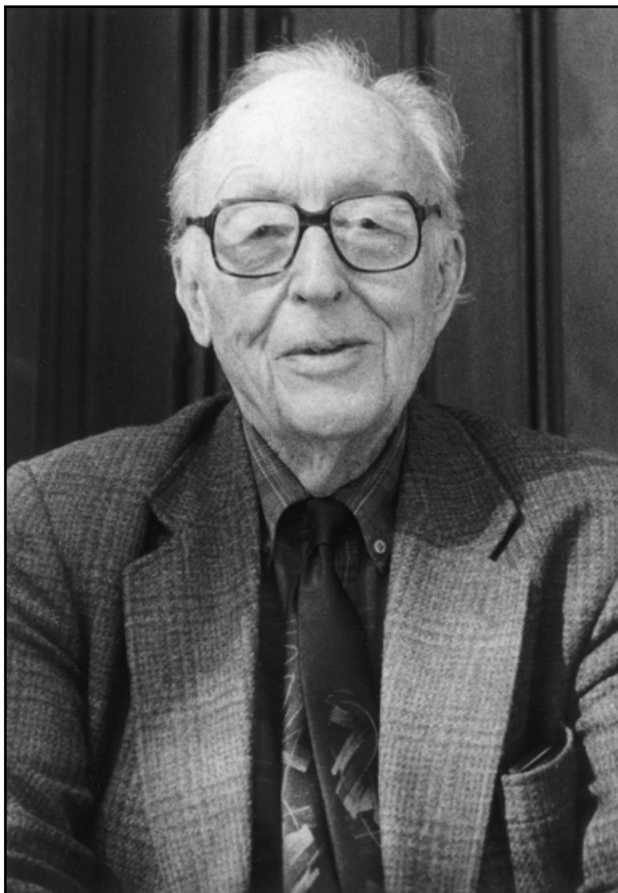
SAUNDERS MAC LANE



4 AUGUST 1909 · 14 APRIL 2005

SAUNDERS MAC LANE was one of the most influential mathematicians of the twentieth century. The sweep of his long and distinguished career includes contributions to the fields of logic, number theory, algebra, and topology as well as the philosophy, history, education, and administration of mathematics. Together with longtime collaborator Samuel Eilenberg, he invented category theory, a contribution to the foundations of mathematics of deep and lasting importance.

The son of a Congregational minister, Mac Lane had an almost evangelical zeal for all aspects of mathematics, studying it as a devoted disciple, traveling and teaching it like a missionary, delivering fiery, sermon-like lectures, and tending the mathematical flock like a pastor. After degrees from Yale (B.S., 1930), Chicago (M.S., 1931), and Göttingen (Ph.D., 1934), and stints at Yale, Harvard (twice), and Cornell, he settled at the University of Chicago, where he remained for more than fifty years. He succeeded Marshall Stone as chair of mathematics at Chicago in the 1950s, when that department was perhaps the world's best. He was at different times president of the American Mathematical Society and the Mathematical Association of America and vice president of the National Academy of Sciences and of the American Philosophical Society, to which he was elected in 1949. He published more than a hundred research papers and six books, and supervised some fifty dissertations. His many contributions and achievements were justly recognized in the form of numerous honorary degrees, prizes, fellowships, and awards, including in 1989 the National Medal of Science, our nation's highest honor for achievement in the sciences.

As a student in David Hilbert's Göttingen of the early 1930s, Mac Lane had learned the new style of abstract algebra in the lectures of Emmy Noether. His first textbook, *A Survey of Modern Algebra* (Macmillan, 1941; coauthored with Garrett Birkhoff), brought the abstract style home and became a classic. The invention of category theory shortly thereafter was driven by his research in algebraic topology. Under the influence of Noether, Birkhoff, and Marshall Stone, he and Eilenberg essentially invented the new field of "homological algebra," which intended to describe geometric objects and relations in terms of associated algebraic ones with which one could calculate—a task at which Mac Lane was particularly skilled. This description required the development of an entirely new language and set of tools, i.e., category theory, the basics of which were first published in 1945 (Eilenberg and Mac Lane, "General Theory of Natural Equivalences," *Trans. Am. Math. Soc.* 58:231–94).

By definition, a *category* consists of "objects" $A, B, C, \ldots$ and "arrows" $f : A \to B$, $g : B \to C, \ldots$ with each arrow associated formally as indicated to a pair of objects as its "domain" and "codomain," and

with each object A having an "identity" arrow $1_A : A \to A$, and each composable pair of arrows having a formal "composite" $g \circ f : A \to C$. ← Q: ∘ symbol correct?
These operations of composition and identity arrows are postulated to satisfy the familiar associativity and unit laws,

$$h \circ (g \circ f) = (h \circ g) \circ f$$
$$f \circ 1_A = f = 1_B \circ f$$

for any $f : A \to B$, $g : B \to C$, $h : C \to D$, making the entire collection of objects and arrows something akin to an abstract group. A simple example of a category has all finite sets as the objects and all functions between them as the arrows, with the usual identity function and composition of functions as the operations. The definition is general enough, however, to capture all manner of different structures and compare them (via "functors," which are mappings between categories) as regards those constructions and properties that can be specified in this "impoverished" language of objects and arrows—and thus only in terms of mappings between objects with the same structure, i.e., in the same category. Here we recognize the legacy of Felix Klein's *Erlanger Programm* (via Mac Lane's training at Göttingen), according to which a species of mathematical structure is determined by preservation under a preselected system of mappings.

The real power of these methods is hardly evident from the definitions, however, and the early reaction to category theory was rather mixed; there were complaints about the reformulation of known results in a jazzy new language. The practical success of this language eventually dispelled any skepticism, however, and today wide swaths of mathematics cannot even be formulated without it. Indeed, its use has revolutionized the foundations and practice of modern algebra and related fields like algebraic topology, geometry, and number theory.

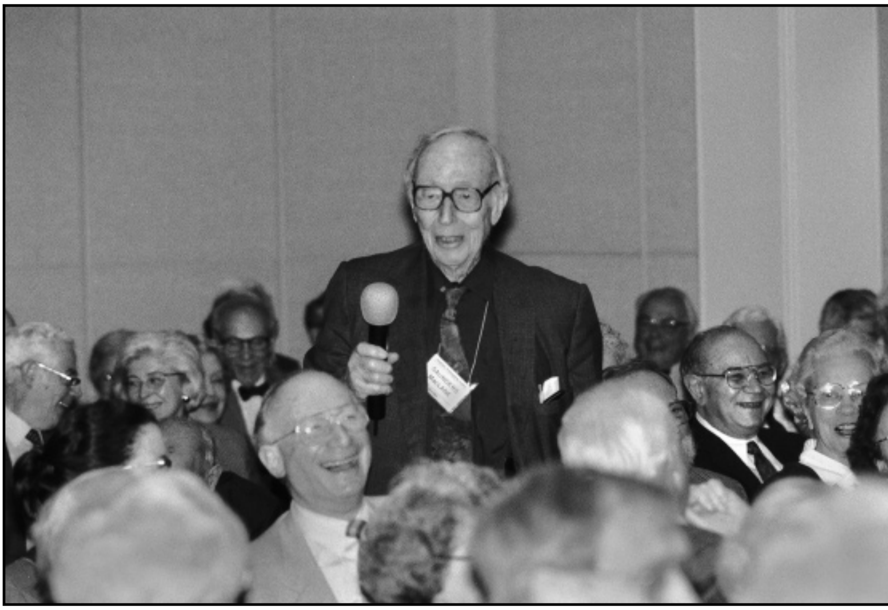
If the area under the curve of Mac Lane's career represents the bulk of his research in the various branches of mathematics, the long arc of it begins and ends in logic. His doctoral thesis in logic was supervised by Paul Bernays and Hermann Weyl. It was completed in a rush after Hitler's rise to power and during the disintegration of the German university, as Jews were dismissed from their posts, and many of their colleagues either resigned in protest or fled in caution. Bernays was dismissed in 1933, and Mac Lane switched to Weyl in order to finish. (See Mac Lane, "Mathematics at Göttingen under the Nazis," *Notices AMS* 42 [1995]: 1134–38.) When Bernays rejected his first proposals, Mac Lane wrote home of his frustration with the "utter lack of philosophic grasp of the local professors toward my thesis," saying that he was "prepared to transfer to the University of Vienna, where there are many who think as I do"—presumably a reference to the "Vienna Circle" logicians Rudolf Carnap and Kurt Gödel.

Bernays was eventually persuaded after a meeting in which Mac Lane "told him of the philosophical aim" of the thesis.[1] The only evidence of this "aim" in the dissertation, entitled "Abbreviated Proofs in Logical Calculus," is in the concluding section, where he briefly discusses the notion of a "guiding idea," which is something like an overarching conception that is both precise and general, and which, he says, should serve as the basis for the determination of the particulars, not only of proofs, but of "all mathematical processes," such as theories and operations. This conception, he says, "suggests a vast and important field of study for mathematical logic: the study of the structure of the elements of mathematics and the determination of this structure through guiding ideas." Moreover, such an investigation, he says, "should strengthen the connection between mathematical Logic and Mathematics; for a successful consideration of the structure of Mathematics must obviously begin with Mathematics itself. From this point of view, mathematical Logic should not become a separate and highly specialized field in itself " (*Abgekürzte Beweise im Logikkalkul* [Göttingen, 1934], 61). It hardly requires pointing out that these remarks could now easily be taken as applying to Category Theory.

At his first posts at Yale and Harvard, while still looking for a permanent position, Mac Lane found that Göttingen-style algebra was in much greater demand than Göttingen-style logic, and he was quick to oblige. He maintained an active interest in logic, however, and wrote numerous reviews of important recent works. In one such review, of Carnap's *Logical Syntax of Language* (*Bulletin AMS*, 1938), he observed that there was a crucial flaw in the all-important definition of "logical validity." Carnap had attempted to define the "logical symbols" as the largest collection of symbols such that every sentence constructed only from them is logically determinate, and Mac Lane observed that there need be no unique such maximal set. The situation is, of course, similar to one arising often in algebra, as he surely recognized. Mac Lane concluded, "Such technical points might raise doubts as to the philosophical thesis Carnap wishes to establish here: that in any language whatsoever one can find a uniquely defined 'logical' part of the language, and that 'logic' and 'science' can be clearly distinguished" (p. 174; see also pp. 173–75). This point would later be the nub of an influential critique of Carnap by Mac Lane's Harvard colleague and friend W. V. O. Quine, leading to much logical research and philosophical debate. Mac Lane kept a close eye on developments in logic

---

[1] The foregoing three quotations are from *Saunders Mac Lane: A Mathematical Autobiography* (Wellesley, Mass.: A. K. Peters, 2005).



FRANK MARGESON

throughout his career, supervising several dissertations in the field, serving on the Council of the Association for Symbolic Logic, and, as a member of the National Academy of Sciences, nominating Gödel for the National Medal of Science. He even accepted the award for him from President Ford when Gödel was too ill to accept it in person. A late chapter in Mac Lane's involvement with logic was opened in the 1960s by F. W. Lawvere's pioneering work on applying category theory to logic, thereby relating it to other fields like algebraic geometry. Mac Lane took a keen interest in this work and became a great supporter of it: conducting research; organizing seminars; writing expositions; and lecturing on the subject around the world. He continued (into his eighties!) to pursue it with burning interest, supervising research (including my own), and eventually writing a book on toposes (with I. Moerdijk), which has become the standard textbook in the field. Thus Mac Lane concluded his career with a masterful return to the subject in which he began it, wielding the mathematical tools he had wrought in the interim to make a final contribution to logic. No biographical note about Saunders Mac Lane would be complete without a few words about the great man's expansive character: his dignified and yet warm demeanor, his curious and generous intellect. He could be a stern and principled auditor, terrifying to many a young lecturer. But he also delighted in amusing listeners with tales of brushing up against Hitler at Weimar, and heckling Russell at Harvard. And there was the occasional song and, yes, the colorful jacket. He was an energetic swimmer, skier, and hiker, leading the troops of conference mathematicians through the countryside (and graduate students up the four flights of stairs to his office—well into his eighties!). In addition to being a writer of lively prose, mathematical and otherwise, he was a lover of poetry and something of a poet, reciting Byron or Shelley, or one of his own "ditties" about a fellow mathematician.

So here's one for him:

> to Saunders Mac Lane, of the plaid sport coat,
> with his thingamajigs and the *Homs* he wrote,
> in Mathematics he stood alone,
> in a category of his own!

Elected 1949; Vice President 1968–71; Councillor 1960–63; Committees: Council Nominees 1970–71; Membership I 1952–55; 1959–63; 1997–2002; Nomination of Officers 1962–63

Steve Awodey
Associate Professor of Philosophy
Carnegie Mellon University

Professor Awodey received his Ph.D. from the University of Chicago in 1996 as Mac Lane's last student.