonstrates that unique equilibrium are attainable when the model features both aggregate risk and policies must respect sequential service constraints. Section 4 illustrates features of optimal exchange rate policies in economies when ex post efficient policies are also incentive compatible. Finally in Section 5, we implement the exchange rate policy on the Ethereum Network.

2 A Model of Currency Crises without Aggregate Risk

In this section, we describe a theoretical model of currency crises in the spirit of Obstfeld (1996) and Morris and Shin (1998). We demonstrate that the existence of an equilibrium resembling a speculative attack is an artifact of an ad hoc restriction on the policy space. When the currency issuer is permitted to use state-contingent policies to defend an exchange rate peg, then the speculative attack equilibrium does not exist.

To fix ideas, we describe our environment as one where an issuer of a cryptocurrency stabilizes the value of the cryptocurrency relative to a reserve currency ("dollars"). As we describe in Section 5, a related interpretation is that of an issuer of a new token on the Ethereum network (an ERC-20 token) stabilizing the value of their tokens with respect to units of ETH, the native unit of account on the Ethereum network.

2.1 Model Environment

The model economy lasts for two periods and features a continuum of measure one of traders. Each trader owns one unit of a *cryptocurrency*. While we describe their fiat endowments at cryptocurrencies, for the purpose of our theory one may equivalently think of these as units of paper currency. The economy features two assets: cryptocurrency and dollars. We normalize the price of the cryptocurrency to be 1 while the price of dollars in period $t = 0, 1$ is e_t units of cryptocurrency. Think of \$36,599 per BTC or \$1,191 per ETH. So an appreciation of the cryptocurrency price corresponds to a higher value of e_t.

At the beginning of period 0, each person realizes a privately observed, uninsurable type. With probability $\mu_D > 0$, traders are of type D, whom we refer to as *dollar* (consumption) traders and with probability $\mu_C = 1 - \mu_D$ traders are of type C, whom we refer to as *crypto* traders. Traders types are identically and independently distributed across agents. A trader's type determines her preferences over asset positions and time.[5]

A dollar trader desires only dollars in period 0. A crypto trader desires (mostly) cryptocurrency in period 1. Specifically, we assume that for any holdings of cryptocurrency x in period 1, a crypto trader retains a portion $\lambda x > 0$ of her cryptocurrency and must convert a portion $(1 - \lambda)x$ to acquire $(1 - \lambda)xe_1$ dollars. We interpret the source of risk here as independent of the true value of the crypto currency—it simply reflects independent shocks to traders' needs. In this sense, these shocks are akin to the liquidity shocks modeled in many theories of banking. Instead, we interpret the parameters, μ_C, μ_D and λ as reflective of the underlying fundamental value of the cryptocurrency.

Facing a path of exchange rates e_0 and e_1, traders may convert cryptocurrency into dollars in period 0 (at price e_0), store the dollars until period 1, and convert back into cryptocurrency (at price $1/e_1$) in period 1 as a form of speculation. We discuss in Section 2.2 when these prices are issuer-determined or market-determined prices. This speculation bears a fixed cost $\tau > 0$ denominated in units of cryptocurrency.[6]

Formally, let $d_t^i \in \{0, 1\}$ denote the units of cryptocurrency trader i requests to convert into dollars in period t, let $d_t = (d_t^i)_{i\in[0,1]}$.[7] Since dollar traders only care about period 0 consumption, each such trader will always submit $d_0^i = 1$. Crypto traders may wish to speculate by submitting a conversion request of $d_0^i = 1$ depending on their perceptions of the path of exchange rates. Note that given (e_0, e_1), a crypto trader who submits conversion demand d_0^i in period 0 will enter period 1 with

$$x_1^i = 1 - d_0^i + \frac{e_0}{e_1} d_0^i - \tau \mathbb{1}_{[d_0^i = 1]} \tag{1}$$

units of cryptocurrency (with $\mathbb{1}$ as an indicator function). The amount x_1^i reflects the unconverted cryptocurrency, $1 - d_0^i$, the period 1 value of cryptocurrency converted in period

[5]As our definition of preferences in (2) makes clear, in this model traders enjoy utility directly from owning assets at a given point in time. This modeling choice aligns our theory closest to the literature on speculative attacks as in Obstfeld (1996).

[6]Given that a speculator's preferences imply she will ultimately require some dollars, one could equivalently think of a speculator as converting only a portion of her period 1 dollars back into cryptocurrency. Given our normalization of the transaction costs of speculating, however, she is indifferent between selling all of her dollars to cryptocurrency and then re-acquiring some dollars.

[7]The restriction of traders' strategies to $\{0, 1\}$ plays a critical role in simplifying the model with aggregate uncertainty studied later in the appear. We make the restriction here to be consistent, though it is more innocuous.

0 (denominated in cryptocurrency), $\frac{e_0}{e_1} d_0^i$, less any transaction cost. Our assumption on preferences implies that such a trader will submit a conversion demand $d_1^i = (1-\lambda)x_1^i$ in period 1.

Given a path of exchange rates $e = (e_0, e_1)$ and dollar demands $d = (d_0, d_1)$, each trader $i \in [0,1]$ has utility function of the form

$$U(d_0^i; e, d) = \begin{cases} u(e_0 d_0^i) & \text{if } i \text{ is of type } D \\ u\left(\left[(1-\lambda)e_1 + \lambda\right] x_1^i\right) & \text{if } i \text{ is of type } C \end{cases} \tag{2}$$

where x_1^i is given by (1).

We assume traders' preferences satisfy the following properties:

Assumption 1*:* (i) u is strictly increasing, continuously twice differentiable and strictly concave and (ii) u satisfies the Inada conditions $\lim_{c\to 0} u'(c) = \infty$ and $\lim_{c\to\infty} u'(c) = 0$.

2.2 Optimal Policy

We envision a currency issuer that chooses an optimal exchange rate policy given an initial endowment of R_0 dollar reserves. One interpretation is that the issuer is a traditional currency board that represents a sovereign government; an alternative interpretation, which we explore in Section 5 is that the issuer is the creator of a new cryptocurrency token on the Ethereum blockchain. As with conventional currency boards or many current blockchain-based stable-coin proposals, we assume the issuer offers to convert cryptocurrency into dollars or dollars into cryptocurrency at specified rates in period 0 and period 1. In addition, we assume the presence of a private market for trading cryptocurrency into dollars at a market rate e^f. Note that if this market rate were to be above the issuer-set rate, arbitrageurs would want to buy cryptocurrency from the issuer (at a low dollar price) and sell them in the private market (at a high market price) earning a risk-free arbitrage. This logic suggests that e^f should always lie below the issuer-offered rate. We assume that as long as the issuer has strictly positive reserves R, then the market rate e^f is the issuer rate, e_1. However, when the issuer has 0 reserves, then traders may buy or sell cryptocurrency for dollars at the floating (and low) rate e^f.

Let $d_t = (d_t^i)_{i\in[0,1]}$ denote the set of traders' conversion demands in period t. Then the issuer's policy choice is an *exchange rate policy* $(e_0(d_0), e_1(d_0, d_1), \phi_0(d_0), \phi_1(d_0, d_1))$ that specifies exchange rates, e_t, and fractional conversion rates, $\phi_t \in [0,1]$, in each period as a function of the relevant history. Note that we restrict the issuer to offer a constant conversion rate to all traders in each period. The exchange rate policies must be feasible.

Definition 1*:* An exchange rate policy is *feasible*, if and only if for all d_t,

$$e_0(d_0)\phi_0(d_0)\int d_0^i di + e_1(d_0,d_1)\phi_1(d_0,d_1)\int d_1^i di \leq R_0. \tag{3}$$

Optimal Policy with Limited Contingency Policies. Motivated by Obstfeld (1996), we now place a restriction on the class of policies the issuer may consider and demonstrate that under this restriction, the optimal exchange rate policy admits a speculative attack equilibrium. Consider first an issuer that defends an exchange rate peg—a fixed e_0 in our model—until it runs out of reserves in which case it converts demand uniformly in period 0 and allows the exchange rate to float at e^f in period 1. We refer to such a policy as a *limited contingency policy*.

Definition 2*:* An exchange rate policy is a *limited contingency policy* if it satisfies

$$e_0(d_0) = \bar{e}_0 \quad \forall d_0 \tag{4}$$

$$\phi_0(d_0) = 1 \quad \forall d_0 \textit{ such that } \bar{e}_0\int d_0^i di \leq R_0 \tag{5}$$

$$\phi_0(d_0) = R_0/[\bar{e}_0\int d_0^i di] \quad \forall d_0 \textit{ such that } \bar{e}_0\int d_0^i di > R_0. \tag{6}$$

We note two important observations about limited contingency policies. First, if the issuer is not able to defend the peg against a given level of dollar demand, then it necessarily exhausts its supply of dollar reserves and allows the exchange rate to float in period 1 (or, for all such d_0, it follows that $\phi_1(d_0,d_1) = 0$). Second, when the issuer is not able to defend the peg, it treats all depositors who demand conversion equally. These two features of limited contingency policies play a key role in allowing for the possibility of speculative equilibria.

Consider next the problem of the issuer choosing among limited contingency exchange rate policies to maximize the expected utility of the traders subject to feasibility constraints and a no-speculation constraint which ensures crypto traders prefer to submit a bid $d_0^i = 0$ rather than $d_0^i = 1$. In such a case, it is immediate that the issuer will choose $\phi_0(d_0) = \phi_1(d_0,d_1) = 1$ when $\int d_0^i di = \mu_D$ and $\int d_1^i di = (1-\lambda)\mu_C$ and will set $\bar{e}_0$ and $\bar{e}_1$ to solve

$$\max \mu_D u(\bar{e}_0) + \mu_C u((1-\lambda)\bar{e}_1 + \lambda) \tag{7}$$

subject to the feasibility constraint (3) and the no-speculation constraint,

$$u((1-\lambda)\bar{e}_1 + \lambda) \geq u\left([(1-\lambda)\bar{e}_1 + \lambda]\left[\frac{\bar{e}_0}{\bar{e}_1} - \tau\right]\right). \tag{8}$$

The no-speculation constraint here may be written compactly as

$$1 \geq \frac{\bar{e}_0}{\bar{e}_1} - \tau \tag{9}$$

implying that optimal policies discourage speculation as long as the currency does not depreciate too much ($\bar{e}_1$ cannot be too much smaller than $\bar{e}_0$).

Since the feasibility constraint (3) necessarily binds, if the no-speculation constraint is slack, an optimum exchange rate policy is characterized by the optimality condition,

$$u'(\bar{e}_0) = u'\left(\frac{R_0 - \mu_D \bar{e}_0}{\mu_C} + \lambda\right) \tag{10}$$

which implies $\bar{e}_0 = R_0 + \mu_C \lambda$. To verify this value of $\bar{e}_0$ is optimal, one need only verify the feasibility and no-speculation constraint, which yields the following proposition.

Proposition 1: If $R_0 - \mu_D \lambda \geq 0$ and

$$(1 - \lambda)\frac{R_0 + \mu_C \lambda}{R_0 - \mu_D \lambda} \leq 1 + \tau \tag{11}$$

then the optimal limited contingency policy satisfies $\bar{e}_0 = R_0 + \mu_C \lambda$ and $\bar{e}_1 = (R_0 - \mu_D \lambda)/(1 - \lambda)$ where $\bar{e}_1$ is defined for d_1 such that $\int d_1^i di = (1 - \lambda)\mu_C$.

Notice that for $\mu_D < R_0$, condition (11) of Proposition 1 is satisfied as $\lambda \to 1$. When this is the case, if traders believe that crypto traders will not speculate, then traders rationally anticipate at worst a mild depreciation of the cryptocurrency. As a result, a single crypto trader will find speculation to be an unprofitable strategy. Since each crypto trader's best response to the belief that other crypto traders will not speculate is to also not speculate, no speculation is an equilibrium of the optimal limited contingency policy.

Moreover, note that when $\mu_D < R_0$, the issuer's policy satisfies $\mu_D \bar{e}_0 < R_0 < \bar{e}_0$. This inequality implies that while the issuer chooses a policy that will not exhaust its reserves if no crypto traders demand currency in period 0, this limited contingency policy will necessarily exhaust all of the issuer's reserves if all crypto traders demand conversion. In this sense, the exchange rate policy is less than 100% backed by reserves. Recall that when the issuer's reserves are exhausted, traders may buy or sell cryptocurrency at the market price e^f.

Consider then a crypto trader's incentives to speculate when she believes all other crypto traders will also speculate (choose $d_0^i = 1$). Under these beliefs, the crypto trader rationally anticipates $\phi_0(d_0) = R_0/\bar{e}_0$ and $\phi_1(d_0, d_1) = 0$. As a result, her payoffs from not speculating are given by $u((1-\lambda)e^f + \lambda)$ and from speculating (with $d_0^i = 1$) are given by

$$u\left([(1-\lambda)e^f + \lambda]\left[1 - \frac{R_0}{\bar{e}_0} + \frac{\bar{e}_0}{e^f}\frac{R_0}{\bar{e}_0} - \tau\right]\right). \tag{12}$$

It follows that whenever $(R_0/e^f) - (R_0/\bar{e}_0) \geq t$, the crypto trader will find it optimal to speculate. We have proved the following lemma.

Lemma 2: If

$$\frac{R_0}{e^f} - \frac{R_0}{R_0 + \mu_C \lambda} \geq \tau \tag{13}$$

(or e^f is sufficiently small), then the optimal limited contingency policy admits an equilibrium where all crypto traders speculate.

We have shown that limited contingency policies suffice to deliver efficient insurance arrangements against traders uncertain needs for dollars. However, as in Obstfeld (1996), such policies allow for too much volatility in exchange rates in the sense that they also admit other equilibria. Note that if the floating exchange rate e^f is sufficiently small, then small changes in e^f would induce no change in the optimal exchange rate policy—its solution under Proposition 1 is independent of the floating rate for such values. However, under any equilibrium selection that admits the speculative equilibrium as an outcome (e.g. under a sunspot selection criteria), the model would feature variation in the floating rate price of cryptocurrency.

We now examine optimal policies without the ad hoc restriction on policy contingencies and demonstrate that such polices can eliminate the possibility of speculative equilibrium.

Optimal Policy with Contingent Policies. Consider next the unrestricted problem of choosing any feasible exchange rate policy to maximize ex ante expected utility of the traders. Clearly, in an equilibrium in which only dollar traders submit demand for dollars in period 0, the outcomes $\bar{e}_0$ and $\bar{e}_1$ from Proposition 1 are the same. The only difference with arbitrarily contingent policies is that the issuer may now change the period 0 exchange rate it offers when total demand in period 0 differs from μ_D.

For example, consider the following policy:

$$e_0(d_0) = \begin{cases} R_0 + \mu_C \lambda & \text{if } \int d_0^i di = \mu_D \\ e^f & \text{if } \int d_0^i di \neq \mu_D \end{cases}, e_1(d_0, d_1) = \begin{cases} \frac{1}{1-\lambda}(R_0 - \mu_D \lambda) & \text{if } \int d_1^i di = \mu_C \\ e^f & \text{if } \int d_1^i di \neq \mu_C \end{cases}, \tag{14}$$

and

$$\phi_0(d_0) = \min\left\{\frac{R_0}{e_0(d_0)\int d_0^i di}, 1\right\}, \quad \phi_1(d_0, d_1) = \min\left\{\frac{R_0 - e_0(d_0)\phi_0(d_0)\int d_0^i di}{e_1(d_0, d_1)\int d_1^i di}, 1\right\}. \quad (15)$$

Under this policy, if no crypto traders plan to speculate, Proposition 1 implies that each crypto trader prefers not to speculate. Alternatively, if any strictly positive fraction of crypto traders are believed to speculate, then each crypto trader expects to obtain utility $u((1-\lambda)e^f + \lambda)$ should she not speculate and expects to obtain utility $u([(1-\lambda)e^f + \lambda][1-\tau])$ should she choose to speculate. Hence, the policy trivially rules out alternative equilibria. More generally, there is a large class of policies which implement the efficient outcome in this economy while admitting a unique (no speculation) equilibrium. We state this result in the following lemma.

Lemma 3: There exist exchange rate policies (with arbitrary contingencies) that implement the efficient outcome uniquely.

Our analysis closely parallels related analysis from Diamond and Dybvig (1983). The Diamond and Dybvig intermediary uses limited resources to provide insurance to agents against idiosyncratic liquidity shocks (demand for early consumption). Our currency issuer uses limited dollar reserves to insure agents against idiosyncratic currency demand shocks. In the banking context, because long term assets grow, it is cheaper to provide real consumption to agents who do not realize a liquidity shock (patient types) and so optimal consumption is larger for for those agents than those who do realize liquidity shocks (impatient types). In the desired (truth-telling) equilibrium, this force ensures patient types do not wish to mimic impatient types.

The key insurance motive in our model is somewhat different. In the limiting case, when $\lambda = 1$, crypto traders do not desire or value dollars at all. In this case, the issuer would want to distribute all of its dollars to the dollar traders in period 0. The only insurance motive, and hence the only motive to fix an exchange rate for the issuer is to equate the dollar consumption of dollar traders in period 0. In this sense, our model provides a motive for exchange rate stability (relative to dollars). A concern of such a policy is that by fully depleting its dollar reserves with dollar traders, the issuer may provide incentives to crypto traders to speculate. When λ is strictly less than but close to 1, the issuer is able to offer most of its dollars to dollar types in period 0 while also credibly maintaining a high exchange rate in period 1 when only dollar types trade in period 0. The problem is that at such fixed exchange rates, the issuer cannot defend its peg when more than the expected number of dollar traders demand conversion just as a Diamond and Dybvig intermediary is bankrupt when too many depositors redeem their deposits before long-term assets mature.

We view Lemma 3 then as an exchange rate policy analogue to the usefulness of suspension contracts in the literature on bank runs beginning with Diamond and Dybvig (1983). In that literature, the anticipation that the bank will suspend convertibility (of deposits to cash) provides patient depositors with the knowledge that their deposits are safe; the anticipation of suspension, therefore, removes the incentives to run. In our model, the anticipation that the issuer will abandon its peg before exhausting its dollar reserves removes the incentives of crypto traders to speculate.

Notice that parametric assumptions contained in Proposition 1 and Lemma 2 are needed only to make the problem interesting. That is, these conditions ensure that the limited contingency policy—the canonical peg policy—admits both a no speculation and a speculation equilibrium. Only under these conditions is it interesting to ask if a richer policy can eliminate speculative attacks. Lemma 3 shows that under these conditions, richer exchange rate policies do eliminate the speculative attack equilibrium. As we have shown, sufficient (though not necessary) conditions that imply the conditions contained in Proposition 11 and Lemma 2 are that $\lambda \to 1$, μ_D relatively small, and e^f sufficiently small. Given traders' preferences, this assumption implies that a crypto trader derives most of her utility from actually owning the cryptocurrency. In other words, crypto types expect to derive most their value of the asset from the asset directly (either as a medium of exchange or as a means to acquire a given service from a platform). Under these conditions, the optimal exchange rate (per unit of cryptocurrency) appreciates because crypto traders will demand a negligible amount of reserves. Second, the measure of dollar traders cannot be too large. That is, there must be a thick enough market for agents who value the cryptocurrency. We will maintain these assumptions below when we consider an environment with aggregate uncertainty about the fundamental relative value of cryptocurrency.

Much like suspension contracts are not efficient when there is aggregate liquidity risk in Diamond and Dybvig (1983), one concern with our analysis is that contingent exchange rate policies may not function well when there is aggregate risk over the need for dollars, or the reserve currency. We explore contingent exchange rate policies when there is aggregate risk in Sections 3 and 4 next.

3 Exchange Rate Policy with Aggregate Risk

In this section, we show that optimal exchange rate policies are immune to fluctuations driven purely by traders' speculative motives even when the issuer faces aggregate uncertainty over fundamental demand for dollars. We show this to be the case even when the issuer's policies are required respect a form of sequential service. While imposing sequential service adds constraints to the issuer of our model, Calomiris and Kahn (1991) emphasize how such constraints may play a socially useful role in related environment by incentivizing traders to screen the issuer. Hence, we view an understanding of the ability to offer stable exchange rates under sequential service as important for understanding proposals to stabilize cryptocurrency prices.

3.1 Model Environment with Aggregate Risk

The model environment is essentially the same as that in Section 2 except for two modifications. First, we modify the number of traders so that there are $I \geq 3$ traders (instead of a continuum of measure 1). As before, at the beginning of period 0, traders' types are i.i.d and with $\mu_C > 0$ and $\mu_D > 0$ representing the probability that a given trader is a crypto or dollar type respectively.

This first modification implies that the model now features aggregate risk to the number of crypto (and dollar) traders. One natural interpretation of this risk in our context is that it reflects aggregate uncertainty in the fundamental demand for cryptocurrencies. Critically, this source of aggregate risk implies that after observing a large volume of dollar demand (perhaps larger than expected), an issuer cannot determine whether this demand reflects speculative demand in anticipation of a depreciation or a fundamental shock to demand for cryptocurrency. Our aim in the rest of this section is to analyze the implications of this uncertainty for the robustness of the issuer's optimally chosen exchange rate policies.

Our second modification changes the timing and information of actions in the game: we assume that each trader $i \in I$ chooses a strategy of dollar demand $d_0^i \in \{0, 1\}$ sequentially with the knowledge of the history of actions chosen by previous traders $j < i$. As before, a choice of $d_0^i = 0$ reflects a choice by the trader to not demand dollars—to not speculate—and a choice of $d_0^i = 1$ reflects a choice to demand dollars, or speculate. Equivalently, we treat a reported demand $d_0^i = 1$ as a report that the trader is a dollar trader and a reported demand $d_0^i = 0$ as a report that the trader is a crypto trader.

Given this timing and information modification, it is natural to examine how optimal exchange rate policies perform when the issuer has to convert cryptocurrency into dollars for traders sequentially. Below, we explicitly define policies that respect a sequential service constraint. We view this modification as critical to examining the robustness of our results given that the most straightforward policy to eliminate speculative attacks in the model without aggregate risk exploited full knowledge of total dollar demand.

3.2 Optimal Policy

Since the issuer serves traders sequentially given a remaining stock of dollar reserves and we allow for history-contingent exchange rate policies, we may define an exchange rate policy simply as plans for period 0 and period 1 exchange rates. To define history-dependent exchange rate policies, we first define the history of dollar demands, $D_0^i = (d_0^1, \ldots, d_0^i)$ for all $i \in \{1, \ldots, I\}$. An exchange rate policy is a set of functions $\{e_0^1(D_0^I), e_0^2(D_0^I), \ldots, e_0^I(D_0^I), e_1(D_0^I)\}$.

Definition 3: An exchange rate policy satisfies *sequential service* if and only if for all i, $e_0^i(D_0^I)$ is measurable with respect to D_0^i.

In other words, exchange rate policies satisfy sequential service as long as for each trader i, the exchange rate they receive depends only on the trader's reported dollar demand and the reported demands of traders who have previously submitted demands to the issuer. To emphasize this measurability restriction, we write $e_0^i(D_0^i)$.

Given an initial stock of dollar reserves and an exchange rate policy, the dollar reserves remaining after the ith trader submits their demand in period 0, d_0^i, satisfies

$$R_0^i(D_0^i) = R_0^{i-1}(D_0^{i-1}) - d_0^i e_0^i(D_0^i). \tag{16}$$

The issuer chooses an exchange rate policy to maximize the expected discounted value of the traders,

$$\mathbb{E} \sum_{i=1}^{I} \left[d_0^i u(e_0^i(D_0^i)) + (1 - d_0^i) u\left((1-\lambda) e_1(D_0^I) + \lambda\right) \right] \tag{17}$$

subject to the reserve transition equations, (16), the feasibility constraints

$$\forall i \in \{1, \ldots, I\} \text{ and } D_0^{i-1}, \quad e_0^i(D_0^i) \leq R_0^{i-1}(D_0^{i-1}) \tag{18}$$

$$\forall D_0^I, \quad (1-\lambda) e_1(D_0^I) \sum_{i=1}^{I} (1 - d_0^i) \leq R_0 - \sum_{i=1}^{I} d_0^i e_0^i(D_0^i) \tag{19}$$

and the no-speculation incentive constraints for crypto traders,

$$\forall D_0^i, \quad \mathbb{E}\left[u\left((1-\lambda)e_1(D_0^I)+\lambda\right)\middle| D_0^i\right] \geq \mathbb{E}\left[u\left(\left[(1-\lambda)e_1(\hat{D}_0^I)+\lambda\right]\left[\frac{e_0^i(\hat{D}_0^i)}{e_1(\hat{D}_0^I)}-\tau\right]\right)\middle| D_0^i\right] \tag{20}$$

where the expectations in (20) are with respect to D_0^I or $\hat{D}_0^I$ and where $\hat{D}_0^i = (d_0^1, \ldots, d_0^{i-1}, 1)$ and $\hat{D}_0^i = (d_0^1, \ldots, d_0^{i-1}, 1, d_0^{i+1}, \ldots, d_0^I)$.

3.3 A Uniqueness Result

We now prove that any incentive-feasible exchange rate policy—a policy satisfying the feasibility constraints ((16) and (18)-(19)) and the incentive constraints (20)—admits a unique perfect Bayesian equilibrium.

Formally, a strategy for agent i is $s_i : D_0^{i-1} \times \{0,1\} \to \{0,1\}$ where the first argument is the vector of reports of those traders who arrive before trader i and the second is the true type of trader i. For a crypto trader, $s_i = 0$ is a choice to not speculate and $s_i = 1$ is a choice to speculate. Let $s^I = (s_1, s_2, \ldots, s_I)$. Let $D_{0,i+1}^I = (d_0^{i+1}, d_0^{i+2}, \ldots, d_0^I)$ denote the vector of types of those in line after trader i and s_{i+1}^I the strategies of these traders. Let $\gamma(D_{0,i+1}^i|s^{i-1}, d_0^i)$ denote trader i's belief. These beliefs represent the probability of the outcome $D_{0,i+1}^I$ conditional on earlier reports and trader i's true type. In principle, these beliefs may differ from the probability of $D_{0,i+1}^I$ given traders' true types, which we denote $\tilde{\gamma}(D_{0,i+1}^i|s^{i-1}, d_0^i)$.

For any strategy, a crypto trader i's final asset position is

$$\begin{aligned} a(s^{i-1}, s_i, s_{i+1}^I) =&(1-s_i)\left[(1-\lambda)e_1(s^{i-1}, s_i, s_{i+1}^I)+\lambda\right] \\ &+ s_i\left[(1-\lambda)e_1(s^{i-1}, s_i, s_{i+1}^I)+\lambda\right]\left[\frac{e_0(s^{i-1}, s_i, s_{i+1}^I)}{e_1(s^{i-1}, s_i, s_{i+1}^I)}-\tau\right] \end{aligned} \tag{21}$$

Since dollar traders have a dominant strategy, $s_i = 1$, we define an equilibrium considering only the incentives of crypto traders.

Definition 4: A strategy s^I and a belief γ is a perfect Bayesian equilibrium if

$$\sum_{D_{0,i+1}^I} \gamma(D_{0,i+1}^I|s^{i-1}, t_i)u\left(a(s^{i-1}, s_i, s_{i+1}^I)\right) \geq \sum_{D_{0,i+1}^I} \gamma(D_{0,i+1}^I|s^{i-1}, t_i)u\left(a(s^{i-1}, \tilde{s}_i, s_{i+1}^I)\right) \tag{22}$$

for all i, D_0^I and $\tilde{s}_i \in \{0,1\}$ and γ is consistent with Bayes' rule whenever possible.

We then have the following proposition.

Theorem 4: Suppose an exchange rate policy satisfies the feasibility constraints (18)-(19) *and the incentive constraints* (20)*. Then, the strategy* $s_i(D_0^{i-1}, d_0^i) = d_0^i$ *and belief* $\tilde{\gamma}(D_{0,i+1}^i | s^{i-1}, d_0^i)$ *is the unique perfect Bayesian equilibrium.*

The proof mirrors exactly that of Andolfatto, Nosal, and Wallace (2007) and depends critically on the assumption that traders' types are independent. The logic follows a basic backward induction argument. First, observe that the last trader has degenerate beliefs about the full set of reports that will determine exchange rates in period 1. Hence, it is immediately follows from (20) that not speculating is a dominant strategy for trader I when she is a crypto-type. Now, suppose trader i believes all subsequent traders will redeem cryptocurrency for dollars if and only if they are dollar types. Because traders' types are independent, trader i's belief about $D_{0,i+1}^I$ is independent of the reporting strategies of traders $1, 2, \ldots, i-1$. Moreover, any reporting strategy of traders $1, 2, \ldots, i-1$ is equivalent to some realization of their true types, D_0^{i-1}. And, for each such realization, incentive compatibility of the exchange rate policy, (20), implies that not speculating is optimal for trader i in this state. Hence, not speculating is also a dominant strategy for trader i.

Notice, when trader's types are correlated, this result may no longer hold. The reason is that the reporting strategies of traders $1, 2, \ldots, i-1$ influence the beliefs of trader i about the types of future traders, $D_{0,i+1}^I$. When this source of complementarity between traders' actions is strong enough, incentive-feasible policies no longer rule out the possibility of multiple equilibria. This result is analogous to similar results that arise in models of bank runs when depositors have correlated types. For example, Ennis and Keister (2009b) shows that when depositors' preference shocks are correlated, direct mechanisms that implement the efficient allocation may continue to yield a run equilibrium when there are more than 3 depositors. In such cases, implementing efficient arrangements without admitting the possibility of multiple equilibria may require the use of indirect mechanisms (see Cavalcanti and Monteiro (2016) specificaly, and Andolfatto, Nosal, and Sultanum (2017), and Payne and Weiss (2020) more generally for recent results on the usefulness of indirect mechanisms in prevent bank runs).

4 Optimal Exchange Rates

In this section, we examine features of the optimal exchange rates that emerge from our model and demonstrate conditions under which this optimal policy resembles an exchange rate peg even though it responds dynamically to traders' demands for dollars. In general, a full characterization of the solution to the program (17)–(20) is challenging. Instead, we examine cases where the ex post efficient outcome—optimal policy assuming traders' types are observable—satisfies the incentive constraints. In such cases, characterizing optimal policies is straightforward. We begin with a tractable example that admits a closed-form solution for optimal policy and interpretable sufficient conditions to ensure the policy is incentive compatible. We then generalize these findings by way of numerical examples.

4.1 The Ex Post Efficient Optimal Exchange Rate Policy

For our theoretical and numerical results below, we begin by examining the solution to the optimal exchange rate policy program assuming the incentive constraints are all slack at the solution. This approach is equivalent to characterizing properties of the ex post efficient policy. Below, we obtain conditions that allow us to verify that the incentive constraints are indeed slack at this optimum.

Under this conjecture, we may solve for the optimal exchange rate policy by way of a straightforward backward induction. In period 1, for any D_0^I and outstanding dollar reserves R, the optimal period one exchange rate $e_1(D_0^I)$ solves

$$W(\Theta(D_0^I); R) = \max \Theta(D_0^I) u\left((1-\lambda)e + \lambda\right), \tag{23}$$

where $\Theta(D_0^i) = \sum_i (1 - d_0^i)$ represents the number of crypto traders, subject to the feasibility constraint, $\Theta(D_0^I)(1-\lambda)e \leq R$. Since there is no reason to retain reserves beyond period 1, it is immediate that for all D_0^I, $e(D_0^I) = R/[\Theta(D_0^I)(1-\lambda)]$ and

$$W(\Theta(D_0^I); R) = \Theta(D_0^I) u\left(\frac{R}{\Theta(D_0^I)} + \lambda\right). \tag{24}$$

We proceed in similar fashion to define the issuer's value function in period 0 for each possible trader's position, i. This value function depends on the outstanding reserves of the issuer, R, and the total number of previously reported crypto-types, $\Theta(D_0^{i-1})$. We omit the dependence of Θ on D_0^{i-1} to simplify notation. The issuer's value function in period 0 for trader i then satisfies

$$V_0^i(\Theta; R) = \max_{e \leq R} \mu_D \left[u(e) + V_0^{i+1}(\Theta; R - e)\right] + \mu_C V_0^{i+1}(\Theta + 1; R) \tag{25}$$

with the convention $V_0^{I+1}(\Theta; R) = W(\Theta; R)$.

The program described by (23)-(25) is straightforward to solve given functional forms for preferences and a given I either analytically or computationally.

4.2 The Three Trader Economy

Consider a specific case of our economy where $I = 3$.[8] Analysis, detailed in Appendix A.1 reveals the following optimal policy.

Proposition 5: Suppose $I = 3$ and $u(c) = -\exp(-\alpha c)$. Then,

$$\begin{aligned}
e_0^1(R) &= \frac{R}{3} - \frac{2}{3\alpha}\log D \\
e_0^2(\Theta;R) &= \frac{R}{2+\Theta} - \frac{1+\Theta}{\alpha(2+\Theta)}\log\left[\mu_D \exp\left(-\alpha\lambda\frac{\Theta}{1+\Theta}\right) + \mu_C \exp\left(-\alpha\lambda\right)\right] \\
e_0^3(\Theta;R) &= \frac{R}{1+\Theta} + \frac{\Theta}{1+\Theta}\lambda \\
e_1(\Theta;R) &= \mathbb{1}_{[\Theta\geq 1]}\frac{R}{\Theta(1-\lambda)} + \mathbb{1}_{[\Theta=0]}e^f
\end{aligned}$$

where D is a function of the fundamental parameters μ_C, μ_D, λ and α that satisfies $D \leq 1$.

It is straightforward to show that as $\mu_C \to 1$, the period 0 exchange rates satisfy $e_0^i \to (R+2\lambda)/3$. For a relatively high fraction of crypto traders, then, the issuer's optimal policy "pegs" the exchange rate for traders in period 0. Note also that $e_0^3(0;R) = R$ so that in the case where traders 1 and 2 both report they are dollar types, the issuer plans to exhaust its remaining reserves for the final trader in case she also reports she is a dollar trader. In other words, in some states, the issuer is prepared to allow the exchange rate to float.

We now derive conditions such that this policy satisfies traders' incentive constraints, (20). First note that not speculating, or choosing $d_0^3 = 0$ for trader 3 is a dominant strategy if and only if for all Θ and R,

$$(1-\lambda)e_1(\Theta+1;R)+\lambda \geq \underbrace{\left[(1-\lambda)e_1(\Theta;R-e_0^3(\Theta;R))+\lambda\right]}_{\text{Consump. per Cryptocurrency}} \underbrace{\left[\frac{e_0^3(\Theta;R)}{e_1(\Theta;R-e_0^3(\Theta;R))} - \tau\right]}_{\text{Spec. Profit}}. \quad (26)$$

Since trader 3 is last, there is no residual uncertainty about the total number of dollar and crypto traders. As a result, this trader simply compares the payoff she receives from not speculating to that she receives from speculating. The left-hand side of (26) represents the consumption (of dollars and cryptocurrency) of the last trader if she does not speculate where she understands that the period 1 exchange rate will be given by $e_1(\Theta+1;R)$.

[8] Note that our analysis here parallels that of Green and Lin (2000) who also considered a three trader case of the model in Diamond and Dybvig (1983) as this version of the model most transparently reveals the nature of the sequential service constraints.

The right-hand side represents her period 1 consumption payoff per unit of cryptocurrency multiplied by her speculative profit. Notice that this trader recognizes that by speculating, she influences the period 1 exchange rate. In particular, if $\Theta = 0$ so that the first two traders have reported that they are dollar traders, trader 3 recognizes that by speculating, the issuer will exhaust its reserves in period 0 and allow the exchange rate to float at rate e^f in period 1. Also note that when she speculates, she is a net seller of dollars to the issuer hence the issuer can afford to pay her as long as the issuer policy is active—i.e. when the issuer has remaining reserves.

Using the optimal policies from Proposition 5, one may show that (26) holds when $\Theta \geq 1$ if and only if

$$\frac{R}{1+\Theta} + 1 \geq -\tau \left[\frac{R}{1+\Theta} + 1 - \frac{1}{1+\Theta}\right] \tag{27}$$

and holds when $\Theta = 0$ if and only if

$$R + \lambda \geq \left[(1-\lambda)e^f + \lambda\right]\left[\frac{R}{e^f} - \tau\right]. \tag{28}$$

Notice that as $\lambda \to 1$, (27) requires $(R+\Theta)(1+\tau) + 1 \geq 0$ which always holds while (28) requires

$$1 + \tau \geq R\left[\frac{1}{e^f} - 1\right]. \tag{29}$$

The inequality (29) places a lower bound on the floating rate that we will impose at $R = R_0$ to guarantee the third trader always prefers to report truthfully. Notice that the lower bound in (29) necessarily lies below the upper bound on e^f implied by the assumption in Lemma 2.

Consider next the incentives of trader 2 to speculate or not when she is a crypto trader. These incentives depend critically on the reported type of trader 1 along with the remaining reserves available to the issuer. When trader 1 reports she is a crypto trader (type C), the expected utility trader 2 obtains from not speculating ($U^{\texttt{NoSpec.}}_{2,\texttt{crypto}}$) is

$$U^{\texttt{NoSpec.}}_{2,\texttt{crypto}} = \mu_D u\left[(1-\lambda)e_1(2; R_0 - e_0^3(2; R_0)) + \lambda\right] + \mu_C u\left[(1-\lambda)e_1(3; R_0) + \lambda\right] \tag{30}$$

while the expected utility she obtains from speculating ($U^{\texttt{Spec.}}_{2,\texttt{crypto}}$) is

$$\begin{aligned}U^{\texttt{Spec.}}_{2,\texttt{crypto}} =&\mu_D u\Bigg(\left[(1-\lambda)e_1(1;R_0-e_0^2(1;R_0)-e_0^3(1;R_0-e_0^2(1;R_0)))+\lambda\right]\times \\ &\qquad\left[\frac{e_0^2(1;R_0)}{e_1(1;R_0-e_0^2(1;R_0)-e_0^2(1;R_0-e_0^2(1;R_0)))}-\tau\right]\Bigg) \\ &+\mu_C u\left(\left[(1-\lambda)e_1(2;R_0-e_0^2(1;R_0))+\lambda\right]\left[\frac{e_0^2(1;R_0)}{e_1(2;R_0-e_0^2(1;R_0))}-\tau\right]\right). \end{aligned} \tag{31}$$

In both (30) and (31), the probabilities μ_D and μ_C represent the objective probability that trader 3 is a dollar or crypto trader. Given not speculating is a dominant strategy for trader 3 (when she is a crypto type), any perfect Bayesian equilibrium requires trader 2 to evaluate the payoffs associated with her strategies using these objective probabilities. Note that trader 2 faces risk in the period 1 exchange rate she will receive arising from the possible (truthful) reports of trader 3. The same is true when trader 2 speculates although in this case the trader also bears risk in her speculative profits.

Similarly, when trader 1 is a dollar trader, the expected utility trader 2 obtains from not speculating ($U^{\texttt{NoSpec.}}_{2,\texttt{dollar}}$) is

$$U^{\texttt{NoSpec.}}_{2,\texttt{dollar}} = \mu_D u\left[(1-\lambda)e_1(1;R_1-e_0^3(1;R_1))+\lambda\right]+\mu_C u\left[(1-\lambda)e_1(2;R_1)+\lambda\right] \tag{32}$$

while the expected utility she obtains from speculating ($U^{\texttt{Spec.}}_{2,\texttt{dollar}}$) is

$$\begin{aligned}U^{\texttt{Spec.}}_{2,\texttt{dollar}} =&\mu_D u\left(\left[(1-\lambda)e^f+\lambda\right]\left[\frac{e_0^2(0;R_1)}{e^f}-\tau\right]\right) \\ &+\mu_C u\left(\left[(1-\lambda)e_1(1;R_1-e_0^2(0;R_1))+\lambda\right]\left[\frac{e_0^2(0;R_1)}{e_1(1;R_1-e_0^2(0;R_1))}-\tau\right]\right). \end{aligned} \tag{33}$$

In Appendix A.1, we prove that as $\lambda\to 1$ and $\mu_C\to 1$, both $U^{\texttt{NoSpec.}}_{2,\texttt{crypto}} \geq U^{\texttt{Spec.}}_{2,\texttt{crypto}}$ and $U^{\texttt{NoSpec.}}_{2,\texttt{dollar}} \geq U^{\texttt{Spec.}}_{2,\texttt{dollar}}$. Hence, not speculating is a dominant strategy for trader 2 when she is a crypto type. Since traders 2 and 3 have dominant strategies, we may determine the first trader's incentives. Towards this end, the expected utility associated with not speculating for Trader 1 when she is a crypto type, $U_1^{\texttt{NoSpec.}}$ is given by

$$\begin{aligned}U_1^{\texttt{NoSpec.}} =&\mu_C^2 u\left((1-\lambda)e_1(3;R_0)+\lambda\right) \\ &+\mu_D\mu_C u\left((1-\lambda)e_1\left(2;R_0-e_0^2(1;R_0)\right)+\lambda\right) \\ &+\mu_C\mu_D u\left((1-\lambda)e_1\left(2;R_0-e_0^3(2;R_0)\right)+\lambda\right) \\ &+\mu_D^2 u\left((1-\lambda)e_1\left(1;R_0-e_0^2(1;R_0)-e_0^3(1;R_0-e_0^2(1;R_0))\right)+\lambda\right). \end{aligned} \tag{34}$$

while the expected utility associated with speculation, $U_1^{\text{Spec.}}$ is given by

$$\begin{aligned}
U_1^{\text{Spec.}} =& \mu_C^2 u\left(\left[(1-\lambda)e_1(2;R_0-e_0^1(R_0))+\lambda\right]\left[\frac{e_0^1(R_0)}{e_1(2;R_0-e_0^1(R_0))}-\tau\right]\right) \\
&+\mu_D\mu_C u\Bigg(\left[(1-\lambda)e_1(1;R_0-e_0^1(R_0)-e_0^2(0;R_0-e_0^1(R_0)))+\lambda\right]\times \\
&\qquad\left[\frac{e_0^1(R_0)}{e_1(1;R_0-e_0^1(R_0)-e_0^2(0;R_0-e_0^1(R_0)))}-\tau\right]\Bigg) \\
&+\mu_C\mu_D u\Bigg(\left[(1-\lambda)e_1(1;R_0-e_0^1(R_0)-e_0^3(1;R_0-e_0^1(R_0)))+\lambda\right]\times \\
&\qquad\left[\frac{e_0^1(R_0)}{e_1(1;R_0-e_0^1(R_0)-e_0^1(1;R_0-e_0^1(R_0)))}-\tau\right]\Bigg) \\
&+\mu_D^2 u\left(\left[(1-\lambda)e^f+\lambda\right]\left[\frac{e_0^1(R_0)}{e^f}-\tau\right]\right)
\end{aligned} \tag{35}$$

As above, in Appendix A.1, we show that $U_1^{\text{NoSpec.}} \geq U_1^{\text{Spec.}}$ as $\lambda \to 1$ and $\mu_C \to 1$. We then have the following result.

Proposition 6: For λ and μ_C in a neighborhood of $\lambda = \mu_C = 1$ and $1+\tau \geq R_0[\frac{1}{e^f}-1]$, the optimal policy described in Proposition 5 is incentive compatible.

Using Theorem 4, Proposition 6 implies that crypto traders not speculating is the unique perfect Bayesian equilibrium of the game among traders. The two limits in the proposition play independent, but complementary roles in the proof of Proposition 6. Notice from Proposition 5 that as $\lambda \to 1$, $e_1(\Theta;R) \to \infty$ as long as $R > 0$. Whenever $\Theta > 0$, the issuer's optimal policy retains reserves into period 1 so that in any such state crypto traders expect the exchange rate to actually *appreciate* from their opportunity to trade in period 0 until period 1. This expected appreciation reduces their incentives to speculate since they expect to end up with a negative holding of cryptocurrency after bearing the transaction costs of speculating.

This logic fails, however, in the state where all traders report they are dollar traders—an event which occurs with strictly positive probability in the finite trader economy. As a result, a crypto trader in an early position, such as the first trader, may expect to earn speculative profits from a depreciation in the event that traders 2 and 3 happen to be dollar traders. As $\mu_C \to 1$, the likelihood of this profitable event tends to 0, though, and the speculative losses the trader earns in other states of the economy dominate leading the trader to prefer not to speculate. These assumptions jointly ensure that all traders expect the cryptocurrency to not depreciate too much.

As we show below by way of numerical examples, however, these limiting results are not particularly special. That is, we show that the optimal exchange rate policy is dominant strategy incentive compatible even if on average, the optimal policy features no appreciation between periods 0 and period 1.

Under the conditions of Proposition 6, we are able to verify that traders' incentive constraints are satisfied for the efficient exchange rate policy. Generalizing this result—that the ex post optimal exchange rate policy satisfies this large set of incentive constraints—to more types is difficult theoretically and numerically due to the fact that it requires verifying $2^I - 1$ incentive constraints (where I is the number of traders).

Moreover, imposing these constraints may be costly (ex ante) to the extent these constraints bind and limit the set of incentive-feasible allocations. Relaxing this set of constraints, as in Green and Lin (2003)—so traders have some, imperfect information about their order in the sequence—may allow for a more general proof of uniqueness with i.i.d. types and many traders. However, in the banking context, as shown by Peck and Shell (2003) and Ennis and Keister (2009b), relaxing these further—so traders have no information about their order in the sequence—may imply that uniqueness is unattainable for some parameter values.[9] These results suggest the possibility of an interesting tradeoff between the nature of traders' information, the existence of speculative attack equilibria, and the value of (constrained) efficient exchange rate policy. As we describe in Section 5, the nature of information that traders may access in blockchain-based protocols is a choice of the protocol designer and understanding these trade-offs is likely to be an important avenue for future research.

4.3 Numerical Illustrations

Figure 2 illustrates an example with $I = 3$ trader. Here we solve the dynamic program of the currency issuer specified in equations (24) and (25) numerically. We solve the straightforward dynamic program and then check that the no-speculation incentive constraints are slack. The figure shows the full state-contingent dynamic policy for the exchange rate policy. For trader $i = 1$, the currency issuer offers an exchange rate of $e_0^1 = 0.707$. The two branches are for trader of crypto type (C) and dollar type (D). Assuming the no

[9] In Appendix A.2 we consider a related environment with finitely many traders who must demand conversion simultaneously and without imposing sequential service constraints. There we derive conditions such that the ex post efficient allocation rule is incentive compatible and, moreover, admits a unique (Bayes Nash) equilibrium. When the conditions for those results are violated, we conjecture that it may be possible to construct multiple equilibria. Again, in such cases indirect mechanisms may play an important role in eliminating unwanted equilibria (Andolfatto, Nosal, and Sultanum (2017)).

speculation incentive condition holds, only the type D trades at this rate and the policy adapts for $i = 2$—lower if the $i = 1$ trader was a dollar (D) type. The final column on the right is the period one exchange rate e_1. In the (unlikely) case where all three of the traders are dollar (D) type, the optimal exchange rate is not defined—there are no C types at period $t = 1$. Here, the realized exchange rate is the floating rate, e^f. As seen in Table 1, we calibrated this example so that it has little expected appreciation or depreciation in the exchange rate. To achieve that we set $\lambda = 0.25$. This implies a large demand for dollars in period one which implies that the ex post efficient exchange rate does appreciate too much. The example also has initial reserves $R_0 = 1.7$ that are moderate; absent a dynamic exchange rate policy, a run equilibrium is admissible. That is if all three traders demand conversion at an unconditional level of 0.7, the currency issuer will exhaust its reserves.

With three traders, there are seven no speculation incentive conditions to check. Here, and at all the other nodes with a C trader, the expected utility from speculating is less than the expected utility from not. So all the incentive constraints are slack. For example, look at the top node of Figure 2 (yellow dot) where for $i = 2$, the C-type considers her options facing an exchange rate of $e_0^2 = 0.717$. If she does not speculate she ends up with period 1 utility from $\lambda + (1 - \lambda)e_1$. In this example, we have $\lambda = 0.25$ so the values of $e_1 = 0.756$ and $e_1 = 0.647$ (blue dots) are important. Alternatively, she might speculate by selling her one unit of cryptocurrency and hoping for depreciation when she converts back to cryptocurrency at e_1. Notice that if she chooses to speculate, the policy reacts. So, she contemplates converting dollars back to cryptocurrency at either $e_1 = 0.655$ a 1% gain net of a transaction cost $\tau = 0.08$ ($e_1/e_0 - \tau = 0.717/0.655 - 0.08 = 1.01$) or $e_1 = 0.494$ a 37% gain ($0.717/0.494 - 0.08 = 1.37$). These gains are offset by the lower values of e_1 for the $1 - \lambda$ portion of utility that comes from owning dollars. Since, in this example, traders are more likely to be crypto types, $\mu_C = .85$ and the trader is risk averse (CARA preferences with $\alpha = 2$ risk aversion) this trade is not attractive.[10]

Numerically, there is nothing special about $I = 3$ case. Figure 3 is an example with $I = 10$ traders, initial reserves $R_0 = 7.5$, and with other parameters remaining the same. The figure shows all paths for the optimal dynamic exchange rate policy, e_0^i for traders $i = 1, ...10$ and then the value for e_1. The thick line is the unconditional mean for e_0^i and e_1. With $\mu_C = 0.85$, the most likely paths are near the mean. The red shaded area highlights the 90–10 percent inter-quantile range. As in the prior example, we check that all the no-speculation incentive constraints hold. The computational burden is higher here as

[10]Recall, the no speculation constraint is: $E[u((1-\lambda)e_1 + \lambda)] \geq E[u((e_0/e_1 - t)((1-\lambda)e_1 + \lambda))]$ For λ values close to one, the trade profit term $(e_0/e_1 - t)$ drives the inequality. Here, we set $\lambda = 0.25$. It is feasible to generate examples with λ closer to one.

there are now $2^I - 1 = 1023$ constraints to check and, in this example, they are all satisfied. Notice in this example, there is a modest (about 1%) expected depreciation in the currency value from period one to two.

Figure 4 illustrates how the sequential trading yields a unique equilibrium that rules out a speculative attack equilibrium. The black straight line is the unconditional mean value for for e_0 (as in Figure 3). The other lines report the mean value of e_0^i and e_1 conditional on the behavior of selected traders. The solid gold (top) line is the mean exchange rate policy conditional on traders $i = 2, 3$ both reporting type C. The solid blue (bottom) line is the mean conditional on traders $i = 2, 3$ both reporting type D. Notice that the impact to a report of D is larger as this is less likely ($\mu_C = 0.85$). Importantly, notice the conditional mean following traders $i = 2, 3$ is flat (does not continue to depreciate much). Trader $i = 4$, for example, does not have any additional motivation to speculatively trade given the $i = 2, 3$ reports. The dashed lines are the same conditional mean, except here we alter the actions of traders $i = 7, 8$. The policy is more sensitive to the trades from D types as their are fewer remaining opportunities for the traders to be dollar type.

5 Implementing on a Blockchain

The model environment we have presented is not specific to the currencies involved. However, the conditional exchange rate policy is well suited to a blockchain settings. In particular, some blockchains facilitate "smart contracts." Smart contracts are not legal contracts that require court enforcement. Instead, the smart contracts are scripts and associated data (or states) that are stored and executed on a distributed platform. They are contracts in that they are commitments enforced by a distributed-ledger blockchain technology. In this section we show how our dynamic exchange rate peg can be implemented using Solidity code on the Ethereum Network. The code and instructions for this example are available at `https://github.com/azetlinjones/cryptopeg`.

The Ethereum Network is the largest (measured by market capitalization) blockchain setting with a smart contracting environment. The coding environment for Ethereum that creates the smart contracts is called Solidity. Importantly, the language is designed for the distributed setting so that executed code yields the same result for everyone on the distributed network who might update the blockchain. It is also Turing Complete, meaning it offers a rich set of possible contracts. The language includes, as a standardized object, ERC20 tokens.[11] ERC20 tokens have standard properties of an asset in that they can be owned and transferred. In this example, we mint a new ERC20 coin, the "CryptoPegCoin" or CPC. We then implement our our dynamic exchange rate policy to peg the CPC coin price to the Ether (ETH), the primary cryptocurrency on Ethereum's network. Relative to our running example, CPC is the cryptocurrency and Ether is the dollar. We return to consider how to incorporate non-blockchain currency like U.S. dollars below.

5.1 Coded Example

We build the demonstration with three tools. First we use Truffle Suite's Ganache[12] to create a local instance of an Ethereum Blockchain. This local test-network lets us develop and test our code without using any real Ether. Ganache also provides some user accounts (public and private key pairs) whose personas we adopt to be our policy maker and traders. Second, we use the Remix[13] tool to edit and compile the Solidity code. The Remix tool also lets us deploy or post the contract to our test Ethereum network as well as run the functions we code into the contract. Lastly, we have webpage (some HTML and Javascript) that acts as a user interface our traders use to trade. Our goal here is to explore how a dynamic peg policy can work in a blockchain setting, so the HTML interface is very primitive. Screen-shots for each of these tools is in Figure 5.

In this example we have four accounts to consider. Alice is the policy maker who will create and deploy the smart contract that implements the dynamic peg. This example has three traders, $I = 3$. We call the traders Bob, Charlie, and Donna.[14]. To initiate things, Alice compiles the Solidity code and deploys it to the blockchain. This means that the human-readable text code is converted into byte-code and that byte-code is what is added to the blockchain.[15] Alice can make her Solidity code public on, say, her policy maker webpage. Importantly, anyone can compile that code and verify that it is the same as the compiled code stored on the blockchain.[16] The Solidity code is a "script" in that it does

[11]The term "ERC20" stands for Ethereum Request For Comments with the 20 to distinguish it from other standards discussions.

[12]`https://www.trufflesuite.com/`

[13]`https://remix.ethereum.org/`

[14]The names Alice and Bob are a tradition introduced in the seminal paper in cryptography that presented the RSA algorithm that is now central to blockchain technology. See Rivest, Shamir, and Adleman (1978)

[15]This step often called "mining" and is the decentralized record-keeping enigmatic to blockchain. Each transactions on Ethereum has a transaction cost called "gas". The gas cost is roughly proportional to the complexity of the transaction. In this small example, these costs are small. Exploring gas cost in a large economy setting is an interesting extension.

[16]Making the human-readable code public is a choice. In our setting, the policy maker is committed to

not run automatically. In her code are several functions (the key ones we discuss below) that are called by her or others. These calls to the functions and the result also happen as transactions on the blockchain.

Initially, Alice "owns" this contract and some of the functions can only be run by the contract owner. In our economic setting, the cryptocurrency is already outstanding and we do not model the "Initial Coin Offering" or ICO. For this example, we have Alice mint 30 coins, transfer ownership of the coins to the contract, and then offer them for sale at 0.9 ETH/CPC. Next, we have our traders buy the coins. The reasons that our traders might buy these CPC are outside our model.[17] With this, we are at date $t = 0$ of our model. Each trader owns 10 of CPC. The contract has reserves R=27 ETH. Alice sets the initial peg price to 1.0 ETH/CPC – note the CPC price is not fully backed. To implement the dynamic exchange rate peg she places the traders in an order. Here, we happen to use the order Bob, Charlie, and Donna.[18] Finally, Alice transfers ownership of the contract to the contract itself. This means that, credibly, Alice as the policy maker has no remaining control or input to the smart contract.

Next our traders have one sequential opportunity to trade. We implement the sequential order with the heavy-handed rolling of "freezing" and "un-freezing" of the account that can trade. Bob is first and he can poll the contract to see the e_0 price is set at 1.0 ETH/CPC. He chooses to trade 0 or 10 coins. Once this transaction is recorded, Charlie has the opportunity to trade. If for example, Bob did trade his CPC for ETH, the contract will offer a lower price to Charlie. And so on to Donna.

5.2 Discussion

As we discussed earlier, the specific nature of information provided to the traders for their decision trade decision is important to determining when an efficient allocation is imple-

the policy and the traders know it. In Ethereum smart contracts, it is common to publish the Solidity code. The rational is akin to legal advice not to sign contracts you have not or cannot read.

[17]ERC20 tokens are typically the "coins" that are created and sold when a new blockchain-related business on the Ethereum Network does an "Initial Coin Offering" (ICO). Often these tokens of the ICO are like tickets that give access to the soon-to-be-created service or application. Filecoin, as an example, is used to rent file storage space. It is a separate and interesting question as to why these businesses are choosing to use a separate token for access as opposed to just pricing in terms of, say, Ethereum's Ether. Presumably, some motive for the "pre-sale" of tokens is capital structure (Davydiuk, Gupta, and Rosen (2019) and Garratt and Van Oordt (2019)). Other motives are coordination and commitment in the industrial organization of the business (Lee and Parlour (2019), Li and Mann (2018), and Goldstein, Gupta, and Sverchkov (2019)).

[18]We do this manually simply because it was easier to code. Assigning traders to a spot in line with some randomness could be done. In general, generating pseudo-random numbers in the distributed environment of Ethereum requires care but is feasible. Gambling, for example, using smart contract is a common application.

mentable with an unique equilibrium (Ennis and Keister (2009b)). In our Solidity, code we mirrored the environment of our model closely. In particular, we placed the traders into an order that was public. We coded a function in our Solidity that lets anyone poll the contract to see the public key of trader i. That is straightforward; it is not necessary. Different code could have accepted all the trade requests and then pseudo-randomly order the traders and executed trades.[19] More sophisticated information structures where traders information about the queue is noisy, as in Green and Lin (2003), are also feasible.

Our dynamic exchange rate policy assumes that the currency issuer is committed to implement the proposed policy. Results from Ennis and Keister (2009a), who study a related framework in a banking context suggest that a currency issuer would not abide by this policy when she lacks commitment. Blockchain then may play the role of a commitment technology to mitigate the potential consequences of lack of commitment. The Ethereum blockchain environment facilitates commitment by giving contracts a public key address. This means that contract can own ETH and other ERC20 coins. Notice we sent the ETH from the purchase of our coins to the contract. The contract owns these resources in that it (and it alone) can send the ETH to our traders. Second, Ethereum allows contracts to "own themselves." So when Alice transferred ownership of the dynamic peg policy contract to the contract she effectively gives up the ability to run any of the "owner-specific" functions. In our specific case, transferring ownership means no longer having the ability to mint additional CPC coin, change the order of traders, or transfer any ETH beyond what is specified in the dynamic peg policy. Like the other aspects of the Solidity code, this is a choice of the policy maker when implementing in the blockchain environment.

In this example, we supported the price of our CPC using ETH. These are both tokens native to the Ethereum Network. Ownership and control is entirely contained on the Ethereum blockchain and tradable through our smart contract. To extend our example to use off-chain assets, US dollars for example, we need a legal framework to link the dollars to a token on the blockchain. For example, we could use our R US Dollars and issue R ERC20 tokens that are a legal claim to the off-chain dollars; fully backed one-for-one. This is analogous to the Tether stable coin policy. We could then use the R tokens to support our CPC coin as in the example with less than one-to-one reserves. More generally, the challenges in the interface between a blockchain and external physical or financial assets is an area of active fintech development.[20]

[19] For example, Solidity code to run an auction is a common topic of Solidity tutorials.

[20] For example, "What is a corporate blockchain?" `https://www.ibm.com/blogs/blockchain/2019/01/what-is-a-corporate-blockchain`.

6 Conclusion

We have shown that the classical problem of speculative attacks against an under-collateralized currency peg arises from an ad hoc restriction that exchange rate policy be unconditional. We have shown that the optimal conditional policy that considers traders in sequence adjusts the conversion rate based on demand-to-date. The optimal conditional policy eliminates the speculative attack since traders late in the sequence have a dominant strategy not to speculate.

A significant concern with the optimal conditional policy we derive is the required degree of trust that it requires. Specifically, one must believe that policymakers will abide by the specified, complicated policy (ex post). Implementing this policy using blockchain-based smart contracts removes the scope for moral hazard by the policymaker and therefore persuades individuals that the specified conditional policy will actually be implemented. In this context, the key value of blockchain is in its ability to generate trust that policies will be implemented as specified by policymakers.

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A Proofs

A.1 Analysis of the Three Trader Economy

This appendix contains proofs of results regarding the environment with $I = 3$ traders.

A.1.1 Proof of Proposition 5

In this section, we solve the finite trader model when there are 3 traders with utility function $u(c) = -\exp(-\alpha c)$. We solve the model using backward induction in closed form. The value of the currency board of entering period 1 with reserves R and θ crypto-traders to be paid is

$$W(\theta, R) = \theta u\left(\frac{R}{\theta} + \lambda\right).$$

where we have used the fact in period 1, the government always uses up all of its resources, or$e_1(\theta; R) = \frac{R}{\theta(1-\lambda)}$.

Period 0 exchange rates for the 3rd Trader. Suppose 2 traders have already arrived, θ of them have reported they are crypto-traders, and the government has R reserves outstanding. If the trader reports she is a crypto-trader, then the currency board pays nothing out and obtains utility $W(\theta + 1, R)$. If the trader reports she is foreign, then the currency board chooses $e_0^3(\theta; R)$ to solve

$$\max_{e \leq R} u(e) + W(\theta, R - e).$$

Assuming the resource constraint ($e \leq R$) is slack, this maximization requires $e_0^3(\theta; R)$ to satisfy

$$u'(e) = u'\left(\frac{R - e}{\theta} + \lambda\right),$$

or

$$e_0^3(\theta; R) = \frac{R}{1 + \theta} + \frac{\theta}{1 + \theta}\lambda.$$

Note, $e_0^3(\theta; R) \leq R$ as long as $\lambda \leq R$. We may define a value function for the government as

$$V_3(\theta, R) = \mu_F[u(e_0^3(\theta; R)) + W(\theta, R - e_0^3(\theta; R))] + \mu_C W(\theta + 1, R).$$

Using the optimal policy, $e_0^3(\theta; R)$, we find

$$V_3(\theta, R) = \mu_F(1 + \theta)u\left(\frac{R + \theta\lambda}{1 + \theta}\right) + \mu_C(1 + \theta)u\left(\frac{R + (1 + \theta)\lambda}{1 + \theta}\right).$$

Period 0 exchange rates for the 2nd trader. We find $e_0^2(\theta; R)$ as the solution to

$$\max_{e \leq R} u(e) + V_3(\theta, R - e).$$

Using the solution to $V_3(\theta, R)$, we may find $e_0^2(\theta; R)$ as the exchange rate that satisfies

$$u'(e) = \mu_F u' \left(\frac{R - e + \theta\lambda}{1+\theta} \right) + \mu_C u' \left(\frac{R - e + (1+\theta)\lambda}{1+\theta} \right).$$

Given CARA utility, this implies

$$\exp(-\alpha e) = \mu_F \exp \left(-\alpha \left[\frac{R + \lambda\theta}{1+\theta} - \frac{e}{1+\theta} \right] \right) + \mu_C \exp \left(-\alpha \left[\frac{R + (1+\theta)\lambda}{1+\theta} - \frac{e}{1+\theta} \right] \right).$$

Solving for e, we conclude

$$e_0^2(\theta; R) = \frac{R}{2+\theta} - \frac{1+\theta}{\alpha(2+\theta)} \log \left[\mu_F \exp \left(-\alpha\lambda \frac{\theta}{1+\theta} \right) + \mu_C \exp\left(-\alpha\lambda\right) \right].$$

As above, we may define a value function for the government facing the 2nd trader in period 0:

$$V_2(\theta, R) = \mu_F[u(e_0^2(\theta; R)) + V_3(\theta, R - e_0^2(\theta; R))] + \mu_C V_3(\theta + 1, R).$$

Period 0 exchange rates for the 1st trader. We find e_0^1 as the solution to

$$\max_{e \leq R} u(e) + V_2(0, R - e).$$

Note that

$$e_0^2(0; R) = \frac{R}{2} - \frac{1}{2\alpha} \log\left[\mu_F + \mu_C \exp(-\alpha\lambda)\right].$$

If we let C denote the constant,

$$C = \mu_F + \mu_C \exp(-\alpha\lambda)$$

then $e_0^2(0; R) = R/2 - \frac{1}{2\alpha}\log C$ and $R - e_0^2(0; R) = \frac{R}{2} + \frac{1}{2\alpha}\log C$. Then

$$V_2(0, R) = \mu_F \left[u\left(\frac{R}{2} - \frac{1}{2\alpha}\log C \right) + V_3 \left(0, \frac{R}{2} + \frac{1}{2\alpha}\log C \right) \right] + \mu_C V_3(1, R).$$

Using the closed form for $V_3(\theta, R)$, V_2 satisfies

$$\begin{aligned} V_2(0, R) =& \mu_F \left[u\left(\frac{R}{2} - \frac{1}{2\alpha}\log C \right) + \mu_F u\left(\frac{R}{2} + \frac{1}{2\alpha}\log C \right) + \mu_C u\left(\frac{R}{2} + \frac{1}{2\alpha}\log C + \lambda \right) \right] \\ &+ \mu_C \left[2\mu_F u\left(\frac{R+\lambda}{2} \right) + 2\mu_C u\left(\frac{R}{2} + \lambda \right) \right]. \end{aligned}$$

The optimal exchange rate policy satisfies

$$u'(e_0^1) = \frac{d}{dR}\hat{V}_2(0, R - e_0^1),$$

or

$$u'(e_0^1) = \mu_F \left[\frac{1}{2} u' \left(\frac{R - e_0^1}{2} - \frac{1}{2\alpha} \log C \right) + \frac{1}{2} \mu_F u' \left(\frac{R - e_0^1}{2} + \frac{1}{2\alpha} \log C \right) + \frac{1}{2} \mu_C u' \left(\frac{R - e_0^1}{2} + \frac{1}{2\alpha} \log C + \lambda \right) \right]$$
$$+ \mu_C \left[\mu_F u' \left(\frac{R - e_0^1 + \lambda}{2} \right) + \mu_C u' \left(\frac{R - e_0^1}{2} + \lambda \right) \right].$$

Using the functional form of $u(\cdot)$, the optimal policy satisfies

$$\exp(-\alpha e_0^1) = \exp\left(-\alpha \left[\frac{R - e_0^1}{2} \right] \right) \left\{ \mu_F \left[\frac{1}{2} C^{\frac{1}{2}} + \frac{1}{2} \mu_F C^{\frac{-1}{2}} + \frac{1}{2} \mu_C C^{\frac{-1}{2}} \exp(-\alpha\lambda) \right] \right.$$
$$\left. + \mu_C \left[\mu_F \exp\left(-\alpha \frac{\lambda}{2} \right) + \mu_C \exp(-\alpha\lambda) \right] \right..$$

Let D denote the constant,

$$D = \mu_F \left[\frac{1}{2} C^{\frac{1}{2}} + \frac{1}{2} \mu_F C^{\frac{-1}{2}} + \frac{1}{2} \mu_C C^{\frac{-1}{2}} \exp(-\alpha\lambda) \right] + \mu_C \left[\mu_F \exp\left(-\alpha \frac{\lambda}{2} \right) + \mu_C \exp(-\alpha\lambda) \right].$$

Since $C = \mu_F + \mu_C \exp(-\alpha\lambda)$, it follows that

$$D = \mu_F B^{\frac{1}{2}} + \mu_C \left[\mu_F \exp\left(-\frac{\alpha\lambda}{2} \right) + \mu_C \exp(-\alpha\lambda) \right].$$

Then,

$$-\alpha e_0^1 = -\frac{\alpha R}{2} + \frac{\alpha e_0^1}{2} + \log D$$

or

$$e_0^1 = \frac{R}{3} - \frac{2}{3\alpha} \log D.$$

We have shown

$$\begin{aligned}
e_0^1(R) &= \frac{R}{3} - \frac{2}{3\alpha} \log D \\
e_0^2(\theta; R) &= \frac{R}{2+\theta} - \frac{1+\theta}{\alpha(2+\theta)} \log \left[\mu_F \exp\left(-\alpha\lambda \frac{\theta}{1+\theta} \right) + \mu_C \exp(-\alpha\lambda) \right] \\
e_0^3(\theta; R) &= \frac{R}{1+\theta} + \frac{\theta}{1+\theta} \lambda \\
e_1(\theta; R) &= \frac{R}{\theta(1-\lambda)}.
\end{aligned}$$

We also have the transition function for reserves. Let $R_i^0(\theta)$ denote the reserves left to the government when the ith trader arrives in period 0 and θ previous traders have declared they are crypto-traders. Then,

$$\begin{aligned}
R_1^0 &= R_0 \\
R_2^0(\theta) &= R_0 - (1-\theta) e_0^1
\end{aligned}$$

and, remaining reserves at the third person depend on the specific history since it is possible that

$e_0^1(R_0) \neq e_0^2(1; R_0)$. We have

$$R_3^0(\theta) = \begin{cases} R_0 & \text{if } \theta = 2 \\ R_0 - e_0^1(R_0) & \text{if } \theta_1 = 0 \text{ and } \theta_2 = 1 \\ R_0 - e_0^2(1; R_0) & \text{if } \theta_1 = 1 \text{ and } \theta_2 = 0 \\ R_0 - e_0^1(R_0) - e_0^2(0; R_0 - e_0^1(R_0)) & \text{if } \theta = 0 \end{cases}$$

Notice, as $\mu_C \to 1$ (so that all agents are crypto with high probability), we find

$$e_0^1(R) \to \frac{R + 2\lambda}{3}, e_0^2(1; R) \to \frac{R + 2\lambda}{3}, e_0^3(2; R) = \frac{R + 2\lambda}{3}$$

and in this sense the exchange rate is "pegged."

A.1.2 Proof of Theorem 6

Incentives for the 3rd Trader. The incentive constraint when $\theta \geq 1$ requires

$$(1 - \lambda)e_1(\theta + 1; R) + \lambda \geq \left[(1 - \lambda)e_1(\theta; R - e_0^3(\theta; R)) + \lambda\right] \left[\frac{e_0^3(\theta; R)}{e_1(\theta; R - e_0^3(\theta; R))} - \tau\right]$$

where

$$e_1(\theta; R) = \frac{R}{\theta(1 - \lambda)} \quad \text{and } e_0^3(\theta; R) = \frac{R + \theta\lambda}{1 + \theta}.$$

Hence,

$$R - e_0^3(\theta; R) = R - \frac{R + \theta\lambda}{1 + \theta} = \frac{\theta(R - \lambda)}{1 + \theta}$$

and

$$e_1(\theta; R - e_0^3(\theta; R)) = \frac{R - \lambda}{(1 + \theta)(1 - \lambda)}.$$

These results imply the incentive constraint may be re-written as

$$\frac{R}{1 + \theta} + \lambda \geq \left[\frac{R - \lambda}{1 + \theta} + \lambda\right] \left[\frac{(R + \theta\lambda)(1 - \lambda)}{R - \lambda} - \tau\right].$$

We show that as $\lambda \to 1$ this incentive constraint necessarily holds. Taking limits of both sides (assuming $R \neq 1$, we have

$$\frac{R}{1 + \theta} + 1 \geq -\tau \left[\frac{R}{1 + \theta} + 1 - \frac{1}{1 + \theta}\right].$$

Multiplying by $1 + \theta$ and simplifing, we have

$$(R + \theta)(1 + \tau) + 1 \geq 0$$

which holds for any $R \geq 0$, $\theta \geq 1$ and $t \geq 0$. When $\theta = 0$, we require

$$(1 - \lambda)e_1(1; R) + \lambda \geq \left[(1 - \lambda)e_0^3(0; R) + \lambda\frac{e_0^3(0; R)}{e^f}\right] - \tau \left[(1 - \lambda)e^f + \lambda\right].$$

Since $e_1(1;R) = R/(1-\lambda)$ and $e_0^3(0;R) = R$, this constraint requires

$$R\lambda + \lambda + \tau[(1-\lambda)e^f + \lambda] \geq \lambda \frac{R}{e^f}$$

As $\lambda \to 1$, this condition requires

$$1 + \tau \geq R\left[\frac{1}{e^f} - 1\right].$$

Since this should hold for all R and R is necessarily (weakly) decreasing in the sequence of traders, is suffices to impose

$$1 + \tau \geq R_0\left[\frac{1}{e^f} - 1\right].$$

We then have for all R and θ that for λ sufficiently close to 1, the last trader has a dominant strategy to report truthfully.

Incentives for the 2nd Trader. Suppose first that trader 1 is a crypto-trader so that $\theta = 1$. The incentive constraint is

$$\begin{aligned}
&\mu_F u\left[(1-\lambda)e_1(2;R_0 - e_0^3(2;R_0)) + \lambda\right] + \mu_C u\left[(1-\lambda)e_1(3;R_0) + \lambda\right] \\
&\geq \mu_F u\left(\left[(1-\lambda)e_1(1;R_0 - e_0^2(1;R_0) - e_0^3(1;R_0 - e_0^2(1;R_0))) + \lambda\right]\left[\frac{e_0^2(1;R_0)}{e_1(1;R_0 - e_0^2(1;R_0) - e_0^3(1;R_0 - e_0^2(1;R_0)))} - \tau\right]\right) \\
&\quad + \mu_C u\left(\left[(1-\lambda)e_1(2;R_0 - e_0^2(1;R_0)) + \lambda\right]\left[\frac{e_0^2(1;R_0)}{e_1(2;R_0 - e_0^2(1;R_0))} - \tau\right]\right).
\end{aligned}$$

To simplify this constraint, note first that one may show

$$\begin{aligned}
e_1(2;R_0 - e_0^3(2;R_0)) &= \frac{1}{3(1-\lambda)}\left[R_0 - \lambda\right], \\
e_1(3;R_0) &= \frac{1}{3(1-\lambda)}R_0.
\end{aligned}$$

Hence, the left-hand side of the incentive constraint is simply

$$\mu_F u\left(\frac{1}{3}R_0 + \frac{2}{3}\lambda\right) + \mu_C u\left(\frac{1}{3}R_0 + \lambda\right).$$

Second, note that

$$e_0^2(1;R_0) = \frac{1}{3}R_0 - \frac{2}{3}\frac{1}{\alpha}\log B$$

where B is a constant that satisifes

$$B = \mu_F \exp\left(-\frac{1}{2}\alpha\lambda\right) + \mu_C \exp(-\alpha\lambda).$$

Then, one may show

$$\begin{aligned}
e_0^3(1; R_0 - e_0^2(1; R_0)) &= \frac{1}{3}R_0 + \frac{1}{3}\frac{1}{\alpha}\log B + \frac{1}{2}\lambda, \\
e_1(1; R_0 - e_0^2(1; R_0) - e_0^3(1; R_0 - e_0^2(1; R_0))) &= \frac{1}{(1-\lambda)}\left[\frac{1}{3}R_0 + \frac{1}{3}\frac{1}{\alpha}\log B - \frac{1}{2}\lambda\right], \\
e_1(2; R_0 - e_0^2(1; R_0)) &= \frac{1}{(1-\lambda)}\left[\frac{1}{3}R_0 + \frac{1}{3}\frac{1}{\alpha}\log B\right].
\end{aligned}$$

Using these results, we may re-write the right-hand side of the incentive constraint as

$$\begin{aligned}
&\mu_F u\left(\frac{1}{3}R_0 + \frac{2}{3}\lambda\right) + \mu_C u\left(\frac{1}{3}R_0 + \lambda\right) \\
&\geq \mu_F u\left(\left[\frac{1}{3}R_0 + \frac{1}{3}\frac{1}{\alpha}\log B + \frac{1}{2}\lambda\right]\left[\frac{(1-\lambda)\left[\frac{1}{3}R_0 - \frac{2}{3}\frac{1}{\alpha}\log B\right]}{\frac{1}{3}R_0 + \frac{1}{3}\frac{1}{\alpha}\log B - \frac{1}{2}\lambda} - \tau\right]\right) \\
&\quad + \mu_C u\left(\left[\frac{1}{3}R_0 + \frac{1}{3}\frac{1}{\alpha}\log B + \lambda\right]\left[\frac{(1-\lambda)\left[\frac{1}{3}R_0 - \frac{2}{3}\frac{1}{\alpha}\log B\right]}{\frac{1}{3}R_0 + \frac{1}{3}\frac{1}{\alpha}\log B} - \tau\right]\right)
\end{aligned}$$

Using the concavity of $u(\cdot)$, it suffices (to prove the incentive constraint holds) to show

$$\begin{aligned}
\frac{1}{3}R_0 + \frac{2}{3}\lambda \geq \mu_F &\left[\frac{1}{3}R_0 + \frac{1}{3}\frac{1}{\alpha}\log B + \frac{1}{2}\lambda\right]\left[\frac{(1-\lambda)\left[\frac{1}{3}R_0 - \frac{2}{3}\frac{1}{\alpha}\log B\right]}{\frac{1}{3}R_0 + \frac{1}{3}\frac{1}{\alpha}\log B - \frac{1}{2}\lambda} - \tau\right] \\
&+ \mu_C\left[\frac{1}{3}R_0 + \frac{1}{3}\frac{1}{\alpha}\log B + \lambda\right]\left[\frac{(1-\lambda)\left[\frac{1}{3}R_0 - \frac{2}{3}\frac{1}{\alpha}\log B\right]}{\frac{1}{3}R_0 + \frac{1}{3}\frac{1}{\alpha}\log B} - \tau\right]
\end{aligned}$$

Using tedious, but straightforward algebra, one may show that as $\lambda \to 1$, the right hand side tends to

$$-\tau\left[\frac{1}{3}R_0 + \frac{1}{3}\frac{1}{\alpha}\log B(1)\right] - \tau\frac{1+\mu_C}{2}.$$

Hence, the necessary inequality condition (as $\lambda \to 1$) requires

$$\frac{1}{3}R_0 + \frac{2}{3} \geq -\tau\left[\frac{1}{3}R_0 + \frac{1}{3}\frac{1}{\alpha}\log B(1)\right] - \tau\frac{1+\mu_C}{2}$$

or

$$\frac{1}{3}(1+\tau)R_0 + t\frac{1+\mu_C}{2} \geq -\frac{\tau}{3}\frac{1}{\alpha}\log B.$$

Since $B \to \mu_F \exp(-\alpha/2) + \mu_C \exp(-\alpha)$ as $\lambda \to 1$, $\lim_{\lambda\to 1} B \geq \exp(-\alpha)$. It follows that $-\tau \log B/3\alpha \leq \tau/3$. Since $\tau/2 \geq \tau/3$, for all R_0,

$$\frac{1}{3}(1+\tau)R_0 + \tau\frac{1}{2} + \frac{\tau\mu_C}{2} \geq \tau\frac{1}{3} \geq -\frac{\tau}{3}\frac{1}{\alpha}\log B$$

so that the required incentive constraint holds for all R_0, α.

Suppose next that Trader 1 is foreign so that $\theta = 0$. The incentive constraint is

$$
\begin{aligned}
&\mu_F u\left[(1-\lambda)e_1(1;R_1 - e_0^3(1;R_1)) + \lambda\right] + \mu_C u\left[(1-\lambda)e_1(2;R_1)+\lambda\right] \\
&\geq \mu_F u\left(\left[(1-\lambda)e^f + \lambda\right]\left[\frac{e_0^2(0;R_1)}{e^f} - \tau\right]\right) \\
&\quad + \mu_C u\left(\left[(1-\lambda)e_1(1;R_1 - e_0^2(0;R_1)) + \lambda\right]\left[\frac{e_0^2(0;R_1)}{e_1(1;R_1 - e_0^2(0;R_1))} - \tau\right]\right)
\end{aligned}
$$

As above, we use the optimal exchange rate policies to express the incentive constraint in terms of reserves and fundamentals. Note that

$$
\begin{aligned}
e_1(1;R_1 - e_0^3(1;R_1)) &\frac{1}{2(1-\lambda)}\left[R_1 - \lambda\right], \\
e_1(2;R_1) &= \frac{1}{2(1-\lambda)}R_1,
\end{aligned}
$$

so that the left-hand side of the incentive constraint is

$$
\mu_F u\left(\frac{1}{2}R_1 + \frac{1}{2}\lambda\right) + \mu_C u\left(\frac{1}{2}R_1 + \lambda\right).
$$

Similarly, letting the constant C satisfy $C = \mu_F + \mu_C \exp(-\alpha\lambda)$,

$$
\begin{aligned}
e^2(0;R_1) &\frac{R_1}{2} - \frac{1}{2\alpha}\log C, \\
e_1(1;R_1 - e_0^2(0;R_1)) &= \frac{1}{1-\lambda}\left[\frac{R_1}{2} + \frac{1}{2\alpha}\log C\right].
\end{aligned}
$$

Then the right-hand side of the incentive constraint is

$$
\mu_F u\left(\left[(1-\lambda)e^f + \lambda\right]\left[\frac{\frac{R_1}{2} - \frac{1}{2\alpha}\log C}{e^f} - \tau\right]\right) + \mu_C u\left(\left[\frac{R_1}{2} + \frac{1}{2\alpha}\log C + \lambda\right]\left[\frac{(1-\lambda)\left[\frac{R_1}{2} - \frac{1}{2\alpha}\log C\right]}{\frac{R_1}{2} + \frac{1}{2\alpha}\log C} - \tau\right]\right)
$$

We proceed as in the previous case and prove

$$
\begin{aligned}
&\frac{1}{2}R_1 + \frac{1}{2}\lambda \\
&\geq \mu_F\left(\left[(1-\lambda)e^f + \lambda\right]\left[\frac{\frac{R_1}{2} - \frac{1}{2\alpha}\log C}{e^f} - \tau\right]\right) + \mu_C\left(\left[\frac{R_1}{2} + \frac{1}{2\alpha}\log C + \lambda\right]\left[\frac{(1-\lambda)\left[\frac{R_1}{2} - \frac{1}{2\alpha}\log C\right]}{\frac{R_1}{2} + \frac{1}{2\alpha}\log C} - \tau\right]\right)
\end{aligned}
$$

As $\lambda \to 1$, straightforward algebra reveals that this inequality holds if

$$
\frac{1}{2}R_1 + \frac{1}{2} \geq \mu_F\left[\frac{\frac{R_1}{2} - \frac{1}{2\alpha}\log C}{e^f} - \tau\right] + \mu_C\left[-\tau\left(\frac{1}{2}R_1 + \frac{1}{2}\frac{1}{\alpha}\log C\right) - \tau\right].
$$

Noting that $-\log C \leq \alpha$ (as $\lambda \to 1$), we have

$$
RHS \leq \mu_F\left[\frac{\frac{R_1}{2} + \frac{1}{2}}{e^f} - \tau\right] + \mu_C\left[-\tau\left(\frac{1}{2}R_1 - \frac{1}{2}\right) - \tau\right].
$$

The incentive constraint holds, therefore, as long as

$$\frac{R_1}{2}\left[1-\frac{\mu_F}{e^f}+\mu_C\tau\right]+\frac{1}{2}-\frac{1}{2}\frac{\mu_F}{e^f}+\tau\left[1-\frac{1}{2}\mu_C\right]\geq 0.$$

Note that if

$$1-\frac{\mu_F}{e^f}+\mu_C\tau\geq 0,$$

then

$$\frac{1}{2}-\frac{1}{2}\frac{\mu_F}{e^f}+\tau\left[1-\frac{1}{2}\mu_C\right]\geq 0.$$

Hence, suppose the first inequality holds (which is a restriction on μ_C given t, e^f that holds whenever $\mu_C \to 1$). Then, at $R_1 = 0$, the inequality holds and raising R_1 relaxes the incentive constraint. Hence, for all $R_1 \in [0, R_0]$ the inequality must also hold.

Incentives for the 1st Trader. Given truthful reporting is dominant for traders 2 and 3, the expected utility associated with truthful reporting for Trader 1 is given by

$$\begin{aligned}U^{tt} &= \mu_C^2 u\left((1-\lambda)e_1(3;R_0)+\lambda\right)+\mu_F\mu_C u\left((1-\lambda)e_1\left(2;R_0-e_0^2(1;R_0)\right)+\lambda\right)\\&+\mu_C\mu_F u\left((1-\lambda)e_1\left(2;R_0-e_0^3(2;R_0)\right)+\lambda\right)+\mu_F^2 u\left((1-\lambda)e_1\left(1;R_0-e_0^2(1;R_0)-e_0^3(1;R_0-e_0^2(1;R_0))\right)+\lambda\right).\end{aligned}$$

The expected utility associated with speculation is given by

$$\begin{aligned}U^{spec} &= \mu_C^2 u\left(\left[(1-\lambda)e_1(2;R_0-e_0^1(R_0))+\lambda\right]\left[\frac{e_0^1(R_0)}{e_1(2;R_0-e_0^1(R_0))}-\tau\right]\right)\\&+\mu_F\mu_C u\left(\left[(1-\lambda)e_1(1;R_0-e_0^1(R_0)-e_0^2(0;R_0-e_0^1(R_0)))+\lambda\right]\left[\frac{e_0^1(R_0)}{e_1(1;R_0-e_0^1(R_0)-e_0^2(0;R_0-e_0^1(R_0)))}-\tau\right]\right)\\&+\mu_C\mu_F u\left(\left[(1-\lambda)e_1(1;R_0-e_0^1(R_0)-e_0^3(1;R_0-e_0^1(R_0)))+\lambda\right]\left[\frac{e_0^1(R_0)}{e_1(1;R_0-e_0^1(R_0)-e_0^1(1;R_0-e_0^1(R_0)))}-\tau\right]\right)\\&+\mu_F^2 u\left(\left[(1-\lambda)e^f+\lambda\right]\left[\frac{e_0^1(R_0)}{e^f}-\tau\right]\right)\end{aligned}$$

Tedious algebra reveals that

$$\begin{aligned}U^{tt} = \mu_C^2 u\left(\frac{R_0}{3}+\lambda\right)&+\mu_F\mu_C u\left(\frac{R_0}{3}+\frac{1}{3}\frac{1}{\alpha}\log B+\lambda\right)\\&+\mu_C\mu_F u\left(\frac{R_0}{3}+\frac{2}{3}\lambda\right)+\mu_F^2 u\left(\frac{R}{3}+\frac{1}{3}\frac{1}{\alpha}\log B+\frac{1}{2}\lambda\right)\end{aligned}$$

and

$$\begin{aligned}
U^{spec} &= \mu_C^2 u\left(\left[\frac{R_0}{3}+\lambda+\frac{1}{3}\frac{1}{\alpha}\log D\right]\left[\frac{e_0^1(R_0)}{e_1(2;R_0-e_0^1(R_0))}-\tau\right]\right)\\
&+\mu_F\mu_C u\left(\left[\frac{R_0}{3}+\lambda+\frac{1}{3}\frac{1}{\alpha}\log D+\frac{1}{2}\frac{1}{\alpha}\log C\right]\left[\frac{e_0^1(R_0)}{e_1(1;R_0-e_0^1(R_0)-e_0^2(0;R_0-e_0^1(R_0)))}-\tau\right]\right)\\
&+\mu_C\mu_F u\left(\left[\frac{R_0}{3}+\frac{1}{2}\lambda+\frac{1}{3}\frac{1}{\alpha}\log D\right]\left[\frac{e_0^1(R_0)}{e_1(1;R_0-e_0^1(R_0)-e_0^1(1;R_0-e_0^1(R_0)))}-\tau\right]\right)\\
&+\mu_F^2 u\left(\left[(1-\lambda)e^f+\lambda\right]\left[\frac{e_0^1(R_0)}{e^f}-\tau\right]\right)
\end{aligned}$$

where

$$\begin{aligned}
B &= \mu_F\exp\left(-\frac{\alpha\lambda}{2}\right)+\mu_C\exp(-\alpha\lambda)\\
C &= \mu_F+\mu_C\exp(-\alpha\lambda)\\
D &= \mu_F C^{\frac{1}{2}}+\mu_C B
\end{aligned}$$

We know that $U^{tt}\geq u\left(\frac{R}{3}+\frac{1}{3}\frac{1}{\alpha}\log B+\frac{1}{2}\lambda\right)$ and that U^{spec} is bounded above by the utility of the convex combination of speculative consumption. Hence, it suffices to prove that

$$\begin{aligned}
&\frac{R}{3}+\frac{1}{3}\log B+\frac{1}{2}\lambda\\
&\geq \mu_C^2\left[\frac{R_0}{3}+\lambda+\frac{1}{3}\frac{1}{\alpha}\log D\right]\left[\frac{e_0^1(R_0)}{e_1(2;R_0-e_0^1(R_0))}-\tau\right]\\
&\quad+\mu_F\mu_C\left[\frac{R_0}{3}+\lambda+\frac{1}{3}\frac{1}{\alpha}\log D+\frac{1}{2}\frac{1}{\alpha}\log C\right]\left[\frac{e_0^1(R_0)}{e_1(1;R_0-e_0^1(R_0)-e_0^2(0;R_0-e_0^1(R_0)))}-\tau\right]\\
&\quad+\mu_C\mu_F\left[\frac{R_0}{3}+\frac{1}{2}\lambda+\frac{1}{3}\frac{1}{\alpha}\log D\right]\left[\frac{e_0^1(R_0)}{e_1(1;R_0-e_0^1(R_0)-e_0^1(1;R_0-e_0^1(R_0)))}-\tau\right]\\
&\quad+\mu_F^2\left[(1-\lambda)e^f+\lambda\right]\left[\frac{e_0^1(R_0)}{e^f}-\tau\right].
\end{aligned}$$

As $\lambda\to 1$, this inequality tends towards

$$\begin{aligned}
&\frac{R_0}{3}+\frac{1}{3}\frac{1}{\alpha}\log B+\frac{1}{2}\\
&\geq -\tau\mu_C^2\left[\frac{R_0}{3}+1+\frac{1}{3}\frac{1}{\alpha}\log D\right]-\tau\mu_F\mu_C\left[\frac{R_0}{3}+1+\frac{1}{3}\frac{1}{\alpha}\log D+\frac{1}{2}\frac{1}{\alpha}\log C\right]\\
&\quad-\tau\mu_C\mu_F\left[\frac{R_0}{3}+\frac{1}{2}+\frac{1}{3}\frac{1}{\alpha}\log D\right]+\mu_F^2\left[\frac{\frac{R_0}{3}-\frac{2}{3}\frac{1}{\alpha}\log D}{e^f}-\tau\right].
\end{aligned}$$

or

$$\begin{aligned}
&\frac{R_0}{3}\left[1-\frac{\mu_F^2}{e^f}+\tau\left(1-\mu_F^2\right)\right]+\frac{1}{3}\frac{1}{\alpha}\log B+\frac{1}{2}+\frac{\mu_F^2}{e^f}\frac{2}{3}\frac{1}{\alpha}\log D\\
&\quad+\tau\left[\mu_C^2\left(1+\frac{1}{3}\frac{1}{\alpha}\log D\right)+\mu_C\mu_F\left(\frac{3}{2}+\frac{2}{3}\frac{1}{\alpha}\log D+\frac{1}{2}\frac{1}{\alpha}\log C\right)+\mu_F^2\right]\geq 0
\end{aligned}$$

Using the fact that $C^{\frac{1}{2}} \geq D \geq B \geq \exp(-\alpha)$, the above inequality holds as long as

$$\frac{R_0}{3}\left[1-\frac{\mu_F^2}{e^f}+\tau\left(1-\mu_F^2\right)\right]+\frac{1}{6}-\frac{2}{3}\frac{\mu_F^2}{e^f}+\tau\left[\frac{2}{3}\mu_C^2+\frac{1}{3}\mu_C\mu_F+\mu_F^2\right] \geq 0.$$

Notice, this inequality is necessarily satisfied as $\mu_C \to 1$.

A.2 A Model with Many Traders without Sequential Service

Here, we characterize optimal mechanisms in large finite economies without sequential service. We derive conditions such that the optimal mechanism admits a unique dominant strategy (Bayes-Nash) equilibrium in the game among traders.

Suppose there are $I < \infty$ traders. The possible states of nature—traders' types—may be represented as $\Omega \in \{0,1\}^I$. Given $\omega \in \Omega$, we let ω_i denote trader i's type (with 0 indicating the trader is a dollar trader and 1 indicating the trader is a crypto trader). We characterize the ex-post efficient allocation and then prove this allocation satisfies traders' incentive constraints for λ sufficiently close to 1 (as assumed through the paper). Let $\theta(\omega)$ denote the number of crypto-traders in state ω. The ex-post efficient allocation solves

$$\max(I-\theta(\omega))u(e_0(\omega))+\theta(\omega)u((1-\lambda)e_1(\omega)+\lambda) \tag{36}$$

subject to

$$(I-\theta(\omega))e_0(\omega)+\theta(\omega)(1-\lambda)e_1(\omega) \leq R_0. \tag{37}$$

The optimality conditions are (37) and

$$u'(e_0(\omega)) = u'((1-\lambda)e_1(\omega)+\lambda). \tag{38}$$

Consequently, since $u(\cdot)$ is strictly concave, we may solve for $e_0(\omega), e_1(\omega)$ explicitly as

$$e_0(\omega) = \frac{1}{I}\left(R_0+\theta(\omega)\lambda\right)$$
$$e_1(\omega) = \frac{1}{I(1-\lambda)}\left(R_0-(I-\theta(\omega))\lambda\right)$$

We now show the following result.

Proposition 7: Suppose that for all ω, $e_0(\omega)/e_1(\omega)-\tau \leq 1$. Then the optimal exchange rate policy admits a unique (Bayes-Nash) equilibrium in the game among traders choosing whether to report their types truthfully (and receiving the specified exchange rates in the appropriate stage) or misreporting and speculating.

Note that when $R_0 \geq I$ and $\lambda \to 1$, the condition of the proposition is necessarily satisfied. Moreover, except when $\theta(\omega)=0$, this policy is under-collateralized in the sense that it cannot afford to deliver $Ie_0(\omega)$ to *all* agents.

We prove the proposition by showing that for any reporting and speculation strategy of the other agents (for any state), trader i prefers to report truthfully and not speculate. If trader i is a dollar trader, then she does not value period 2 consumption and hence it is trivially optimal for her to report truthfully independent of the strategies of other traders. We thus focus on the incentives of trader i when she is a crypto type ($\omega_i = 1$). Formally, let $\sigma_i(\omega_i)$ denote trader i's reporting strategy when she is of type i and let $\eta = \sum_{j \neq i} \sigma_j(\omega_j)$. We prove that for all η,

$$(1-\lambda)e_1(\eta+1) + \lambda \geq [(1-\lambda)e_1(\eta) + \lambda]\left[\frac{e_0(\eta)}{e_1(\eta)} - \tau\right]. \tag{39}$$

Towards proving (39), first note that by assumption,

$$(1-\lambda)e_1(\eta) + \lambda \geq [(1-\lambda)e_1(\eta) + \lambda]\left[\frac{e_0(\eta)}{e_1(\eta)} - \tau\right] \tag{40}$$

since $e_0/e_1 - \tau \leq 1$. Next, we show that $e_1(\eta+1) \geq e_1(\eta)$ so that

$$(1-\lambda)e_1(\eta+1) + \lambda \geq (1-\lambda)e_1(\eta) + \lambda \tag{41}$$

which combined with (40) implies (39).

To see that $e_1(\eta+1) \geq e_1(\eta)$, we prove the more general claim that the optimal mechanism satisfies $e_1(\eta)$ is increasing in η. Consider the program

$$\max_{\gamma} (I-\eta) u\left(\frac{\gamma}{I-\eta}\right) + \eta u\left(\frac{R_0 - \gamma}{\eta} + \lambda\right) \tag{42}$$

and let $\Gamma(\eta)$ denote the solution. Of course, the solution to this program coincides with the the solution to the optimal mechanism by setting $\eta = \theta(\omega)$ and $e_0(\omega) = \Gamma(\eta)/(I-\eta)$ and $e_1(\omega) = (R_0 - \Gamma(\eta))/\eta(1-\lambda)$.

We will show

$$\frac{d}{d\eta}\frac{R_0 - \Gamma(\eta)}{\eta} \geq 0 \tag{43}$$

which requires

$$\Gamma'(\eta) + \frac{R_0 - \Gamma(\eta)}{\eta} \leq 0. \tag{44}$$

To prove (44), note that from (42), $\Gamma(\eta)$ satisfies

$$u'\left(\frac{\Gamma(\eta)}{I-\eta}\right) - u'\left(\frac{R_0 - \Gamma(\eta)}{\eta} + \lambda\right) = 0. \tag{45}$$

Totally differentiating this optimality condition with respect to η implies that both $\Gamma'(\eta) + \Gamma(\eta)/(I-\eta)$ and $\Gamma'(\eta) + (R_0 - \Gamma(\eta))/\eta$ equal 0 or they are of opposite sign. As a result, to show (44), it suffices to show

$$\frac{R_0 - \Gamma(\eta)}{\eta} \leq \Gamma(\eta)/(I-\eta) \tag{46}$$

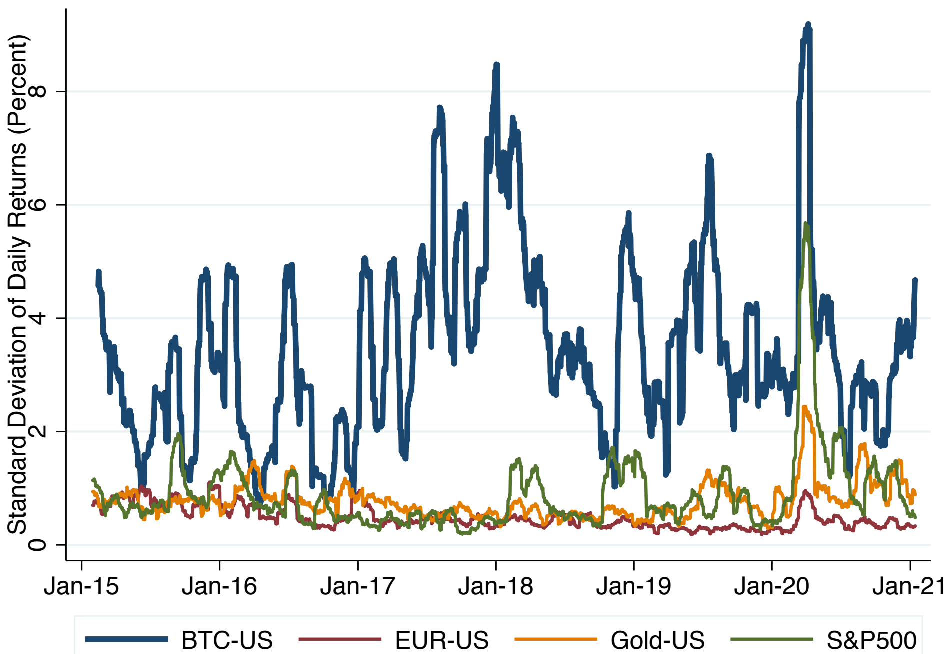
which is guaranteed using (45), strict concavity of $u(\cdot)$ and $\lambda > 0$.

Table 1: **Dynamic Exchange Rate Policy – $I = 3$ Traders**

	trader	Ee_t	$sd(e_t)$	Median
$t = 0$	1	0.707	0.000	0.707
	2	0.699	0.042	0.717
	3	0.692	0.068	0.729
T=1	4	0.707	0.070	0.756

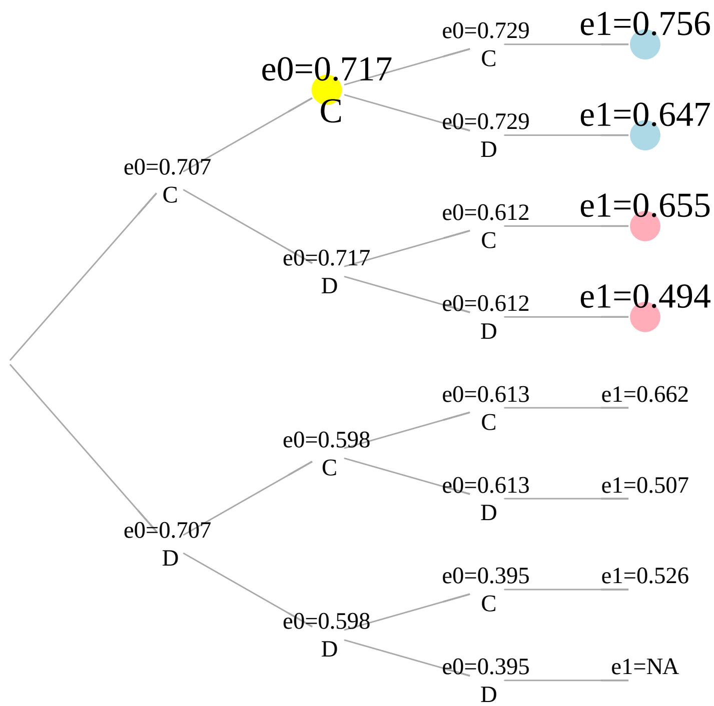
Moments of the optimal dynamic exchange rate policy in the case of $I = 3$. The parameters of the example include, $\lambda = 0.25$, CARA utility with risk aversion $\alpha = 2$, and probability of type C is $\mu = 0.85$, transaction cost $\tau = 0.08$, initial reserves are $R_0 = 1.7$, and a floating rate $e^f = 0.45$.

Figure 1: Daily Volatility



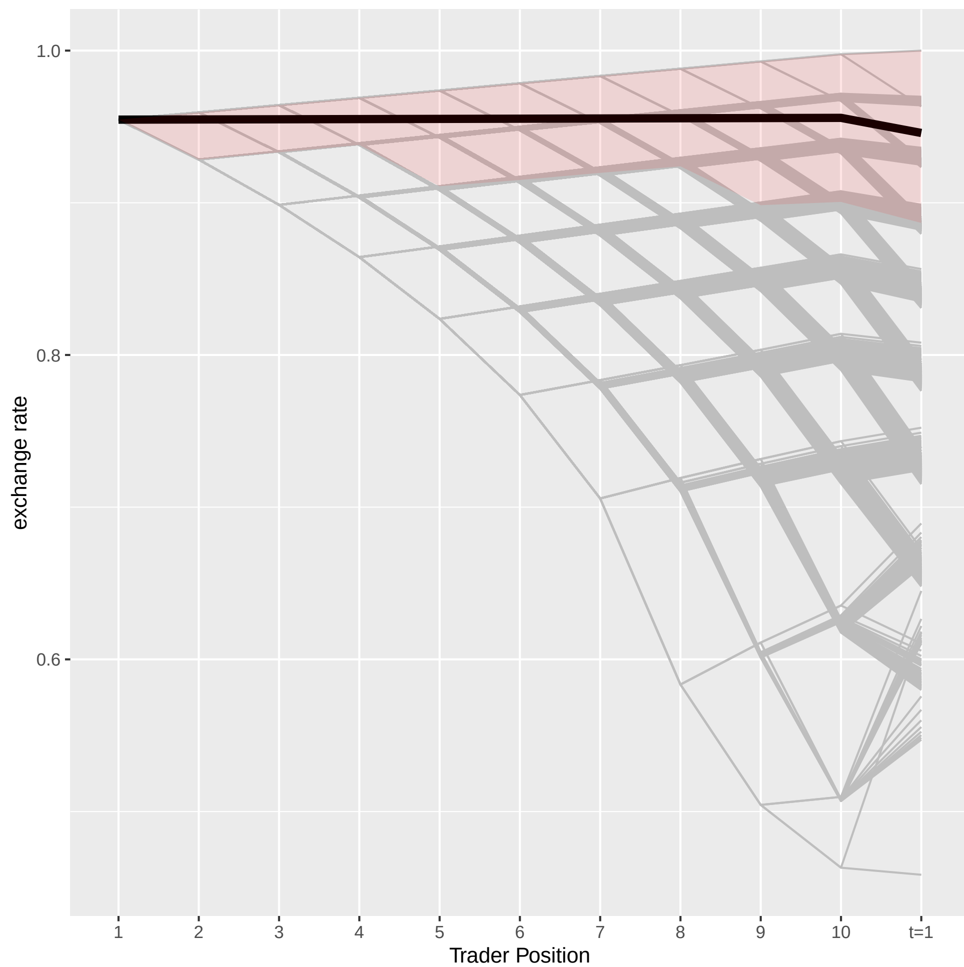
The 30-day (rolling) standard deviation of daily USD price changes of Bitcoin (BTC-UD), Euro (EUR-US), S&P500 stock market index (S&P500), and Gold (Gold-US). All data is from FRED, Federal Reserve Bank of St. Louis. 2015-01-15 to 2021-01-13

Figure 2: Dynamic Exchange Rate Policy – $I = 3$ Traders



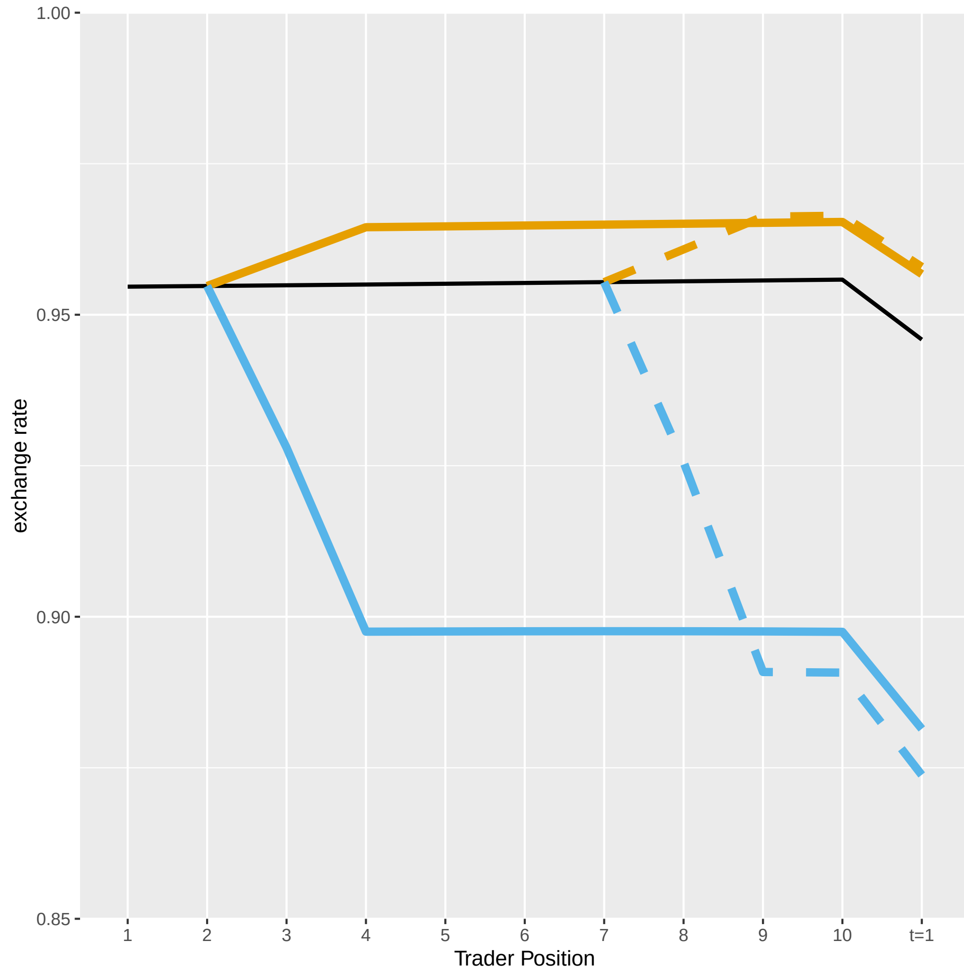
The optimal dynamic exchange rate policy in the case of $I = 3$ traders. The C denotes crypto-type. The D denotes dollar type. The colored dots illustrate speculative trade. Should a type-C trader at the pink dot choose to trade at $e_0^2 = 0.717$, she repurchases cryptocurrency at the uncertain e_1 values highlighted by the blue dot. On the bottom path where all traders are D, the policy e_1 is not defined and the exchange rate is at the floating rate (Here, the parameter $e^f = 0.39$) The parameters of the example include, $\lambda = 0.25$, CARA utility with risk aversion $\alpha = 2$, and probability of type C is $\mu = 0.85$, initial reserves $R_0 = 1.7$, and transaction cost $\tau = 0.08$.

Figure 3: Dynamic Exchange Rate Policy $-$ $I = 10$ Traders



The optimal dynamic exchange rate policy in the case of $I = 10$ traders. Plotted is the period echange rate e_0^i for each the traders' positions $i = 1, .., 10$ and then the period one exchange rate e_1. Each line is a realization of a path for C and D for each of the i traders. The dark black line is the mean. The red shaded region indicates outcomes in the 10 – 90 percentile range. The parameters of the example include, $\lambda = 0.25$, CARA utility with risk aversion $\alpha = 2$, and probability of type C is $\mu = 0.85$, transaction cost $\tau = 0.08$, initial reserves are $R_0 = 7.5$, and a floating rate $e^f = 0.45$.

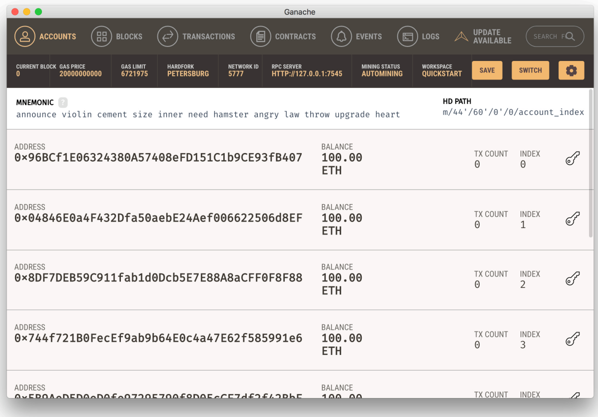
Figure 4: Dynamic Exchange Rate Policy – $I = 10$ Traders - Conditional Means



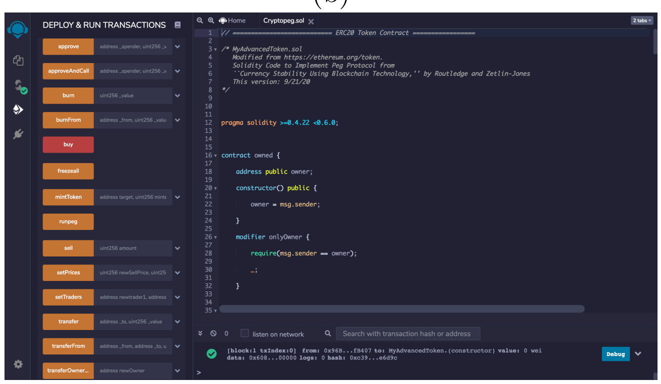
The optimal dynamic exchange rate policy in the case of $I = 10$. The x-axis shows the traders' positions $i = 1, .., 10$ and then the $t = 1$ policy rate. Each line is a realization of a path for C and D for each of the i traders. The dark black line is the mean. The unconditional mean value of e_0 is the black (straight) line. The gold, solid line is the mean value for e_0 conditional on trader $i = 2$ and $i = 3$ being of type C. The blue, solid line is the mean value for e_0 conditional on trader $i = 2$ and $i = 3$ being of type D. The gold, dashed line is the mean value for e_0 conditional on trader $i = 7$ and $i = 8$ being of type C. The blue, dashed line is the mean value for e_0 conditional on trader $i = 7$ and $i = 8$ being of type D. The parameters of the example include, $\lambda = 0.25$, CARA utility with risk aversion $\alpha = 2$, and probability of type C is $\mu = 0.85$, transaction cost $\tau = 0.08$, initial reserves are $R_0 = 7.5$, and a floating rate $e^f = 0.45$.

Figure 5: Implementing on a Blockchain - Tools

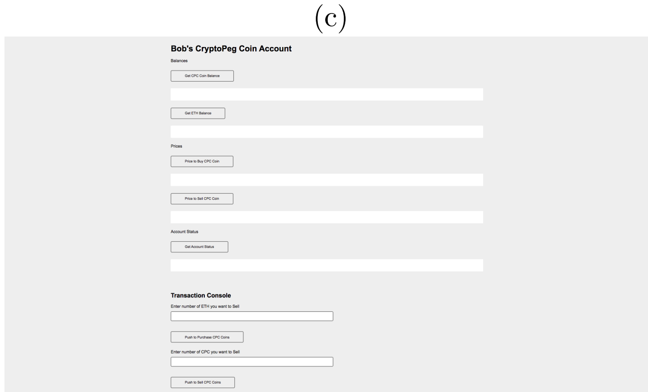
(a)



(b)



(c)



Screen-shots of the tools used to implement the dynamic exchange policy on the Ethereum Network. (a) Truffle Suite's Ganache `https://www.trufflesuite.com/`. (b)Remix `https://remix.ethereum.org/` tool to edit and compile the Solidity code. (c) HTML "wallet" trader interface.